The Rational valued characters of the Group ($Q_{2m} \times C_5$) When m is prime number

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1- Abstract

The main purpose of this paper is to find the rational valued characters table of the group ($Q_{2m} \times C_5$) , When m is prime number , where Q_{2m} is denoted to Quaternion group , and C_5 is the cyclic group of order 5 .
In this paper .we find the general form of t the rational valued characters table of the group ($Q_{2m} \times C_5$) is the tensor product of the rational valued characters of Q_{2m} and C_5

المستخلص

الهدف الرئيسي لهذا البحث هو ايجاد جدول الشواخص ذات القيم النسبيه للزمره $Q_{2m}\times C_5$,عندما m عدد اولي ,حيث يرمز للزمره الرباعيه بالرمز Q_{2m},ويرمز للزمره الدائريه ذات الرتبه 5 بالرمز C_5 .
في هذا البحث وجدنا جدول الشواخص ذات القيم النسبيه للزمره $Q_{2m}\times C_5$ باستخدام ضرب tensor للشواخص ذات القيم النسبيه لـQ_{2m} و C_5 .

2- Introduction

Let G be a finite group , two elements of G are said to be Γ – conjugate if the cyclic subgroups they generate are equivalence relation on G . Its classes are called Γ – classes the Z – valued class function on the group G , which is constant on the Γ – classes forms a finitely generated abelian group $cf (G , Z)$ of rank equal to the number of Γ – classes .
The intersection of $cf (G , Z)$ with the group of all generalized characters of G , $R (G)$ is a normal subgroup of $cf (G , Z)$ dented by $R (G)$ Each element in $R (G)$ can be written as
$u_1\theta_1 + u_2 \theta_2 + \dots\dots + u_l \theta_l$, where l is the number of Γ – classes , $u_1 , u_2 \dots\dots u_l \in Z$ and
$\theta_i = \sum_{\sigma \in Gal (Q (\lambda i) / Q)} \sigma (\lambda_i)$

Where λi is an irreducible character of the group G and σ is any element in Galios group Gal ($Q (\lambda i) / Q$) .
Let $\equiv^* (G)$ denotes the $l \times l$ matrix which the rows corresponds to the θi 's and columns corresponds to the Γ classes of G .
In 1995 N. R. Mahamood [3] studied the factor group cf (Q_{2m} , Z) / R (Q_{2m}) . The aim of this paper is to find $\equiv^*(Q_{2m} \times C_5)$ and determine general (m + 3× m + 3) matrix form of the rational valued characters table of the group ($Q_{2m} \times C_5$) .

3- Preliminaries

Definition (3.1) [3]

For each positive integer m , the generalized Quaternion Group Q_{2m} of order 4 m with two generators x and y satisfies
$Q_{2m} = \{ x^h y^k , 0 \leq h \leq 2m - 1 , k = 0.1 \}$
Which has the following properties
$\{ x^{2m} = y^4 = 1, y \times^m y^{-1} = x^{-m} \}$

Definition (3.2) [3]

Table (1)

	X^k	$X^k y$
ψ_1	1	1
ψ_2	1	– 1
ψ_3	$(-1)^k$	$i(-1)^k$
ψ_4	$(-1)^k$	$i(-1)^{k+1}$

There are two types of irreducible characters one of them is the character of the linear representations R_1 , R_2 , R_3 and R_4 which are denoted by ψ_1 , ψ_2 , ψ_3 and ψ_4 . respectively as in the table(1)
where $0 \leq k \leq 2m - 1$
The rest characters of irreducible representations T_h of degree 2 are denoted by λ_h such that :
$\lambda_h(x^k) = \omega^{hk} + \omega^{-hk} = e^{\pi ihk/m} + e^{\pi ihk/m} = 2\cos(\pi hk / m)$
we are denoted to $\omega^{hk} + \omega^{-hk}$ by V_{hk} , thus
$V_{hk} = V_{2m-hk}$, $V_m = -2$, $V_{2m} = 2$
also we will write $V_{J(hk)}$ such that
J (hk) = min { hk (mod 2m) , 2m – hk (mod 2m) } , in the character table of the quaternion group Q_{2m} when m is prime number , where $V_{J(hk)} = 2\cos(\pi J(hk)/m)$, $\lambda h(\chi^k y) = 0$
Where $0 \leq k \leq 2m - 1$, $1 \leq h \leq m - 1$ and $\omega = e^{2\pi i/2m}$
So there are m + 3 irreducible characters of Q_{2m} .

Theorem (3.3) [1]
Let $T_1 : G_1 \longrightarrow GL(n, k)$ and $T_2 : G_2 \longrightarrow GL(m, k)$ are two irreducible representation of the group G_1 and G_2 with characters λ_1 and λ_2 respectively , then $T_1 \otimes T_2$ is irreducible representation of the group $G_1 \times G_2$ with character $\lambda_1 . \lambda_2$.
Proposition (3.4) [2]
The rational valued characters $\theta_i = \sum_{\sigma \in Gal(Q(\lambda i)/Q)} \sigma(\lambda_i)$
Form basis for $R(\overline{G})$, where λ_i are the irreducible characters of G and their numbers are equal to the number of all distinct Γ– classes of G .

The rational character table of the quaternion group Q_{2m} when m is prime number(3.5) [3]
The rational characters table of Q_{2m} when m is prime number is given in the following table :
$\equiv^*(Q_{2m}) =$

Table(2)

Γ-classes	[I]	[x^2]	[x^m]	[x]	[y]
θ_1	1	1	1	1	1
θ_2	m-1	-1	m-1	-1	0
θ_3	1	1	1	1	-1
θ_4	m-1	-1	-(m-1)	1	0
θ_5	2	2	-2	-2	0

The Group $Q_{2m} \times C_5$ (3.6)

The direct product group $Q_{2m} \times C_5$, where C_5 is a cyclic group of order 5 then $| Q_{2m} \times C_5 | =$ 20 m since , the irreducible representation of the group $Q_{2m} \times C_5$ are the tensor products of those of Q_{2m} and those of C_5 . The group C_5 has five irreducible representation , their characters λ_1 , λ_2 , λ_3 , λ_4 and λ_5 are given in the table (2) :

Table (3)

CLa	I	r	r^2	r^3	r^4
| CLa |	1	1	1	1	1
λ_1'	1	1	1	1	1
λ_2'	1	ϵ	ϵ^2	ϵ^3	ϵ^4
λ_3'	1	ϵ^2	ϵ^4	ϵ	ϵ^3
λ_4'	1	ϵ^3	ϵ	ϵ^4	ϵ^2
λ_5'	1	ϵ^4	ϵ^3	ϵ^2	ϵ

According to theorem (3.3) , each irreducible characters λ_i of Q_{2m} defines five irreducible characters λ_{i1} , λ_{i2} , λ_{i3} , λ_{i4} and λ_{i5} such that $\lambda_{i1} = \lambda_i \lambda_1$, $\lambda_{i2} = \lambda_i \lambda_2$, $\lambda_{i3} = \lambda_i \lambda_3$,

$\lambda_{i4} = \lambda_i \lambda_4$ and $\lambda_{i5} = \lambda_i \lambda_5$ of $Q_{2m} \times C_5$ then

$$\equiv (Q_{2m} \times C_5) = \equiv (Q_{2m}) \otimes \equiv (C_5)$$

Example (3.7)

To find characters table $Q_{14} \times C_5$ from (3.3) we have the characters table of $\equiv (Q_{14})$

clα	[I]	[x^2]	[x^4]	[x^6]	[x^7]	[x]	[x^3]	[x^5]	[y]	[xy]
| clα |	1	2	2	2	1	2	2	2	7	7
ψ_1	1	1	1	1	1	1	1	1	1	1
λ_2	2	V_4	V_6	V_2	2	V_2	V_6	V_4	0	0
λ_4	2	V_6	V_2	V_4	2	V_4	V_2	V_6	0	0
λ_6	2	V_2	V_4	V_6	2	V_6	V_4	V_2	0	0
ψ_2	1	1	1	1	1	1	1	1	- 1	-1
λ_1	2	V_2	V_4	V_6	- 2	V_1	V_3	V_5	0	0
λ_3	2	V_6	V_2	V_4	- 2	V_3	V_5	V_1	0	0
λ_5	2	V_4	V_6	V_2	- 2	V_5	V_1	V_3	0	0
ψ_3	1	1	1	1	- 1	- 1	- 1	- 1	i	- i
ψ_4	1	1	1	1	- 1	- 1	- 1	- 1	- i	i

Table (4)

, where $V_i = 2 \cos (\pi i / 7)$.

