AN INVESTIGATION OF THE DYNAMIC RESPONSE OF THREE STORIES BUILDING USING NONLINEAR SPRING

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ABSTRACT

A novel analysis of three stories - building, using finite element method via ANSYS software with nonlinear spring is presented. The investigation is carried out to show the natural frequencies and the response of the building under nonlinear effect of spring. The free and force vibration of a building with a non-linear spring–mass system has been investigated. The non-linear spring appears in the form of quadratic, exponential and logarithmic equation. The transient and modal solutions are done by using ANSYS to the non-linear system. The results of the linear frequencies and the response matched well with those obtained in the mathematical model. Subsequent non-linear study indicates that there is a pronounced effect of the non-linear spring and its mass. The exponential equation of nonlinear stiffness gives the highest magnitude of three modes of the natural frequencies with 0.0371% ,0.014% and 0.07% respectively of other cases of form equation (quadratic and logarithmic).

KEYWORDS: vibration beam, non-linear vibration, non-linear spring–mass system, Ansys

الخلاصة :

يتطرق هذا البحث لتحليل بنايه من ثلاث طوابق باستخدام طريقه العناصر المحدده وباستخدام برنامج . Ansysلقد تم البحث لغرض التحقق من قيم الترددات الطبيعيه والاستجابه للبنايه تحت التاثير الغير خطى لحركه النابض ,حيث تم استخدم البنايه كمنظومه غير خطيه من خلال نموذج (كتله– نابض غير خطي) لدراسه الاهتزازات واستخدام المصفوفات للنموذج الحسابي.

لقد تم تمثيل الحالة الغير خطية للنابض باستخدام معادلات غير خطيه مثل المعادلة التربيعية ، الاسية و اللوغارتمية للحركه الاهتزازيه حيث تم تطبيق برنامج Ansysللنظام الغير خطى لغرض ايجاد حلول للنموذج .

اظهرت النتائج للترددات والاستجابات للحركات الغير خطيه وللمعادلات بانواعها التى استخدمت بان هناك تاثير واضح وملموس للنابض الغير خطى والكتل التى اختيرت كنموذج من خلال الجداول والمخططات .

SYMBOLS:

U_1, U_2, U_3	Kinetic energy for one ,two and three stories
T_1, T_2, T_3	Potential energy for one ,two and three stories
ρ	Mass density
E	Elastic modulus
A	Cross-sectional area
I	Area moment of inertia
i	Transverse number
ω	Frequency
w	Deflection
K	Spring stiffness of each story

INTRODUCTION

The dynamic characteristics of a structure play a significant role in the overall performance and design of various engineering systems. The determination of the mode shape and the natural frequencies of such dynamic structures has been a topic of primary importance and, as such, has received considerable attention from various researchers. The non-linear response of a simply supported beam with an attached spring–mass system was investigated by Pakdemirli and Nayfeh (1994). Nayfeh and Nayfeh (1994) obtained the non-linear modes and frequencies of a simply supported Euler–Bernoulli beam resting on an elastic foundation having quadratic and cubic non-linearity. Pohit et al. (2004) have modeled the characteristics of an elastomeric material and investigated the effect of non-linear elastomeric constraint on a rotating blade. Kelly (2010) studied the free and forced vibrations of elastically connected structures when the structures are identical, uniform or non uniform. Most studies on non-linear spring have focused primarily on the linearized dynamic analysis. Few informations are available on the influence of the elastomeric on the structural dynamic characteristics. So the major objectives of the present paper is to study the influence of the non-linear constraint on non-linear frequencies and response.

FORMULATION

The problem is considered that of three structural elements in parallel and connected by elastic non linear spring. Investigators have examined the free and forced response of elastically connected strings (Oniszczuk, 2000). Kelly and Srinivas (2009) developed a Rayleigh- Ritz method for elastically connected stretched structures. In the this study, there are three stories building with a non-linear springs of stiffness K_1 and K_2 , as shown in **Fig.1.**

The other end of the springs are attached to a rigid beams. The deformation of the spring–mass system depends on the deflection of the beam. The expressions for the kinetic energy and the potential energy of the structure are given respectively as (Nayfeh and Balachandran,2010):

$$U_1 = \frac{1}{2}\int_0^l E_1\, I_1 (\bar{\bar{w}}_1)^2 dx \quad + \frac{1}{2} K_1 \left(w_{1_{(a)}} - w_{2_{(a)}} \right)^2 \tag{1}$$

$$U_1 = \frac{1}{2}\int_0^l \left[E_1 I_1 (\bar{\bar{w}}_1)^2 + K_1 \left(w_{1_{(x)}} - w_{2_{(x)}} \right)^2 \int (x-a) \right] dx \tag{2}$$

$$U_2 = \frac{1}{2}\int_0^l \left[E_2 I_2 (\bar{\bar{w}}_2)^2 + \left\{ K_1 \left(w_{2_{(a)}} - w_{1_{(a)}} \right)^2 + K_2 \left(w_{2_{(a)}} - w_{3_{(a)}} \right)^2 \right\} \right] dx \tag{3}$$

$$U_3 = \frac{1}{2}\int_0^l \left[E_3 I_3 (\bar{\bar{w}}_3)^2 + K_1 \left(w_{3_{(a)}} - w_{2_{(a)}} \right)^2 \right] dx \tag{4}$$

$$T_1 = \frac{1}{2}\int_0^l \rho_1\, A_1 (\dot{w}_1)^2 dx \tag{5}$$

$$T_2 = \frac{1}{2}\int_0^l \rho_2\, A_2 (\dot{w}_2)^2 dx\text{v} \tag{6}$$

$$T_3 = \frac{1}{2}\int_0^l \rho_3\, A_3 (\dot{w}_3)^2 dx \tag{7}$$

$$Q_i = F_o w_i \tag{8}$$

Hence , $F_o(x,t)$ is distributed load (N/m), then (MATLAB, 1993):

