
INTERFERENCE OF SHALLOW MULTIPLE STRIP FOOTINGS ON SAND

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ABSTRACT

In the present study, the effects of multiple strip footing configurations on the bearing capacity and load-settlement curves of sandy soil were investigated using nonlinear finite element analysis. Two and three strip footings resting on sand were analyzed. In each case, different footing distances were applied for the purposes of comparison among all of the results. From these results, it was observed that the load responses of multiple strip footings are similar to those of the single strip footing at distances greater than four times the footing width. Also, a design equation necessary for quantifying the values of the bearing capacity ratio for the different multiple strip footing configurations were derived. Experimental test results from the literature were selected to verify the used nonlinear finite element method.

Keywords : Finite elements, interference, sand, shallow, strip footings.

الخلاصة

في الدراسة الحالية جرى تحري تأثير ترتيب الأُسس الشريطية المتعددة على قابلية تحمل التربة ومنحنيات الحمل-الهطول للترب الرملية باستعمال طريقة العناصر المحددة اللاخطية. تضمنت الدراسة تحليل أساسين وثلاثة أسس شريطية مسندة على الرمل. في كل حالة تم تغيير المسافات بين الأُسس لغرض المقارنة بين النتائج. من هذه النتائج، لوحظ بأن استجابة الأُسس الشريطية المتعددة للأحمال تكون مشابهة لتصرف الأُسس الشريطي المنفرد إذا كانت المسافات بين الأُسس أكبر من أربع مرات عرض الأُسس. كذلك تم وضع معادلة تصميمية لحساب قابلية تحمل التربة في حالة الأُسس الشريطية المتعددة. تم اختبار النتائج المستخلصة من طريقة العناصر المحددة المستخدمة في هذه الدراسة مع الفحوصات العملية لباحثين آخرين لبيان مدى دقة الطريقة المستخدمة.

INTRODUCTION

A number of theories are available which deal with the calculation of bearing capacity of a single footing. However, only limited information is available to determine the interfering effect on the ultimate bearing capacity of two closely spaced footings. Stuart 1962 had theoretically studied the interference effect of two strip footings on their ultimate bearing capacity. A number of small-scale model tests have also been performed by different researchers to study the interference effect of the two footings [Das and Larbi-Cherif 1983, Kumar and Saran 2003]. By using an upper bound limit analysis in combination with finite elements and linear programming, the effect of the spacing of two interfering rough rigid strip footings on their ultimate bearing capacity was investigated. The footings were considered to be placed on the surface of a cohesionless medium [Kumar and Kouzar 2008].

The study of Graham et al. 1984 is based on the method of characteristics. It has been demonstrated that the ultimate bearing capacity of a group of multiple strip footings placed on sand increases continuously with decrease in spacing among the footings; the increase in the bearing capacity becomes quite extensive for very small spacing. It is not an easy task to perform experiments dealing with the interference of simultaneously loaded multiple strip footings.

Griffiths et. al. 2006 reported the results of parametric studies relating to soil variability and spatial correlation, on the bearing capacity of two parallel rough rigid strip footings on a weightless soil with randomly varying undrained shear strength. For this probabilistic study, a plane strain, nonlinear elastic-perfectly plastic -Tresca finite element analysis is combined with random field theory using Monte Carlo simulation.

In a recent attempt, Kumar and Ghosh 2007 analyzed the bearing capacity of two close footings, and proposed a slip mechanism and bearing capacity solution for multiple footing conditions. Most of them concern only two or three strip footing configurations under 2D plane-strain conditions.

Ghazavi and Lavasan 2008 examined numerically the bearing capacity ratio for rough square footings located at the surface of a homogeneous sandy soil reinforced with a geogrid. The failure stage in the sand was controlled using the Mohr–Coulomb criterion and a non-associated flow rule. Numerical results were compared with those obtained from other experiments and were found to be in good agreement. A parametric study revealed the role of the distance between reinforcing layers and footings and the width and depth of reinforcing layers on the bearing capacity.

In the study of Kumar and Bhoi 2008, an experimental program was planned to carry out a few tests to determine the load-deformation of a number of multiple strip footings placed on sand. Rather than using all the footings, only one footing was employed in the experiments and the vertical plane of symmetry between two adjacent footings was simulated by placing vertically a smooth high strength glass sheet at equal distances on either side of the footing; this vertical glass sheet was fully restrained against any possible horizontal movement. On this basis, a number of experiments were carried

out by varying the horizontal distance between the footing and the glass sheet, on either side of the footing, so as to examine the effect of spacing among footings on the results. In addition, the variation of friction angle of sand on the results was also examined. The obtained results were also compared with the theory of Graham et al. 1984.

Lee and Eun 2009 studied the effects of multiple footings in sand on bearing capacity for both strip and spread footings. In order to evaluate the effects of multiple footings in various configurations, finite element analyses to simulate the various multiple-footing conditions were performed. Both 2D plane-strain (for strip footings) and 3D (for spread footings) conditions were considered. Based on all of the results obtained, bearing capacity ratios and a design equation for multiple-footing conditions are proposed.

FAILURE MECHANISM FOR MULTIPLE FOOTING CONDITION

Effect of Multiple-Footing Conditions on Limit Bearing Capacity

According to the limit equilibrium slip mechanism [Terzaghi 1943] and [Meyerhof 1965], the failure zones of soil underneath a footing include the active (zone A in Fig. 1), the radial transition (zone R in Fig. 1), and the passive (zone P in Fig. 1) zones. If an additional footing exists and is loaded within a distance given by zone P (as indicated by the dashed line in Fig. 1), the mechanical interaction of the footings would render the stress distribution and slip mechanism shown in Fig. 1 invalid.

