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Utilizing Operation in Certain Types of Open Sets

A Review

Jamil Mahmoud Jamil*

College of science, University of Diyala, jamil291078@yahoo.com, Baquaba, Diyala.
*Corresponding author email: jamil291078@yahoo.com

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Abstract

The idea of operation was initiated for the first time by Kasahara[1], and he investigated the properties of α-continuous functions and α-closed graph of operation μ. After then Krishnan and Balachandran[2], [3] applied operation μ to define a new type of semi-open and pre-open namely μ semi-open and μ pre-open sets. Consequently, Hariwan[4] gave an overview of μb-open set and μb-continuous functions along with shown some general characteristics of various separation axioms. In 2001, Maki and Noiri [5] used bi-operation notion to study $[\mu,\mu^*]$-open sets which several characterizations are obtained. Since that many researchers have identified various kinds of open sets regarding different topologies by using operation.

Conclusion:

The notion of operation has an essential position to describe new concepts in topology such as continuity, compactness, and separation axioms. In this work, we recalled the definitions of various types of open sets regarding operation and overview the relations among these sets and compare the results through previous studies. Later, many researchers were interested to establish new types of open sets regarding to bi-operation instead of using operation. Comparisons of different open sets regarding operation and bi-operation have been made. Furthermore, we summarized some applications for these sets for operation and bi-operation and explained their relationships. Finally, it would be useful to use this principle to research these features in soft and micro topology in the future.

Key words:

Operation, bi-operation, μ semi-open set, μ pre-open set, $sp[\mu,\mu^*]$-open set.

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استخدام عامل التشغيل في انواع محددة من المجموعات المفتوحة

مقالة

جميل محمود جميل*

1 كلية العلوم، جامعة ديالى، jamil291078@yahoo.com، ديالى، العراق

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الخلاصة

مقدمة:

مفهوم عامل التشغيل تم تعريفه لأول مرة من قبلkasahara حيث قام بدراسة خواص الدوال المستمرة من النمط α والرسم البياني المغلق من النمط α بواسطة عامل التشغيلμ بعد ذلك قام كل منKrishnan و Balachandran بأستخدام عامل التشغيل μ لتعريف نوع جديد من المجموعات شبه المفتوحه و المجموعات المفتوحه اوليا اسماها المجموعات شبه المفتوحه μ و المجموعات المفتوحه اولياμ في عام 2001 استخدم كل من Maki و Noiri مفهوم ثنائي عامل التشغيل لدراسة المجموعة المفتوحه بحيث تم الحصول على تمييزات عدة ومن ذاك قام العديد من الباحثين بدراسة وتحقيق انواع مختلفة من المجموعات المفتوحة– $[\mu, \mu^*]$ وذلك من خلال استخدام ثنائي عامل التشغيل .

الاستنتاجات:

مفهوم عامل التشغيل لعب دورا مهما ورئيسيا في وصف عدة مفاهيم في توبولوجيا مثل الاستمرارية و التراص وبديهيات الفصل .في هذا العمل قمنا بأستدعاء تعاريف عدة حول انواع متعددة من المجموعات المفتوحة باستخدام عامل التشغيل حيث قمنا بمراجعة العديد من العلاقات بين هذه المجموعات المختلفة و مقارنة النتائج المختلفة في الدراسات السابقة بعد ذلك اهتم العديد من الباحثين بدراسة المجموعات المفتوحة باستخدام ثنائي عامل التشغيل بدلا من احادي عامل التشغيل ولذلك قمنا ببحث العلاقات بين المجموعات المفتوحة باستخدام احادي عامل التشغيل و المجموعات المفتوحة بأستخدام ثنائية عامل التشغيل واخيرا يمكن استخدام هذا المفهوم في دراسات مستقبلية وعلى سبيل المثال دراسة خصائصه في توبولوجي الدقيق و الناعم.