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{w}_i}\right) - \left(\frac{\partial L}{\partial w_i}\right) = Q_i \quad , \quad [\text{i=1, 2, 3}] \tag{9}$$

$$\int_0^l \left[\rho_1 A_1 (\ddot{w}_1)_{xx} + E_1 I_1 (\overline{\overline{w}}_1)_{xx} + K_1 \left(w_{1_{(a)}} - w_{2_{(a)}}\right)\right] dx = Q_1 \tag{10}$$

$$\int_0^l [\rho_2 A_2 (\ddot{w}_2)_{xx} + E_2 I_2 (\overline{\overline{w}}_2)_{xx} + (w_2(k_1+k_2) - k_1 w_1 - k_2 w_3)]\, dx = Q_2 \tag{11}$$

$$\int_0^l [\rho_3 A_3 (\ddot{w}_3)_{xx} + K_2 (w_3 - w_2)]\, dx = Q_3 \tag{12}$$

$$\begin{bmatrix} E_1 I_1 \frac{\partial^4}{\partial^4 x} + k_1 & -k_1 & 0 \\ -k_1 & E_2 I_2 \frac{\partial^4}{\partial^4 x} + (k_1+k_2) & -k_2 \\ 0 & -k_2 & E_3 I_3 \frac{\partial^4}{\partial^4 x} + k_2 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} + \begin{bmatrix} \rho_1 A_1 & 0 & 0 \\ 0 & \rho_2 A_2 & 0 \\ 0 & 0 & \rho_3 A_3 \end{bmatrix} \begin{Bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} \tag{13}$$

where $\frac{EI}{Unit\ length} = \emptyset$ and $\frac{\rho A}{Unit\ length} = u$, thus

$$\begin{bmatrix} \emptyset_1 \frac{\partial^4}{\partial^4 x} + k_1 & -k_1 & 0 \\ -k_1 & \emptyset_2 \frac{\partial^4}{\partial^4 x} + (k_1+k_2) & -k_2 \\ 0 & -k_2 & \emptyset_3 \frac{\partial^4}{\partial^4 x} + k_2 \end{bmatrix} \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix} + \begin{bmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{bmatrix} \begin{Bmatrix} \ddot{w}_1 \\ \ddot{w}_2 \\ \ddot{w}_3 \end{Bmatrix} = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} \tag{14}$$

$$k_i = \begin{bmatrix} \emptyset_1 \widetilde{w}_k^2 + k_1 & -k_1 & 0 \\ -k_1 & \emptyset_2 \widetilde{w}_k^2 + (k_1+k_2) & -k_2 \\ 0 & -k_2 & \emptyset_3 \widetilde{w}_k^2 + k_2 \end{bmatrix} \tag{15}$$

$$M_i = \begin{bmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{bmatrix} \quad (16)$$

hence, Eq.(14) can be written as

$$k_i w_i \ + M_i \ddot{w}_i = Q_{i_{(x,t)}} \quad (17)$$

Here, $(i = 1, 2,3)$ is the transverse deflection at the three segments of the beams.

The above derivation is to demonstrate the Eq.(17) to treat the nonlinear case, in which the derivation of this equation is ambiguous in Ansys software. The solution of Eq.(17) is nontrivial solution, i.e. eigenproblem solution. So Transient analysis is used to evaluate the response and Modal analysis is used to evaluate the natural frequencies of the building via Ansys code (Appendices A and B) .

MULTI MASS NON LINEAR SPRING MODEL

In this study the three stories building to be analyzed, the weights of the floors and walls are in indicated in the **Fig.2** and are assumed to include the structural weight as well (Mario Paz, 1983). The building consists of a series of frames spaced 15 ft apart. It is further assumed that the structural properties are uniform along the length of the building and, therefore the analysis to be made of an interior frame yields the response of entire building. The building is modeled by representing by the spring- mass system as shown in **Fig.3**.

