NUMERICAL SOLUTION OF THE INTEGRAL DIFFRACTION EQUATION FOR REAL AND PERFECT LENSES

Noori H. Al-Hashimi and Shaker. N. Sharqi[1]
Department of physics, College of Education, Basra University, Basra Iraq
[1]Department of science, college of Basic Education, Misan University, Misan, Iraq

Abstract:
Numerical model for the distribution of laser intensity through lenses exhibiting spherical aberration is proposed. The diffraction integral equation is modified for the case of CO_2 laser in a gas discharge plasma. A multiple distinct collinear regions of intense ionization along the optic axis near the lens focus with equidistance separations are observed, and the distance between them is measured. The results are compared with some available experimental data, and the comparison shows a good agreement between them.

حل عددي لمعادلة الحيود التكاملية للعدسات الحقيقية والعدسات المثالية

نوري حسين نور الهاشمي وشاكر نعيم شرقي[1]
قسم الفيزياء- كلية التربية- جامعة البصرة- بصرة-عراق
[1]قسم العلوم- كلية التربية الأساسية- جامعة ميسان- ميسان- العراق

الملخص:
تم اقتراح نموذج عددي لدراسة توزيع شدة الليزر خلال العدسات التي تعاني من زيغ كروي. لقد تم تعديل معادلة الحيود التكاملية ليتسنى لنا تطبيقها على ليزر ثاني أوكسيد الكاربون CO_2 المستخدم في بلازما التفريغ الغازي. أعطت النتائج عدد من حالات التأين الشديدة على طول المحور الضوئي بالقرب من نقطة تركيز العدسة وبمسافات متساوية، حيث تم قياس المسافة بينها. قورنت النتائج النظرية مع النتائج العملية المتوفرة وقد أظهرت المقارنة تطابقا جيدا بين الاثنين.

Keywords: optical discharge; laser; plasma

Introduction:
The physical processes involved in the initiation and growth of the ionization in gases under the influence of the electromagnetic field in the range of the optical-frequency created by focused of Q-switched laser beams have been studied extensively, see for example Evans and Morgan [pp1099,1968,1], Evans[pp691,1983,2], Morgan [pp94,1986,3], Gamaly [pp944,1992,4 and pp516,1993,5], Rode [pp3676,1997,6], Al-Hashmiy [pp77,1996, 7 and pp13,2001,8], Al-Kelly [pp23,1999,9] and Sharqi [pp104,1996,10]. Non of

these studies take care of the real distribution of the laser beam through the gas discharge plasma. However the practical observation of the plasma created by using Q-switched ruby laser with relatively large laser beam diameter (0.5-1.0) cm. and short lens focal length (1.0-5.0) cm. and the steady-state computation of the interference pattern, based on the integral diffraction equation, for Gaussian beams focused by a simple biconvex spherical-surfaced lens with varying degrees of spherical aberration function, reveal the presence of several zones of high intensity separated by regions of lower intensity lying along the beam axis and the movement of these regions towards the lens.

This study presents results of a numerical analysis of the intensity distribution of the diffraction pattern for CO_2 laser in the neighborhood of the lens focus. The effect of the spherical aberration function on the intensity distribution is taken into account.

Quantitative calculations of the separation between the discrete plasmas and the data are presented in fig(5), and fig(6).

THEORY:

A perfect optical system must surely be one in which every point in an object space corresponds precisely to a point in an image space, being connected to it by the same rays passing through all points of the optical path. The intensity distribution in the region of the Gaussian image point of a perfect, aberration-free lens is then, described by the diffraction pattern of the converging spherical wave front produced by the lens aperture. According to simple geometrical optics, the energy density in the focused image of a plane wave front would be infinite at the focal point. So, if the lens is perfect, the intensity will be a maximum at the Gaussian image point. However, in a real lens this will not generally be the case. In real lens systems, the convergent wave front is always slightly distorted from the ideal spherical shape and this distortion is called *aberration phenomenon,* which can be represented mathematically by an aberration function Φ. This is defined as the number of wavelengths displacement between the perfect undistorted spherical wave front and the distorted wave front at any point on the exit pupil. Therefore, for a given aberration function, the value of the intensity I at any point in the region of the focus of a given lens and a single-mode laser system giving a circular beam of radius *(a)* with a Gaussian electric field intensity profile of the form $E(r) = E_0 \exp(-r^2/2a^2)$, for $0 \le r \le a$, can be computed from the integral diffraction equation, which is given by [9]:

$$\frac{I(z,r)}{\alpha} = \left| \int_0^1 \exp\left(-\frac{\rho^2}{2}\right) \exp\left[i\left\{-0.25kBa^4\rho^4 - 0.5\Gamma_1 z\rho^2\right\}\right] J_0(\Gamma_2 r\rho)\rho d\rho \right|^2 \qquad (1)$$