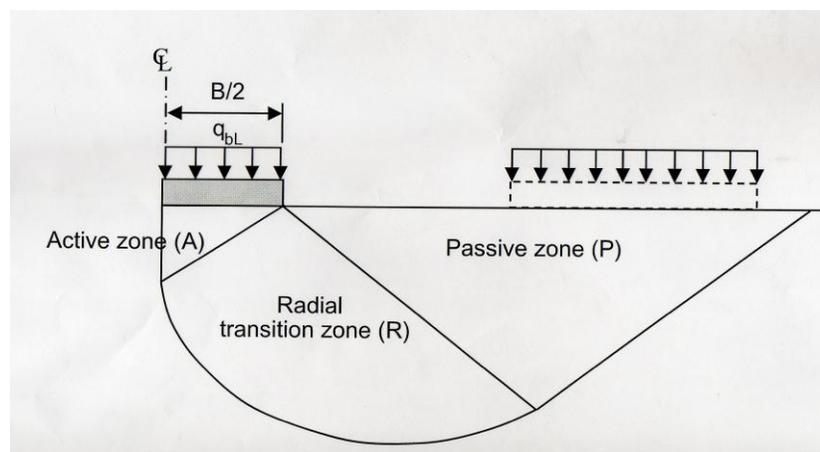


Fig. 1 Failure mechanism of single footing [Lee and Eun 2009].

The theoretical derivations were typically based on stress redistribution due to multiple-footing configuration or modifications of the conventional failure mechanism. In an early example, Stuart 1962 referred to the interaction effects between footings as

“interference”, and proposed the following modified bearing capacity equation for two surface footings on sands:

$$q_u = 0.5B \cdot \gamma \cdot \xi_\gamma \cdot N_\gamma \quad (1)$$

where q_u is the limit bearing capacity; γ = unit weight of foundation soil; B = footing width; ξ_γ = bearing capacity efficiency factor; and N_γ = bearing capacity factor. According to Stuart 1962, ξ_γ varies as a function of the soil friction angle ϕ and footing distance.

The results of the earlier investigations indicate that q_u under the multiple-footing condition is typically greater than that under single-footing condition. According to Graham et al. 1984, the q_u of two close footings at an edge-to-edge footing distance (s) equal to the footing width (B) is approximately 150% greater than that of a single footing for soils with $\phi = 35^\circ$. If the footing distance increases, however, the q_u decreases to a point approaching that of a single footing.

Failure Mechanism for Multiple-Footing Conditions

Figure 2 illustrates the key aspects of the modified failure mechanism for multiple footings adopted in the above-mentioned investigations. In Fig. 2a–c, three typical cases of two parallel footings are illustrated, representing the in-contact, intermediate, and isolated conditions, respectively. The in-contact condition with ($s = 0$) (Fig. 2a) can be treated in the same way as a single-footing case with the footing width twice that of single footing. The failure mechanism and mobilization of q_u , therefore, would be identical to those of single- footing cases.

The intermediate condition (Fig. 2b) represents multiple footings at a certain footing distance (s). In this case, as (s) increases, the internal radial zone expands, whereas the outer radial and passive zones contract. If the footing distance further increases, there can be no interaction between footings, and the mechanical behavior, thus, would come eventually to be the same as that of the isolated single footing shown in Fig. 2c. It is important to note that the modified failure mechanism described in Fig. 2 is valid essentially for two or three footing configurations under 2D plane-strain conditions.

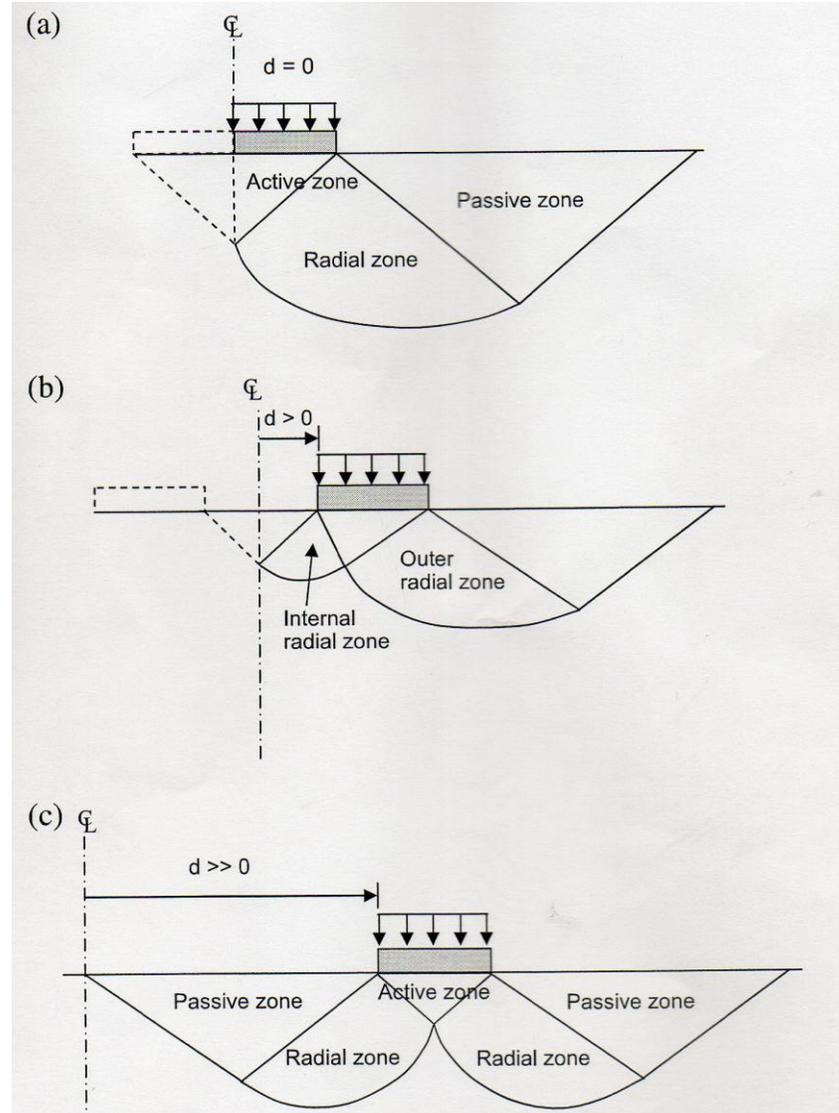


Fig. 2 Failure mechanisms of multiple footings for (a) in-contact condition; (b) intermediate condition; and (c) isolated condition [Lee and Eun 2009].