The concentrated weights which are each taken as the total floor weight plus that of the tributary walls are computed as follows:

W_1= 100*30*15+20*12.5*15*2=52,500 lb
m_1= 136 lb.sec^2 /in
W_2= 50*30*15+20*5*15*2=25,500 lb
m_2 = 66 lb.sec^2/in
m_3= 136 lb.sec^2 /in

The stiffness (spring constant) of each story is given by (Nayfeh and Balachandran,2010):

$$K_o= 24EI/L^3 \quad (18)$$

for linear analysis and the individual values for the steel column sections indicated are thus 30700 lb/in, and 44300 lb/in. The author represented the nonlinear by the relation

$$K= K_o+ K_1 \quad (19)$$

where K_1 represented the nonlinear term

Hence, for nonlinear spring, the author investigated that the nonlinear taken the form quadratic, exponential and logarithmic equation as

$$K= 30700+44300\omega^2 \quad (20)$$
$$K=30700+44300\ 2^{\omega} \quad (21)$$
$$K=30700+44300\ \log\omega \quad (22)$$
$$K= 30700+44300\ e^{\omega} \quad (23)$$

The above stiffness represented the nonlinear spring and its conclusion from Nayfeh and Balachandran,2010.

In ANSYS software the direct generation method is used, which is to determine the location of every node and size, shape and connectivity of every element prior to defining these entities in ANSYS model. In the current study, one dimensional 2-node nonlinear spring element (combin39) and one node mass element (mass21) are used to model the three story building [10]. **Fig.4** shows the element used.

RESULTS AND DISCUSSION

The natural frequencies and the response of the building under linear and nonlinear effect of spring is carried out and appeared. The results shown in **Table 1** which gives three modes of frequencies for non-linear effect in the form of quadratic, exponential and logarithmic equation, by applying ANSYS to the non-linear system.(Appendix-A- represented the main commands used to evaluate the frequencies – Modal Analysis) .

When comparing the results, it is observed that the form of equations has pronounced effect on the mode of the natural frequencies, the last equation of K in table gives the highest magnitude of three modes of the natural frequencies with 0.0371%, 0.014% and 0.07% respectively of other cases of form equation of K.

The transient analysis of the building is used to evaluate the response under linear effect of spring shown in **Fig.5** , the response under non-linear effect of spring for Eqs.(20),(21),(22) and (23) and the result of the first three modes of vibration are shown in **Fig.6 to Fig.9.** In all modes of vibration, as the amplitude of vibration increases, the effect of non-linear spring becomes more prominent.(Appendix-B- represented the main commands used to evaluate the response- Transient Analysis)

Fig.10 shows the relation between natural frequencies and amplitude of non-linear cases, the first three non-linear frequencies with tip amplitudes for different form equations of K.

Structures are usually designed on the assumption that the structure is linearly elastic and that it remains linear elastic when subjected to any expected dynamic excitation . However the structure has to be designed for an eventual excitation of large magnitude such as strong motion earthquake or effect of nuclear explosion. So the structures will not remain linearly elastic, it must assume an elastic-plastic behavior beyond the elastic limit. Hence to solve the matrix equation (Eq.(17)) of this type of motion are tedious when perform by hand , so the Ansys code is used. **Fig. 11** shows the compassion of the response for linear spring model, good agreement is evident between the Ansys results and published results.

CONCLUSIONS

The formulation of the building structure of three stories with linear and non-linear constraint has been investigated as masses connected with non-linear spring. The Lagrange principal equation applied to the partial differential equation and boundary conditions to investigate the non-linear frequency which can be reduced to a coupled set of differential equation write as matrix form**,** the transient and modal solutions are done by applying ANSYS to the linear and the non-linear system. The results of the linear frequencies and response match well with those obtained in the mathematical model. Subsequent non-linear study indicates that there is a pronounced effect of the non-linear spring and its mass when non-linear spring appears in the form of quadratic, exponential and logarithmic equation.

The last equation of K gives the highest magnitude of three modes of the natural frequencies with 0.0371% ,0.014% and 0.07% respectively of other cases of form equation .

APPENDIX-A- Represented the main commands used to evaluate the frequencies- Modal Analysis

```
/PREP7
ET,1,COMBIN39,,,2
ET,2,MASS21,,,4
R,1,0.0,30700,1,119300,2,207900  ! SPRING DATA
RMORE,3,385100,4,739500,5,1448300
RMORE,6,2865900,7,5701100,8,11371500
RMORE,9,22712300,10,45393900
R,2,136
R,3,66
! spring mesh
N,1,0,0
N,2,0,-1
N,3,0,-2
E,1,2
E,2,3
! mass mesh
TYPE,2
REAL,2
E,1
type,2
real,3
E,2
E,3
! Boundary Condition
D,1,ux
D,2,UX
D,3,UX
KBC,1                    ! STEP LOADING
SAVE
FINISH
/SOLU
antype,modal
modopt,subsp,3
mxpand,3
solve
finish
```

