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ON Q-SMARANDACHE COMPLETELY CLOSED IDEAL WITH RESPECT TO AN ELEMENT OF A SMARANDACHE BH-ALGEBRA

المثاليةQ- سمرندش المغلقة تماماً بالنسبة الى عنصر في جبر سمرندش -BH

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Abstract

In this paper, we define the concepts of a Q-Smarandache completely closed ideal of a Smarandache BH- algebra and a Q-Smarandache completely closed ideal with respect to an element of a Smarandache BH-algebra.We stated and proved some theorems which determine the relationships between these notions and some types of Q-Smarandache ideals of a Smarandache BH-algebra.

الخلاصة

عرفنا في هذا البحث مفهومي المثاليةQ- سمرندش المغلقة تماما , المثالية Q- سمرندش المغلقة تماما بالنسبة الى عنصر في جبر- سمرندش.BH واعطينا وبرهنا بعض المبرهنات التي تحدد العلاقة بين هذين المفهومين والمثاليات الاخرى في جبر- سمرندش BH.

INTRODUCTION

The notion of BCK-algebra was formulated first in 1966 by Y .Imai and K.Iseki as a generalization of the concept of set-theoretic difference and propositional calculus [5].In 1983,Q.P.Hu and X.Li introduced the notion of BCH-algebra which are generalizetion of BCK/BCI-algebras[7]. In 1998 , Y. B. Jun, E. H. Roh and H .S .Kim introduced the notion of BH-algebra ,which is a generalization of BCH-algebra [10]. In 2000,S.S. Ahn and H.S. Kim discussed positive implactive in BH-algebra [9] . In 2005, Y. B .Jun introduced the notion of Smarandache BCI-algebra , Smarandache ideal of Smarandache BCI-algebra[11].After year,Y.B.Jun introduced the notion of a Smarandache fantastic ideal of Smarandache BCI-algebra [12].In 2009, A.B. Saeid and A. Namdar , introduced the notion of a Smarandache BCH-algebra and Q-Smarandache ideal of a Smarandache BCH-algebra [1] . In 2012, H. H .Abass introduced the notion of Q-Smarandache closed ideal and Q-Smarandache fuzzy closed Ideal with respect to an element of a Q-Smarandache BCH –algebra [3].In the same year, H. H. Abass and H. A. Dahham introduced the notion of completely closed ideal with respect to an element of BH-algebra [4] . In this paper, we define the concepts of a Q-Smarandache completely closed ideal of a Smarandache BH-algebra and a Q-Smarandache completely closed ideal with respect to an element of a Smarandache BH-algebra .We stated and proved some theorems which determine the relationships between these notions and some types of Q-Smarandache ideals of a Smarandache BH-algebra.

1.PRELIMINARIES

In this section, we give some basic concept about a BCK-algebra ,a BCI-algebra a BCH-algebra , (a branch subset , BCA-part , a medial part , a subalgebra , an ideal , a completely closed ideal , a completely closed ideal with respect to an element of a BH-algebra) of a BH-algebra and a Q-Smarandache (fantastic ideal , ideal , closed ideal with respect to an element of a marandache BH-algebra) of a Smarandache BH-algebra with some theorems, propositions and examples.

Definition (1.1) :[5]

A BCI-algebra is an algebra (X,*,0) of type (2,0) , where X is a nonempty set, * is a binary operation and 0 is a constant , satisfying the following axioms:

i. $((x*y)*(x*z))*(z*y) = 0$, for all x, y, z$\in$ X,

ii. $(x*(x*y))*y = 0$, for all x, y$\in$ X,

iii. $x * x = 0$, for all x$\in$ X,

iv. $x * y =0$ and $y * x = 0$ imply $x = y$. for all x, y$\in$ X.

Definition (1.2) :[11]

A ***BCK-algebra*** is a BCI-algebra satisfying the axiom:

v. $0 * x = 0$ for all $x \in X$.