Where $\Gamma_1 = k(\frac{a}{f})^2$, $\Gamma_2 = k(\frac{a}{f})$, $\rho = \frac{r}{a}$, $\alpha = 3.17\Gamma_1 P$, and

$$B = \frac{1}{8f^3}\left[\frac{n^2}{(n-1)^2} - \frac{n}{n+2} + \frac{4(n+1)^2}{n(n+2)}\right],$$

$\Phi = -0.25Br^4$

Here $J_0(\Gamma_2 r\rho)$ is called the first order *Bessel function*, f is the lens focal length in cm., λ is the laser beam wavelength in cm., P is the laser beam power in watt, n is the refractive index of the lens material, and, $k=2\pi/\lambda$, is the wave number in cm^{-1}.

Results and Discussion:

It seems difficult to solve equation (1) analytically for the case of aberrations that are not small compared to a wavelength due to the rapidly oscillating nature of the integral and the introducing of the extra term which is $exp(-\rho^2/2)$. So, in order to calculate the intensity distribution in the region of the focus for a wide range of the primary spherical aberration values, it is necessary to solve this equation numerically. Consequently, we have developed numerical solution for uniform plane waves; moreover, we take into account in this solution the spatial variation of the beam intensity across the wave front. The mesh size used in this calculation are chosen so that convergence of the integration is insured to one part in million at successive steps. This occurs by running the program a number of times at few points by using a steadily decreasing step length until two solutions using consecutive lengths are converged. However, for a reason of the rapidly oscillating nature of the integral, it is necessary to use very small step lengths.

The numerical results first are checked with the experimental data obtained by Evans and Morgan [1], for the case of lenses exhibit spherical aberration. In our comparison we used the ruby laser which is the same type of the laser used in Evans's work [2] together with the typical experimental values of focal lengths (4.0 and 1.8) cm.

A demonstration of our theoretical results compared with those obtained experiment- ally for ruby laser can be seen in Table (1).

Table (1): A comparison between the theoretical and observed results for ruby laser, Where f is the lens focal length, a is the beam diameter, Φ is the aberration function, z is the shift of the principle focus from the Gaussian one and d is the displacement between any two successive maxima of the real lens.

f cm.	a cm.	Φ λ		z cm.		d cm.	
		Exp.	Theo.	Exp.	Theo.	Exp.	Theo.
4.0	0.5	0.49	0.43	0.023	0.021	0.030	0.025
1.8	0.5	6.00	5.89	0.067	0.056	0.010	0.005

Figure (1) shows the intensity distribution of the uniform incident wave front with a perfect lens, i.e. without primary spherical aberration, the focal length and the radius of the lens is 4.0 cm. and 0.5 cm. respectively. The type of the laser used is a CO_2 laser. One can see from this figure that, the Gaussian focus and the diffraction focus are coincided, and we can see a symmetrical distribution about the focal plane. This condition gives good agreement with our assumption which assumes that if there is no aberrations occur, the intensity at the focus will be a Gaussian. The other diffraction maxima lie on the optic axis on either side of the Gaussian focus and they are very week. The value of the intensity at the principal diffraction focus is 1.14×10^{11} watt cm^{-2}.

Figure (2) shows a uniform incident wave front for a lens with parameters similar to those used to obtain the data of figure (1), but in this case, the effect of the spherical aberration is taken into account. From this figure one can conclude that for a small amount of the primary spherical aberration, the principal diffraction focus is no longer coincides with the Gaussian focus. In addition, the off-axis maxima appear larger than those of the perfect lens, and the intensity distribution about the focal plane is no longer symmetric. Furthermore, the maximum intensity is smaller than that of the aberration free lens for the same laser power, lens focal length and diameter of laser beam.

The calculations show that the value of the intensity at the principal diffraction focus is 1.1x1011 watt cm-2, and the value of the displacement from the Gaussian focus is 0.03 cm. The primary spherical aberration value for this lens is 0.34λ.

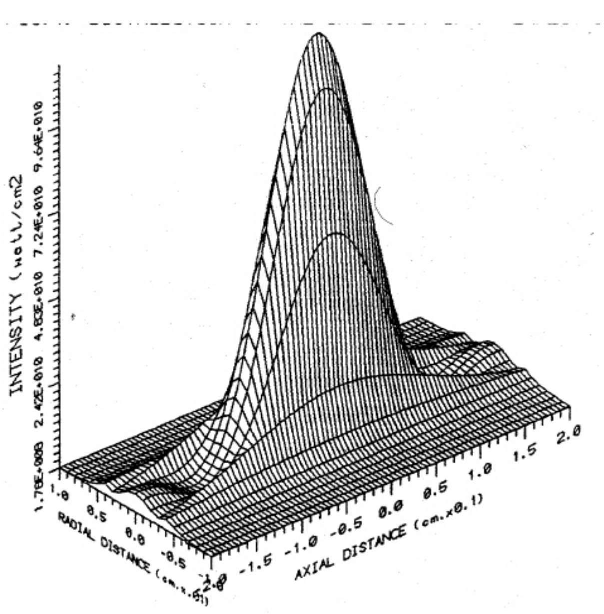

Figure (1): Laser beam intensity distribution through a perfect lens with f=4 cm.