APPENDIX-B- Represented the main commands used to evaluate the response- Transient Analysis

```
/PREP7
ANTYPE,TRANS             ! NONLINEAR TRANSIENT DYNAMIC ANALYSIS
ET,1,COMBIN39,,,2
ET,2,MASS21,,,4
R,1,0.0,30700,1,119300,2,207900  ! SPRING DATA
RMORE,3,385100,4,739500,5,1448300
RMORE,6,2865900,7,5701100,8,11371500
```

```
RMORE,9,22712300,10,45393900
R,2,136              ! MASS DATA
R,3,66
! spring mesh
N,1,0,0
N,2,0,-1
N,3,0,-2
E,1,2
E,2,3
! mass mesh
TYPE,2
REAL,2
E,1
type,2
real,3
E,2
E,3
! Boundary Condition
D,1,ux
D,2,UX
D,3,UX
IC,3,UY,-1
KBC,1                     ! STEP LOADING
SAVE
FINISH
/SOLU
TRNOPT, , , , , ,HHT
SOLCONTROL,0
CNVTOL,F,1,1E-4           ! FORCE CONVERGENCE CRITERIA
OUTRES,NSOL,1
NSUBST,5
OUTPR,BASIC,NONE
TIME,.0002                ! TIME TO ALLOW INITIAL CHANGE IN ACCELERATION
LSWRITE                   ! WRITE LOAD STEP FILE 1
NSUBST,40
OUTPR,BASIC,LAST
TIME,0.18                 ! TIME ARBITRARILY SELECTED
LSWRITE                   ! WRITE LOAD STEP FILE 2
LSSOLVE,1,2,1             ! READ IN 2 LOAD STEPS AND SOLVE
FINISH
/POST26
TIMERANGE,.003,.18
NSOL,2,3,U,Y,3UY
PRVAR,2                   ! PRINT DISPLACEMENTS
Plvar,2          ! Plot DISPLACEMENTS
*GET,PER,VARI,2,EXTREM,TMIN
*status,parm
FINISH
```

Table 1 The first three modes of natural frequencies (Hz) for all form equation of non-linear spring.

Case	First mode	Second mode	Third mode
$K= 30700+44300\omega^2$	0.365	4.521	8.938
$K=30700+44300\ 2^{\omega}$	Nil	5.702	11.272
$K=30700+44300\ \log\omega$	0.137	2.892	5.718
$K=30700+44300\ \ln\omega$	0.137	2.892	5.718
$K= 30700+44300\ e^{\omega}$	0.429	6.417	12.667

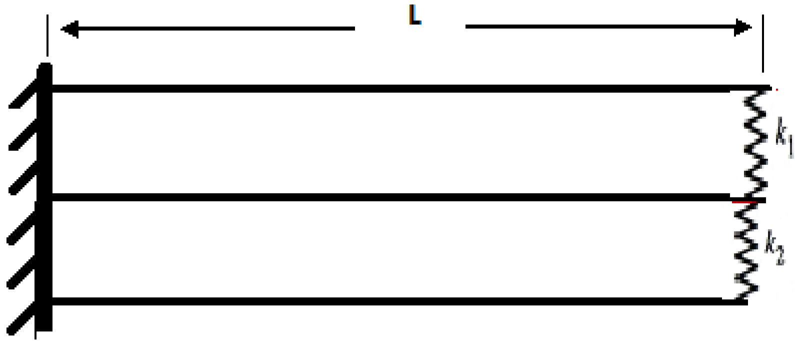
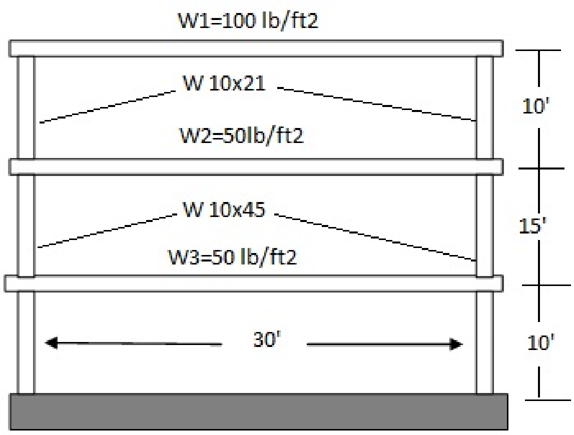


Fig.1 represent the three stories building with nonlinear springs



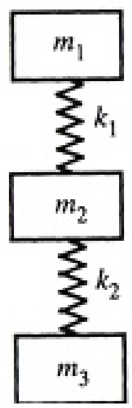
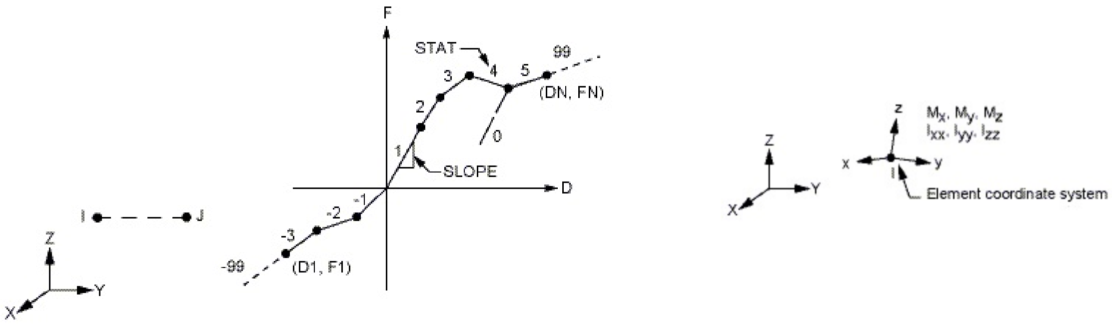


