تبريد بدرجة حرارة ثابتة، وقد عزلت الاسطح العليا والسفلى من الحيز الدراسة تضمنت الحل العددي لمعادلات نايفير-ستوك والطاقة بأستخدام طريقة الفروقات المحددة. صيغة دالة الانسياب استخدمت في الموديل الرياضي. لوحظ أن الحل العددي قادر على حساب دالة الانسياب، الدوامية ودرجات الحرارة للمجال الحسابي. استخدم برنامج حاسوبي بلغة (FORTRAN 90) لحل المعادلات العددية. المسألة تم تحليلها والمعادلات اللابعدية الحاكمة تم حلها بأستخدام طريقة الفروقات المحددة. الحيز مملوء بالهواء وبرقم براندل يساوي 0.71 . رقم رايلي يتغير بمدى ( $^{5}$ 10 × 5 ≥ 28 × 60) ونسبة باعية (2 ≥ 28 × 60) . لقد وجد انه لقيم رايلي القليلة فأن انتقال الحرارة يكون بالتوصيل وانه يبدأ بالتحول الى الحمل مع ازدياد عدد رايلي . من اجل مصداقية الحل العددي، تم مقارنة النتائج التي الحصول عليها من تغيير رقم نسلت الموقعي، والعلاقة بين معدل رقم نسلت ورقم رايلي مع بحث سابق، لحيز هوائى مربع الشكل ،(2) وات

# **1.1 Introduction**

Natural convection in enclosures has been extensively studied both experimentally and numerically, since it has considerable interest in many engineering and sciences applications, e.g., collection of solar energy, operation and safety of nuclear reactors, energy efficient design of buildings and rooms, effective cooling of electronic components and machinery (Oosthuzen and Naylor, David, 1999).

# **1.2 Research Objective**

The objectives of this research are listed below:

- i. Study the phenomena of natural convection inside a two-dimensional enclosure, when one of its vertical walls is heated partially and the reminder of the wall and opposite wall is cooled, while the horizontal walls are adiabatic.
- ii. Find the optimal position for the partial heating.
- iii. Study the effect of aspect ratio on the flow pattern.
- iv. Built a program based on a finite difference method (FDM) and validate the applied numerical method for the classical two-dimensional rectangular cavity.
- v. The results will be in form of contours (stream function, vorticity and isotherm) and a plot between Nusselt number versus Rayleigh number will be done.
- vi. Comparisons with other investigations will be introduced to make sure from the present work results.

# **1.3 Literature Survey**

Natural convection in rectangular enclosure has been studied for many years. In this item, the previous related literatures dealt with the subject of the research will be presented. Some of studies made a study on laminar natural convection in rectangular enclosure with partially heated. (Orhan Aydin and Wen-Jei Yang, 2000) studied numerical investigation of natural convection of air in a two-dimensional, rectangular with localized heating from below and symmetrical cooling from the sides, localized heating is simulated by a centrally located heat source on the bottom wall, and four different values of the dimensionless heat source length 1/5, 2/5, 3/5 and 4/5 are considered. Solution were obtained for Rayleigh number  $10^3$  to  $10^6$ . (Abdulhaiy , Radhwan and Zaki, 2000) examined numerically natural convection heat transfer in a square air-filled enclosure

