Charge density distributions for odd-*A* of 2*s*-1*d* shell nuclei

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Abstract:

An analytical expression for the charge density distributions is derived based on the use of occupation numbers of the states and the single particle wave functions of the harmonic oscillator potential with size parameters chosen to reproduce the observed root mean square charge radii for all considered nuclei. The derived expression, which is applicable throughout the whole region of $2s-1d$ shell nuclei, has been employed in the calculations concerning the charge density distributions for odd-A of $2s-1d$ shell nuclei, such as ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. It is found that introducing an additional parameters, namely α, α_1 and α_2, which reflect the difference of the occupation numbers of the states from the prediction of the simple shell model leads to obtain a remarkable agreement between the calculated and experimental results of the charge density distributions throughout the whole range of r.

Key words: charge density distributions for odd-A of $2s-1d$ shell nuclei; root mean square charge radii; occupation numbers of higher states for ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei.

Introduction:

The charge density distributions (CDD) and form factors are the most important quantities in the nuclear structure which were well studied experimentally over a wide range of nuclei. This interest in the *CDD* are related to the basic bulk nuclear characteristics such as the shape and size of nuclei, their binding energies, and other quantities which are connected with the *CDD*. Besides, the density distribution is an important object for experimental and theoretical investigations since it plays the role of a fundamental variable in nuclear theory. The *CDD* can be determined experimentally from the scattering of high-energy electrons by the nucleus. By measuring the elastic cross sections one obtains information about the distribution of the charges within the nucleus. Various theoretical methods [1, 2] are used for calculations of *CDD*, among them the theory of finite Fermi system, the Hartree-Fock method with skyrme effective interaction. A phenomenological method was applied that is based on the natural orbital representation to construct the ground state one body density matrix [1]. This method describes correctly both density and momentum distributions in closed shell nuclei ^4He, ^{16}O and ^{40}Ca. Here, the parameters of the matrix are fixed by a best fit to the experimental density distribution and to the correlated nucleon momentum distribution. An analytical expression were derived for the one and two body terms in the cluster expansion of *CDD* and elastic form factors of $1s-1p$ and $2s-1d$ shell nuclei [3, 4]. This expression was used for the systematic study of the effect of the short-range correlations on the *CDD*. Their study depends on the harmonic oscillator size parameter (b) and the correlation

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parameter β, where these parameters were determined by fitting the theoretical charge form factors to the experimental one. Many particle shell model wave functions were used to calculate the form factors of the even-even nuclei with mass number $A = 20 - 36$ that have the positive parity states [5]. Many theoretical works (with various assumptions) concerning the calculations of *CDD* and elastic form factors have been carried out [6-10] for even-A of $2s-1d$ shell nuclei. The *CDD* of $2s-1d$ shell nuclei [11] was calculated with the assumption that there is an inert core of filled $1s$ and $1p$ shells and the proton numbers in $2s$ and $1d$ shells are equal to $2-\alpha$ and $Z-10+\alpha$, respectively. Here, α represents the deviation of the shell charges from the prediction of the simple shell model and Z is the proton number (total charge of the nucleus). In general, the calculated *CDD* were in good agreement with those of experimental data for all considered $2s-1d$ shell nuclei.

In the present work, the method of Gul'karov et al. [11] is followed but with including some higher shells into consideration with the aim of driving an analytical expression for the *CDD*, based on the use of the single particle wave functions of the harmonic oscillator potential and the occupation numbers of the states, applicable through out the whole region of $2s-1d$ shell nuclei. The derived expression is then employed for determination of *CDD* for odd-A of $2s-1d$ shell nuclei, such as ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. The calculated *CDD* demonstrate a remarkable agreement with those of experimental data throughout the whole range of the distance r.

Theory

In the simple shell model, the *CDD* is evaluated in terms of the radial part of the harmonic oscillator wave functions $R_{nl}(r)$ as

$$\rho(r) = \frac{1}{4\pi}\sum_{nl} 2(2l+1)\left|R_{nl}(r)\right|^2 \quad \ldots(1)$$

where the form of $R_{nl}(r)$ is very well known [11, 12] while n and l are the single particle principal and orbital angular momentum quantum numbers, respectively. In the simple shell model, the $2s-1d$ shell nuclei are considered as an inert core of filled $1s$ and $1p$ shells and the proton numbers in $2s$ and $1d$ shells are equal to 2 and $Z-10$, respectively. According to the assumption of the simple shell model of eq. (1), an analytical expression for the *CDD* of $2s-1d$ shell nuclei is obtained as