$V_{2m} = 2$, $V_m = -2$.

By Theorem (3.3) the characters table of $Q_{14} \times C_5$ can be written as follows :

$\equiv (Q_{14} \times C_5) = \equiv (Q_{14}) \otimes \equiv (C_5)$, Then ≡ ($Q_{14} \times C_5$) is given in the table (14)

4 . The main results

Example (4.1)

To construct the rational valued characters table of ($Q_{2m} \times C_5$) when have to do the following from table(14) we have the characters table of ($Q_{14} \times C_5$) By the definition of ($Q_{2m} \times C_5$) :then,

$\equiv (Q_{14} \text{ x } C_5) = \equiv (Q_{14}) \otimes \equiv (C_5)$

To calculate the rational valued characters table of ($Q_{14} \times C_5$)

$\theta_{11} = \psi_{11} , \theta_{12} = \psi_{12} + \psi_{13} + \psi_{14} + \psi_{15}$

$\theta_{31} = \psi_{21} , \theta_{32} = \psi_{22} + \psi_{23} + \psi_{24} + \psi_{25}$

$\theta_{51} = \psi_{31} + \psi_{41} , \theta_{52} = \psi_{32} + \psi_{33} + \psi_{34} + \psi_{35} + \psi_{42} + \psi_{43} + \psi_{44} + \psi_{45}$

The elements of Gal (λ_{1i}) / Q are :

$\{ \sigma_{1i} , \sigma_{3i} , \sigma_{5i} \}$

$\sigma_{1i} (\lambda_{1i}) = \lambda_{1i} , \sigma_{3i} (\lambda_{1i}) = \lambda_{3i} , \sigma_{5i} (\lambda_{1i}) = \lambda_{5i}$

where i = 1 , 2 , 3 , 4 , 5

By proposition (3.4)

1- (I) if i = 1

$\theta_{41} = \sigma_{11} (\lambda_{11}) + \sigma_{31} (\lambda_{11}) + \sigma_{51} (\lambda_{11}) = \lambda_{11} + \lambda_{31} + \lambda_{51}$

$\theta_{41} ([I , I]) = 2 + 2 + 2 = 6$

$\theta_{41} ([I , r]) = 2 + 2 + 2 = 6$

$\theta_{41} ([x^2 , I]) = V_2 + V_6 + V_4 = -1$

$\theta_{41} ([x^2 , r]) = V_2 + V_6 + V_4 = -1$

$\theta_{41} ([x^7 , I]) = (-2) + (-2) + (-2) = -6$

$\theta_{41} ([x^7 , r]) = (-2) + (-2) + (-2) = -6$

$\theta_{41} ([x , I]) = V_1 + V_3 + V_5 = 1$

$\theta_{41} ([x , r]) = V_1 + V_3 + V_5 = 1$

$\theta_{41} ([y , I]) = 0 + 0 + 0 = 0$

$\theta_{41} ([y , r]) = 0 + 0 + 0 = 0$

1- (II) if i = 2 , 3 , 4 , 5

$\theta_{42} = \sigma_{12} (\lambda_{12}) + \sigma_{13} (\lambda_{13}) + \sigma_{14} (\lambda_{14}) + \sigma_{15} (\lambda_{15}) + \sigma_{32} (\lambda_{12}) + \sigma_{33} (\lambda_{13})$
$+ \sigma_{34} (\lambda_{14}) + \sigma_{35} (\lambda_{15}) + \sigma_{52} (\lambda_{12}) + \sigma_{53} (\lambda_{13}) + \sigma_{54} (\lambda_{14}) + \sigma_{55} (\lambda_{15})$

$\theta_{42} ([I , I]) = 2 +2 +2 +2 +2 +2 +2 +2 +2 +2 +2 +2 = 24$

$\theta_{42} ([I , r]) = 2 \epsilon + 2 \epsilon^2 + 2 \epsilon^3 + 2 \epsilon^4 + 2 \epsilon + 2 \epsilon^2 + 2 \epsilon^3 + 2 \epsilon^4 + 2 \epsilon + 2 \epsilon^2 + 2 \epsilon^3 + 2 \epsilon^4$

$= 2 (\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + 2 (\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + 2 (\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4)$

$= 2 (-1) + 2 (-1) + 2 (-1) = -6$

$\theta_{42} ([x^2, I]) = V_2 + V_2 + V_2 + V_2 + V_6 + V_6 + V_6 + V_6 + V_4 + V_4 + V_4 + V_4$

$= (V_2 + V_6 + V_4) + (V_2 + V_6 + V_4) + (V_2 + V_6 + V_4) + (V_2 + V_6 + V_4)$

$= -1 -1 -1 -1 = -4$

$\theta_{42} ([x^2 , r]) = \epsilon V_2 + \epsilon^2 V_2 + \epsilon^3 V_2 + \epsilon^4 V_2 + \epsilon V_6 + \epsilon^2 V_6 + \epsilon^3 V_6 + \epsilon^4 V_6 + \epsilon V_4 + \epsilon^2 V_4 + \epsilon^3 V_4 + \epsilon^4 V_4$

$= V_2 (\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + V_6 (\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4) + V_4 (\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4)$

$= -V_2 - V_6 - V_4 = - (V_2 + V_6 + V_4) = - (-1) = 1$

$\theta_{42} ([x^7 , I]) = (-2) +(-2) +(-2) +(-2) +(-2) +(-2) +(-2) +(-2) + (-2) +(-2) +(-2) +(-2) = -24$

$\theta_{42} ([x^7 , r]) = (-2 \epsilon) +(-2 \epsilon^2) +(-2 \epsilon^3) +(-2 \epsilon^4) +(-2 \epsilon) +(-2 \epsilon^2) +(-2 \epsilon^3) +(-2 \epsilon^4) +(-2 \epsilon) +(-2 \epsilon^2) +(-2 \epsilon^3) +(-2 \epsilon^4)$

$= (-2)(-1) + (-2)(-1) + (-2)(-1) = 6$

$\theta_{42} ([x , I])=V_1 + V_1 + V_1 + V_1 + V_3 + V_3 + V_3 + V_3 + V_5 + V_5 +V_5 +V_5$

