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The J-integral and KIc as a Measure of Fracture Toughness of Steel Fibre Concrete

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Abstract:

By using a simple maximum load failure criterion, material toughness can be determined from load-deflection curves measured during testing of notched flexural specimens. The J-integral (Jc) and the stress intensity factor (Kc) results as a measure of toughness have been determined using four point loading edge notched beams. Generally, there is a good agreement between these two results which describe the fracture characteristics of steel fibre reinforced concrete (SFRC). The fibre content varies between 0% and 1.5% by volume with perpendicular, parallel and random orientations to load application were adopted in this paper. Increasing the fibre content results in increased kIc results and also results of JIc. The enhancement of cracking strength, ultimate strength and fracture toughness for the previous fibre content and orientation are reported.

الخلاصة:

بأستعمال خاصية التحمل القصوى للفشل فأن قساوة المادة يمكن أن تقاس من منحنيات الحمل- الأنحناء من خلال فحص نماذج الأنحناء التي تحتوي على شق في جانب الشد. عموما هناك علاقة جيدة بين (Jc)(معامل رايس) و كذلك معامل شدة الجهد(kc) والتي توضح خاصية التشقق بالنسبة الى الخرسانة المسلحة بالألياف الحديدية .أن نسب الالياف تتراوح بين (0-1.5%) و بأتجاهات عمودية، افقية و عشوائية على اتجاه الثقل المسلط. ان زيادة نسبة الألياف الحديدية نتج عنة زيادة في KIc وكذالك زيادة في JIc. أن الزيادة في المقاومة للتشقق وكذالك شدة المقاومة للتكسر بالعلاقة لنسب الالياف واتجاهاتها قد درست في البحث الحالي

Introduction

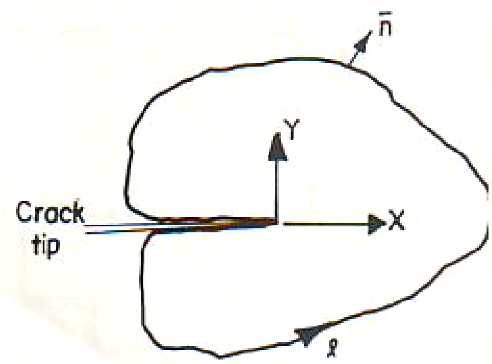
In the last three decades many researchers observed that the increased toughness, or ductility, provided by including fibre reinforcement is probably the most important material property of fibre content, specially when these fibres are deformed or provided with end anchorage[**Halvorsen, T.G.1980**]. Their research programs were devoted on appropriate way to measure this increase of toughness. Different parameters were proposed to describe the fracture behaviour in concrete subjected to mode I deformation such as fracture kIc, critical strain energy release rate GIc, critical value of J-integral (JIc), critical tip-opening displacement CTODc, crack resistance R and toughness index .

Much effort is being devoted to develop fracture mechanics methods for the analysis of cracked concrete structures. The fracture mechanics deals of with the forces associated with the rupture of a solid body and, in particular with the balance energies involved with propagation of cracks[**Tada *et al* 2000**]. It describes the interrelationship between stress energy and crack length, and thus is a useful tool to estimate quantitatively the propagation of a crack in a given system. The stress and displacement fields can be expressed in terms of a stress intensity factor, k , which is a function of load and geometry[**Naus, D.J. and Lott, J.L., 1969**]. At the onset of rapid unstable crack growth, the critical stress intensity factor, kIc , is assumed to be a material property, fracture toughness. This factor was independent of the geometry of the beam specimen [**Valazco, G. *et al* 1980, Jenq, Y.S. and Shah, S.P.,1985**].

The linear-elastic fracture mechanics (LEFM) has been applied to Portland cement pastes, mortars and concretes since the early 1960s[**Shah, S. P. and McGarry, F.J. 1971**, **Kesler, C.E.,*et al* 1972**]. However, these concepts cannot be successfully described the behaviour of mortars and concretes. The interest was renewed in the topic of fibre reinforced concrete after the appearance of that the function of the fibres was to provide a crack arrest mechanism. The early investigators attempted to apply fracture mechanics theory to such materials[**Romualdi, J. P., and Batson, G.B., 1963**, **Romualdi, J. P.**, **Mandel, A. J., 1964**, **Jenq, Y. S. and Shah, S.P., 1986**], they concluded that the function of steel fibres is effectively reduce the kI.