with one discrete flush heating in a vertical enclosure, where a finite difference solution was presented for a  $(10^2 < \text{Ra} < 10^6)$ , they founded that the optimum location over the range of Rayleigh number was for the heater mounted at the center of wall. (Raos, 2001) investigated of the laminar natural convection, two dimensional phenomena in enclosed space rectangular with differentially heated sides and adiabatic horizontal walls, had been defined in order to predict good enough results. (Al-Bahi, Radhwan and Zaki, 2002) modeled numerical study for laminar natural convection heat transfer in an air filled vertical square cavity differentially heated with a single isoflux discrete heater on one wall with top and bottom adiabatic surfaces. The coupled unsteady 2-D conservation equations were solved, average Nusselt number variation for S/L= 0.25 to 0.75 and Rayleigh number from10<sup>3</sup>to 10<sup>6</sup>. (Al-Bahi, Al-Hazmy and Zaki, 2005) studied numerically the effect of inclination angle for laminar free convection in a rectangular enclosure (aspect ratio = 5), which was discretely heated by an isoflux flush mounted small heater (L / H = 0.125). The local and average Nusselt numbers were compared at inclination angles from 0 (bottom heating) to 180° (top heating), for modified Rayleigh numbers  $10^2 \le \text{Ra} \le 10^6$ . The maximum Nusselt number was found close to the vertical orientation while the minimum was at the horizontal position with fluid heated from the top for which convection was effectual and the average Nusselt number was greater than unity. (Ambarita, Kishinami, Daimaruya and Saitoh, 2006) studied numerically a differentially heated square cavity, which was formed by horizontal adiabatic walls and vertical isothermal walls. Heat transfer by natural convection for dry air was studied by solving the governing equation. (T. Basak, S.Roy, A.R.Balakrishnan, 2006) investigated numerical study for steady laminar natural convection flow in a square cavity with uniform and non-uniformly heated bottom wall, and adiabatic top wall maintain constant temperature of cold vertical walls, the finite-element method had been used to solve the governing equations. The range of the Rayleigh number from  $10^3$ to $10^5$  and prantel number ( $0.7 \le Pr \le 10$ ). They founded that non-uniform heating of the bottom wall produces greater heat transfer rates at the center of the bottom wall than the uniform heating case for all Rayleigh numbers, average Nusselt numbers show overall lower heat transfer rates for the non-uniform heating case. Critical Rayleigh numbers for conduction dominant heat transfer cases had been obtained and for convection dominated regimes, power law correlations between average Nusselt number and Rayleigh numbers were presented. (N.H.Saeid and Y.Yaacob, 2006)Two dimensional natural convection heat transfers in a two-dimensional square cavity with non-uniform side-wall temperature was studied numerically. They founded that the average Nusselt number varies based on the hot-wall temperature. (M.Sathiyamoorthy, T. Basak, S.Roy and I.Pop, 2007) numerically investigated natural convection flow in a square cavity with linear heated vertical walls and uniformly heated bottom wall. (Sidik, 2009) studied natural convection in a square cavity with localized heating from below and symmetrical cooling from the sides had been studied using finite difference double-distribution function thermal lattice Boltzmann model. The evolution of lattice Boltzmann equation had been discredited using the third order accuracy finite difference upwind scheme. The flow pattern including vortices and thermal boundary layer could be seen clearly. The results obtained demonstrate that this approach is very efficient procedure to study flow and heat transfer in a differentially heated cavity flow. The extension to 3D computations and high Rayleigh numbers was the subject of further investigations within ongoing research. (A.Sarkar, S.K.Mahapatra, S.K.Behera, 2009) steadied laminar natural convection in air-

filled, 2-D rectangular enclosure heated from below and cooled from above was studied numerically. The equations were solved by SIMPLER algorithm. Simulations were performed for several values of both the width-to-height aspect ratio of the enclosure in the range between 0.25 and 1, and the Rayleigh number based on the cavity height in the range between 1.00e3 and 5.00e5, whose influence upon the flow patterns, the temperature distributions and the heat transfer rates are analyzed and discussed. (Moghimi and Mirgolbabaei, 2009)investigated numerical study for steady laminar natural convection in air-filled, 2-D rectangular enclosure heated from below and cooled from above, the solution were obtained for aspect ratio (0.25 and 1), and Rayleigh number based on the cavity height in the range between  $10^3$  and  $5 \times 10^5$ . The study of natural convection flow along isothermal plates and in channels using diffusion velocity method done by (Dare and Petinrin, 2010), they found that as the wall temperature increase while keeping the mainstream fluid temperature constant, the thermal boundary layer thickness increases. In this study, natural convection heat transfer in a rectangular enclosure with uniform partial heating from the left side wall at (Th), the reminder of the wall and the opposite is cooled at uniform temperature (Tc), the top and bottom walls are adiabatic. It is suggested here to introduce successive heating and cooling on the same vertical wall of the enclosure, for steady state numerical solution and finding the optimal position for the highest heat transfer amount, and study the effect of aspect ratio on the flow with partial heating. The effect of Rayleigh number on the flow patterns and the resulting heat transfer is determined for the value of Rayleigh numbers from  $10^3$  to  $5 \times 10^5$ . The numerical technique based on the finite difference method (FDM) is generally applied in the computations of a uniform grid size. The results are presented as a form of the average Nusselt number,  $\overline{Nu}$ , which represents the rate of heat being transferred. In addition, the results are also presented in graphical forms of stream function and isotherms contours, which demonstrate the fluid flow and thermal distributions inside the enclosure.