$= (V_1 + V_3 + V_5)+(V_1 + V_3 + V_5)+(V_1 + V_3 + V_5)+(V_1 + V_3 + V_5)$

$= 1 + 1 + 1 + 1 = 4$

$\theta_{42}([x, r]) = \epsilon V_1 + \epsilon^2 V_1 + \epsilon^3 V_1 + \epsilon^4 V_1 + \epsilon V_3 + \epsilon^2 V_3 + \epsilon^3 V_3 + \epsilon^4 V_3$
$+ \epsilon V_5 + \epsilon^2 V_5 + \epsilon^3 V_5 + \epsilon^4 V_5 = (-1) V_1 + (-1) V_3 + (-1) V_5$
$= -1 (V_1 + V_3 + V_5) = (-1)(1) = -1$
$\theta_{42}([y, I]) = 0+0+0+0+0+0+0+0+0+0+0+0 = 0$
$\theta_{42}([y, r]) = 0+0+0+0+0+0+0+0+0+0+0+0 = 0$
Also $\sigma_{1i}(\lambda_{2i}) = \lambda_{2i}$, $\sigma_{3i}(\lambda_{2i}) = \lambda_{4i}$, $\sigma_{5i}(\lambda_{2i}) = \lambda_{6i}$ where $i = 1, 2, 3, 4, 5$
By proposition (3.4)
1- (I) if $i = 1$
$\theta_{21} = \sigma_{11}(\lambda_{21}) + \sigma_{31}(\lambda_{21}) + \sigma_{51}(\lambda_{21}) = \lambda_{21} + \lambda_{41} + \lambda_{61}$
$\theta_{21}([I, I]) = 2+2+2 = 6$
$\theta_{21}([I, r]) = 2+2+2 = 6$
$\theta_{21}([x^2, I]) = V_4 + V_6 + V_2 = -1$
$\theta_{21}([x^2, r]) = V_4 + V_6 + V_2 = -1$
$\theta_{21}([x^7, I]) = 2+2+2 = 6$
$\theta_{21}([x^7, r]) = 2+2+2 = 6$
$\theta_{21}([x, I]) = V_2 + V_4 + V_6 = -1$
$\theta_{21}([x, r]) = V_2 + V_4 + V_6 = -1$
$\theta_{21}([y, I]) = 0+0+0 = 0$
$\theta_{21}([y, r]) = 0+0+0 = 0$

(II) if $i = 2, 3, 4, 5$
$\theta_{22} = \sigma_{12}(\lambda_{22}) + \sigma_{13}(\lambda_{23}) + \sigma_{14}(\lambda_{24}) + \sigma_{15}(\lambda_{25}) + \sigma_{32}(\lambda_{22}) + \sigma_{33}(\lambda_{23})$
$+ \sigma_{34}(\lambda_{24}) + \sigma_{35}(\lambda_{25}) + \sigma_{52}(\lambda_{22}) + \sigma_{53}(\lambda_{23}) + \sigma_{54}(\lambda_{24}) + \sigma_{55}(\lambda_{25})$
$= \lambda_{22} + \lambda_{23} + \lambda_{24} + \lambda_{25} + \lambda_{42} + \lambda_{43} + \lambda_{44} + \lambda_{45} + \lambda_{62} + \lambda_{63} + \lambda_{64} + \lambda_{65}$
$\theta_{22}([I, I]) = 2+2+2+2+2+2+2+2+2+2+2+2 = 24$
$\theta_{22}([I, r]) = 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 +$
$2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 = 2(-1) + 2(-1) + 2(-1) = -6$
$\theta_{22}([x^2, I]) = (V_4 + V_4 + V_4 + V_4) + (V_6 + V_6 + V_6 + V_6) + (V_2 + V_2 + V_2 + V_2) = (-1) + (-1) + (-1) + (-1) = -4$
$\theta_{22}([x^2, r]) = \epsilon V_4 + \epsilon^2 V_4 + \epsilon^3 V_4 + \epsilon^4 V_4 + \epsilon V_6 + \epsilon^2 V_6 + \epsilon^3 V_6 + \epsilon^4 V_6$
$+ \epsilon V_2 + \epsilon^2 V_2 + \epsilon^3 V_2 + \epsilon^4 V_2 = V_4(-1) + V_6(-1) + V_2(-1)$
$= -(V_4 + V_6 + V_2) = -1(-1) = 1$
$\theta_{22}([x^7, I]) = 2+2+2+2+2+2+2+2+2+2+2+2 = 24$
$\theta_{22}([x^7, r]) = 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon + 2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 + 2\epsilon +$
$2\epsilon^2 + 2\epsilon^3 + 2\epsilon^4 = 2(-1) + 2(-1) + 2(-1) = -6$
$\theta_{22}([x, I]) = V_2 + V_2 + V_2 + V_2 + V_4 + V_4 + V_4 + V_4 + V_6 + V_6 + V_6 + V_6$
$= (-1) + (-1) + (-1) + (-1) = -4$
$\theta_{22}([x, r]) = \epsilon V_2 + \epsilon^2 V_2 + \epsilon^3 V_2 + \epsilon^4 V_2 + \epsilon V_4 + \epsilon^2 V_4 + \epsilon^3 V_4 + \epsilon^4 V_4$
$+ \epsilon V_6 + \epsilon^2 V_6 + \epsilon^3 V_6 + \epsilon^4 V_6 = V_2(-1) + V_4(-1) + V_6(-1)$
$= -(V_2 + V_4 + V_6) = -1(-1) = 1$
$\theta_{22}([y, I]) = 0$, $\theta_{22}([y, r]) = 0$

The elements $[I, r], [I, r^2], [I, r^3], [I, r^4]$ are in the same Γ- conjugate and $[x^2, I], [x^4, I], [x^6, I]$ are in the same Γ- conjugate and $[x^2, r], [x^2, r^2], [x^2, r^3], [x^2, r^4], [x^4, r], [x^4, r^2], [x^4, r^3], [x^4, r^4], [x^6, r], [x^6, r^2], [x^6, r^3], [x^6, r^4]$ are in the same Γ- conjugate and $[x^7, r], [x^7, r^2], [x^7, r^3], [x^7, r^4]$
are in the same Γ- conjugate and $[x, I], [x^3, I], [x^5, I]$ are in the same Γ- conjugate and $[x, r], [x, r^2], [x, r^3], [x, r^4], [x^3, r], [x^3, r^2], [x^3, r^3], [x^3, r^4], [x^5, r], [x^5, r^2], [x^5, r^3], [x^5, r^4]$ are in the same Γ- conjugate and $[y, I], [xy, I]$ are in the same Γ- conjugate and $[y, r], [y, r^2], [y, r^3], [y, r^4], [xy, r], [xy, r^2], [xy, r^3], [xy, r^4]$ are in the same Γ- conjugate .