Fig.2 represent the weights of the floors and walls **Fig.3 model of Fig.2**



Combin39 mass21

Fig.4 Elements used in the analysis

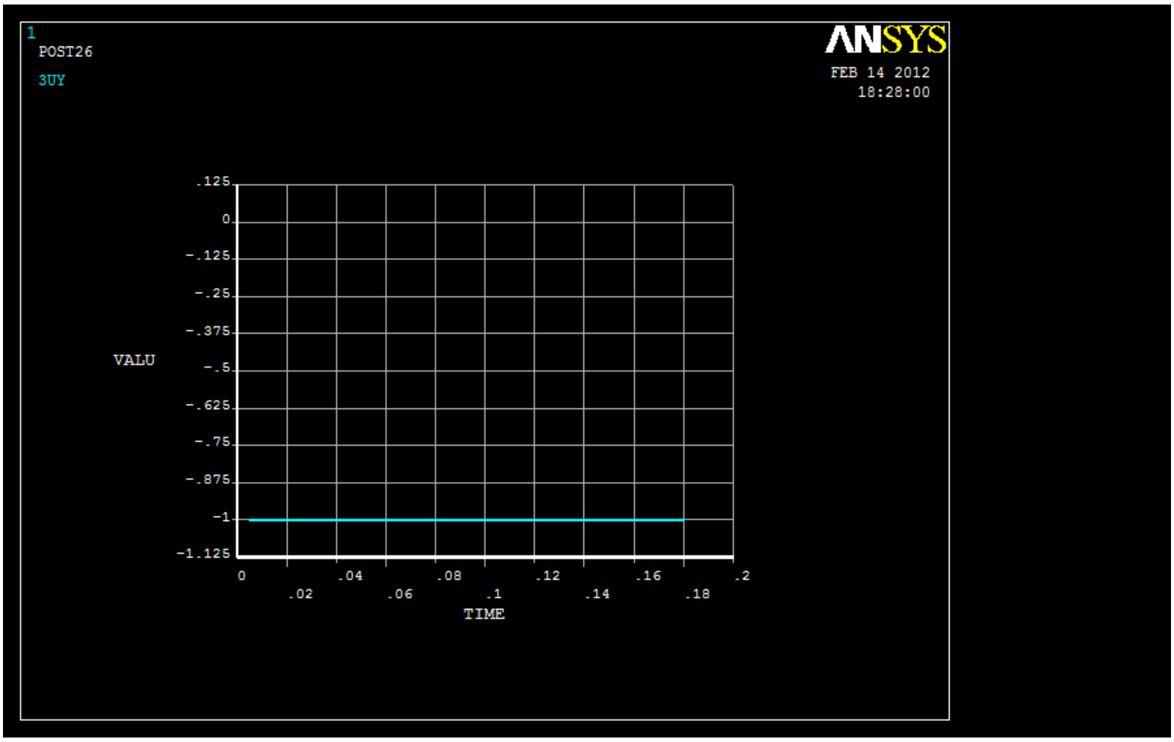


Fig.5 Response in case of linear