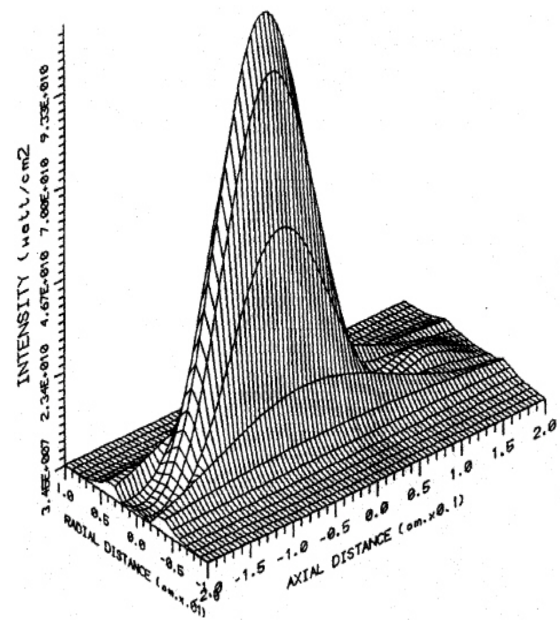

Figure (2): Laser beam intensity distribution in a real lens with f=4 cm.

Figure (3) shows the intensity distribution of the uniform incident wave front through a perfect lens without primary spherical aberration. We assume the lens parameters such that f = 1.8 cm. , a = 0.5 cm. and the laser type supposed is a CO_2 laser. The Gaussian focus and the diffraction focus coincide and there are symmetric distribution about the focal plane. The value of the intensity at the principal diffraction focus is 5.62×10^{11} watt cm^{-2}, and it appears more concentrated at the Gaussian focus. Figure (4) shows a departure from symmetry around the focal plane and still further displacement of the axial maxima toward the focusing lens. The value of the intensity at the principal diffraction focus for the same parameters values of figure (3) is 1.34×10^{11} watt

cm^{-2}, and the amount of displacement from the Gaussian focus is 0.05 cm. The calculated primary spherical aberration for this lens is 1.0λ.

For ruby laser, our calculated values of the separation *d* between maxima is approximately 0.025 cm. for the case of lens focal length 4.0 cm., and 0.005 cm. for a lens of focal length 1.8 cm, these values are in excellent agreement with those values measured experimentally by Lee and Wilson (1969), which are 0.03 cm. and 0.01 cm. respectively. For CO_2 Laser, the calculated separation *d* between maxima is approximately 0.083 cm. for the case of lens focal length is 4.0 cm. as shown in Figure (5), and 0.03 cm. for a lens with focal length 1.8 cm. as shown in figure (6).

Based on our knowledge, there are no experimental data for CO_2 laser to compare with. Finally, the analysis shows that for lenses exhibiting spherical aberration, the intensity distribution is highly distorted from that obtained from a perfect lens. The distribution is not symmetric about the focal plane and the position of maximum intensity is displaced toward the lens and appreciable axial and off-axis maxima appear.

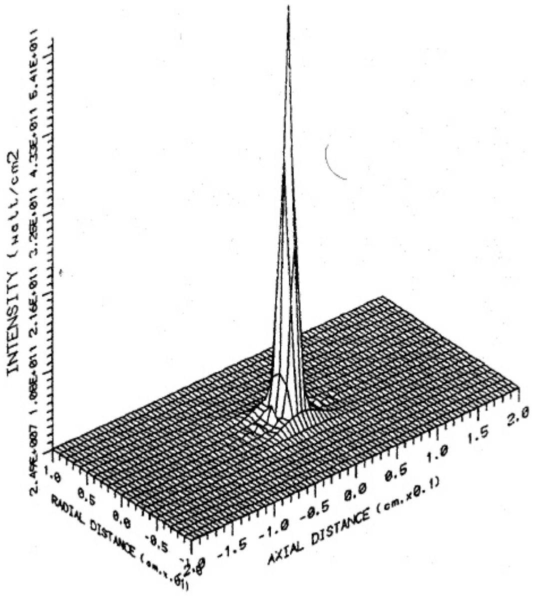

Figure (3): Laser beam intensity distribution through a perfect lens with f=1.8 cm.