$\equiv^{*}$ ($Q_{14} \times C_5$)

cl_α	[I , I]	[I , r]	[x^2 , I]	[x^2 , r]	[x^7, I]	[x^7 , r]	[x , I]	[x , r]	[y , I]	[y , r]
$\lvert cl_\alpha \rvert$	1	1	2	2	1	1	2	2	7	7
θ_{11}	1	1	1	1	1	1	1	1	1	1
θ_{12}	4	-1	4	-1	4	-1	4	-1	4	-1
θ_{21}	6	6	-1	-1	6	6	-1	-1	0	0
θ_{22}	24	-6	-4	1	24	-6	-4	1	0	0
θ_{31}	1	1	1	1	1	1	1	1	-1	-1
θ_{32}	4	-1	4	-1	4	-1	4	-1	-4	1
θ_{41}	6	6	-1	-1	-6	-6	1	1	0	0
θ_{42}	24	-6	-4	1	-24	6	4	-1	0	0
θ_{51}	2	2	2	2	-2	-2	-2	-2	0	0
θ_{52}	8	-2	8	-2	-8	2	-8	2	0	0

Table (5)

Theorem (4.2)
The rational valued characters table of the group ($Q_{2m} \times C_5$) when m is prime number is given as follows :
$\equiv^{*} (Q_{2m} \times C_5) = \equiv^{*} (Q_{2m}) \otimes \equiv^{*} (C_5)$
Proof : Since $C_5 = \{ I , r , r^2 , r^3 , r^4 \}$

since

$\equiv C_5=$

cl_α	I	R	r^2	r^3	r^4
λ_1	1	1	1	1	1
λ_2	1	ϵ	ϵ^2	ϵ^3	ϵ^4
λ_3	1	ϵ^2	ϵ^4	ϵ	ϵ^3
λ_4	1	ϵ^3	ϵ	ϵ^4	ϵ^2
λ_5	1	ϵ^4	ϵ^3	ϵ^2	ϵ

Table(6)

and ,$\equiv^{*}$ (C_5) =

Γ-classes	I	r
θ_1	1	1
θ_2	4	– 1

Table(7)

Then , $\lambda_1 (I) = \theta_1 (I) = 1$
$\lambda_1 (r) = \lambda_1(r^2) = \lambda_1(r^3) = \lambda_1 (r^4) = \theta_1 (r^i) = 1 , i = 1 , \ldots , 4$
$\lambda_2 (I) + \lambda_3(I) + \lambda_4 (I) + \lambda_5 (I) = \theta_2 (I) = 1 + 1 + 1 + 1 = 4$
$\lambda_2 (r) + \lambda_3(r) + \lambda_4 (r) + \lambda_5 (r) = \theta_2 (r) = \epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4 = – 1$
Where $\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4 = – 1$
From the definition of ($Q_{2m} \times C_5$) theorem (3.3)
$\equiv (Q_{2m} \times C_5) = \equiv (Q_{2m}) \otimes \equiv (C_5)$
each element in $Q_{2m} \times C_5$
Let $u \in Q_{2m} \times C_5$ then u = (q , c) when

$q \in Q_{2m}$ and $c \in C_5$, $C = r^i$, $i = 0 , \ldots , 4$
$q = x^s y^k$, $0 \le s \le 2m$, $k = 0 , 1$
and each irreducible character of $Q_{2m} \times C_5$ is
$\lambda_{ij} = \lambda_i \lambda_j'$
where λ_i is an irreducible character of Q_{2m}
and λ_j' is an irreducible character of C_5 , then

$$\lambda_{ij}(u) = \lambda_{ij}(q,c) = \lambda_i(q) . \lambda_j'(c) = \begin{cases} \lambda_i(q) & \text{if } j = 1 \text{ for all } c \\ 4\lambda_i(q) & \text{if } j = 2,3,4,5 \text{ and } c = I \\ -\lambda_i(q) & \text{if } j = 2,3,4,5 \text{ and } c \neq I \end{cases}$$

From proposition (3.4)

$$\theta_{ij} = \sum_{\sigma \in Gal(Q(\lambda_{ij})/Q)} \sigma(\lambda_{ij})$$

where θ_{ij} is the rational valued of characters table of ($Q_{2m} \times C_5$) then ,

$$\theta_{ij}(u) = \sum_{\sigma \in Gal(Q(\lambda_{ij}(u))/Q)} \sigma(\lambda_{ij}(u)) = \sum_{\sigma \in Gal(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q) . \lambda_j(c))$$

(I) if $j = 1$, $\lambda_j'(c) = 1$

$$\theta_{ij}(u) = \sum_{\sigma \in Gal(Q(\lambda_i(u))/Q)} \sigma(\lambda_i(q)) . \quad \lambda_1'(c) = \theta_i(q) . 1 = \theta_i(q') . \theta_j(I)$$

where θ_i is the rational valued of characters table of Q_{2m}
(II) a – if $j = 2 , 3 , 4 , 5$ and $c = I$

$$\theta_{ij}(u) = \sum \sigma(\lambda_i(q) . \lambda_j(I)) = \sum_{\sigma \in Gal(Q(\lambda_{ij})/Q)} \sigma(\lambda_i(q) \quad . \quad [\sum \sigma \lambda_j(I)])$$

$$= \sum \sigma(\lambda_i(q)[1+1+1+1] = \theta_i(q) . 4 = \theta_i(q) . \theta_j(I)$$

'b – if $j = 2 , 3 , 4 , 5$ and $c \neq I$

$$\theta_{ij}(u) = \sum \sigma(\lambda_i(q) . [\sum \sigma \lambda_j(c)])$$

$$= \sum \sigma(\lambda_i(q)[\epsilon + \epsilon^2 + \epsilon^3 + \epsilon^4)]$$

$$= -\sum \sigma(\lambda_i(q) = \sum \sigma(\lambda_i(q) . -1 = \theta_i(q) . \theta_j(c)$$

From (I) and (II) we have $\theta_{ij} = \theta_i \ \theta_j$ then

$$\equiv^*(Q_{2m} \times C_5) = \equiv^*(Q_{2m}) \times \equiv^*(C_5)$$

Example (4.3)
To find $\equiv^*(Q_{14} \times C_5)$ by using theorem(4.2).from

$\equiv^*(Q_{14}) =$

Γ-classes	[I]	$[x^2]$	$[x^7]$	[x]	[y]
θ_1	1	1	1	1	1
θ_2	6	-1	6	-1	0
θ_3	1	1	1	1	-1
θ_4	6	-1	-6	1	0
θ_5	2	2	-2	-2	0

Table (8)

and $\equiv^{*}(C_5)$=

Γ-classes	[I]	[r]
θ_1	1	1
θ_2	4	-1

Table(9)

By theorem (4.2):
$\equiv^{*}(Q_{14} \times C_5) = \equiv^{*}(Q_{14}) \times \equiv^{*}(C_5)$

$\equiv^{*}(Q_{14}) \times \equiv^{*}(C_5)$=

cl_α	[I , I]	[I , r]	[x^2 , I]	[x^2 , r]	[x^7, I]	[x^7 , r]	[x , I]	[x , r]	[y , I]	[y , r]
$\vert cl_\alpha \vert$	1	1	2	2	1	1	2	2	7	7
θ_{11}	1	1	1	1	1	1	1	1	1	1
θ_{12}	4	-1	4	-1	4	-1	4	-1	4	-1
θ_{21}	6	6	-1	-1	6	6	-1	-1	0	0
θ_{22}	24	-6	-4	1	24	-6	-4	1	0	0
θ_{31}	1	1	1	1	1	1	1	1	-1	-1
θ_{32}	4	-1	4	-1	4	-1	4	-1	-4	1
θ_{41}	6	6	-1	-1	-6	-6	1	1	0	0
θ_{42}	24	-6	-4	1	-24	6	4	-1	0	0
θ_{51}	2	2	2	2	-2	-2	-2	-2	0	0
θ_{52}	8	-2	8	-2	-8	2	-8	2	0	0

Table(10)

Which is same rational valued character table in example (4.1)

Corollary(4.4):

The general form of the rational valued characters table of the group $Q_{2m} \times C_5$ is given by theorem (4.2)

Proof:

$\equiv^{*}(Q_{2m})$ =

Γ-classes	[I]	[x^2]	[x^m]	[x]	[y]
θ_1	1	1	1	1	1
θ_2	m-1	-1	m-1	-1	0
θ_3	1	1	1	1	-1
θ_4	m-1	-1	-(m-1)	1	0
θ_5	2	2	-2	-2	0