الكلمات المفتاحية:

عامل التشغيل, ثنائي عامل التشغيل, مجموعة شبه المفتوحةμ, مجموعة المفتوحة اوليا μ, مجموعة المفتوحة $sp[\mu, \mu^*]$.

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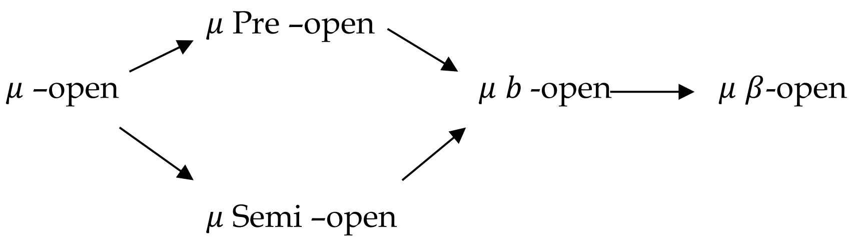
INTRODUCTION

We recalled the definition of operation μ in sense of Kasahara[1] on topological space (X, τ)as a function from topology τ into a power set of X in which $M \subseteq M^{\mu}$ for all $M \in \tau$, where M^{μ} represented the value of μ at M. Later, Ogata[6] used the notion of operation μ to define γ-open set. A subset M of X is called μ-open , if for any $p \in M$, there is an open set G such that $p \in G$ and $G^{\mu} \subseteq M$. Furthermore, a subset M of X is named $[\mu, \mu^*] -$ open if there are two open sets $K$and L containing p such that $K^{\mu} \cap L^{\mu^*} \subseteq M$.

2- The recent researches regarding operation on different types of open sets:

We will discuss several studies about different types of open sets regarding to the conception of operation. Gosh [7] stated the notion of μ pre –open sets and found that μ pre –open and pre-open sets are independent. However, there are identical in μ-regular space. In addition, the researcher showed that the union of any μ pre –open sets is also μ pre –open while the intersection of finite members of μ pre –open sets may not be μ pre –open. Basu, Afsan, and Gosh [8]concluded that the above statements are valid for μ β-open sets. Meanwhile, Hussain and others [9] identified general properties about μ semi –open set. Hariwan [4] determined the relation among μ b-open with other open sets regarding operation μ.

Diagram (1)



Basu, Afsan , and Gosh [8] had founded the notions of the functions namely μ continuous and μ –βcontinuous functions . It was indicated that any μ continuous function is μ –βcontinuous. Generally, the reverse direction may not be true. Moreover, the concepts of μ –β anti closed, μ –β anti open, μ –β closed, μ –βopen , and μ –β irresolute functions are delineated. It was stated for a function f from μ –β regular space (X, τ) with operation μ onto any space (Y, τ^*) is μ –β anti closed, μ –β anti open, and μ –β continuous, then (Y, τ^*) is regular. Similarly, a function f from a regular space (X, τ) onto any space (Y, τ^*) with operation $\mu^*$is μ –β closed, μ –β open, and μ –β continuous, then (Y, τ^*) is μ –β regular space. Hariwan [4] utilized the concept of μ –β open sets to establish various class of functions such as μ –b continues, and μ -b irresolute. New class of graph of function had been found namely μ -b closed graph such that many characterizations are given. Later, Baravan, and Nazihah [10] used the operation μ to define new open set namely semi-generalized μ-open set and investigated some fundamental properties about these sets.

3 The recent studies dealing with bi-operation regarding topology

We will address studies that have established different forms of open set for topology by utilizing bi-operation. Focusing on some of its fundamental basic facts.

3-1 The relationships among the various types of $[\mu, \mu^*]$ −open sets:

Carpintero, Rajesh, and Rosas[11] introduced and investigated the notions of $[\mu, \mu^*]$ − semiopen, $[\mu, \mu^*]$ − preopen, $[\mu, \mu^*]$ − α open, and $[\mu, \mu^*]$ − semipre open sets. The following diagram explains the relationships among them:

Diagram(2)

$$
[\mu,\mu^*] - \alpha\text{ open} \Rightarrow \begin{matrix} [\mu,\mu^*] - \text{semiopen} \not\Leftrightarrow [\mu,\mu^*] - \text{pre open} \\ \Downarrow \\ [\mu,\mu^*] - \text{semipre open} \end{matrix} \Longrightarrow [\mu,\mu^*] - \text{b open}
$$

Some of the reverse directions of above diagram were discussed in [10].