Definition (1.3):[7]

A ***BCH-algebra*** is an algebra (X,*,0), where X is a nonempty set ,"*" is a binary operation and 0 is a constant , satisfying the following axioms:

i. $x*x=0$, $\forall$ $x\in X$.

ii. $x*y=0$ and $y*x=0$ imply $x=y$, $\forall$ $x,y\in X$.

iii. $(x*y)*z=(x*z)*y$, $\forall$ $x,y,z\in X$.

Definition (1.4):[10]

A ***BH-algebra*** is a nonempty set X with a constant 0 and a binary operation"*" satisfying the following conditions:

i. $x*x=0$, $\forall$ $x\in X$.

ii. $x*y=0$ and $y*x =0$ imply $x = y$, $\forall$ $x, y\in X$.

iii. $x*0 =x$, $\forall$ $x\in X$.

Remark (1.5) :[10]

i. Every BCK-algebra is a BCI-algebra.

ii. Every BCK-algebra is a BCH-algebra.

iii. Every BCK-algebra is a BH-algebra.

iv. Every BCI\BCH-algebra is a BH-algebra.

Definition (1.6) :[8]

Let X be a BH-algebra. Then the set $med(X)=\{x\in X|0*(0*x)=x\}$ is called the ***medial part*** of X

Definition (1.7) :[6]

Let X be a BCH-algebra and $a\in med(X)$.$B(a) = \{x \in X : a*x=0\}$ is called ***a branch subset*** of X. ***determined by a***

We generalize the concept of a ***BH-algebra***

Definition (1.8) :

Let X be a BH-algebra and $a\in med(X)$.$B(a) = \{x \in X : a*x=0\}$ is called ***a branch subset*** of X. ***determined by a***

Definition (1.9):[3]

Let X be a BH-algebra. Then the set $X_+=\{x\in X:0*x=0\}$ is called the ***BCA-part*** of X .

Definition (1.10):[2]

A BCK-algebra X is called commutative if $x*(x*y) = y*(y*x)$, for all x, y$\in$X.

Lemma(1.11):[2]

In BCI-algebra X the following conditions
are equivalent:

i. $x * y = x *(y *(y *x))$ $\forall$ $x,y \in X$.

ii. $x*(y*(y*x))=y*(x*(x*y))$ $\forall$ $x,y\in X$.

iii. X is a commutative BCK-algebra

Definition (1.12) :[8]

Let X a BH-algebra and S ⊆ X.Then S is called a ***subalgebra*** of X if x*y∈ S for all x,y∈ S.

Remark(1.13):[9]

Let (X,*,0) and (Y,# ,0΄) be BH-algebras. A mapping f:X→Y is called a ***homomorphism*** if f(x*y)=f(x)#f(y) for any x,y∈X. A homomorphism f is called a ***monomorphism*** (resp., ***epimorphism***) if it is injective (resp**., surjective**). If f : X→ Y is a homomorphism of BH-algebras, then f(0) =.0΄

Definition (1.14):[10]

Let X be a BH-algebra and I ⊆ X. Then I is called an ***ideal*** of X if it satisfies:

i. 0∈ I.

ii. x*y ∈ I and y∈ I imply x∈ I.

Definition (1.15): [4]

An ideal I of a BH-algebras is called a ***completely closed ideal*** if x ∗ y ∈ I, ∀ x, y ∈ I.

Definition (1.16):[4]

Let I be an ideal of a BH-algebras X and b ∈X. Then I is called a ***completely closed ideal*** with respect to b(denoted by ***b-completely closed ideal***) if b*(x*y)∈I, ∀x,y∈I.

Definition(1.17):[1]

A ***Smarandache BCH\BCI-algebra*** is defined to be a BCH-algebra X in which there exists a proper subset Q of X such that

i.0 ∈ Q and |Q| ≥ 2 .

ii. Q is a BCK-algebra under the operation of X.