Table (11)

$\equiv^* (C_5) =$

Γ-classes	[I]	[r]
θ_1	1	1
θ_2	4	-1

Table(12)

By theorem (4.2):
$\equiv^* (Q_{2m} \times C_5) = \equiv^* (Q_{2m}) \times \equiv^* (C_5) =$

Γ-classes	[I , I]	[I , r]	[x^2 , I]	[x^2 , r]	[x^m, I]	[x^m,r]	[x , I]	[x , r]	[y , I]	[y , r]
$\lvert cl_\alpha \rvert$	1	1	2	2	1	1	2	2	m	m
θ_{11}	1	1	1	1	1	1	1	1	1	1
θ_{12}	4	-1	4	-1	4	-1	4	-1	4	-1
θ_{21}	m-1	m-1	-1	-1	m-1	m-1	-1	-1	0	0
θ_{22}	4(m-1)	-(m-1)	-4	1	4(m-1)	-(m-1)	-4	1	0	0
θ_{31}	1	1	1	1	1	1	1	1	-1	-1
θ_{32}	4	-1	4	-1	4	-1	4	-1	-4	1
θ_{41}	m-1	m-1	-1	-1	-(m-1)	-(m-1)	1	1	0	0
θ_{42}	4(m-1)	-(m-1)	-4	1	-4(m-1)	m-1	4	-1	0	0
θ_{51}	2	2	2	2	-2	-2	-2	-2	0	0
θ_{52}	8	-2	8	-2	-8	2	-8	2	0	0

Table (13)

$\equiv (Q_{14}\times C_5) =$

CL_α	[I,I]	[I,r]	[I,r^2]	[I,r^3]	[I,r^4]	[x^2,I]	[x^2,r]	[x^2,r^2]	[x^2,r^3]	[x^2,r^4]	[x^4,I]	[x^4,r]	[x^4,r^2]	[x^4,r^3]	[x^4,r^4]	[x^6,I]	[x^6,r]	[x^6,r^2]	[x^6,r^3]	[x^6,r^4]	[x^7,I]	[x^7,r]	[x^7,r^2]	[x^7,r^3]	[x^7,r^4]
$\Psi_{(1,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\Psi_{(1,2)}$	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4
$\Psi_{(1,3)}$	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3
$\Psi_{(1,4)}$	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2
$\Psi_{(1,5)}$	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ
$\chi_{(2,1)}$	2	2	2	2	2	V_4	V_4	V_4	V_4	V_4	V_6	V_6	V_6	V_6	V_6	V_2	V_2	V_2	V_2	V_2	2	2	2	2	2
$\chi_{(2,2)}$	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$	V_4	ϵV_4	ϵ^2V_4	ϵ^3V_4	ϵ^4V_4	V_6	ϵV_6	ϵ^2V_6	ϵ^3V_6	ϵ^4V_6	V_2	ϵV_2	ϵ^2V_2	ϵ^3V_2	ϵ^4V_2	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$
$\chi_{(,3)}$	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$	V_4	ϵ^2V_4	ϵ^4V_4	ϵV_4	ϵ^3V_4	V_6	ϵ^2V_6	ϵ^4V_6	ϵV_6	ϵ^3V_6	V_2	ϵ^2V_2	ϵ^4V_2	ϵV_2	ϵ^3V_2	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$
$\chi_{(2,4)}$	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$	V_4	ϵ^3V_4	ϵV_4	ϵ^4V_4	ϵ^2V_4	V_6	ϵ^3V_6	ϵV_6	ϵ^4V_6	ϵ^2V_6	V_2	ϵ^3V_2	ϵV_2	ϵ^4V_2	ϵ^2V_2	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$
$\chi_{(2,5)}$	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ	V_4	ϵ^4V_4	ϵ^3V_4	ϵ^2V_4	ϵV_4	V_6	ϵ^4V_6	ϵ^3V_6	ϵ^2V_6	ϵV_6	V_2	ϵ^4V_2	ϵ^3V_2	ϵ^2V_2	ϵV_2	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ
$\chi_{(4,1)}$	2	2	2	2	2	V_6	V_6	V_6	V_6	V_6	V_2	V_2	V_2	V_2	V_2	V_4	V_4	V_4	V_4	V_4	2	2	2	2	2
$\chi_{(4,2)}$	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$	V_6	ϵV_6	ϵ^2V_6	ϵ^3V_6	ϵ^4V_6	V_2	ϵV_2	ϵ^2V_2	ϵ^3V_2	ϵ^4V_2	V_4	ϵV_4	ϵ^2V_4	ϵ^3V_4	ϵ^4V_4	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$
$\chi_{(4,3)}$	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$	V_6	ϵ^2V_6	ϵ^4V_6	ϵV_6	ϵ^3V_6	V_2	ϵ^2V_2	ϵ^4V_2	ϵV_2	ϵ^3V_2	V_4	ϵ^2V_4	ϵ^4V_4	ϵV_4	ϵ^3V_4	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$
$\chi_{(4,4)}$	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$	V^6	ϵ^3V^6	ϵV^6	ϵ^4V^6	ϵ^2V^6	V^2	ϵ^3V^2	ϵV^2	ϵ^4V^2	ϵ^2V^2	V^4	ϵ^3V^4	ϵV^4	ϵ^4V^4	ϵ^2V^4	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$
$\chi_{(4,5)}$	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ	V^6	ϵ^4V^6	ϵ^3V^6	ϵ^2V^6	ϵV^6	V^2	ϵ^4V^2	ϵ^3V^2	ϵ^2V^2	ϵV^2	V^4	ϵ^4V^4	ϵ^3V^4	ϵ^2V^4	ϵV^4	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ
$\chi_{(6,1)}$	2	2	2	2	2	V_2	V_2	V_2	V_2	V_2	V_4	V_4	V_4	V_4	V_4	V_6	V_6	V_6	V_6	V^6	2	2	2	2	2
$\chi_{(6,2)}$	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$	V_2	ϵV_2	ϵ^2V_2	ϵ^3V_2	ϵ^4V_2	V_4	ϵV_4	ϵ^2V_4	ϵ^3V_4	ϵ^4V_4	V_6	ϵV_6	ϵ^2V_6	ϵ^3V_6	ϵ^4V_6	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$
$\chi_{(6,3)}$	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$	V_2	ϵ^2V_2	ϵ^4V_2	ϵV_2	ϵ^3V_2	V_4	ϵ^2V_4	ϵ^4V_4	ϵV_4	ϵ^3V_4	V_6	ϵ^2V_6	ϵ^4V_6	ϵV_6	ϵ^3V_6	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$
$\chi_{(6,4)}$	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$	V_2	ϵ^3V_2	ϵV_2	ϵ^4V_2	ϵ^2V_2	V_4	ϵ^3V_4	ϵV_4	ϵ^4V_4	ϵ^2V_4	V_6	ϵ^3V_6	ϵV_6	ϵ^4V_6	ϵ^2V_6	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$
$\chi_{(6,5)}$	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ	V_2	ϵ^4V_2	ϵ^3V_2	ϵ^2V_2	ϵV_2	V_4	ϵ^4V_4	ϵ^3V_4	ϵ^2V_4	ϵV_4	V_6	ϵ^4V_6	ϵ^3V_6	ϵ^2V_6	ϵV_6	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ
$\Psi_{(2,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
$\Psi_{(2,2)}$	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4
$\Psi_{(2,3)}$	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3
$\Psi_{(2,4)}$	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2
$\Psi_{(2,5)}$	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ

Table (14)

$\chi_{(1,1)}$	2	2	2	2	2	V_2	V_2	V_2	V_2	V_2	V_4	V_4	V_4	V_4	V_4	V_6	V_6	V_6	V_6	V^6	-2	-2	-2	-2	-2
$\chi_{(1,2)}$	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$	V_2	ϵV_2	$\epsilon^2 V_2$	$\epsilon^3 V_2$	$\epsilon^4 V_2$	V_4	ϵV_4	$\epsilon^2 V_4$	$\epsilon^3 V_4$	$\epsilon^4 V_4$	V_6	ϵV_6	$\epsilon^2 V_6$	$\epsilon^3 V_6$	$\epsilon^4 V_6$	-2	-2ϵ	$-2\epsilon^2$	$-2\epsilon^3$	$-2\epsilon^4$
$\chi_{(1,3)}$	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$	V_2	$\epsilon^2 V_2$	$\epsilon^4 V_2$	ϵV_2	$\epsilon^3 V_2$	V_4	$\epsilon^2 V_4$	$\epsilon^4 V_4$	ϵV_4	$\epsilon^3 V_4$	V_6	$\epsilon^2 V_6$	$\epsilon^4 V_6$	ϵV_6	$\epsilon^3 V_6$	-2	$-2\epsilon^2$	$-2\epsilon^4$	-2ϵ	$-2\epsilon^3$
$\chi_{(1,4)}$	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$	V_2	$\epsilon^3 V_2$	ϵV_2	$\epsilon^4 V_2$	$\epsilon^2 V_2$	V_4	$\epsilon^3 V_4$	ϵV_4	$\epsilon^4 V_4$	$\epsilon^2 V_4$	V_6	$\epsilon^3 V_6$	ϵV_6	$\epsilon^4 V_6$	$\epsilon^2 V_6$	-2	$-2\epsilon^3$	-2ϵ	$-2\epsilon^4$	$-2\epsilon^2$
$\chi_{(1,5)}$	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ	V_2	$\epsilon^4 V_2$	$\epsilon^3 V_2$	$\epsilon^2 V_2$	ϵV_2	V_4	$\epsilon^4 V_4$	$\epsilon^3 V_4$	$\epsilon^2 V_4$	ϵV_4	V_6	$\epsilon^4 V_6$	$\epsilon^3 V_6$	$\epsilon^2 V_6$	ϵV_6	-2	$-2\epsilon^4$	$-2\epsilon^3$	$-2\epsilon^2$	-2ϵ
$\chi_{(3,1)}$	2	2	2	2	2	V_6	V_6	V_6	V_6	V^6	V_2	V_2	V_2	V_2	V_2	V_4	V_4	V_4	V_4	V_4	-2	-2	-2	-2	-2
$\chi_{(3,2)}$	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$	V_6	ϵV_6	$\epsilon^2 V_6$	$\epsilon^3 V_6$	$\epsilon^4 V_6$	V_2	ϵV_2	$\epsilon^2 V_2$	$\epsilon^3 V_2$	$\epsilon^4 V_2$	V_4	ϵV_4	$\epsilon^2 V_4$	$\epsilon^3 V_4$	$\epsilon^4 V_4$	-2	-2ϵ	$-2\epsilon^2$	$-2\epsilon^3$	$-2\epsilon^4$
$\chi_{(3,3)}$	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$	V_6	$\epsilon^2 V_6$	$\epsilon^4 V_6$	ϵV_6	$\epsilon^3 V_6$	V_2	$\epsilon^2 V_2$	$\epsilon^4 V_2$	ϵV_2	$\epsilon^3 V_2$	V_4	$\epsilon^2 V_4$	$\epsilon^4 V_4$	ϵV_4	$\epsilon^3 V_4$	-2	$-2\epsilon^2$	$-2\epsilon^4$	-2ϵ	$-2\epsilon^3$
$\chi_{(3,4)}$	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$	V_6	$\epsilon^3 V_6$	ϵV_6	$\epsilon^4 V_6$	$\epsilon^2 V_6$	V_2	$\epsilon^3 V_2$	ϵV_2	$\epsilon^4 V_2$	$\epsilon^2 V_2$	V_4	$\epsilon^3 V_4$	ϵV_4	$\epsilon^4 V_4$	$\epsilon^2 V_4$	-2	$-2\epsilon^3$	-2ϵ	$-2\epsilon^4$	$-2\epsilon^2$
$\chi_{(3,5)}$	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ	V_6	$\epsilon^4 V_6$	$\epsilon^3 V_6$	$\epsilon^2 V_6$	ϵV_6	V_2	$\epsilon^4 V_2$	$\epsilon^3 V_2$	$\epsilon^2 V_2$	ϵV_2	V_4	$\epsilon^4 V_4$	$\epsilon^3 V_4$	$\epsilon^2 V_4$	ϵV_4	-2	$-2\epsilon^4$	$-2\epsilon^3$	$-2\epsilon^2$	-2ϵ
$\chi_{(5,1)}$	2	2	2	2	2	V_4	V_4	V_4	V_4	V_4	V_6	V_6	V_6	V_6	V^6	V_2	V_2	V_2	V_2	V_2	-2	-2	-2	-2	-2
$\chi_{(5,2)}$	2	2ϵ	$2\epsilon^2$	$2\epsilon^3$	$2\epsilon^4$	V_4	ϵV_4	$\epsilon^2 V_4$	$\epsilon^3 V_4$	$\epsilon^4 V_4$	V_6	ϵV_6	$\epsilon^2 V_6$	$\epsilon^3 V_6$	$\epsilon^4 V_6$	V_2	ϵV_2	$\epsilon^2 V_2$	$\epsilon^3 V_2$	$\epsilon^4 V_2$	-2	-2ϵ	$-2\epsilon^2$	$-2\epsilon^3$	$-2\epsilon^4$
$\chi_{(5,3)}$	2	$2\epsilon^2$	$2\epsilon^4$	2ϵ	$2\epsilon^3$	V_4	$\epsilon^2 V_4$	$\epsilon^4 V_4$	ϵV_4	$\epsilon^3 V_4$	V_6	$\epsilon^2 V_6$	$\epsilon^4 V_6$	ϵV_6	$\epsilon^3 V_6$	V_2	$\epsilon^2 V_2$	$\epsilon^4 V_2$	ϵV_2	$\epsilon^3 V_2$	-2	$-2\epsilon^2$	$-2\epsilon^4$	-2ϵ	$-2\epsilon^3$
$\chi_{(5,4)}$	2	$2\epsilon^3$	2ϵ	$2\epsilon^4$	$2\epsilon^2$	V_4	$\epsilon^3 V_4$	ϵV_4	$\epsilon^4 V_4$	$\epsilon^2 V_4$	V_6	$\epsilon^3 V_6$	ϵV_6	$\epsilon^4 V_6$	$\epsilon^2 V_6$	V_2	$\epsilon^3 V_2$	ϵV_2	$\epsilon^4 V_2$	$\epsilon^2 V_2$	-2	$-2\epsilon^3$	-2ϵ	$-2\epsilon^4$	$-2\epsilon^2$
$\chi_{(5,5)}$	2	$2\epsilon^4$	$2\epsilon^3$	$2\epsilon^2$	2ϵ	V_4	$\epsilon^4 V_4$	$\epsilon^3 V_4$	$\epsilon^2 V_4$	ϵV_4	V_6	$\epsilon^4 V_6$	$\epsilon^3 V_6$	$\epsilon^2 V_6$	ϵV_6	V_2	$\epsilon^4 V_2$	$\epsilon^3 V_2$	$\epsilon^2 V_2$	ϵV_2	-2	$-2\epsilon^4$	$-2\epsilon^3$	$-2\epsilon^2$	-2ϵ
$\Psi_{(3,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
$\Psi_{(3,2)}$	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$
$\Psi_{(3,3)}$	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$
$\Psi_{(3,4)}$	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$
$\Psi_{(3,5)}$	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$
$\Psi_{(4,1)}$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1
$\Psi_{(4,2)}$	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$
$\Psi_{(4,3)}$	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$
$\Psi_{(4,4)}$	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$
$\Psi_{(4,5)}$	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$