# 2.1 Mathematical Analysis and Assumptions

The geometry under investigation is an air-filled rectangular enclosure, which is considered insulated at the top and bottom walls. The vertical left side wall of the enclosure is heated partially with length S, the ratio between the length of heater S to the length of enclosure H (S/H=0.5) (heating half of the wall H/2) and in multi positions localized at (lower, center and upper ), while the reminder of the wall and other vertical right side wall is maintained at a uniform cold temperature ( $T_c$ ).

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Fig.1. Schematic diagram of the physical model and boundary



Fig 2 Uniform grid distribution  $(41 \times 41)$  for the rectangular enclosure, AR=1.



Fig 3 Effect of number of grid on average Nusselt number for  $Ra=10^4$ , AR=1.

The geometry under consideration and boundary conditions is shown schematically in **Fig1**. The working fluid is chosen to be air with Prandtl number (Pr =0.71). The fluid inside the enclosure is assumed incompressible, Newtonian with density variation only pertinent to temperature changes. The governing continuity, Navier-Stokes(x and y directions) and energy equations for steady state buoyancy driven fluid flow are (Diaz and Winston, 2008):

**Continuity Equation:** 

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

x-direction momentum equation:  

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \mu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right]$$
(2)

y-direction momentum equation:

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial P}{\partial y} - \rho g + \mu \left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right]$$
(3)

Energy equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$
(4)

The governing equations are transformed into dimensionless forms under the following non-dimensional variables (Basak et al ,2006):

$$\Psi = \frac{\Psi \operatorname{Pr}}{v}, \Omega = \frac{\omega H^{2} \operatorname{Pr}}{v}$$

$$X = \frac{x}{H}, Y = \frac{y}{H}, AR = \frac{W}{H}$$

$$\theta = \frac{T - T_{c}}{T_{h} - T_{c}}$$

$$Ra = \frac{g\beta(T_{H} - T_{c})H^{3}}{\alpha v} = \frac{g\beta(T_{H} - T_{c})H^{3}}{v^{2}} \operatorname{Pr}$$

$$U = \frac{uH}{\vartheta} = \frac{\partial\Psi}{\partial Y}, V = \frac{vH}{\vartheta} = -\frac{\partial\Psi}{\partial X}$$
(5)

**Eqs.1 to 4** are written in the following dimensionless form (known as steady-state derived variables convection equations):

$$\begin{pmatrix} \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} \end{pmatrix} = -\Omega$$
(6)

$$\left(\frac{\partial\Psi}{\partial Y}\frac{\partial\Omega}{\partial x} - \frac{\partial\Psi}{\partial x}\frac{\partial\Omega}{\partial Y}\right) = Pr\left(\frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial Y^2}\right) + Ra\frac{\partial\theta}{\partial x}$$
(7)

$$\frac{\partial \Psi}{\partial Y}\frac{\partial \theta}{\partial x} - \frac{\partial \Psi}{\partial x}\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{8}$$

## **2.2 Boundary Conditions**

The boundary conditions for velocity and temperature fields which are used in the present study can be arranged as follows (Abdulhaiy et al, 2000):

$$X = 0: U = V = \Psi = 0, -\Omega = \frac{\partial^2 \Psi}{\partial X^2}, \theta = \begin{cases} 1 & \frac{h_1}{h} \le Y \le \frac{h_2}{h} & (a) \\ 0 & elsewhere \end{cases}$$

$$X = 1: U = V = \Psi = 0, -\Omega = \frac{\partial^2 \Psi}{\partial X^2}, \quad \theta = 0 \quad (b)$$

$$\begin{cases} Y = 0 \\ Y = 1 \end{cases}: U = V = \Psi = 0, \quad -\Omega = \frac{\partial^2 \Psi}{\partial X^2}, \quad \frac{\partial \theta}{\partial Y} = 0 \quad (c) \end{cases}$$

$$(9)$$

**Eq.9-a** is written in a general form, whereas for  $h_1 = 0$  and  $h_2 = 1$ , the problem presents the standard enclosure conditions with one side at  $T_{h,i}(\theta = 1)$ , and the opposite side at  $T_{c,i}(\theta = 0)$ .