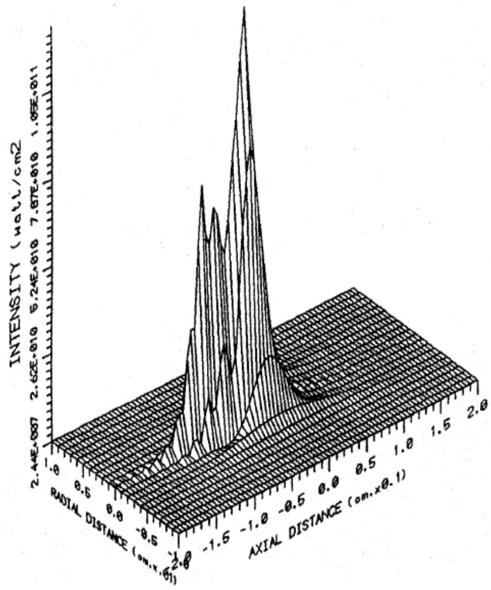

Figure (4): Laser beam intensity distribution through a real lens with f=1.8 cm.

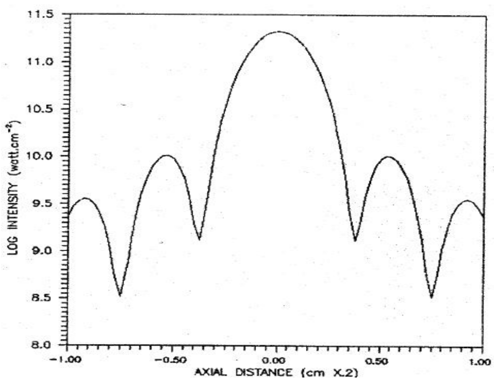

Figure (5): axial distribution of the laser intensity in aberration free lens f=4 cm.

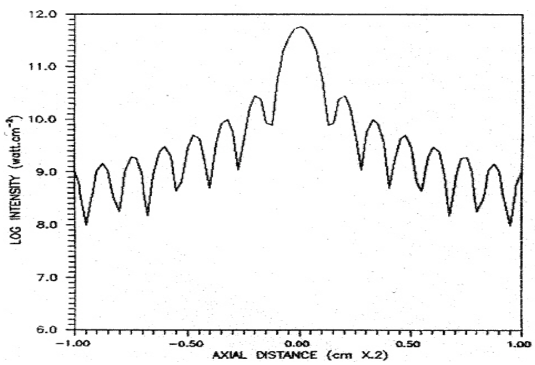

Figure (6): axial distribution of the laser intensity in aberration free lens f =1.8cm.

Conclusion:

The numerical analysis of the diffraction integral predicts the intensity distribution in the region of the focus for the types of lenses which are frequently used in studies of laser-induced plasma. It has been shown that the intensity distribution is critically dependent on the lens aberration function. The analysis shows that for lenses exhibiting spherical aberration, the intensity distribution is highly distorted from that obtained from a perfect lens; the distribution is not symmetrical about the focal plane and the position of maximum intensity is displaced towards the lens. The results obtained in the present investigation confirm the importance of spherical aberration in governing the intensity distribution in focused laser beams. This emphasis the need for reliable evaluation of the focal volume in studies of the loss processes in the initiation of the plasma by laser.

References:

[1] C. R. Evans and C. G. Morgan; (1968); *Phys. Rev. Letter*; **vol. 22**(21) pp1099.

[2] C. L. Evans; (1983); *J Phys D: Appl. Phys*; **vol. 17** pp691.

[3] C. G. Morgan; (1986); *Radiative processes in discharge plasma*; Plenum Publishing Corporation; UK.

[4] E. G. Gamaly; (1992); *Phys. fluid B*; **vol. 5**(3) pp944.

[5] E. G. Gamaly; (1993); *Phys. Rev. E*; **vol. 48**(1) pp516.

[6] A. V. Rode, E. G. Gamaly and D. B. Luther; (1997); *Phys. Plasmas*; **vol. 4**(10) pp3676.

[7] N. H. Al-Hashmiy and A. A. AL-Kelly; (1996); *Basrah J. Science A*; **vol. 14**(1) pp77.

[8] N. H. Al-Hashmiy and A. A. Al-Kelly; (2001); *FSB*; **vol. 14** pp13.

[9] A. A. Al-Kelly; (1999); *"Theoretical study of continuous optical discharge in laser beam focus"*; Ph.D. Thesis; Univ. of Basra; Iraq, pp23.

[10] S. N. Sharqi; (1996); *"Numerical analysis of laser produced COD in flowing gases"*; Ph.D. Thesis; Basra Univ.; Iraq, pp104.

[11] J. H. Lee and J. Wilson; (1969); *Private communication;* Imperial College; London, pp25.