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{5+\left(\frac{4Z}{15}-\frac{4}{3}\right)\left(\frac{r}{b}\right)^4\right\} \ldots(2)$$

and the corresponding mean square charge radii (*MSR*) can be determined by [13]

$$\langle r^2\rangle = \frac{4\pi}{Z}\int_0^\infty \rho(r) r^4 dr \quad \ldots \quad (3)$$

where the normalization condition of the *CDD* is given by [2]

$$Z = 4\pi\int_0^\infty \rho(r) r^2 dr \quad \ldots(4)$$

Introducing eq. (2) into eq. (3) and integrating, the *MSR* of $2s-1d$ shell nuclei is obtained as

$$\langle r^2\rangle = b^2\left(\frac{7}{2}-\frac{10}{Z}\right) \quad \ldots(5)$$

In this study, the higher shells are included in the calculations of *CDD* by using the assumption that there is an inert core of filled $1s$ and $1p$ shells and the proton numbers in the shells $2s, 1d$ and $2p$ are equal to, respectively, $2-\alpha$, $Z-10+\alpha_1$ and α_2 and not to 2, $Z-10$ and 0 as in

the simple shell model of eq. (2). The parameters α, α_1 and α_2 (with $\alpha = \alpha_1 + \alpha_2$) represent the deviation of the shell charges $2s, 1d$ and $2p$, respectively, from the prediction of the simple shell model. Using this assumption with the help of eq. (1), an analytical expression of the *CDD* for the $2s-1d$ shell nuclei is obtained as

$$\rho(r) = \frac{e^{-r^2/b^2}}{\pi^{3/2}b^3}\left\{\begin{array}{l} 5-\frac{3\alpha}{2}+\left[\frac{11\alpha}{3}-\frac{5\alpha_1}{3}\right]\left(\frac{r}{b}\right)^2+ \\ \left(\frac{4Z}{15}-\frac{4}{3}-2\alpha+\frac{8\alpha_1}{5}\right) \\ \left(\frac{r}{b}\right)^4+\frac{4\alpha_2}{15}\left(\frac{r}{b}\right)^6 \end{array}\right\} \quad \ldots(6)$$

and the corresponding *MSR* is

$$\langle r^2\rangle = b^2\left(\frac{7}{2}-\frac{10}{Z}+\frac{\alpha_2}{Z}\right) \quad \ldots (7)$$

The central *CDD*, i.e. at $r=0$, is determined from eq. (6) as

$$\rho(0) = \frac{1}{\pi^{3/2}b^3}\left\{5-\frac{3\alpha}{3}\right\} \quad \ldots \quad (8)$$

In this study, we compare the calculated *CDD* of considered odd-A nuclei with those of two Parameter Fermi ($2PF$) or three Parameter Fermi ($3PF$), which are extracted from the analysis of elastic electron-nuclei scattering experiments, and are given by [13]

$$\rho_{2PF}(r) = \rho_0/(1+e^{\frac{r-c}{z}}) \quad \ldots (9)$$

$$\rho_{3PF}(r) = \rho_0\left(1+\frac{wr^2}{c^2}\right)\Big/\left(1+e^{\frac{r-c}{z}}\right) \ldots (10)$$

where the constant ρ_0 is obtained from the normalization condition of the charge density distribution of eq. (4).

Results and Discussion

Our analytical expression of eq. (6) has been used for the study of *CDD* for odd-A of $2s-1d$ shell nuclei, such as ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. In eq. (6), the harmonic oscillator size parameter b is chosen in such a way that to reproduce the experimental root mean square charge radii $\langle r^2\rangle_{\exp}^{1/2}$ of the considered nuclei, the parameter α is determined by introducing the experimental $\rho_{\exp}(r=0)$ into eq. (8), the parameter α_2 is determined by introducing the experimental *MSR* into eq. (7) and the parameter α_1 is determined from the relation $\alpha_1 = \alpha - \alpha_2$. It is important to remark that when $\alpha = \alpha_1 = \alpha_2 = 0$, eqs. (6) and (7) coincide with those of eqs. (2) and (5), respectively. The calculated *CDD* of considered odd-A nuclei are compared with those of $2PF$ or $3PF$ [13]. In Table (1), we present the parameters ω, c and z required by the experimental *CDD* of $2PF$ and $3PF$ together with their root mean square charge radii $\langle r^2\rangle_{\exp}^{1/2}$ and central charge densities $\rho_{\exp}(r=0)$ for ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. Table (2) displays all parameters needed for calculating $\rho(r)$ of eq. (6), such as the harmonic oscillator size parameter b and the calculated parameters of α, α_1 and α_2 for considered nuclei. Table (3) demonstrates the calculated occupation numbers for $2s, 1d$ and $2p$ shells and the calculated root mean square charge radii $\langle r^2\rangle_{cal}^{1/2}$ obtained by using eqs. (5) and(7).