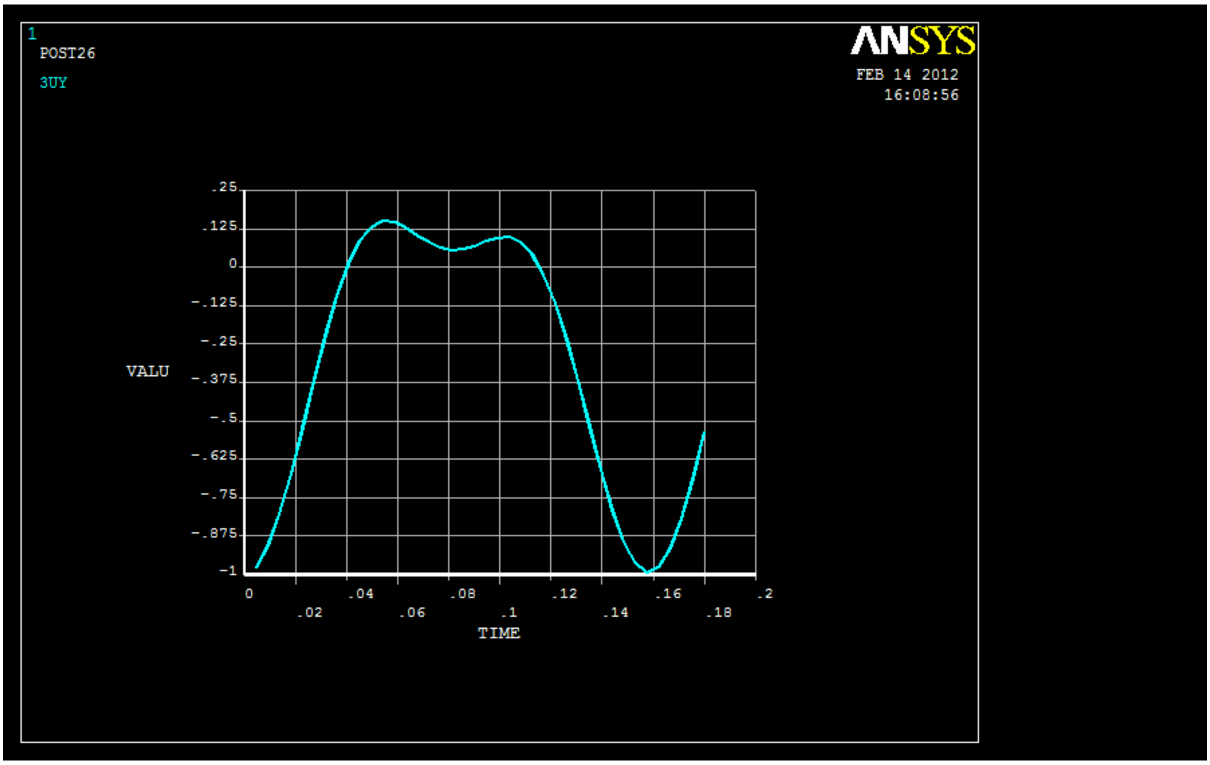


Fig.6 Response in case K= 30700+44300ω²

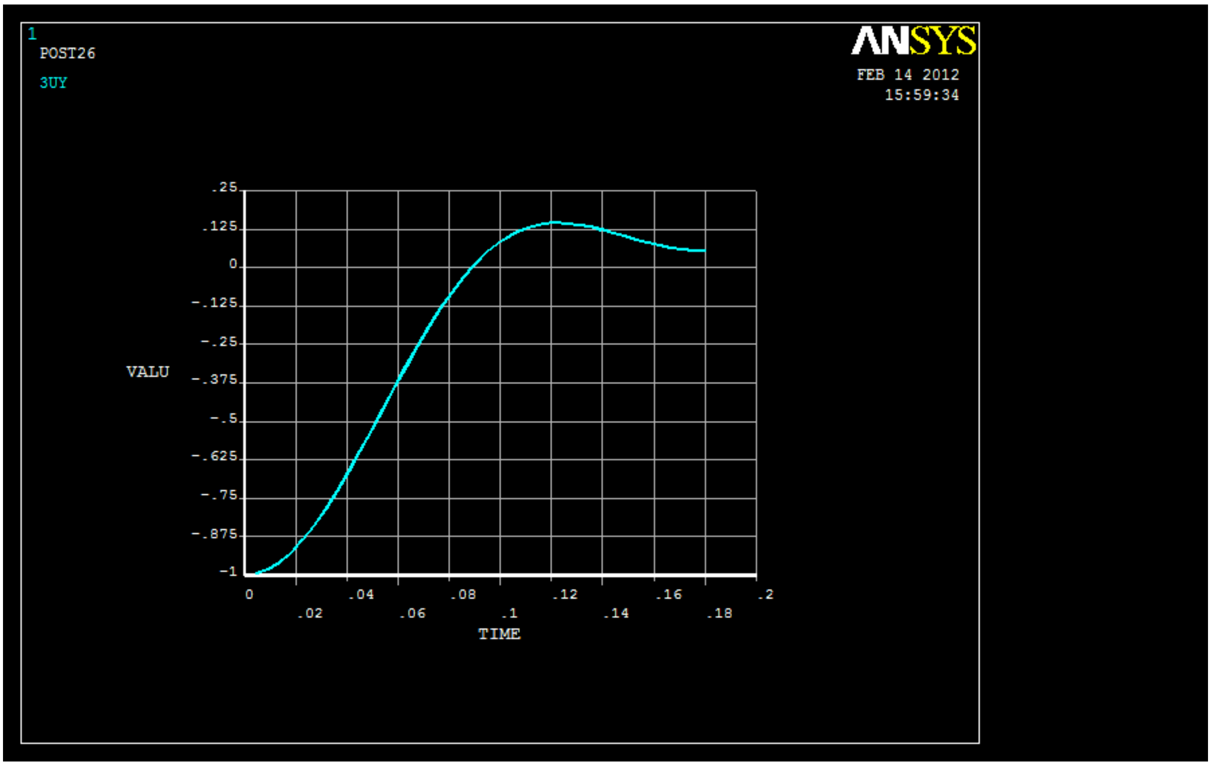


Fig.7 Response in case of K=30700+44300 2^{ω}

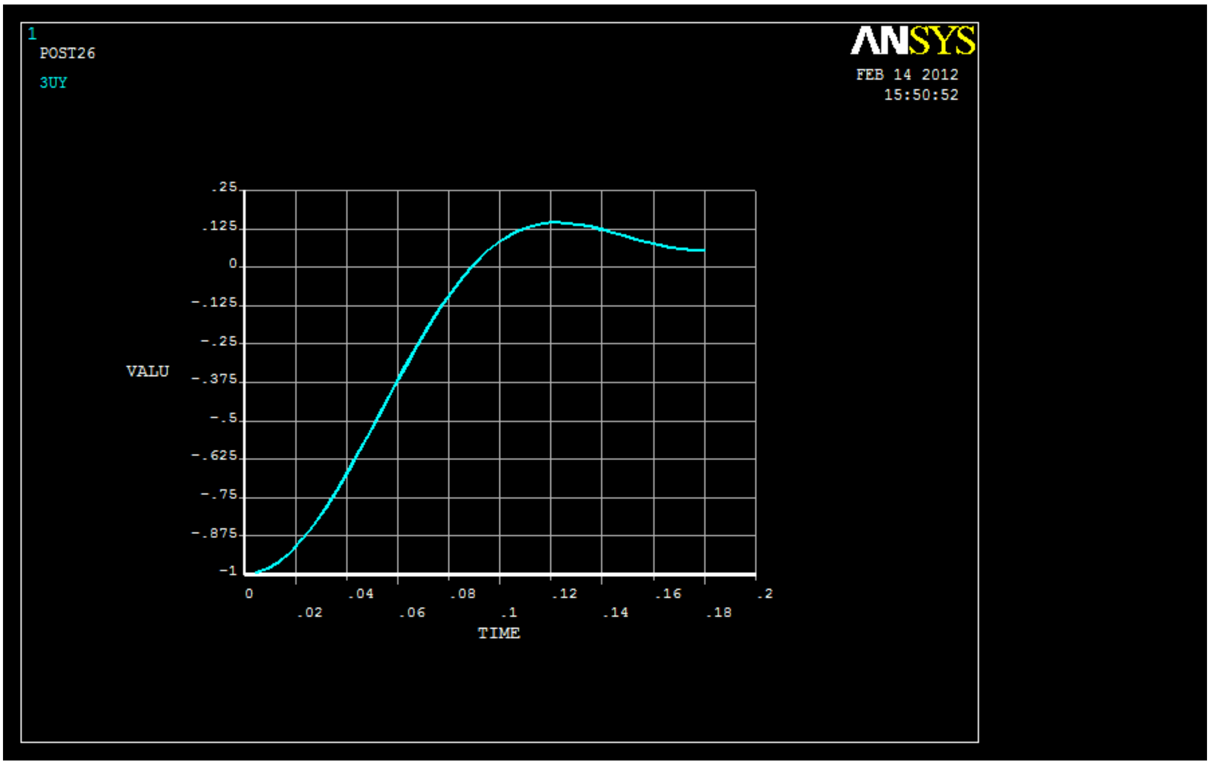