We generalize the concept of a ***Smarandache BH-algebra***

Definition(1.18):

A ***Smarandache BH-algebra*** is defined to be a BH-algebra X in which there exists a proper subset Q of X such that

i.0 ∈ Q and |Q| ≥ 2 .

ii.Q is a BCK-algebra under the operation of X.

Definition(1.19):[11]

Let X be Smarandache BCI-algebra**.** A nonempty subset I of X is called a ***Smarandache ideal of X related to Q*** (or briefly***, Q-Smarandache ideal*** of X) if it satisfy

i.0 ∈ I ,

ii.(∀x ∈Q)(∀y ∈ I)(x*y ∈ I⇒ x ∈ I) .

We generalize the concept of a ***Q-Smarandache ideal*** to the ***Smarandache BH-algebra***

Definition(1.20):

Let X be Smarandache BH-algebra**.** A nonempty subset I of X is called a ***Smarandache ideal of X related to Q*** (or briefly***, Q-Smarandache ideal*** of X) if it satisfy

i.0 ∈ I ,

ii. (∀x ∈Q)(∀y ∈ I)(x*y ∈ ⇒ x ∈ I) .

Theorem(1.21):[1]

Let X be Smarandache BCH-algebra **.**Then any ideal of X is a Q-Smarandache ideal of X.

We generalize the theorem (1.21) to the ***Smarandache BH-algebra***

Theorem(1.22):

Let X be Smarandache BH-algebra .Then any ideal of X is a Q-Smarandache ideal of X.

Theorem (1.23):[1]

Let Q_1 and Q_2 be BCK-algebras contained in a Smarandache BCH algebra X and $Q_1 \subseteq Q_2$. Then every Smarandache ideal of X related to Q_2 is a Smarandache ideal of X related to Q_1.

We generalize the theorem(1.23)to the ***Smarandache BH-algebra.***

Theorem (1.24):

Let Q_1 and Q_2 be BCK-algebras contained in a Smarandache BH algebra X and

$Q_1 \subseteq Q_2$. Then every Smarandache ideal of X related to Q_2 is a Smarandache ideal of X related to Q_1.

Definition(1.25):[12]

Let X be Smarandache BCI-algebra .A nonempty subset I of X is called ***a Smaradache fantastic ideal of X related to Q*** (or briefly, Q-***Smarandache fantastic ideal*** of X) if it satisfies:

i.$0 \in I$

ii.$(x,y \in Q)(z \in I)(x*y)*z \in I \Rightarrow x *(y *(y *x)) \in I$

We generalize concept of a ***Q-Smarandache fantastic ideal*** to the ***Smarandache BH-algebra***

Definition (1.26):

Let X be Smarandache BH-algebra .A nonempty subset I of X is called ***a Smaradache fantastic ideal of X related to Q*** (or briefly ,Q-***Smarandache fantastic ideal*** of X) if it satisfies:

i.$0 \in I$

ii.$(x,y \in Q)(z \in I)(x*y)*z \in I \Rightarrow x *(y *(y *x)) \in I$

Theorem(1.27):[12]

Let X be Smarandache BCI-algebra. Every Q-Smarandache fantastic ideal of X is a Q-Smarandache ideal of X

We generalize the theorem(1.27) to a ***Q-Smarandache fantastic ideal*** to a ***Smarandache BH-algebra.***

Theorem(1.28):

Let X be Smarandache BH-algebra. Every Q-Smarandache fantastic ideal of X is a Q-Smarandache ideal of X

Definition(1.29):[3]

Let X be a BCH-algebra and I be a Q-Smarandache ideal of X . Then I is called a ***Q-Smarandache Closed Ideal with respect to an element b∈X*** (denoted ***Q-Smarandache b-closed ideal***) if $b*(0*x)\in I$, for all $x\in I$.