Carpintero and others stated that every $[\mu, \mu^*]$ − open is $[\mu, \mu^*]$ − preopen set. Also, He concludes for $[\mu, \mu^*]$ −regular space, the notions $[\mu, \mu^*]$ − preopen and pre-open are identical. Hariwan[12] utilized two operations μ and μ^* to represent $\alpha_{(\mu,\mu^*)}$-open and $\alpha_{(\mu,\mu^*)}$-generalized closed sets. Mainly, he showed that the union of any$\alpha_{(\mu,\mu^*)}$-open sets is also $\alpha_{(\mu,\mu^*)}$-open set. However, the intersection of two $\alpha_{(\mu,\mu^*)}$-open sets might not be a $\alpha_{(\mu,\mu^*)}$-open set .This implies that the family of $\alpha_{(\mu,\mu^*)}$-open sets not always be a topology. But he overcome the intersection obstacle when he proved the intersection of two $\alpha_{(\mu,\mu^*)}$-open sets is $\alpha_{(\mu,\mu^*)}$-open set whenever μ and μ^* are α-regular operations. Also, he stated that every $\alpha_{(\mu,\mu^*)}$-open set is α_μ-open while the relationships between $\alpha_{(\mu,\mu^*)}$-open and α-open sets are shown to be independent.

In 2020, Jamil[13] defined and studied the concept $sp[\mu, \mu^*]$-open set. He showed that every $[\mu, \mu^*]$ − open set is $sp[\mu, \mu^*]$-open. An example was given to illustrate that not every $sp[\mu, \mu^*]$-open is $[\mu, \mu^*]$ − open set.

Furthermore, the notions $sp[\mu, \mu^*]$-open set and semi$[\mu, \mu^*]$ − open are independent. Similarly, $sp[\mu, \mu^*]$-open set and pre$[\mu, \mu^*]$ − open are also independent

Also, he proved for extremely disconnected space, $sp[\mu,\mu^*]$-open is weaker than semi$[\mu,\mu^*]$-open. Similarly for pre $[\mu,\mu^*]$-open set.

3-2 On certain types of continuous and irresolute functions regarding bi-operation

An important feature of general topology is the aspect of continuous mapping. In particular, several weak and strong forms have been investigated. The notion of $([\mu,\mu^*],[\zeta,\zeta^*])-$ continuous function was introduced by Maki and Noiri[5]. Moreover, they created the term of $([\mu,\mu^*],[\zeta,\zeta^*])-$ homemorphism function and induced several properties about these functions. Carpintero, Rajesh, and Rosas[10] developed new types of continuity namely $([\mu,\mu^*],[\zeta,\zeta^*])-semi$ continuous, $([\mu,\mu^*],[\zeta,\zeta^*])-pre$ continuous, $([\mu,\mu^*],[\zeta,\zeta^*])-\alpha$ continuous, $([\mu,\mu^*],[\zeta,\zeta^*])-b$ continuous functions. They described certain relationships among these functions as seen in the following diagram.

Diagram (3)

$$([\mu,\mu^*],[\zeta,\zeta^*])- \text{ continuous function}$$
$$\Downarrow$$
$$([\mu,\mu^*],[\zeta,\zeta^*])- \alpha \text{ continuous function}$$
$$\Downarrow$$
$$([\mu,\mu^*],[\zeta,\zeta^*])-pre \text{ continuous function} \quad \not\Leftrightarrow \quad ([\mu,\mu^*],[\zeta,\zeta^*])- \text{ semi continuous function}$$
$$\Downarrow$$
$$([\mu,\mu^*],[\zeta,\zeta^*])-b \text{ continuous function}$$

They believed the reverse directions were definitely not valid. But counter examples are not included. Meanwhile, they identified an $([\mu,\mu^*],[\zeta,\zeta^*])-$ irresolute function and gave several descriptions about this function. In addition, they observed $([\mu,\mu^*],[\zeta,\zeta^*])-$ irresolute function is$([\mu,\mu^*],[\zeta,\zeta^*])-$ semi continuous function and claimed that the backwards direction may not be true.