Table (1): Values of various parameters required by the *CDD* of $2PF$ and $3PF$

Nucleus	Type of *CDD* [13]	w	c	z	$\rho_{\exp}(0)$ [13](in fm^{-3})	$<r^2>_{\exp}^{1/2}$ [13] (in fm)
^{25}Mg	$3PF$	-0.236	3.220	0.580	0.0859	3.003
^{27}Al	$2PF$	-	3.070	0.519	0.0834	3.060
^{29}Si	$2PF$	-	3.170	0.520	0.0827	3.130
^{31}P	$3PF$	-0.173	3.369	0.582	0.0874	3.190

Table (2): Calculated parameters used in eq. (6) for the calculations of the *CDD*

Nucleus	Z	b	α	α_1	α_2
^{25}Mg	12	1.838	1.3517	1.3185	0.0332
^{27}Al	13	1.851	1.3706	1.3232	0.0474
^{29}Si	14	1.871	1.3225	1.1368	0.1857
^{31}P	15	1.890	1.1423	0.9107	0.2316

Table (3): Calculated occupation numbers of $2s$, $1d$ and $2p$ shells and the calculated $\langle r^2 \rangle_{cal}^{1/2}$

Nucleus	Z	Occupation No. of $2s$	Occupation No. of $1d$	Occupation No. of $2p$	$\langle r^2 \rangle_{cal}^{1/2}$ obtained by eq.(5)	$\langle r^2 \rangle_{cal}^{1/2}$ obtained by eq.(7)
^{25}Mg	12	0.6483	3.3185	0.0332	3.0014	3.0030
^{27}Al	13	0.6294	4.3232	0.0474	3.0579	3.0600
^{29}Si	14	0.6775	5.1368	0.1857	3.1225	3.1300
^{31}P	15	0.8577	5.9107	0.2316	3.1813	3.1900

The dependence of the *CDD* (in the unit of fm^{-3}) on r (in the unit of fm) is shown in Fig. 1 for ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. The solid circles (●) are the experimental data of $2PF$ or $3PF$ [13]. The plus symbol distributions are the calculated *CDD* obtained either by eq. (2) or by eq. (6) using the values $\alpha = \alpha_1 = \alpha_2 = 0$. The solid distributions are the calculated *CDD* when the higher shells are included in the calculations and obtained by eq. (6) using the values of α, α_1 and α_2 given in Table (2). It is obvious that the form of the *CDD* represented by eq. (2) or (6) behaves as an exponentially decreasing function as seen by the plus symbol distributions or solid distributions for all considered nuclei of Fig. 1. This figure shows that the probability of finding a proton near the central region ($0 \le r \le 2\ fm$) of $\rho_{ch}(r)$ is larger than the tail region ($r > 2\ fm$). Besides, including the higher shells through introducing the calculated values of α, α_1 and α_2, presented in Table (2), into eq. (6) leads to reducing significantly the central region of $\rho_{ch}(r)$ and increasing slightly the tail region of $\rho_{ch}(r)$ as seen by the solid distributions. This means that the effect of inclusion of higher shells in our calculations tends to increase the probability of transferring the protons from the central region of the nucleus towards its surface and tends to increase the root mean square charge radius $< r^2 >^{1/2}$ of the nucleus, see Table (3), and then makes the nucleus to be less rigid than the case when there is no this effect. Fig. 1 also illustrates that the plus symbol distributions in all considered nuclei are not in good accordance with those of $2PF$ or $3PF$, especially at the central region of $\rho_{ch}(r)$ but once the higher shells are considered in the calculations due to introducing the calculated values of α, α_1 and α_2 given in Table (2) into eq. (6), the results for the *CDD* become in astonishing accordance with those of $2PF$ or $3PF$ throughout the whole values of r.

It is clear from Fig. 1 that the solid distribution of ^{25}Mg deviates slightly from the experimental data of $3PF$,

especially at the region of short distances $0.8 \le r \le 2$ *fm*. In general, considering the higher shells in eq. (6) improves strongly the calculated result of *CDD* in ^{25}Mg, where these higher shells are not enough for resolving completely the problem of slight deviation. This deviation may be attributed to the other reasons or to the necessity of considering other higher shells, such as the $1f$ shell. In this case the occupation number of $1f$ shell must be different from zero. However, this deviation doesn't affect generally the very well accordance with the experimental data of $3PF$ throughout the whole values of r.

This study tends to the conclusion that introducing an additional parameters α, α_1 and α_2, that reflect the difference of the occupation numbers of the states from the prediction of the simple shell model, leads to a very good agreement between the calculated and experimental results of the charge density distributions throughout the whole range of r.