We generalize concept of a ***Q-Smarandache b-closed ideal*** to the ***Smarandache BH-algebra.***

Definition(1.30):

Let X be a BH-algebra and I be a Q-Smarandache ideal of X . Then I is called a ***Q-Smarandache Closed Ideal with respect to an element b∈X*** (denoted ***Q-Smarandache b-closed ideal***) if $b*(0*x)\in I$, for all $x\in I$.

2.THE MAIN RESULTS

In this section we introduced the concepts a Q-Smarandache completely closed ideal of a Smarandache BH-algebra and Q-Smarandache completely closed ideal with respect an element of Smarandache BH-algebra .

Definition (2.1):

A Q-Smarandache ideal I of a Smarandache BH-algebras is called a ***Q-Smarandache completely closed ideal*** if $x * y \in I$, for all $x,y\in I$

Example (2.2):

Consider the Q-Smarandache BH-algebra X. Where X={0,1,2,3} and Q={0,1} with binary operation * defined by:

*	0	1	2	3
0	0	0	0	0
1	1	0	3	3
2	2	0	0	2
3	3	0	0	0

The Q-Smarandache ideal I={0,1,3} is a Q-Smarandache completely closed ideal of X Since:
$0 * 0 = 0 \in I$, $0 * 1 = 1 \in I$, $0 * 3 = 0 \in I$
$1 * 0 = 1 \in I$, $1 * 1 = 0 \in I$, $1 * 3 = 3 \in I$
But the Q-Smarandache ideal J={0,1,2} is not a Q-Smarandache completely closed ideal since:
$1,2 \in J$ but $1 * 2 = 3 \notin J$.

Proposition (2.3):

Let X be Q-Smarandache BH-algebra .Then every Q-Smarandache completely closed ideal of X is a subalgebra.

Proof:

Let I be a Q-Smarandache completely closed ideal of X.
⇒ I is a Q-Smarandache ideal of X. [By definition(2.1)]
Now , let x,y∈I
⇒ x*y∈ I [By definition(2.1)]
Then I is subalgebra of X [By definition(1.12)]
.■

Remark(2.4)

The converse proposition(2.3) is not hold as in the following example.
Consider the Smarandache BH-algebras X ,Where X={0,1,2,3,4} and Q={0,2} with binary operation * defined by

*	**0**	1	2	3	4
0	0	0	0	0	4
1	1	0	0	0	4
2	2	2	0	0	4
3	3	3	3	0	4
4	4	4	4	4	0

The set S={0,1,4} is subalgebra but is not a Q-Smarandache completely closed ideal of a BH-algebra Since: 2∈Q∧ 2*4=4∈ S but 2∉ S

Proposition (2.5):

Let Q_1 andQ_2 be a BCK-algebras contained in a Smarandache BH algebra X and $Q_1 \subseteq Q_2$. Then every Smarandache completely closed ideal of X related to Q_2 is a Smarandache completely closed ideal of X related to Q_1 .

Proof:

Let I be Q_2-Smarandache completely closed ideal of X.

⇒ I is a Q_2-Smarandache ideal of X. [By Definition(2.1)]
⇒ I is Q_1-Smarandache ideal of X. [By Theorem(1.24)]
Now,let x, y ∈ I
⇒x*y∈ I [Since I is a Q_2-Smarandache completely closed ideal of X]
⇒ I is a Q_1-Smarandache completely closed ideal. ■

Proposition (2.6):

Let X be a Smarandache BH-algebra and I be a completely closed ideal .Then I is a Q-Smarandache completely closed ideal.

Proof:

Let I be completely closed ideal of X.
⇒ I is an ideal of X. [By Definition(1.15)]
⇒ I is a Q-Smarandache ideal of X. [By theorem(1.22)]
Now,let x, y ∈ I
⇒x*y∈ I [Since I is a completely closed ideal of X]
⇒ I is a Q-Smarandache completely closed ideal. ■

Proposition (2.7):

Let X be Smarandache BH-algebra, Then every Q-Smarandache ideal which is contained in Q is a Q-Smarandache completely closed ideal.