Nishioka,K. et al.1978 showed that the method of test had no influence on kIc, and they argued that the LEFM can be applied to fibre reinforced concrete. **Mai, Y. W., 1979**], test results of a series of asbestos-cement composites suggest that KIc increases with the fibre content which agrees with the results obtained earlier by [**Brown, J. H.**, **1973**] on glass fibre reinforced concrete beams.

Fibre reinforced materials are not linearly elastic and show post-elastic deformation which encouraged the investigators to apply nonlinear fracture mechanics concepts to fibre reinforced concrete instead of of applying linear elastic fracture mechanics. The J-integral was found for nonlinear-elastic materials to be path- independent energy line integral about a closed loop enclosing a crack tip[**Rice, J. R. , 1968**] and defined with contour C as shown in Figure 1 . For linear- elastic case the value of J-integral is equal to G, the strain energy release rate (from LEFM). The first application of the J-integral to concrete and fibre concrete was by [**Mindess, S., *et al*., 1977**]. They found that JIc is much more sensitive indicator of the effectiveness of the fibre addition than GIc. Further studies on the J-integral were carried by [**Halvorsen, T.G.1980**, **Brandt1980**] and **Velazco et al 1980**]. They reported the applicability of J- integral to fibre reinforced material as fracture toughness parameter.



$$J=\oint_{\ell} W\,dy-\bar{T}\cdot\left(\frac{d\bar{u}}{d\bar{x}}\right)ds$$

Where ℓ = Path of integration, taken counterclockwise around the crack tip;
W = Strain energy density;
$\bar{T}$ = Traction vector defined by the outward normal vector $\bar{n}$ on the path ℓ;
$\bar{u}$ = Displacement vector; and
s = Arc length along the path ℓ, $ds^2 = dx^2+dy^2$.

Fig. 1 Crack tip geometry and line integral

Experimental details

Two series of test specimens (high strength concrete and medium strength concrete) were prepared and tested in four point loading, as illustrated in Fig.2. Each series comprised 90 specimens in which half were notched at the centre of the tension side, with a 3 mm diamond saw, to give a notch depth of 30 mm, as presented in Tables 1and

2. The crack mouth opening displacement CMOD and the acoustic emission rate were measured . The point at which the ascending part deviates from linearity (point of first crack) was determined by the computer program which was capable of detecting deviation of 0.02%.

In the first series, the matrix proportions were 1: 1.01: 2.04 representing cement: fine aggregate: coarse aggregate. The cement used is ordinary Portland cement, the fine aggregate was river sand and the coarse aggregate was crashed granite of 10 mm maximum size. The w/c ratio used in the above mix was 0.38.

In the second series, the matrix proportions were 1: 1.75: 3.18 representing cement: fine aggregate: coarse aggregate. The cement used was again ordinary Portland cement (with fly ash replacing 20% of the cement) the fine aggregate was river sand (as used in the first series) and the coarse aggregate was river gravel of 10 mm maximum size which consisted of limestone and siliceous river sand. The w/c ratio used in this mix was 0.50. Superplasticizer was added to fiber concrete at a rate of 2% per weight of cement used in the mix (for both series).

The concrete was designed to give a 28 days cube compressive strength of 55-58 N/mm² for the first series and 30-34 N/mm² for the second series. In all the mixes straight round steel fibres (diameter 0.25 mm and aspect ratio of 100) were used with a volume fraction varying between 0.25 – 1.5%. All specimens (100x100x500 mm) were prepared by means of vibrating table. The prisms were cast horizontally and the fibre orientation technique was similar to that used by [**Hannant and Spring,1974**]. This orientation can be produced by combining table vibration with a simple mechanical device for alignment. All the specimens were placed in a curing room(90-95% RH) and 20±1°C four four months and after that the specimens were kept at 70±5% relative humidity and 21±2ºC until the time of testing.

All the specimens were tested under controlled deflection by means of an Instron machine (Model 1115) at a nominal room temperature. The crosshead of the machine was driven at 0.5 mm/min in all experiments.