Fig.8 Response in case of K=30700+44300 logω or K=30700+44300 lnω

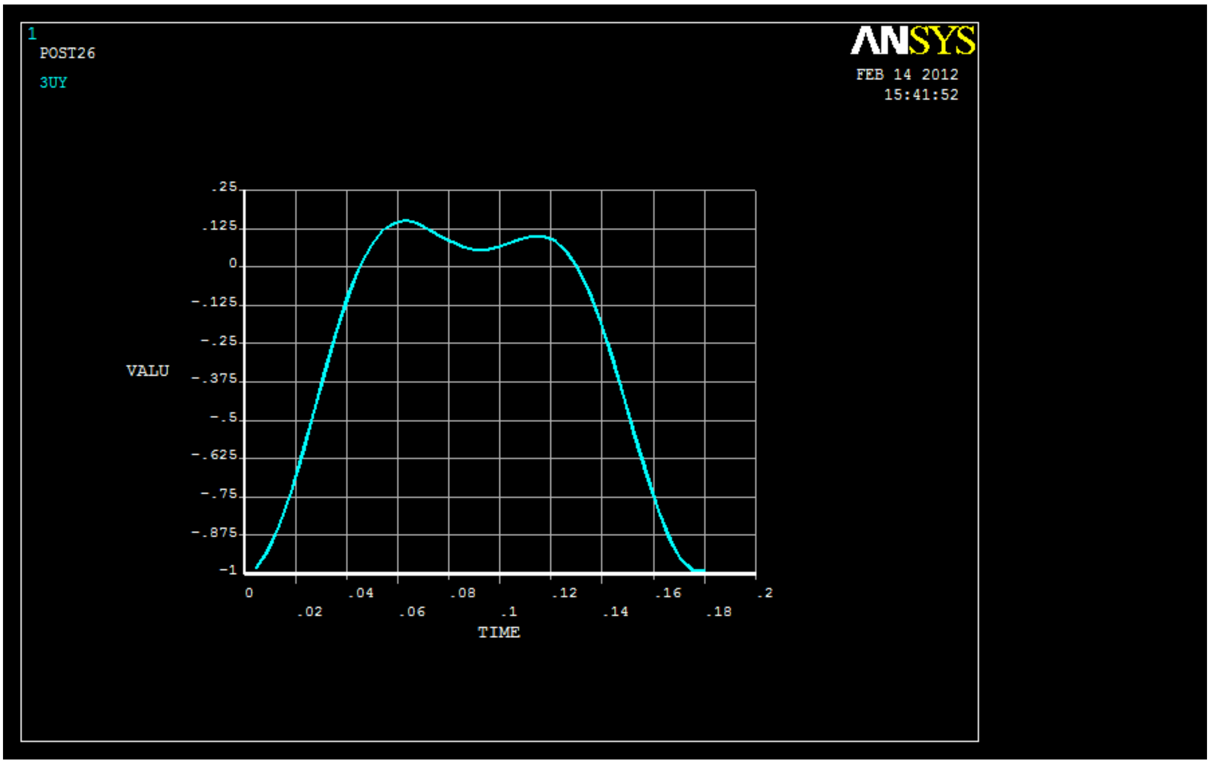


Fig.9 Response in case of K= 30700+44300 e^{ω}

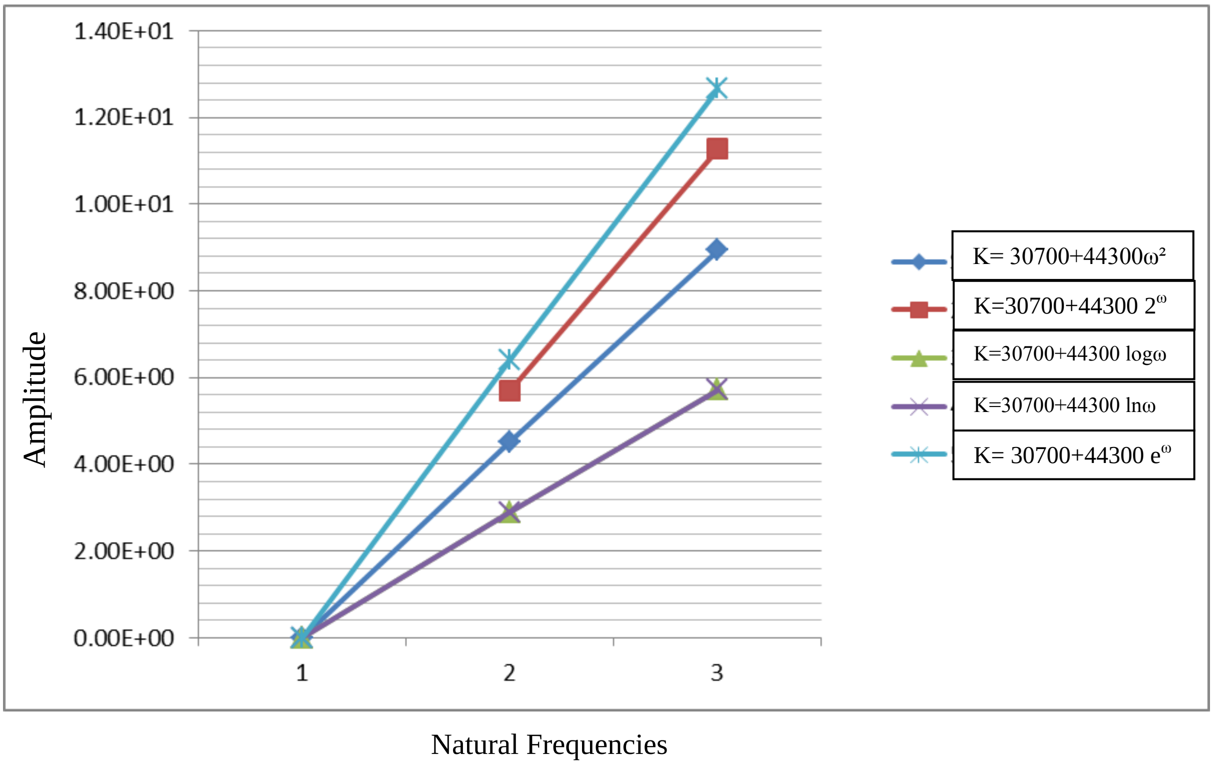


Fig.10 The relation between natural frequencies