Proof:

Let I be Q-Smarandache ideal of X and x ,y ∈ I

⇒ x*y ∈ Q [Since I⊆ Q]

When (x*y)*x=(x*x)*y [Since (x*y)*z =(x*z)*y. By definition(1.3)(iii) and remark(1.5)(ii)]

=0*y [Since x*x =0. By definition(1.2)(iii)]

= 0 ∈ I [Since Q is a BCK-algebra.By definition (1.2)(v)]

⇒(x*y)*x∈ I and x∈I

⇒x*y∈ I [Since I is a Q-Smarandache ideal By.definition(1.20)(ii)]

⇒ I is a Q-Smarandache completely closed ideal.■

Proposition (2.8):

Let X be Smarandache BH-algebra, and I be a Q-Smarandache ideal such that Q is commutative BCK-algebra .Then I is a Q-Smarandache fantastic ideal.

Proof:

1. 0∈I [Since I is a Q-Smarandache ideal. By definition(1.20)(i)]

2.Let x,y ∈ Q , z ∈ I and (x*y)*z ∈ I

⇒x*y∈ I. [Since I is a Q-Smarandache ideal. By definition(1.20)(ii)]

But x*y=x*(y*(y*x)) [Since Q is a commutative BCK-algebra by Lemma(1.11) (iii⇒i)and remark(1.5)(i)]

⇒x*(y*(y*x)) ∈ I

⇒ I is a Q-Smarandache fantastic ideal. [By definition(1.26)]

. ■

Proposition (2.9):

Let X be Smarandache BH-algebra, and I be Q-Smarandache completely closed ideal such that Q is commutative BCK-algebra .Then I is a Q-Smarandache fantastic ideal.

Proof:

Let I be Q-Smarandache completely closed ideal of X.

⇒ I be Q-Smarandache ideal of X. [By definition (2.1)]

By Proposition (2.8),we have

I is a Q-Smarandache fantastic ideal. ■

Proposition (2.10):

Let X be Smarandache BH-algebra. Then every Q-Smarandache fantastic ideal which is contained in Q is a Q-Smarandache completely closed ideal.

Proof:

Let I be Q-Smarandache fantastic ideal of X.

⇒ I be Q-Smarandache ideal of X. [By theorem(1.28)]

By proposition (2.7),we have

I is a Q-Smarandache completely closed ideal.■

Remarks (2.11):

The converse of a Proposition(2.10) is not be true as in the following example

Example (2.12):

Consider the Q-Smarandache BH-algebras X .Where X={0,1,2,3,4,5} and Q={0,1,2,3,4} with binary operation * defined by:

*	0	1	2	3	4	5
0	0	0	0	0	0	5
1	1	0	1	0	1	5
2	2	2	0	2	0	5
3	3	1	3	0	3	5
4	4	4	4	4	0	5
5	5	5	5	5	5	5

The a Q-Smarandache completely closed ideal I, where I={0,1,3} is not a Q-Smarandache fantastic ideal Since:2,4∈Q,3∈I and (2*4)*3=0∈I but 2*(4*(4*2))=2∉I

Theorem (2.13):

Let X be Smarandache BH-algebra and I be a Q-Smarandache ideal such that x*y∉ I for all x ∉ I and y ∈I.Then I is an ideal.

Proof:

Let I be a Q-Smarandache ideal of X and x ∈ X and y ∈ I.

1. 0∈I [Since I is a Q-Smarandache ideal. By definition(1.20)(i)]

2.Let x* y ∈ I and y ∈ I.Then we have two cases.

Case(I): If x∈ Q ⇒ x ∈ I [Since I is a Q-Smarandache ideal. By definition(1.20)(ii)]

Case(II): If x ∉ Q,either x ∈ I or x∉ I

If x ∈ I ⇒I is an ideal

If x∉ I ⇒ x*y ∉ I [By hypothesis]

And this contradiction since x*y ∈I

Therefore, I is an ideal.■

Corollary (2.14):

Let X be Smarandache BH-algebra and I be a Q-Smarandache completely closed ideal such that a*b∉ I for all a ∉ I and b∈I.Then I is a completely closed ideal.