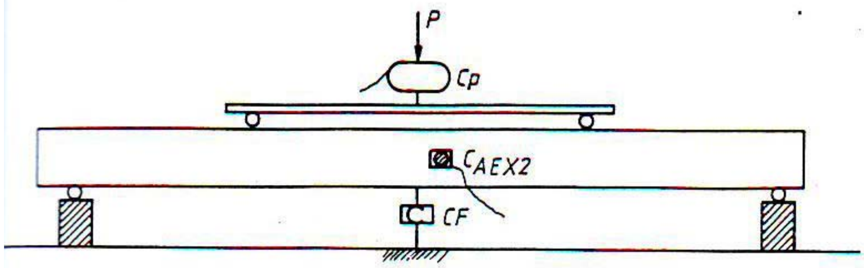


Fig. 2a Location of Strain and AE gauges in flexure test: un-notched beam

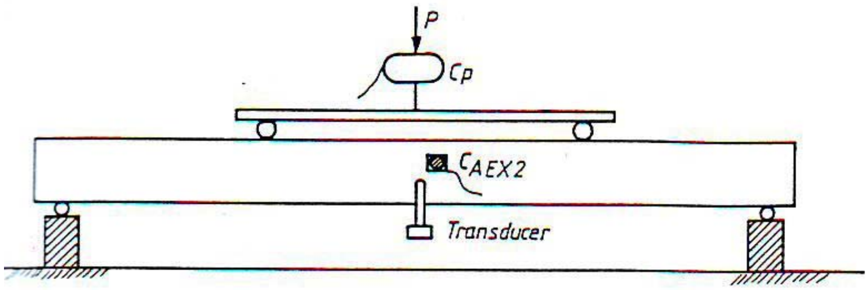


Fig.2b Location of Strain and AE gauges in flexure test· notched beam

The elastic energy absorbed by the beam can be determined from the area under load deflection curve for an identically un-notched beam loaded to the same load level sustained by the notched beam. Therefore the J-integral can be determined in notched beam specimens from the following expression[**Halvorsen, T.G.1980**]:

$$J = \frac{2}{B(D-a)}[AT - AU]$$

Where A = area under load –displacement curve up to a particular load
AT= area under curve up to a particular load for notch beam
AU= area under curve for un-notched beam loaded to the same Load level sustained by notched beam
B = beam width
D = beam depth
a = notch depth

From the notched beam specimens the kIc can be calculated as follows[**Barr, B. I. G., Thomas, W. F. 1986**]:

$$kIc = \frac{3pa^{1/2}(L1-L2)}{2BD} F(a/D)$$

Where p = maximum applied load
a = notched depth
L1= distance between supports
L2= distance between load points

$$F(a/D) = 1.99 - 2.47(a/D) + 12.97(a/D)^2 + 23.17(a/D)^3 + 24.80(a/D)^4$$

Test results

The J-integral values were evaluated at the initiation of slow cracking (JI) and at the termination of slow crack growth (JII), where the curvilinear portion of the deflection – CMOD plot is used . The initiation point of the curvilinear portion where JI is evaluated and the termination point of it where JII is evaluated with an accuracy of 0.02%. These results are shown in Tables 3 and 4. It can be observed from these results that JI is not sensitive to fibre inclusion as is JII which shows that the role of the fibres starts after the formation of the cracks. However, this role is dependent upon fibre orientation and volume. The termination of slow cracking is the stage at which there is a sharp increase in the acoustic emission rate which was noticed during the experimental work. With the help of this technique the point of main crack initiation can be determined. Fig.3 illustrates the relationship between JIc and fibre content for both high and medium strength concrete with randomly and perpendicularly distributed fibres. Fig.3a illustrates the relation between JIc and fibre content in random distribution of high and medium strength concrete. The JIc appears to be more sensitive for high fibre contents. For concrete with perpendicularly aligned fibres the JIc results are observed to reach their maximum value with 1% fibre content for both high and medium strength concretes (see Fig.3b).

The fracture toughness results kIc are summarized in table 5 and 6 and illustrated in Fig.4. The kIc results for high and medium strength fibre concrete did not vary very much for all mixes. The highest values appeared, in general, to be with 1.5% fibre content.