and amplitude of non-linear cases

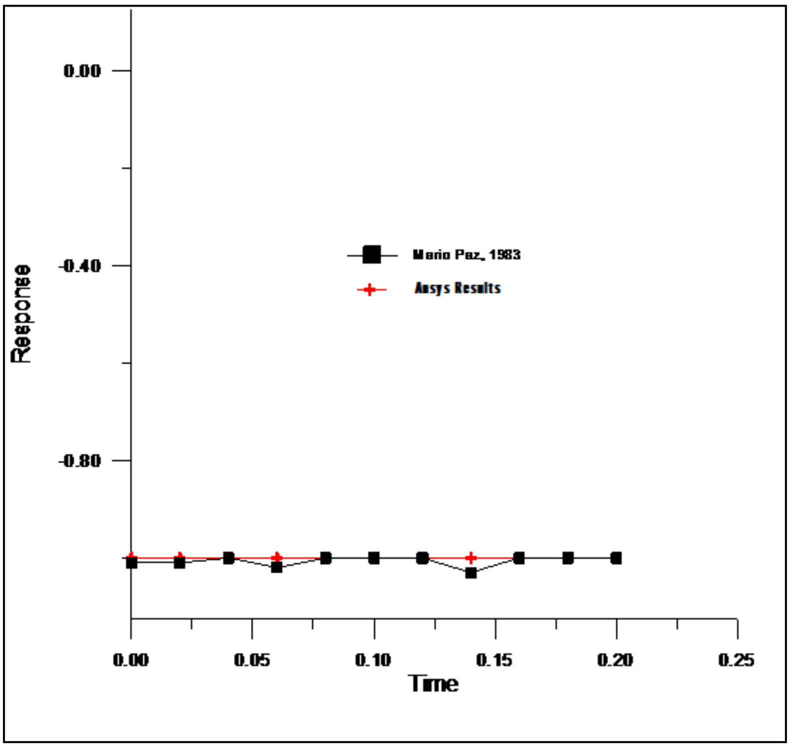


Fig.11 represent published results (Mario Paz,1983) and Ansys results for linear model

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