The rate of heat transfer is expressed in terms of local Nusselt number, Nu, at the heated section as follows:

$$Nu_h = -\frac{\partial \theta}{\partial x}\Big|_{x=0}; Y_1 \le Y \le Y_2$$
(10)

Where  $Y_1 = h_1/h$ , and  $Y_2 = h_2$ 

The average Nusselt number,  $\overline{Nu_h}$ , is defined by:

$$\overline{Nu_h} = \frac{1}{Y_2 - Y_1} \int_{Y_1}^{Y_2} Nu_h dY$$
3.1 Numerical Solution
(11)

The method of numerical solution considered is the central finite difference scheme technique to convert the partial differential equation to an algebraic equation which can be solved numerically. The energy equation is a nonlinear partial differential equation, which has (convection terms) on the left hand side of the **Eq. 8** and (diffusion term) on the right hand side. To covert the convection and diffusion terms to algebraic terms, central difference scheme will be used, as below (Tannehill et al. (1997))

$$\left(\frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{2\Delta Y}\right) \left(\frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta X}\right) - \left(\frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2\Delta X}\right) \left(\frac{\theta_{i,j+1} - \theta_{i,j-1}}{2\Delta Y}\right) = \left(\frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{\Delta X^2}\right) + \left(\frac{\theta_{i,j+1} - 2\theta_{i,j} + \theta_{i,j-1}}{\Delta Y^2}\right)$$
(12)

Similarity, the finite difference form of the vorticity transport equation, i.e., Eq. 7, gives:

$$\left(\frac{\Psi_{i,j+1} - \Psi_{i,j-1}}{2\Delta Y}\right)\left(\frac{\Omega_{i+1,j} - \Omega_{i-1,j}}{2\Delta X}\right) - \left(\frac{\Psi_{i+1,j} - \Psi_{i-1,j}}{2\Delta X}\right)\left(\frac{\Omega_{i,j+1} - \Omega_{i,j-1}}{2\Delta Y}\right) = \Pr\left[\left(\frac{\Omega_{i+1,j} - 2\Omega_{i,j} + \Omega_{i-1,j}}{\Delta X^2}\right) + \left(\frac{\Omega_{i,j+1} - 2\Omega_{i,j} + \Omega_{i,j-1}}{\Delta Y^2}\right)\right] + Ra\left(\frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta X}\right) \quad (13)$$

The finite difference form of the equation relating the dimensionless stream function, i.e., **Eq. 6**, is:

$$\Psi_{i,j} = \left[ \left( \frac{\Psi_{i+1,j} + \Psi_{i-1,j}}{\Delta X^2} + \frac{\Psi_{i,j+1} + \Psi_{i,j-1}}{\Delta Y^2} \right) + \Omega_{i,j} \right] / \left( \frac{2}{\Delta X^2} + \frac{2}{\Delta Y^2} \right)$$
(14)

## **3.2 Solution Procedure**

The governing dimensionless differential equations are discretized to a finite difference form. The computational scheme, based on Successive Over Relaxation, SOR, is arranged to solve the three equations for the nth iteration step. The initial values over the field for  $\theta$ ,  $\Omega$  and  $\psi$  are assumed zero to all internal nodes are taken as initial starting values. Over-relaxation is actually used so the "updated" values of  $\phi_{i,j}$  are actually taken as:

$$\phi_{i,j}^{new} = \phi_{i,j}^{old} + r \left( \phi_{i,j}^{calculated} - \phi_{i,j}^{old} \right)$$
(15)

Where the subscripts i and j refer to a grid node,  $\phi$  is a general dependent variable ( $\theta$ ,  $\Omega$ , or  $\psi$ ). The relaxation parameters,  $\gamma_{\theta} = \gamma_{\omega} = 1$  and  $\gamma_{\psi} = 1.6$  give stable numerical computation Ra  $\leq 10^4$ , over 41×41 grid point with uniform spaced mesh in both x- and y-directions as shown in **Fig 2**. **Fig 3** demonstrates the influence of number of grid points for a test case of fluid confined within the present configuration at Ra=10<sup>4</sup>, AR=1.