The relationship of the kIc to the fibre content and compressive strength are plotted in Fig. 4, and illustrated in Tables 7 and 8. An increasing value of the fracture toughness can be observed with increasing compressive strength, or fibre content. This shows that kIc values increase with the increase of the volume fraction of the steel fibres, which implies that high strength concrete is more tough. However, this may not be the case if one considers the results obtained from Tables 7 and 8,where the compressive strength of high strength concrete is relatively higher than that of medium strength concrete. Thus the kIc could depend on fibre content and fibre distribution in the concrete matrix, more than on the compressive strength. This may lead to the conclusion that high strength concretes are more brittle.

The J-integral results are shown in Fig.5. In general a large increase in JIc values is observed with an increase in fibre volume fraction. In both high and medium strength concrete the J- integral was much more sensitive to the fibre addition than the corresponding conventional critical stress intensity factor kIc (see Figs 4. and 5).

The relationship between kIc and JIc and fibre content for perpendicularly aligned fibres in high and medium strength concrete are shown for clarity in Tables 5 and 6, in which it is observed that the JIc value is a much more sensitive indicator of effectivness of fibre content than kIc.

The coefficient of variation for kIc were relatively small for high and medium strength concrete because they depend only on the maximum load. There was a much larger scatter in the JIc values (which depend on the shape of the load-deflection curve as well as on the maximum load). In general, the coefficient of variation for perpendicularly aligned fibres (in both high and medium strength concretes) shows a smaller variation than in the case of parallel and random fibre orientation. This is due to the fact that parallel aligned fibres have little influence on the behaviour of the load-deflection curve for concrete tested in flexure.

Conclusions:

1. The results presented in this study on the fracture resistance of fibre reinforced concrete show that the fracture characteristics can be represented the two parameters of fracture toughness, the stress intensity facture and the J-integral. Fracture toughness gives the resistance of the concrete matrix to cracking, while the J-integral gives an indication of post cracking performance of the material.
2. At low fibre content, the JIc values are nearly constant for both concretes used in this investigation, but rapidly increase at fibre contents of more than 0.5% by volume.
3. The J-integral was much more sensitive to the fibre addition than the corresponding conventional critical stress intensity factor kIc.
4. The results of these tests show that for both high and medium strength FRC the perpendicularly aligned fibres show better performance than parallel and random distribution of fibres, while the parallel alignment gives the lowest set of results.

Notations:

Kc stress intensity factor

KIc stress intensity factor in mode I
JIc integral of Rice in mode I
CMOD crak mouth opening displacement

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Table 1 Details of high strength concrete beam specimens tested

Vf%	Notched beams tested in flexure		Un-notched beams tested in MOR		Orientation
	Specimen designation	No. of Specimens	Specimen designation	No. of Specimens	
0.0	HN	9	HW	9	
0.25	AN	3	AW	3	Perpendicular
0.50	AN1	3	Aw1	3	
1.00	AN2	3	AW2	3	
1.50	AN3	3	AW3	3	
0.25	BN	3	BW	3	Parallel
0.50	BN1	3	BW1	3	
1.00	BN2	3	BW2	3	
1.50	BN3	3	BW3	3	
0.25	CN	3	CW	3	Random
0.50	CN1	3	CW1	3	
1.00	CN2	3	CW2	3	
1.50	CN3	3	CW3	3	

Table 2 Details of medium strength concrete beam specimens tested

Vf%	Notched beams tested in flexure		Un-notched beams tested in MOR		Orientation
	Specimen designation	No. of Specimens	Specimen designation	No. of Specimens	
0.0	LN	9	LW	9	____________
0.25	DN	3	DW	3	Perpendicular
0.50	DN1	3	DW1	3	
1.00	DN2	3	DW2	3	
1.50	DN3	3	DW3	3	
0.25	EN	3	EW	3	parallel
0.50	EN1	3	EW1	3	

1.00	EN2	3	EW2	3	
1.50	EN3	3	EW3	3	
0.25	FN	3	FW	3	Random
0.50	FN1	3	FW1	3	
1.00	FN2	3	FW2	3	
1.50	FN3	3	FW3	3	

Table 3 J-integral for different fibre contents and orientations (high strength concrete)

Vf%	JI KN/m	JII KN/m	Orientation
O	44.24	134.05	________
0.25	35.05	144.24	perpendicular
0.50	37.10	207.14	
1.00	47.48	235.71	
1.50	40.10	281.43	
0.25	36.57	144.28	Parallel
0.50	42.62	156.86	
1.00	25.71	283.33	
1.50	44.29	180.33	
0.25	32.90	111.05	Random
0.50	57.95	173.71	
1.00	43.67	331.90	
1.50	56.95	258.10	