FINITE ELEMENT MODELING OF MULTIPLE FOOTINGS

In general, construction of shallow foundations requires multiple footings with configurations more complex than simple parallel-placed two- or three-footing situations. In order to investigate the limit bearing capacity for the multiple-footing conditions at various configurations, nonlinear finite element analyses were performed to quantify the multiple strip footings effect. Two cases under the 2D plane-strain were considered in the finite element analyses. For each case, different edge-to-edge footing distances (s) were considered. The PLAXIS finite element computer program was used in the present study. In the finite element analyses, 15-node triangular elements were adopted so as to model soil with elastoplastic behavior and Mohr-Coulomb failure criteria. The concrete footings

were modeled using plate elements with linear elastic behavior. The size of the finite element mesh was 40 m (length) \times 20 m (depth), while that of the footing was 2.0 m width. For the boundary condition, fixed and roller conditions were applied at the bottom and lateral boundaries, respectively. The total number of elements adopted in the analysis was about 225 and based on the convergence of the solution for different cases. Fig. 3 shows example of finite elements mesh used in the analysis for the plane strain conditions.

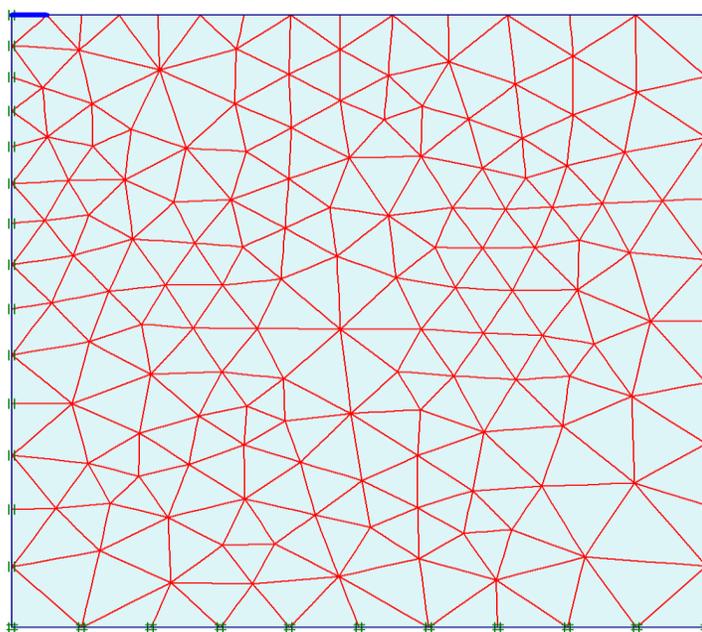


Fig. 3 Finite element mesh for soil and strip footing.

The soils were assumed in all cases to be dense dry sand of the unit weight equal to 16 kN/m^3 . The initial stress state was set as geostatic, with a lateral earth pressure ratio K_0 of 0.5, and the interface angle between the footing base and the soil was 35° .

As the actual design of footings assumes simultaneous application of loads, the loads in this finite element analysis for multiple footing cases were applied by maintaining uniform pressures equally distributed on each footing surface.

TYPES OF ANALYSIS

Two different multiple-footing configurations were considered in the FE analyses, as shown in Fig. 4. Case 1 (Fig. 4a) represents two parallel-placed strip footings under the 2D plane-strain condition, whereas case 2 (Fig. 4b) corresponds to three-strip footings spaced in line at equal (s) distances. In both cases, the footing width was 2 m.

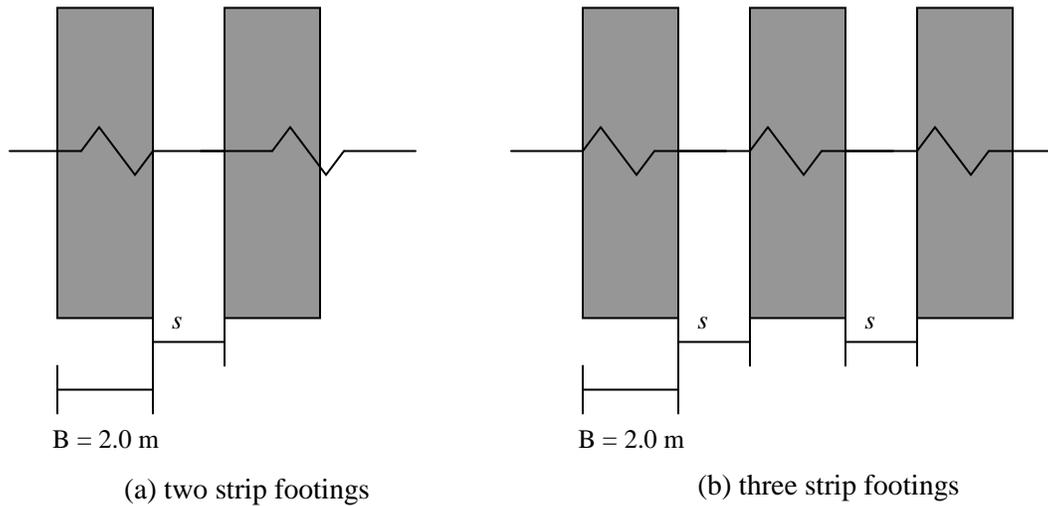


Fig. 4 Multiple-footing conditions adopted in finite element analyses: (a) case 1; (b) case 2.

For each multiple-footing case shown in Fig. 4, different footing distances were considered and used to evaluate the multiple-footing effect as a function of footing distance. For case 2 the central footings were set as the target footings to analyze variations of the limit bearing capacity. Table 1 summarizes the detailed footing conditions considered in the FE analyses.

Table 1 Multiple-footing cases adopted in FE analysis.