Proof:

Let I be Q-Smarandache completely closed ideal of X.

⇒ I is Q-Smarandache ideal of X. [By definition(2.1)]

⇒ I is an ideal of X. [By theorem(2.13)]

Now,let x,y ∈ I

⇒x*y∈ I [Since I is a Q-Smarandache completely closed ideal]

⇒ I is a completely closed ideal.■

Proposition (2.15):

Let f:(X,*,0)→(Y,#, 0´) be a Smarandache BH-epimorphism ,if I is a Q-Smarandache ideal of X. Then f(I) is a f(Q)-Smarandache ideal of Y.

Proof:

Let I be a Q-Smarandache ideal of X.

1) Since 0∈I⇒f(0)= 0´ ∈f(I). [By remrk(1.13)]

2) Let x#y∈f(I) and y∈f(I)

⇒∃a,b∈I such that f(a)=x, f(b)=y,

⇒f(a)#f(b)∈f(I) and f(b)∈f(I),

⇒f(a*b) ∈f(I) and f(b)∈f(I),

$\Rightarrow a*b \in I$ and $b \in I$,
$\Rightarrow a \in I$ [Since I is a Q-Smarandache ideal of X]
$\Rightarrow f(a) \in f(I)$
$\Rightarrow x \in f(I)$.
$\Rightarrow$ f(I) is a Q-Smarandache ideal.■

Proposition (2.16):

Let $f:(X,*,0) \rightarrow (Y,\#,0')$ be a Smarandache BH-epimorphism, if I is a Q-Smarandache completely closed ideal in X. Then f(I)is a f(Q)-Smarandache completely closed ideal in Y.

Proof:

Let I be a Q-Smarandache completely closed ideal of X ,
$\Rightarrow$ I is a Q-Smarandache ideal [By definition(2.1)]
Then f(I) is a Q-Smarandache ideal [By Proposition(2.15)]
Now, let $x , y \in f(I)$
$\Rightarrow \exists a,b \in I$ such that $f(a)=x$, $f(b)=y$,
$\Rightarrow x\#y=f(a)\#f(b)$
$=f(a*b) \in f(I)$ [Since $a*b \in I$ and I is a Q-Smarandache completely closed ideal. By definition(2.1)]
$\Rightarrow$f(I) is a f(Q)-Smarandache completely closed ideal.■

Proposition (2.17):

Let $\{ I_i , i \in \lambda\}$ be a family of a Q-Smarandache ideals of a BH-algebra X. Then $\bigcap_{i \in \lambda} I_i$ is a Q-Smarandache ideal.

Proof:

1)Since $0 \in I_i \ \forall i \in \lambda \Rightarrow 0 \in \bigcap_{i \in \lambda} I_i$

2)Let $x * y \in \bigcap_{i \in \lambda} I_i$, $y \in \bigcap_{i \in \lambda} I_i$

$\Rightarrow$ $x * y \in I_i$ and $y \in I_i$, $\forall i \in \lambda$
$\Rightarrow$ $x \in I_i \quad \forall i \in \lambda$ [since I_i is a Q-Smarandache ideal $\forall i \in \lambda$]
$\Rightarrow x \in \bigcap_{i \in \lambda} I_i$

$\Rightarrow \bigcap_{i \in \lambda} I_i$ is a Q-Smarandache ideal.■

Proposition (2.18) :

Let $\{ I_i , i \in \lambda\}$ be a family of a Q-Smarandache completely closed ideals of a smarandache BH-algebra X. Then $\bigcap_{i \in \lambda} I_i$ is a Q-Smarandache completely closed ideal .