The criterion for convergence is examined according to a realistic condition for each state variable at each node as:

$$\frac{\left|\boldsymbol{\phi}_{i,j}^{new} - \boldsymbol{\phi}_{i,j}^{old}\right|}{\left|\boldsymbol{\phi}_{i,j}^{old}\right|} \le \mathbf{E}_{\max}$$
(16)

A computer program in (Fortran 90) is built to execute the numerical algorithm which is mentioned above; it is general for a natural convection in two-dimensional enclosure, as the following procedure:

- 1. Input data (aspect ratio, Rayleigh number, Prandtl number, dimension of grid,...etc).
- 2. Initial values (guess values).
- 3. Boundary conditions.
- 4. Calculation of stream function (eq.(14)).
- 5. Calculation of temperature field (eq.(12)).
- 6. Calculation of vorticity internal nodes (eq.(13)).
- 7. Calculation of vorticity for boundary nodes.
- 8. Calculation of Nusselt number.
- 9. Check for convergence for temperature field. If no repeated to step 4 until reached to coverage.

10. Write output data.

## 4. NUMERICAL RESULTS VERIFICATION

For the purpose of the present numerical algorithm verification, the results obtained from the present code for variation local Nusselt number and a relation between average Nusselt number and Ra number, are compared with the other study (Lo, D.C., Young, D.L. and Tsai, 2007), for laminar natural convection in square enclosure filled in air (Pr=0.71), with full heating left vertical wall, AR=1. There are a good agreement in results which validate the present computational model as shown in **Figs 4** and **5**.

## **5. RESULTS AND DISCUSSION:**

Numerical simulation are performed for Pr=0.71 ( air is the working fluid), for partial heating from the left side wall and for three positions (lower, center and upper ) and different values of Rayleigh number range  $(10^3 \le \text{Ra} \le 5 \times 10^5)$ , the aspect ratio of enclosure is  $(0.5 \le \text{AR} \le 2)$ .



Fig 4 show comparison between present work and Lo et al. for variation Nu at  $Ra=10^4$ .

Fig 5 show comparison between present work and Lo et al. for relation between Nu and Ra.

The results presented in two forms, the first will focus on flow and temperature fields, which contents isotherms, streamlines and vortices. Another case of results represented the average Nusselt number with Rayleigh number.

## 5.1 Effect of partially heated:

Flow and temperature field are simulated using isotherms, stream lines and vortices for partially heated of the left side wall and for different positions (lower, center, upper), with effect of Rayleigh number for (AR=1), (H=W).

For lower position the development of isotherms, streamlines and vortices at Ra number varies from  $10^3$  to  $5 \times 10^5$  are shown in **Fig 6**. Heat dissipated from the lower wall develops a fluid layer to moves at low velocity for Ra= $10^3$  as indicated by the largely spaced isotherms to thus shown for Ra= $10^4$ , the isotherms are converged near the hot

section, is nearly parallel lines, and this space increase with increase the Rayleigh number and move down according to the gravity. The cold wall above the heating wall, suppresses this buoyancy effects. The hot fluid losses part of its energy to the cold section of the same wall and moves along the adiabatic ceilings then down along the right side wall forming a single central cell with a center shifted from that of the enclosure with  $(|\psi_{min}=0.633|, at Ra=10^3)$ , and the strength of this cell increase with increasing Rayleigh number  $(|\psi_{min}=13.039|, at Ra=10^5)$ . Eddy cell is formed in the upper left corner as a result of the partially heated (upper cold wall) with  $(|\psi_{max}=0.0026|, at Ra=10^3)$  and its magnitude increase with increase the Rayleigh  $(|\psi_{max}=1.521|, at Ra=10^5)$ . The lower vortex, at Ra=5×10<sup>5</sup>, moves clockwise, and the effect of the upper vortex, rotating anticlockwise causes the distortion shown in the stream functions.