Footing condition	Footing distance (s/B)
Case 1 (2D)	0.0, 0.25, 0.5, 1.0, 2.0, 4.0
Case 2 (2D)	0.0, 0.25, 0.5, 1.0, 2.0, 4.0

MODEL VALIDATION

Comparison with the Experimental Work of Kumar and Bhoi 2008

A three dimensional view of the experimental setup of Kumar and Bhoi 2008 was illustrated in Fig. 5. The central vertical plane OC (Fig. 5a) between two adjacent footings becomes the plane of symmetry. Along the plane OC, it is assumed that (i) no

shear stress should act, and (ii) the horizontal deformation of the soil mass at any point should remain equal to zero. Therefore, the placement of a number of multiple footings, which will in turn require a much larger size of the tank, can be simply eliminated provided the plane of symmetry can be appropriately modeled in the experiments [Kumar and Bhoi 2008].

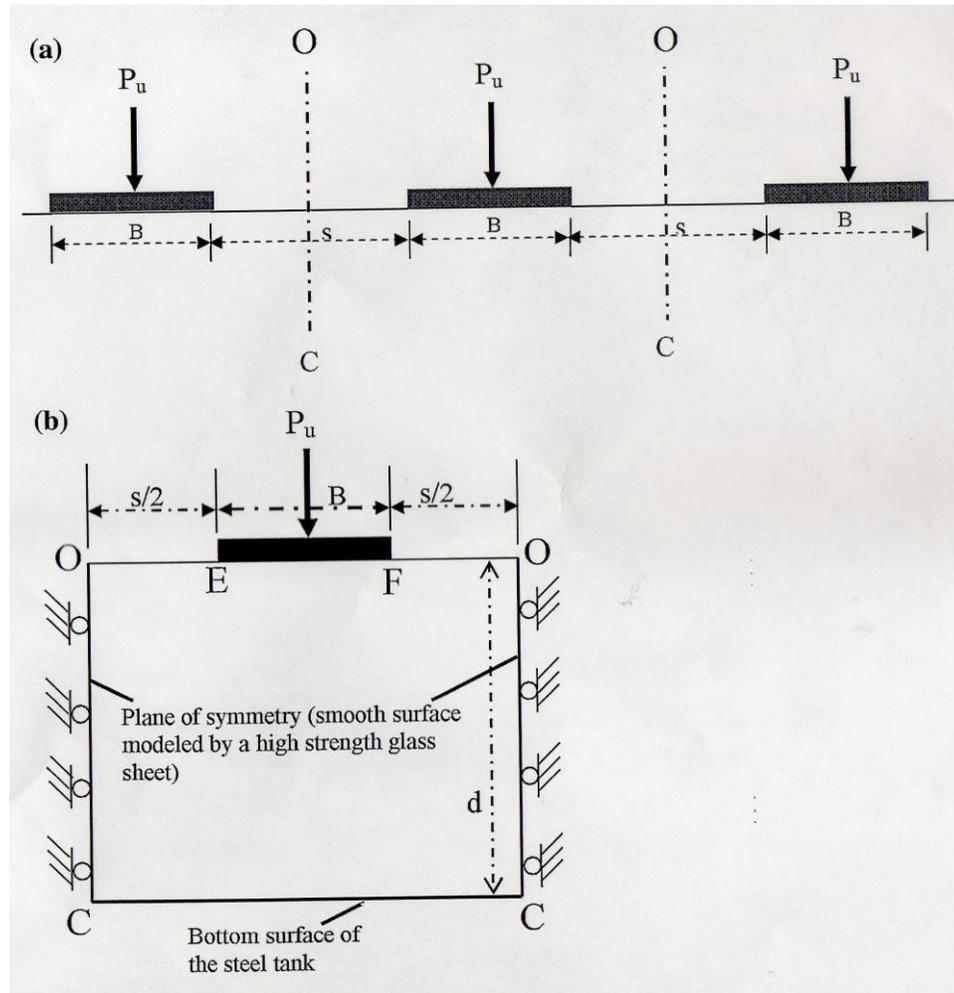


Fig. 5 (a) Definition of problem; and (b) chosen boundary domain for carrying out the experimental work of Kumar and Bhoi 2008.

This was done through the work of Kumar and Bhoi 2008 by fixing a vertical smooth high strength glass sheet exactly at the location of the plane of symmetry. With the simulation of two planes of symmetry, on either side of the footing, only one footing will, therefore, be needed in the experimental setup. The horizontal distance between the glass sheet (OC) and the edge (E) of the footing will become half the clear spacing among the footings. In the present experiments, a rectangular steel tank of size 2.0 m (length), 0.37 m (width), 0.65 m (depth) was used. The tests were conducted on a steel footing of size 7.0 cm (width), 36.0 cm (length), 2.5 cm (thickness). The clear distance

between the fixed glass sheet and the edge of the footing was varied in between 5 and 60 cm; this corresponds to a value of s/B in between 1.43 and 17.14. The location of the footing was always fixed at the centre of the tank. In contrast, the position of the glass sheet was varied to examine the effect of s/B . The footing was vertically loaded by installing a hydraulic jack in between the footing base and a strong horizontal reaction beam as shown in Fig. 6. The load was assumed to be uniformly distributed on the footing in the finite element solution.