Table 4 J-integral for different fibre contents and orientations (medium Strength concrete)

Vf%	JI KN/m	JII KN/m	Orientation
O	41.27	118.04	________
0.25	50.81	205.70	perpendicular
0.50	48.07	322.14	
1.00	21.77	667.86	
1.50	56.76	561.43	
0.25	28.10	103.60	parallel
0.50	35.20	172.00	
1.00	33.60	191.80	
1.50	31.90	156.60	
0.25	14.00	144.30	random
0.50	15.90	132.60	
1.00	95.90	486.00	
1.50	22.90	526.20	

Table 5 Fracture toughness results for high strength FRC beams with Different fibre contents and orientations

Vf%	Linear Fracture toughness kIc $MN/m^{3/2}$	V%	Non-linear fracture toughness JIc kN/m	V%	Orientation
0	0.84	5.7	0.0328	10	
0.25	0.93	9.5	0.158	29	Perpendicular
0.50	1.05	5.2	0.219	8.1	
1.00	1.08	9.9	0.360	3.0	
1.50	1.47	4.6	0.344	23	
0.25	0.88	18.9	0.231	49	Parallel
0.50	0.97	7.7	0.170	12.4	
1.00	1.19	10.3	0.390	52	
1.50	0.99	9.0	0.179	20	
0.25	0.86	12	0.123	24	Random
0.50	1.11	5.11	0.179	31	
1.00	1.20	1.0	0.319	8.2	
1.50	1.35	9.7	0.393	0.393	

Table 6 Fracture toughness results for medium strength FRC beams with Different fibre contents and orientations

Vf%	Linear Fracture toughness kIc $MN/m^{3/2}$	V%	Non-linear fracture toughness JIc kN/m	V%	Orientation
0	0.85	10.8	0.0324	19.9	
0.25	0.79	2.4	0.107	22	Perpendicular
0.50	0.96	9.9	0.204	13.0	
1.00	1.37	10.0	0.879	11	
1.50	1.45	9.3	0.668	24.6	
0.25	0.78	18.1	0.131	19	Parallel
0.50	0.93	12.6	0.171	61	
1.00	0.96	9.7	0.267	21	
1.50	0.96	16.6	0.228	1.8	
0.25	0.81	18.8	0.179	31	Random
0.50	0.91	10.6	0.189	21	
1.00	1.56	9.8	0.598	18.5	
1.50	1.48	8.8	0.691	58	

Table 7 Fracture toughness and compressive strength results for FRC high strength concrete

Vf%	Fracture toughness kIc(KN/m$^{3/2}$	Compressive strength fcu(N/mm^2)	V%	Orientation
0	0.84	66.33	3.9	__________
0.25	0.93	77.29	3.4	perpendicular
0.50	1.05	76.88	3.4	
1.00	1.08	88.22	3.7	
1.50	1.47	92.41	2.2	
0.25	0.88	85.67	4.2	parallel
0.50	0.97	80.97	1.7	
1.00	1.19	91.32	2.5	
1.50	0.99	83.50	3.5	
0.25	0.86	74.67	5.1	Random
0.50	1.11	77.28	3.0	
1.00	1.20	89.67	5.0	
1.50	1.26	88.26	6.4	

Table 8 facture toughness and compressive strength results for FRC medium strength concrete

Vf%	Fracture toughness kIc(KN/m$^{3/2}$	Compressive strength fcu(N/mm^2)	V%	Orientation
0	0.85	49.17	3.1	__________
0.25	0.79	56.74	3.5	perpendicular
0.50	0.96	74.88	4.1	
1.00	1.37	64.40	2.6	
1.50	1.45	73.10	3.2	
0.25	0.78	60.83	6.1	parallel
0.50	0.93	61.66	6.5	
1.00	0.96	63.80	3.0	
1.50	0.96	64.31	4.4	
0.25	0.81	52.77	1.6	Random
0.50	0.91	68.29	2.1	
1.00	1.56	68.56	5.1	
1.50	1.48	65.28	3.1	