For central position the development of isotherms, streamlines and vortices when the heating is placed at the center position at Rayleigh number verifies from  $10^3$  to  $5 \times 10^5$ are shown in **Fig 7**, for the different Rayleigh number it can be notice that the flow forms a main cell with its center of the enclosure that is beyond to the local of the heating in the center of enclosure with ( $|\psi_{min}=0.801|$ , at Ra= $10^3$ ), and the strength of this cell increase with increasing Rayleigh number ( $|\psi_{min}=9.813|$ , at Ra= $10^5$ ). And a small eddy cell appear near in the upper left corner and a new eddy cell appear in the lower left according to the cold walls upper and lower the heating wall with ( $|\psi_{max}=0.0014|$ , at Ra= $10^3$ ) and its magnitude increase with increase the Rayleigh ( $|\psi_{max}=0.586|$ , at Ra= $10^5$ ). The isotherms are converged near the hot central section and the top cold section, but much less at the lower cold section. This because the hot air is flow to the upper cold wall because of its low density and then pass through the enclosure until reach to the lower cold wall according to the gravity.

For upper position when the heating wall is placed at the upper position heat is transferred from the upper heated wall to both of the lower and opposite cold walls, the isotherms, streamlines and vorticity shown in **Fig 8**, as Ra=10<sup>3</sup> to  $5 \times 10^5$ . It can be notice that one central cell appear in the upper half according to location of the heating in the upper half wall with ( $|\psi_{min}=0.581|$ , at Ra=10<sup>3</sup>), and the strength of this cell increase with increasing Rayleigh number ( $|\psi_{min}=7.713|$ , at Ra=10<sup>5</sup>), and one eddy cell at the lower left corner according to the partial heating ( cold lower half wall) with ( $|\psi_{max}=0.0022|$ , at Ra=10<sup>3</sup>) and its magnitude increase with increase the Rayleigh ( $|\psi_{max}=0.949|$ , at Ra=10<sup>5</sup>).

It is clear from above results that the effect of Rayleigh number on the flow is increasing the strength of flow with increasing it, and for partial heating the optimal position is when the heating localized at the lower position.

## 5.2 Effect of aspect ratio

Aspect ratio AR of the rectangular enclosure is important parameter for flow and temperature fields. This changed from 0.5 to 2 along the present study. Thus **Fig 9** shows the effect of AR on the isotherms and streamlines for Rayleigh number (Ra=10<sup>5</sup>), and for lower position. For AR=0.5, there are a big vortex appear in the enclosure with ( $|\psi_{min}=4.69|$ ), and a weak one on the upper left corner. In AR=1 **Fig 6**, Ra=10<sup>5</sup>, there are two vortices, the main with ( $|\psi_{min}=13.039|$ ), and a small vortex in the upper left corner. In

AR=2 Fig 9, three vortices distributed in the enclosure with different strength, ( $|\psi_{min}=14.875|$ ). This phenomena at streamlines which effects on isotherms which are uniform nearly at AR=0.5 and became random and irregular in the enclosure if aspect ratio increase due to vortices.

It is clear from above results when the aspect ratio increases the strength of flow increase, and the amount of heat transfer increase corresponding to increasing the area of heat transfer, and this is clear in **Fig 10**, which shows the logarithm relation between Rayleigh number Ra and average Nusselt number ( $\overline{Nu}$ ) with different aspect ratios, the average Nusselt number ( $\overline{Nu}$ ) increase with increasing aspect ratios.

## 5.3 The Variation of Nusselt Number

Fig 10 show logarithm relation between the average Nusselt number ( $\overline{Nu}$ ) and Rayleigh numbers Ra. First, you can resulting from this figures that average Nusselt number ( $\overline{Nu}$ ) increase with increasing Rayleigh number Ra, and it is indicated a critical value of Rayleigh Ra, mark in the natural way the demarcation point between the conduction mode and the onset of the natural convection mode.

**Fig 10-a** show the optimal position for the partial heating, the highest value is obtained near the lower position and the lowest value is near the upper position. This result is in agreement with the study of (Al-Bahi, Radhwan and Zaki, 2002) where the maximum heat transfer is obtained for heat transfer placed close the bottom of enclosure.