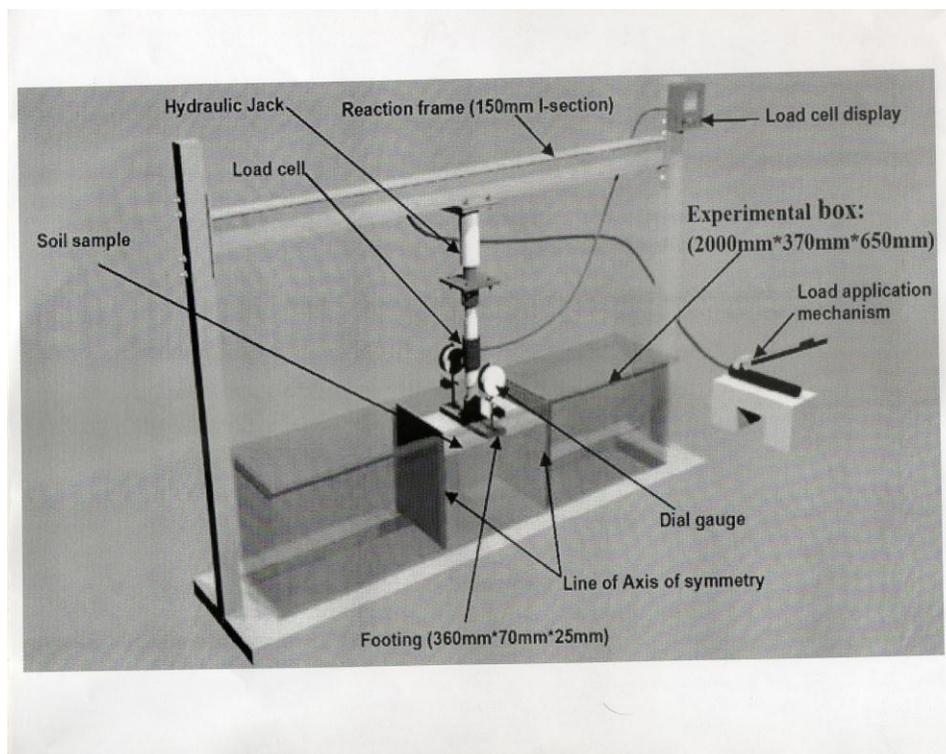


Fig. 6 A sketch of the chosen experimental setup [Kumar and Bhoi 2008].

The average vertical pressure on the footing versus vertical settlement plots for the value of s/B equal 1.43 (unit weight of sand is 16.2 kN/m^3 and the angle of internal friction (ϕ) of soil mass was measured to be 37.4°) given by the experimental work of Kumar and Bhoi 2008 and finite element analysis are shown in Fig. 7. The curves show good agreement between the experimental and the finite element analysis with higher value of failure load. It can be noticed that all the curves remain almost linear and the points of ultimate shear failure were clearly distinct. For the finite element analysis, the magnitude of the ultimate bearing capacity was found to increase continuously with the increase in soil friction angle as shown in Fig. 8.

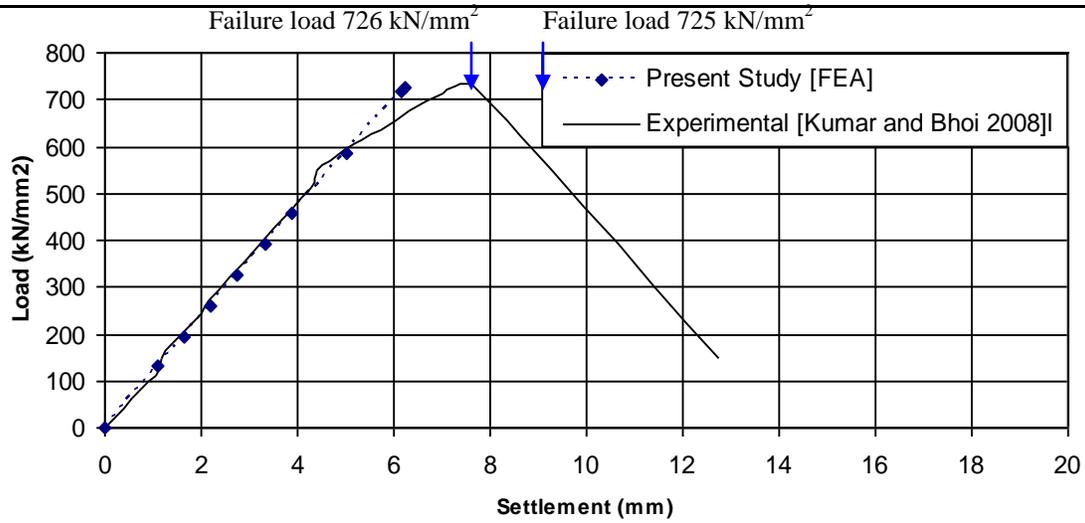


Fig. 7 Comparison of load settlement curves for experimental and finite element analysis.

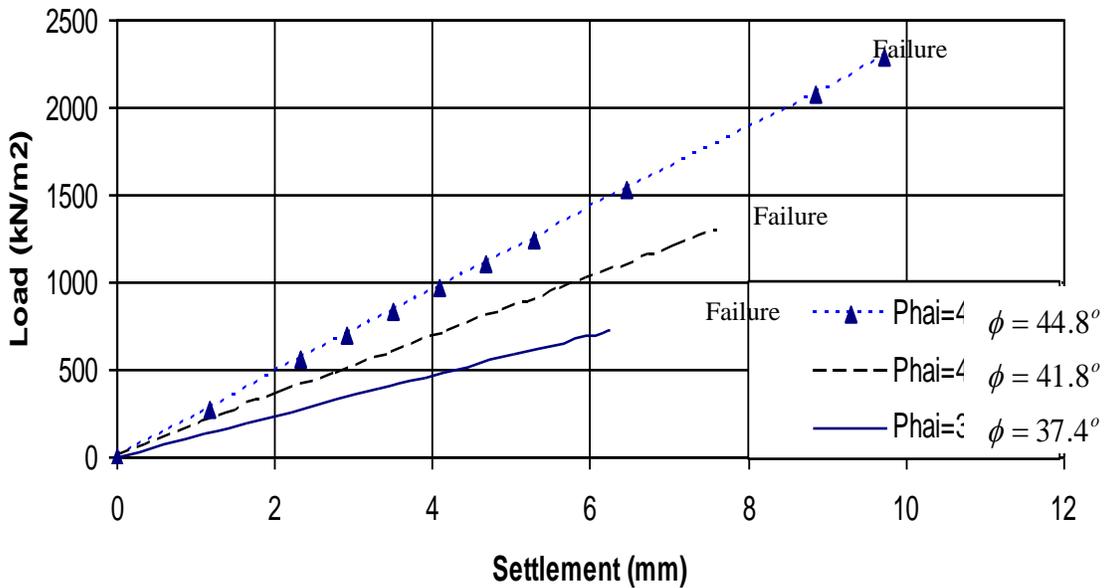


Fig. 8 Load-settlement curves for different angles of friction.