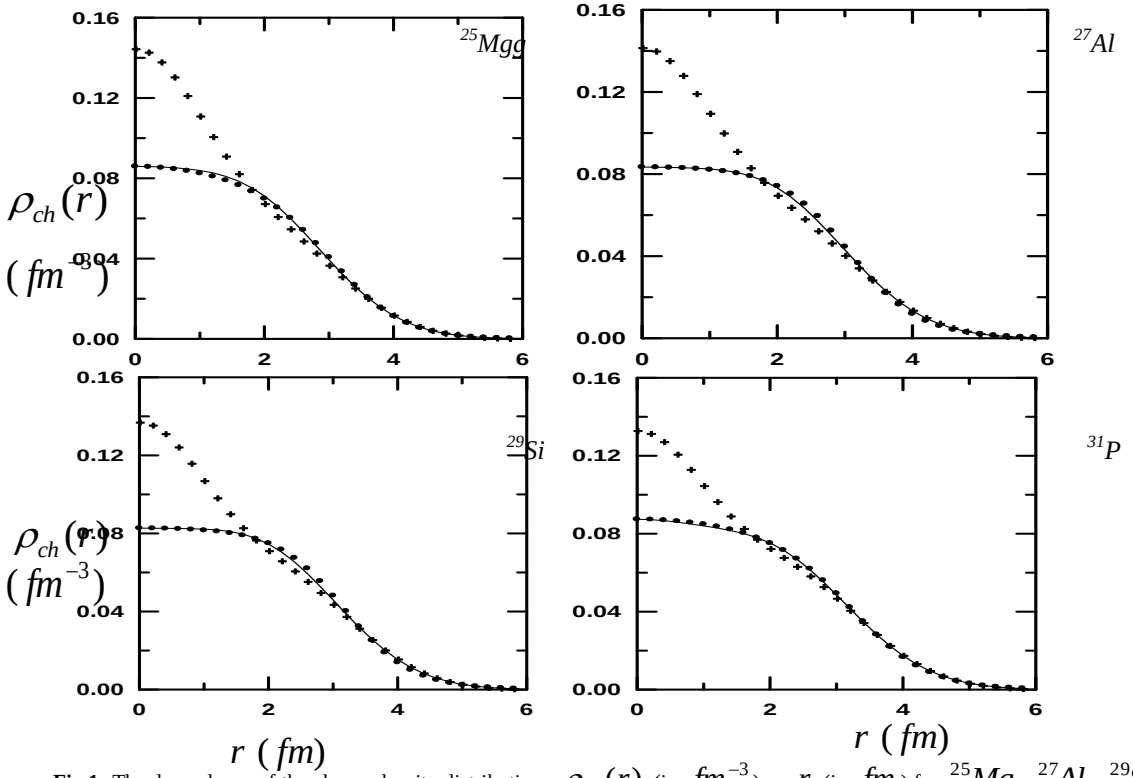

Fig.1: The dependence of the charge density distributions $\rho_{ch}(r)$ (in fm^{-3}) on r (in fm) for ^{25}Mg, ^{27}Al, ^{29}Si and ^{31}P nuclei. The solid circles (●) are the experimental data [13] of $2PF$ (for ^{27}Al and ^{29}Si nuclei) and $3PF$ (for ^{25}Mg and ^{31}P nuclei). The plus symbols (+) are the calculated CDD obtained either by eq. (2) or by eq. (6) using the values $\alpha = \alpha_1 = \alpha_2 = 0$. The solid distributions are the calculated CDD obtained by eq. (6) using the values of α, α_1 and α_2 given in Table (2).

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توزيع كثافة الشحنة النووية لنوى فردية في عددها الكتلي وتقع ضمن القشرة النووية $2s$-$1d$

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الخلاصة:

تم اشتقاق صيغة رياضية تحليلية لتوزيع كثافة الشحنة النووية للنوى الواقعة ضمن القشرة النووية $2s-1d$. الصيغة الرياضية المشتقة تعتمد على كل من اعداد الأشغال للحالات النووية والدوال الموجية للجهد المتذبذب التوافقي ذي أعلومات حجمية اختيرت لكي تعيد انتاج مربع متوسط انصاف اقطارالشحنة لجميع النوى تحت الدراسة. لقد تم استخدام الصيغة الرياضية المشتقة في حساب توزيع كثافة الشحنة النووية للنوى الفردية (^{25}Mg , ^{27}Al , ^{29}Si و ^{31}P) الواقعة ضمن القشرة النووية $2s-1d$. لقد وجد ان ادخال الأعلومات الأضافية (α, α_1 و α_2)، والتي تعكس الفرق بين اعداد الأشغال للحالات النووية قيد الدراسة وبين تلك التي يتنبأ بها أنموذج القشرة البسيط ، تؤدي الى توافق ممتاز بين النتائج المحسوبة والنتائج العملية لتوزيع كثافة الشحنة النووية ولكل قيم المسافة r .