In order to enrich this work, we need to compare $([\mu,\mu^*],[\zeta,\zeta^*])-$ continuous function with $\mu-\zeta$continuous function and investigate their properties.

It is highly recommended that interaction between μ-continuity and $[\mu,\mu^*]$-continuity be clarifying and discuss the notations $[\mu,\mu^*]$-almost continuous, $[\mu,\mu^*]$-weakly continuous, $[\mu,\mu^*]$-closed, and $[\mu,\mu^*]$-open functions in details.

3-3 $[\mu,\mu^*]$-Separation Axioms

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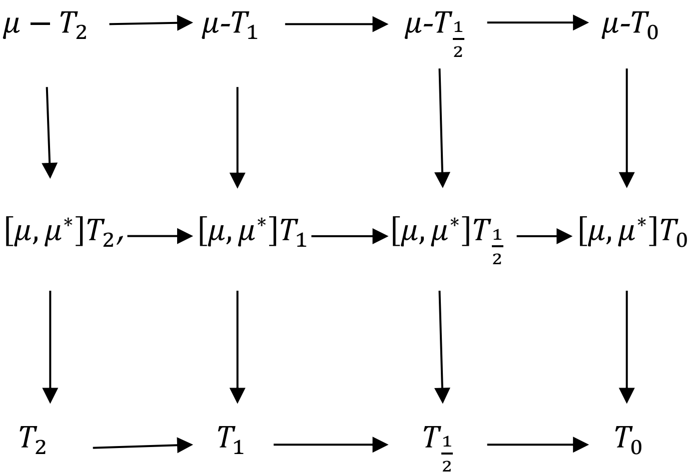
In 2001, Maki and Noiri[5] presented the definitions of $[\mu,\mu^*]T_i$ −spaces for $i = 0, {}^1/_2, 1, 2$ and analyzed interactions between all these spaces. Consequently, they explained how these spaces are connected to μ-T_i –spaces and T_i –spaces for $i = 0, {}^1/_2, 1, 2$ as shown in the illustration below:

Diagram (4)



One of the essential outcomes, being $[\mu,\mu^*]$-T_i space for $i = 0,1,\frac{1}{2},2$ is invariant under homeomorphism. In addition, they have discussed many facets of $[\mu,\mu^*]$-T_i, but they neglect exploring the descriptions of $[\mu,\mu^*]$-regular space and ignoring certain essential characteristics of these spaces.

3-4 $sp[\mu,\mu^*]$- compact spaces

The idea of $sp[\mu,\mu^*]$- compact space and $sp[\mu,\mu^*]$- compact set were created by Jamil [13]. He gave several descriptions of these spaces. We mention some of these significant outcomes. He proved the intersection of $[\mu,\mu^*]$-closed set and $sp[\mu,\mu^*]$-compact is also $sp[\mu,\mu^*]$-compact for μ and μ^* are respectively semi- μ-regular and pre-μ^*regular operators. But he did not discuss if the intersection of any $sp[\mu,\mu^*]$-compact is $sp[\mu,\mu^*]$-compact set or not.

Furthermore, the union of any $sp[\mu,\mu^*]$-compact sets is also $sp[\mu,\mu^*]$-compact. In addition, a topological space X is $sp[\mu,\mu^*]$-compact space if and only if all $[\mu,\mu^*]$-closed sub-sets is $[\mu,\mu^*]$-compact for any two operators μ andμ^*.

Conflict of interests.

There are non-conflicts of interest.

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