Terzaghi Bearing Capacity Equation

In order to validate the FE model employed in this study, the results for the single-footing conditions were compared with those from the bearing capacity equation. In the finite

element analyses, a soil model with constant friction angle following the original Mohr-Coulomb failure criterion were adopted.

For comparative purpose, two different strip footings with widths of $B = 1.0$ m and 2.0 m on sand were considered. The soil was assumed to be dense dry sand with the unit weight of $\gamma = 16 \text{ kN/m}^3$. The friction angle for this sand was assumed to be 35° . The value of N_γ corresponding to this friction angle is 42.4 according to Terzaghi..

Fig. 9 shows the unit load–settlement curves of the two footings obtained from the FE analysis. As can be seen, the q_u values for $B = 1.0$ m and 2.0 m with a constant friction angle ϕ of 35° were found to be approximately 331 and 643 kPa, respectively because of solution divergence. These results are fairly close matches to those calculated from the bearing capacity equation of Terzaghi for $N_\gamma = 42.4$, which are 339.5 and 678.4 kPa. Based on these overall results, it can be concluded that the FE models adopted in this study are valid, in that (1) the bearing capacity for $B = 2.0$ m from FE analysis was approximately two times that for $B = 1.0$ m, as it should be and (2) the FE analysis results show a reasonable agreement with those from the conventional bearing capacity equation.

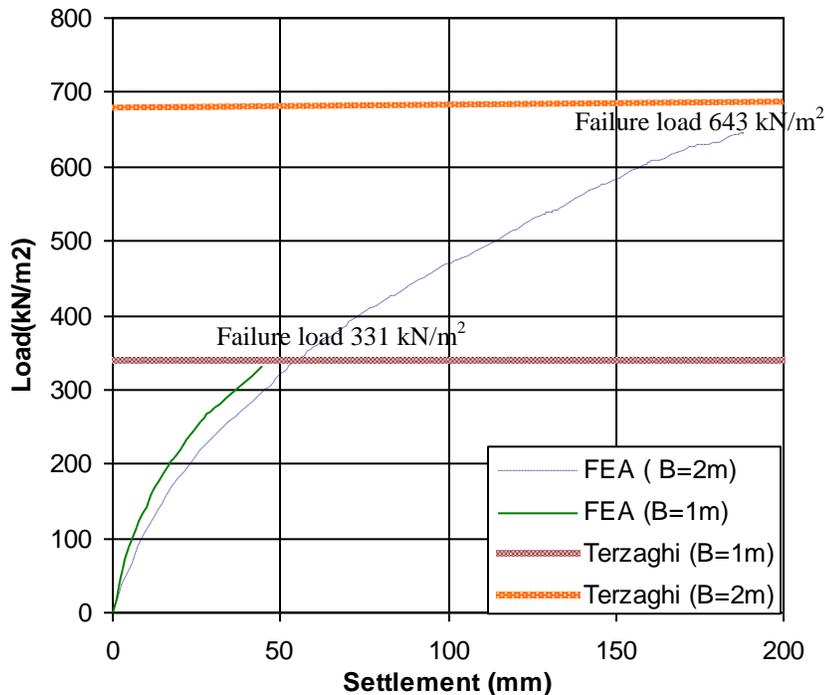


Fig. 9 Load–settlement curves obtained from finite element analysis and Terzaghi bearing capacity equation.

ESTIMATION OF BEARING CAPACITY FOR MULTIPLE FOOTINGS

Load Responses for Multiple-Footing Conditions

In the cases listed in Table 1 FE analyses were performed to obtain load responses and limit bearing capacities under the various multiple-footing configurations. Fig. 10 shows a set of unit load–settlement curves obtained from FE analyses for case 1 (two parallel-placed strip footings) with different footing distances. It can be seen that, according to the footing distance, different load responses and bearing capacities are produced. The values of q_u were defined at the loads on the load–settlement curves where no significantly increased load was observed due to solution divergence.