Proof:

Since I_i is a Q-Smarandache completely closed ideal $\forall i \in \lambda$
$\Rightarrow I_i$ is a Q-Smarandache ideal $\forall i \in \lambda$ [By definition (2.1)]
$\Rightarrow \bigcap_{i \in \lambda} I_i$ is a Q-Smarandache ideal [By Proposition (2.17)]
Now,
Let $x, y \in \bigcap_{i \in \lambda} I_i$
$\Rightarrow$ $x, y \in I_i \ \forall i \in \lambda$
$\Rightarrow x * y \in Ii \ \forall i \in \lambda$ [Since I_i is a Q-Smarandache completely closed ideal $\forall i \in \lambda$]

$\Rightarrow x * y \in \bigcap_{i\in\lambda} I_i$

Therefore, $\bigcap_{i\in\lambda} I_i$ is a Q-Smarandache completely closed ideal.■

Definition (2.19):

Let I be a Q-Smarandache ideal of a BH-algebras X and b ∈X. Then I is called a ***Q-Smarandache completely closed ideal with respect to b*** (denoted by ***Q-Smarandache b-completely closed ideal***) if b*(x*y)∈I, ∀x,y∈I.

Example (2.20):

Consider the Q-Smarandache BH-algebras X .Where X={0,1,2,3,4} and Q={0,1,2,3} with binary operation * defined by:

*	**0**	1	2	3	4
0	0	0	0	0	4
1	1	0	0	0	4
2	2	2	0	0	4
3	3	3	3	0	4
4	4	4	4	4	0

Then Q-Smarandache ideal I={0,1} is a Q-Smarandache 1-completely closed ideal, Since:
1*(0*0)=1∈I , 1*(0*1)=1∈I , 1*(0*3)=1 ∈I , 1*(3*0)=0 ∈I
1*(1*0)=0∈I , 1*(1*1)=1∈I , 1*(3*1)=0 ∈I , 1*(3*3)=1 ∈I
But it is not Q-Smarandache 4-completely closed ideal since:
4*(0*1)=4 ∉ I

Proposition (2.21):

Let X be a Smarandache BH-algebra and b∈X.Then every Q-Smarandache b-completely closed ideal is a Q-Smarandache b-closed ideal.

Proof:

Let I be Q-Smarandache b-completely closed ideal in X.
⇒ I be Q-Smarandache ideal in X. [By definition(2.19)]
Now,let x ∈ I and b ∈X
when x=0
⇒b*(0*y) ∈ I [Since I is a Q-Smarandache completely closed ideal of X]
⇒ I is a Q-Smarandache b-closed ideal.■

Proposition (2.22):

Let X be a Smarandache BH-algebra and let I be a Q-Smarandache ideal of X. Then I is a Q-Smarandache completely closed ideal if and only if I is a Q-Smarandache b-completely closed ideal ∀b∈I.

Proof:

Let x,y∈I and b∈I.
⇒ b*(x*y)∈I [Since I is Q-Smarandache completely closed ideal By definition(2.1)]
⇒I is a Q-Smarandache b-completely closed ideal ∀b∈I.

conversely

Let x,y∈I
⇒ x*y=x*(y*0) [Since X is BH-algebra, by definition(1.4)(iii)]
⇒ x*(y*0)∈I [Since I is Q-Smarandache b-completely closed ideal∀b∈I. . By definition(2.19)]
⇒ x*y∈I
⇒ I is a Q-Smarandache completely closed ideal.■

Theorem (2.23):

Let X be Smarandache BH-algebra, $b \in med(X)$,let I be Q- Smarandache ideal such that $I \subseteq B(b) \cup \{0\}$ and $x*y \in B(b)$ for all $x,y \in X$ .Then I is a Q-Smarandache b-completely closed ideal.

Proof:

Let I be Q-Smarandache ideal in X.