**Fig 10-b** show the effect of aspect ratio on heat transfer rate, the highest value is obtained for the (AR=2) and the lowest value is for (AR=0.5), this beyond to the area of heat transfer , when increasing the length of the left wall (with constant width) the heat transfer rate is increasing and when decreasing the length of the wall the heat transfer rate is decreasing.

## 6. CONCLUSIONS

The following Conclusions can be found from the results of the present work:

- 1- When the values of the Rayleigh number are low, the isotherms are straight lines and approximately parallel referring that the heat transfer is due to conduction. Also, rotating vortices with small magnitudes of the stream function are observed in the rectangular enclosure.
- 2- When the values of the Rayleigh number increases, the air circulation inside the enclosure is greatly dependent on Rayleigh number. From the other hand, the effect of natural convection is also appeared in the temperature contours which being to confuse and move in the direction of the central part of the bottom wall.
- 3- The optimal position for the partial heating, the highest value is obtained near the lower position and the lowest value is near the upper position.
- 4- The heat transfer rate is increase with increasing the aspect ratio (increasing height with constant width), according to increasing the area of the heating.

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Fig 6 (a)Isotherms; (b)streamlines; (c) vorticity contours of partial heated left wall (lower position), right cold wall and adiabatic top and bottom walls inside rectangular enclosure  $_{\Lambda D-1}$ 

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Fig 7 (a)Isotherms; (b)streamlines; (c)vorticity contours of partial heated left wall (center position), right cold wall and adiabatic top and bottom walls inside rectangular enclosure



Fig 8 (a)Isotherms; (b)streamlines; (c) vortices contours of partial heated left wall (upper position), right cold wall and adiabatic top and bottom walls inside rectangular enclosure AR=1.



Fig 9 isotherms and streamlines contours of partial heated left wall (lower position), right cold wall and adiabatic top and bottom walls inside rectangular enclosure (a) AR=0.5, (b) AR=2, and  $Ra=10^5$ .



Fig 10 Logarithm relationship between average Nusselt number and Rayleigh number, (a) Effect of partial heating. (b) Effect of aspect ratio, for lower position.

# NOMENCLATURE

The following symbols are used generally throughout the text. Others are defined as and when used.

<u>Symbols</u>	<u>Meaning</u>	<u>Unit</u>
AR	Aspect ratio (W/H).	
Cp	Specific heat at constant pressure.	kJ/kg.k
E <sub>max</sub>	Maximum error.	
Gr	Grashof number.	
g	Gravitational acceleration.	$m/s^2$
Н	Height of the enclosure.	m
h	Heat transfer coefficient.	$W/m^2.k$
k	Thermal conductivity.	W/m.k
$L_1$	Distance to the lower edge of the heater.	m
$L_2$	Distance to the upper edge of the heater.	m
Nu	Nusselt number.	
$\overline{Nu}$	Average Nusselt number.	
S	Heater length( $S = \frac{1}{2}$ H).	m
Р	Pressure.	N/ $m^2$
Pr	Prandtl number	
Ra	Rayleigh number	
$\mathbf{r}, \mathbf{r}_{\Psi}, \mathbf{r}_{\Omega}, \mathbf{r}_{\theta}$	Relaxation parameter for stream	
	function, vorticity, and temperature	
	respectively	
Т	Temperature.	k
u	Velocity component in x-direction.	m/s
U	Dimensionless Velocity component in x-	
	direction.	
V	Velocity component in y-direction.	m/s
V	Dimensionless Velocity component in y-	
	direction.	
W	Length of the enclosure.	m
x, y	Cartesian space coordinates.	
X,Y	Dimensionless Cartesian space	
	coordinates.	
_	<u>Greek Symbol</u>	
β	Coefficient of thermal expansion.	K <sup>-1</sup>
φ	General dependent variable.	
μ	Molecular dynamic viscosity.	

ν	Kinematic viscosity.	$m^2/s$
ω	Vorticity.	s <sup>-1</sup>
Ω	Dimensionless vorticity.	
ρ	Density of fluid.	kg/m <sup>3</sup>
Ψ	stream function.	$m^2/s$
Ψ	Dimensionless stream function.	
θ	Dimensionless temperature.	
	<u>Subscript Symbols</u>	
c, h	Related to cold and hot side respectively	
(i,j)	Grid nodes in $(x,y)$ direction	