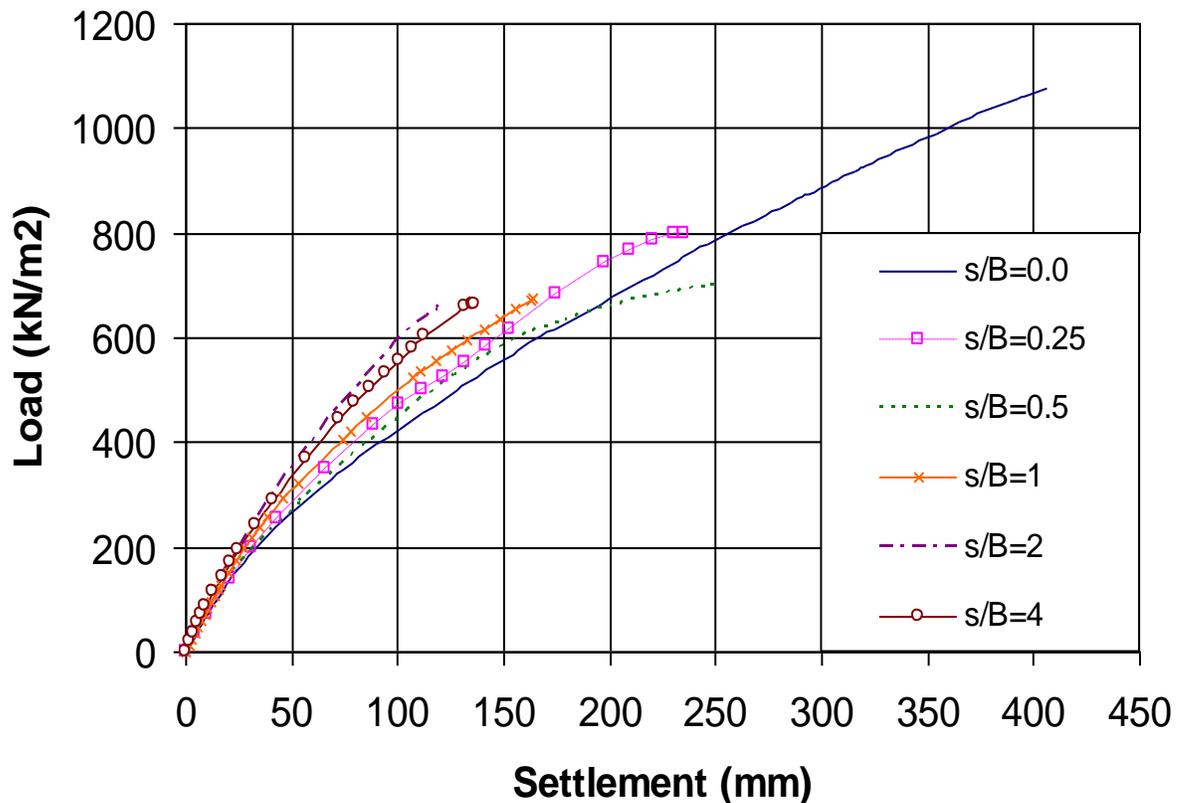


Fig. 10 Load–settlement curves for case 1 (two strip footings).

From Fig. 10, the q_u values for two strip footings with $s = 0, 0.25B, 0.5 B, 1.0B, 2.0B$ and $4.0B$ were found to be 1080, 798, 698, 677, 671 and 665 kPa, respectively. It was also observed that the load response and q_u at $s = 4.0B$ were fairly close to those for the single footing case, indicating that no significant multiple-footing effect had arisen. For the case 2, the obtained load responses corresponding to three strip footings, were plotted

in Fig. 11. Whereas the overall ranges of loads and q_u are higher, than those for case 1, the general trend of q_u variation with footing distance appeared to be similar to that observed for case 1.

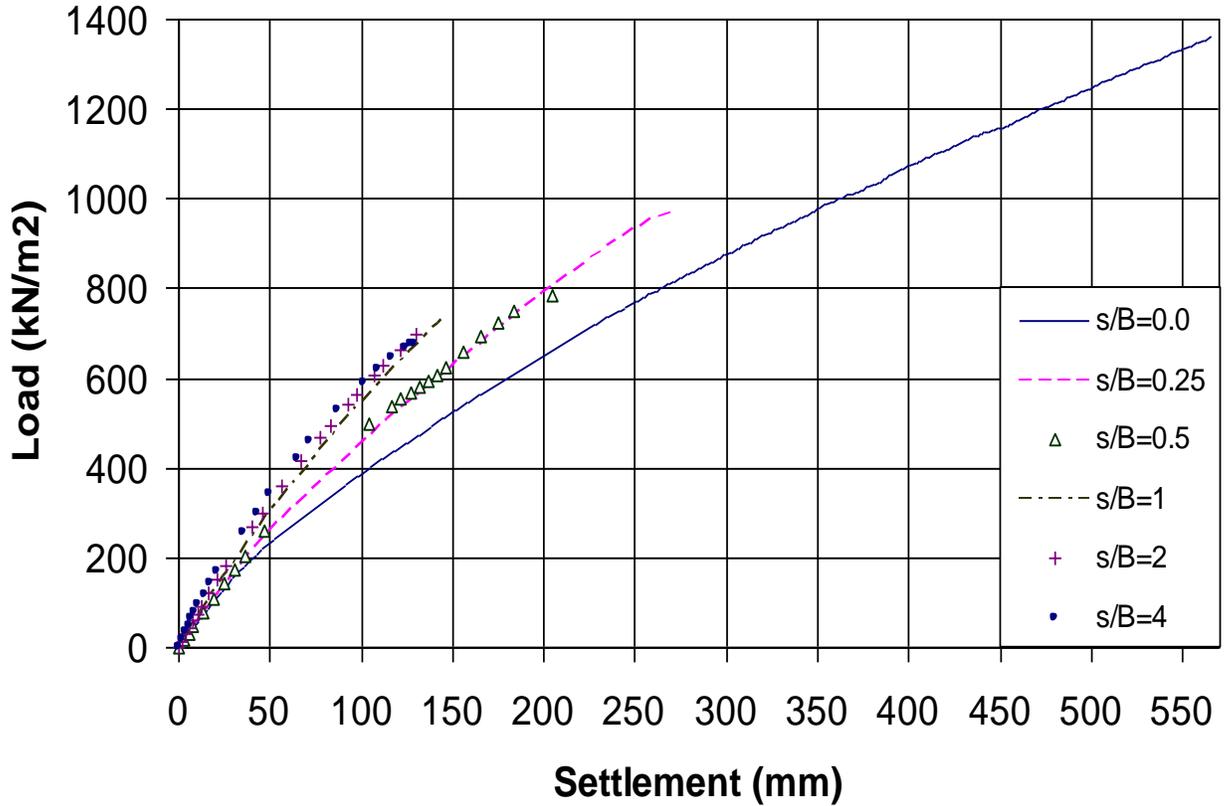


Fig.11 Load–settlement curves for case 2 (three strip footings).

Bearing Capacity Ratio for Multiple-Footing Conditions

Based on the results obtained from the FE analyses, the q_u values for each multiple-footing configuration were obtained and summarized in Table 2. In order to quantify the multiple- footing effects on q_u , a ratio of q_u for multiple footings to that for single footing was introduced as follows:

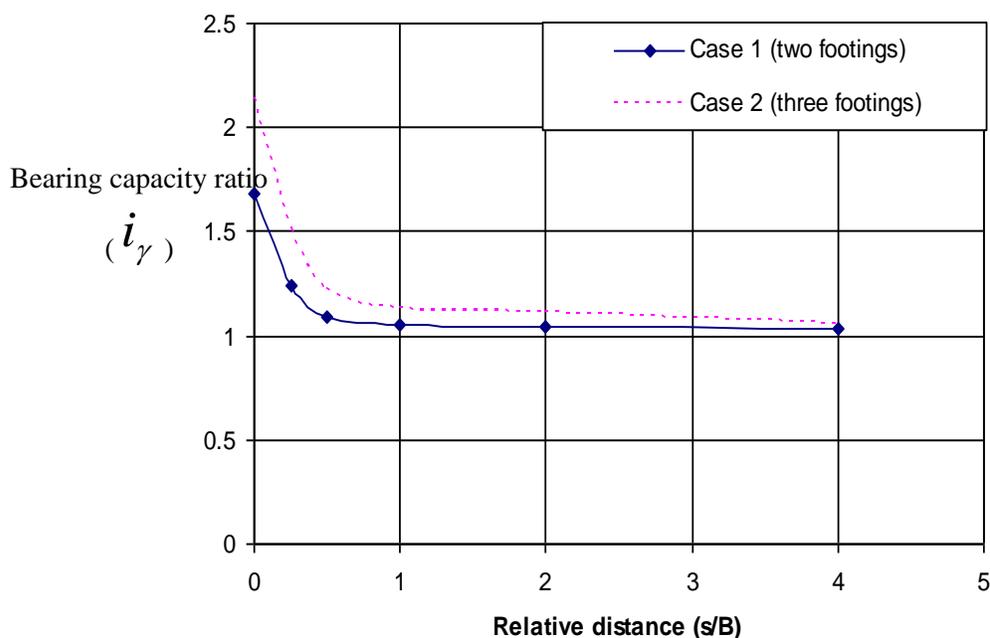
$$i_\gamma = \frac{q_u)_m}{q_u)_s} \tag{2}$$

where i_γ = bearing capacity ratio; $q_u)_m$ = limit bearing capacity for multiple-footing conditions; and $q_u)_s$ = limit bearing capacity for single footing conditions.

Table 2. Values of q_u (kPa) for different footing configurations.

	Single footing	Multiple footings					
		0.0 B	0.25 B	0.5B	1.0 B	2.0 B	4.0 B
Case 1	643	1080	798	698	677	671	665
Case 2	643	1380	971	786	725	712	675

Fig. 12 shows the values of i_γ as a function of the relative footing distance s/B for the two cases 1 and 2. As can be seen, for the two cases, the value of i_γ decreases with increasing s/B . Although, close arrangements of footings would not be common and beneficial in practice, it remains important that the mechanical behavior of multiple footings be fully understood.

**Fig. 12.** Values of i_γ with footing distance for cases 1 and 2.

Under the in-contact condition ($s/B = 0$), the values of i_γ were observed to be smaller than what would be obtained from the conventional bearing capacity equation. For example, the i_γ for case 1 with $s/B = 0$ in Fig.12 is approximately 1.6, which is smaller than 2 from the conventional bearing capacity equation using a footing size of $2B$. This is due to the footing size effect, which arises from the reduction of the friction angle attendant on increasing confining stress. Also, Fig 12 shows that the values of i_γ decreased non-linearly with increasing footing distance approaching 1. This is somewhat to be expected, as individual footings would behave as an isolated footing after a certain footing distance limit, beyond which no interaction between footings exists. This in turn

indicates that for footing distances greater than this limit distance 4, no consideration of the multiple-footing effect would be necessary. If i_γ included in a general bearing capacity equation as an additional influence factor, the modified general bearing capacity equation for shallow multiple footing on sand can be written as:

$$q_u = 0.5B \cdot \gamma \cdot \xi_\gamma \cdot N_\gamma \cdot i_\gamma \quad (3)$$

CONCLUSIONS

In the present study, the effects of multiple strip footing configurations on bearing capacity of sandy soil were investigated. Nonlinear finite element analyses to simulate various multiple footing conditions were performed using elastoplastic model with Mohr-Coulomb failure criterion. Experimental test results from the literature were selected and used in verifying the finite element modeling of the problem. Two cases of strip footings under the 2D plane-strain condition were considered. For each case, different footing distances were applied as the basis of a comparison. It was observed that the load response and the q_u at $s = 4.0B$ were fairly close to those for the single footing case obtained from Terzaghi equation, indicating that no significant multiple strip footings effect had arisen.

In order to quantify the multiple-footing effects on the q_u , the bearing capacity ratio i_γ was evaluated for each case considered in the finite element analyses. Under the in-contact condition, the values of i_γ were observed to be smaller than what would be obtained from the conventional bearing capacity equation due to the size effect.

REFERENCES

Das B. M., Larbi-Cherif S., 1983 "Bearing capacity of two closely-spaced shallow foundations on sand", *Soils and Foundations* ; **23**(1):1–7.

Ghazavi M. and Lavasan A. A. ,2008 "Interference effect of shallow foundations constructed on sand reinforced with geosynthetics", *Geotextiles and Geomembranes Journal* ; **26**: 404-415.

Graham J, Raymond GP and Suppiah A., 1984 "Bearing capacity of three closely-spaced footings on sand", *Geotechnique* ; **34**(2):173–182.

Griffiths D.V., Fenton, G. A. and Manoharan N., 2006 "Undrained Bearing Capacity of Two-Strip Footings on Spatially Random Soil", *International Journal of Geomechanics (ASCE)*; November/December :421-425.

Kumar A. and Saran S., 2003 "Closely spaced footings on geogrid-reinforced sand",

Journal of Geotechnical and Geoenvironmental Engineering (ASCE) ; **129**(7):660–664.

Kumar J. and Bhoi, M. K.,2008 "Interference of Multiple Strip Footings on Sand Using Small Scale Model Tests", *Geotechnical Geological Engineering Journal*;**26** :469-477.

Kumar J. and Ghosh P., 2007 "Ultimate bearing capacity of two interfering rough strip footings.", *International Journal of Geomechanics (ASCE)* ; **7**(1):53–62.

Kumar J. and Kouzer, 2008 "Bearing capacity of two interfering footings", *International Journal for Numerical and Analytical Methods in Geomechanics*;**32**: 251-264.

Lee J. and Eun J., 2009 "Estimation of bearing capacity for multiple footings in sand", *Computer and Geotechnics Journal* ;**36** :1000-1008.

Meyerhof,G., 1965 "Shallow Foundations", *Journal of Soil Mechanics Division, ASCE SM2*, :21-31.

Stuart J.G., 1962 "Interference between foundations, with special reference to surface footings in sand", *Geotechnique* ; **12**(1):15–22.