Table (14)

[x,I]	[x,r]	$[x,r^2]$	$[x,r^3]$	$[x,r^4]$	$[x^3,I]$	$[x^3,r]$	$[x^3,r^2]$	$[x^3,r^3]$	$[x^3,r^4]$	$[x^5,I]$	$[x^5,r]$	$[x^5,r^2]$	$[x^5,r^3]$	$[x^5,r^4]$	[y,I]	[y,r]	$[y,r^2]$	$[y,r^3]$	$[y,r^4]$	[xy,I]	[xy,r]	$[xy,r^2]$	$[xy,r^3]$	$[xy,r^4]$
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4
1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3
1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2
1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ
V_2	V_2	V_2	V_2	V_2	V_6	V_6	V_6	V_6	V^6	V_4	V_4	V_4	V_4	V_4	0	0	0	0	0	0	0	0	0	0
V_2	ϵV_2	ϵ^2V_2	ϵ^3V_2	ϵ^4V_2	V_6	ϵV_6	ϵ^2V_6	ϵ^3V_6	ϵ^4V_6	V_4	ϵV_4	ϵ^2V_4	ϵ^3V_4	ϵ^4V_4	0	0	0	0	0	0	0	0	0	0
V_2	ϵ^2V_2	ϵ^4V_2	ϵV_2	ϵ^3V_2	V_6	ϵ^2V_6	ϵ^4V_6	ϵV_6	ϵ^3V_6	V_4	ϵ^2V_4	ϵ^4V_4	ϵV_4	ϵ^3V_4	0	0	0	0	0	0	0	0	0	0
V_2	ϵ^3V_2	ϵV_2	ϵ^4V_2	ϵ^2V_2	V_6	ϵ^3V_6	ϵV_6	ϵ^4V_6	ϵ^2V_6	V_4	ϵ^3V_4	ϵV_4	ϵ^4V_4	ϵ^2V_4	0	0	0	0	0	0	0	0	0	0
V_2	ϵ^4V_2	ϵ^3V_2	ϵ^2V_2	ϵV_2	V_6	ϵ^4V_6	ϵ^3V_6	ϵ^2V_6	ϵV_6	V_4	ϵ^4V_4	ϵ^3V_4	ϵ^2V_4	ϵV_4	0	0	0	0	0	0	0	0	0	0
V_4	V_4	V_4	V_4	V_4	V_2	V_2	V_2	V_2	V_2	V_6	V_6	V_6	V_6	V^6	0	0	0	0	0	0	0	0	0	0
V_4	ϵV_4	ϵ^2V_4	ϵ^3V_4	ϵ^4V_4	V_2	ϵV_2	ϵ^2V_2	ϵ^3V_2	ϵ^4V_2	V_6	ϵV_6	ϵ^2V_6	ϵ^3V_6	ϵ^4V_6	0	0	0	0	0	0	0	0	0	0
V_4	ϵ^2V_4	ϵ^4V_4	ϵV_4	ϵ^3V_4	V_2	ϵ^2V_2	ϵ^4V_2	ϵV_2	ϵ^3V_2	V_6	ϵ^2V_6	ϵ^4V_6	ϵV_6	ϵ^3V_6	0	0	0	0	0	0	0	0	0	0
V_4	ϵ^3V_4	ϵV_4	ϵ^4V_4	ϵ^2V_4	V_2	ϵ^3V_2	ϵV_2	ϵ^4V_2	ϵ^2V_2	V_6	ϵ^3V_6	ϵV_6	ϵ^4V_6	ϵ^2V_6	0	0	0	0	0	0	0	0	0	0
V_4	ϵ^4V_4	ϵ^3V_4	ϵ^2V_4	ϵV_4	V_2	ϵ^4V_2	ϵ^3V_2	ϵ^2V_2	ϵV_2	V_6	ϵ^4V_6	ϵ^3V_6	ϵ^2V_6	ϵV_6	0	0	0	0	0	0	0	0	0	0
V_6	V_6	V_6	V_6	V^6	V_4	V_4	V_4	V_4	V_4	V_2	V_2	V_2	V_2	V_2	0	0	0	0	0	0	0	0	0	0
V_6	ϵV_6	ϵ^2V_6	ϵ^3V_6	ϵ^4V_6	V_4	ϵV_4	ϵ^2V_4	ϵ^3V_4	ϵ^4V_4	V_2	ϵV_2	ϵ^2V_2	ϵ^3V_2	ϵ^4V_2	0	0	0	0	0	0	0	0	0	0
V_6	ϵ^2V_6	ϵ^4V_6	ϵV_6	ϵ^3V_6	V_4	ϵ^2V_4	ϵ^4V_4	ϵV_4	ϵ^3V_4	V_2	ϵ^2V_2	ϵ^4V_2	ϵV_2	ϵ^3V_2	0	0	0	0	0	0	0	0	0	0
V_6	ϵ^3V_6	ϵV_6	ϵ^4V_6	ϵ^2V_6	V_4	ϵ^3V_4	ϵV_4	ϵ^4V_4	ϵ^2V_4	V_2	ϵ^3V_2	ϵV_2	ϵ^4V_2	ϵ^2V_2	0	0	0	0	0	0	0	0	0	0
V_6	ϵ^4V_6	ϵ^3V_6	ϵ^2V_6	ϵV_6	V_4	ϵ^4V_4	ϵ^3V_4	ϵ^2V_4	ϵV_4	V_2	ϵ^4V_2	ϵ^3V_2	ϵ^2V_2	ϵV_2	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	1	ϵ	ϵ^2	ϵ^3	ϵ^4	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$
1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	1	ϵ^2	ϵ^4	ϵ	ϵ^3	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$
1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	1	ϵ^3	ϵ	ϵ^4	ϵ^2	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$
1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	1	ϵ^4	ϵ^3	ϵ^2	ϵ	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$

Table (14)