Let $x,y \in I \Rightarrow x,y \in X$

$\Rightarrow x*y \in B(b)\ \forall\ x,y \in X$

$\Rightarrow b*(x*y)=0$ [Since $I \subseteq B(b) \cup \{0\}$ and by definition (1.8)]

$\Rightarrow b*(x*y) \in I$

$\Rightarrow$ I is a Q-Smarandache b-completely closed ideal.■

Proposition (2.24):

Let X be a Smarandache BH-algebra such that $X= X_+$. If I is a Q-Smarandache ideal , then I is a Q-Smarandache 0-completely closed ideal .

Proof :

Let $x, y \in I$

$0*(x*y)=0$ [Since $X=X_+$ .By definition (1.9)]

$\Rightarrow 0*(x*y) \in I$

$\Rightarrow$ I is a Q-Smarandache 0-completely closed ideal.■

Proposition (2.25):

Let X be smarandache BH-algebra and I be a Q-Smarandache b-completely closed ideal such that $x*y \notin I$ for all $x \notin I$ and $y \in I$.Then I is a b-completely closed ideal.

Proof:

Let I be Q-Smarandache h-completely closed ideal in X.

$\Rightarrow$ I is Q-Smarandache ideal in X. [By definition(2.19)]

$\Rightarrow$ I is an ideal in X [By theorem(2.13)]

Let $x,y \in I$

$\Rightarrow b*(x*y) \in I$ [Since I is a Q-Smarandache b-completely closed ideal]

Then I is a b-completely closed ideal.■

Proposition (2.26):

Let $f:(X,*,0) \rightarrow (Y,\#, 0')$ is a Smarandache BH-epimorphism . If I is a Q-Smarandache h-completely closed ideal in X, then f(I)is a f(Q)-Smarandache f(h)-completely closed ideal of Y.

Proof:

Let I be a Q-Smarandache h-completely closed ideal of X.

$\Rightarrow$ I is a Q-Smarandache ideal [By definition(2.19)]

$\Rightarrow$ f(I) is a f(Q)-Smarandache ideal of Y [By Proposition (2.15)]

Now,let $f(x),f(y) \in f(I)$

$\Rightarrow f(b)\#((f(x)\#f(y))=f(b)\#(f(x*y))$

$=f(b*(x*y))$

But $b*(x*y) \in I$

$\Rightarrow f(b*(x*y)) \in f(I)$

$\Rightarrow$ f(I)is a f(Q)-Smarandache f(h)-completely closed ideal of Y .■

Corollary (2.27) :

Let $f: (X, *, 0) \rightarrow (Y, \#,0')$ be a BH-epimorphism. If I is Q-Smarandache 0-completely closed ideal of X, then f(I) is f(Q)-Smarandache $0'$-completely closed ideal in Y.

Proof

Is directly from Proposition (2.26) . ■

Proposition (2.28):

Let { I_i , $i\in\lambda$} be a family of a Q-Smarandache b-completely closed ideals of a Smarandache BH-algebra X. Then $\bigcap_{i\in\lambda} I_i$ is a Q-Smarandache b-completely closed ideal.

Proof:

let X be a BH-algebra, and let I_i be a Q-Smarandache b-completely closed ideal

$\Rightarrow$ I_i is a Q-Smarandache ideal $\forall i\in\lambda$ [By definition (2.19)]

$\Rightarrow \bigcap_{i\in\lambda} I_i$ is a Q-Smarandache ideal [By proposition (2.17)]

Now,

Let $x, y \in \bigcap_{i\in\lambda} I_i$

$\Rightarrow x, y \in I_i \quad \forall i \in \lambda$ [Since I_i is a Q-Smarandache b-completely closed ideal $\forall i \in \lambda$]

$\Rightarrow$ $b*(x*y) \in I_i$, $\forall i \in \lambda$

$\Rightarrow$ $b * (x * y) \in \bigcap_{i\in\lambda} I_i$

Therefore, $\bigcap_{i\in\lambda} I_i$ is a Q-Smarandache b-completely closed ideal. ■

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