V_1	V_1	V_1	V_1	V_1	V_3	V_3	V_3	V_3	V_3	V_5	V_5	V_5	V_5	V_5	0	0	0	0	0	0	0	0	0	0
V_1	ϵV_1	$\epsilon^2 V_1$	$\epsilon^3 V_1$	$\epsilon^4 V_1$	V_3	ϵV_3	$\epsilon^2 V_3$	$\epsilon^3 V_3$	$\epsilon^4 V_3$	V_5	ϵV_5	$\epsilon^2 V_5$	$\epsilon^3 V_5$	$\epsilon^4 V_5$	0	0	0	0	0	0	0	0	0	0
V_1	$\epsilon^2 V_1$	$\epsilon^4 V_1$	ϵV_1	$\epsilon^3 V_1$	V_3	$\epsilon^2 V_3$	$\epsilon^4 V_3$	ϵV_3	$\epsilon^3 V_3$	V_5	$\epsilon^2 V_5$	$\epsilon^4 V_5$	ϵV_5	$\epsilon^3 V_5$	0	0	0	0	0	0	0	0	0	0
V_1	$\epsilon^3 V_1$	ϵV_1	$\epsilon^4 V_1$	$\epsilon^2 V_1$	V_3	$\epsilon^3 V_3$	ϵV_3	$\epsilon^4 V_3$	$\epsilon^2 V_3$	V_5	$\epsilon^3 V_5$	ϵV_5	$\epsilon^4 V_5$	$\epsilon^2 V_5$	0	0	0	0	0	0	0	0	0	0
V_1	$\epsilon^4 V_1$	$\epsilon^3 V_1$	$\epsilon^2 V_1$	ϵV_1	V_3	$\epsilon^4 V_3$	$\epsilon^3 V_3$	$\epsilon^2 V_3$	ϵV_3	V_5	$\epsilon^4 V_5$	$\epsilon^3 V_5$	$\epsilon^2 V_5$	ϵV_5	0	0	0	0	0	0	0	0	0	0
V_3	V_3	V_3	V_3	V_3	V_5	V_5	V_5	V_5	V_5	V_1	V_1	V_1	V_1	V_1	0	0	0	0	0	0	0	0	0	0
V_3	ϵV_3	$\epsilon^2 V_3$	$\epsilon^3 V_3$	$\epsilon^4 V_3$	V_5	ϵV_5	$\epsilon^2 V_5$	$\epsilon^3 V_5$	$\epsilon^4 V_5$	V_1	ϵV_1	$\epsilon^2 V_1$	$\epsilon^3 V_1$	$\epsilon^4 V_1$	0	0	0	0	0	0	0	0	0	0
V_3	$\epsilon^2 V_3$	$\epsilon^4 V_3$	ϵV_3	$\epsilon^3 V_3$	V_5	$\epsilon^2 V_5$	$\epsilon^4 V_5$	ϵV_5	$\epsilon^3 V_5$	V_1	$\epsilon^2 V_1$	$\epsilon^4 V_1$	ϵV_1	$\epsilon^3 V_1$	0	0	0	0	0	0	0	0	0	0
V_3	$\epsilon^3 V_3$	ϵV_3	$\epsilon^4 V_3$	$\epsilon^2 V_3$	V_5	$\epsilon^3 V_5$	ϵV_5	$\epsilon^4 V_5$	$\epsilon^2 V_5$	V_1	$\epsilon^3 V_1$	ϵV_1	$\epsilon^4 V_1$	$\epsilon^2 V_1$	0	0	0	0	0	0	0	0	0	0
V_3	$\epsilon^4 V_3$	$\epsilon^3 V_3$	$\epsilon^2 V_3$	ϵV_3	V_5	$\epsilon^4 V_5$	$\epsilon^3 V_5$	$\epsilon^2 V_5$	ϵV_5	V_1	$\epsilon^4 V_1$	$\epsilon^3 V_1$	$\epsilon^2 V_1$	ϵV_1	0	0	0	0	0	0	0	0	0	0
V_5	V_5	V_5	V_5	V_5	V_1	V_1	V_1	V_1	V_1	V_3	V_3	V_3	V_3	V_3	0	0	0	0	0	0	0	0	0	0
V_5	ϵV_5	$\epsilon^2 V_5$	$\epsilon^3 V_5$	$\epsilon^4 V_5$	V_1	ϵV_1	$\epsilon^2 V_1$	$\epsilon^3 V_1$	$\epsilon^4 V_1$	V_3	ϵV_3	$\epsilon^2 V_3$	$\epsilon^3 V_3$	$\epsilon^4 V_3$	0	0	0	0	0	0	0	0	0	0
V_5	$\epsilon^2 V_5$	$\epsilon^4 V_5$	ϵV_5	$\epsilon^3 V_5$	V_1	$\epsilon^2 V_1$	$\epsilon^4 V_1$	ϵV_1	$\epsilon^3 V_1$	V_3	$\epsilon^2 V_3$	$\epsilon^4 V_3$	ϵV_3	$\epsilon^3 V_3$	0	0	0	0	0	0	0	0	0	0
V_5	$\epsilon^3 V_5$	ϵV_5	$\epsilon^4 V_5$	$\epsilon^2 V_5$	V_1	$\epsilon^3 V_1$	ϵV_1	$\epsilon^4 V_1$	$\epsilon^2 V_1$	V_3	$\epsilon^3 V_3$	ϵV_3	$\epsilon^4 V_3$	$\epsilon^2 V_3$	0	0	0	0	0	0	0	0	0	0
V_5	$\epsilon^4 V_5$	$\epsilon^3 V_5$	$\epsilon^2 V_5$	ϵV_5	V_1	$\epsilon^4 V_1$	$\epsilon^3 V_1$	$\epsilon^2 V_1$	ϵV_1	V_3	$\epsilon^4 V_3$	$\epsilon^3 V_3$	$\epsilon^2 V_3$	ϵV_3	0	0	0	0	0	0	0	0	0	0
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	i	i	i	i	i	-i	-i	-i	-i	-i
-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$	i	$i\epsilon$	$i\epsilon^2$	$i\epsilon^3$	$i\epsilon^4$	-i	$-i\epsilon$	$-i\epsilon^2$	$-i\epsilon^3$	$-i\epsilon^4$
-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	i	$i\epsilon^2$	$i\epsilon^4$	$i\epsilon$	$i\epsilon^3$	-i	$-i\epsilon^2$	$-i\epsilon^4$	$-i\epsilon$	$-i\epsilon^3$
-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$	i	$i\epsilon^3$	$i\epsilon$	$i\epsilon^4$	$i\epsilon^2$	-i	$-i\epsilon^3$	$-i\epsilon$	$-i\epsilon^4$	$-i\epsilon^2$
-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$	i	$i\epsilon^4$	$i\epsilon^3$	$i\epsilon^2$	$i\epsilon$	-i	$-i\epsilon^4$	$-i\epsilon^3$	$-i\epsilon^2$	$-i\epsilon$
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-i	-i	-i	-i	-i	I	i	i	i	I
-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$	-1	$-\epsilon$	$-\epsilon^2$	$-\epsilon^3$	$-\epsilon^4$	-i	$-i\epsilon$	$-i\epsilon^2$	$-i\epsilon^3$	$-i\epsilon^4$	I	$i\epsilon$	$i\epsilon^2$	$i\epsilon^3$	$i\epsilon^4$
-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	-1	$-\epsilon^2$	$-\epsilon^4$	$-\epsilon$	$-\epsilon^3$	-i	$-i\epsilon^2$	$-i\epsilon^4$	$-i\epsilon$	$-i\epsilon^3$	I	$i\epsilon^2$	$i\epsilon^4$	$i\epsilon$	$i\epsilon^3$
-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$	-1	$-\epsilon^3$	$-\epsilon$	$-\epsilon^4$	$-\epsilon^2$	-i	$-i\epsilon^3$	$-i\epsilon$	$-i\epsilon^4$	$-i\epsilon^2$	I	$i\epsilon^3$	$i\epsilon$	$i\epsilon^4$	$i\epsilon^2$
-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$	-1	$-\epsilon^4$	$-\epsilon^3$	$-\epsilon^2$	$-\epsilon$	-i	$-i\epsilon^4$	$-i\epsilon^3$	$-i\epsilon^2$	$-i\epsilon$	I	$i\epsilon^4$	$i\epsilon^3$	$i\epsilon^2$	$i\epsilon$

Table (14)

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