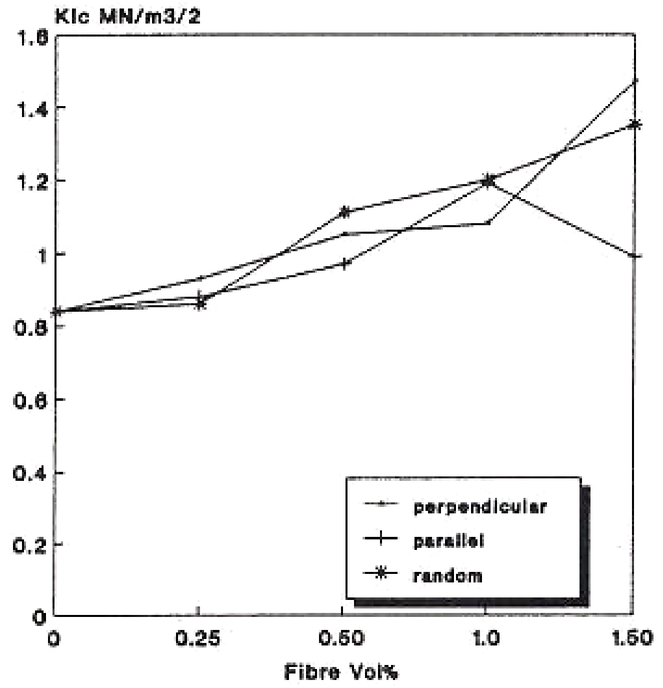

Fig.4. a Relation between Klc and fibre content and their orientation(high strength concrete)

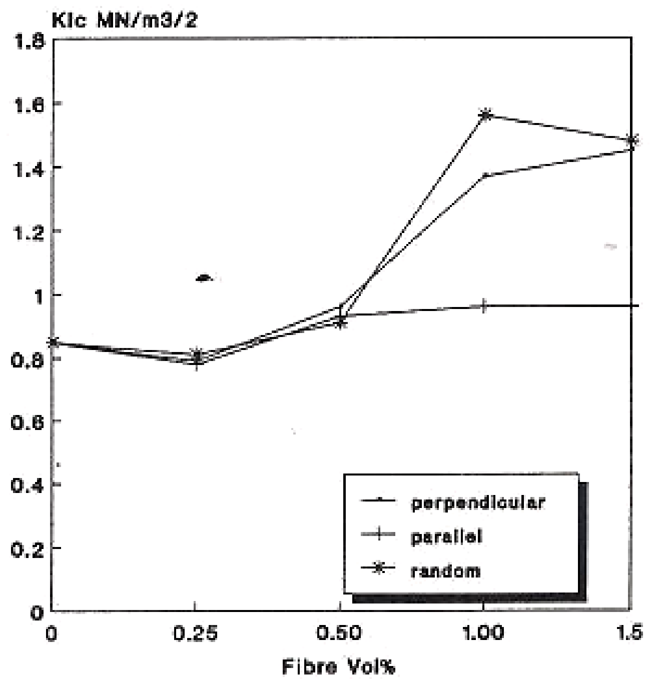

Fig.4. b Relation between Klc and fibre content and their orientation(medium strength concrete)

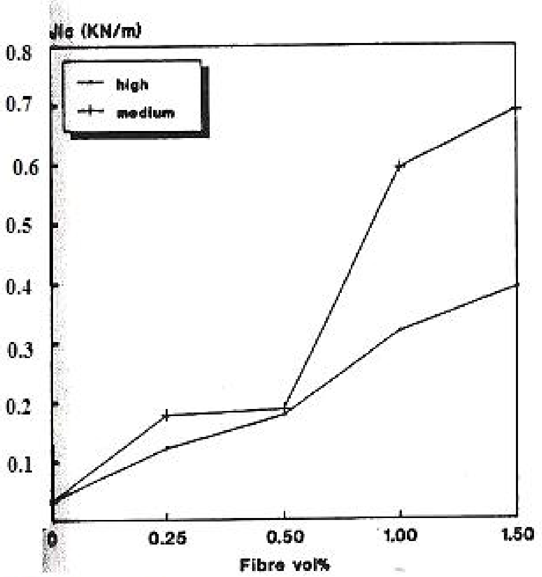

Fig.3a . Relation between JIC and fiber content in random distribution(high and medium strength concrete)

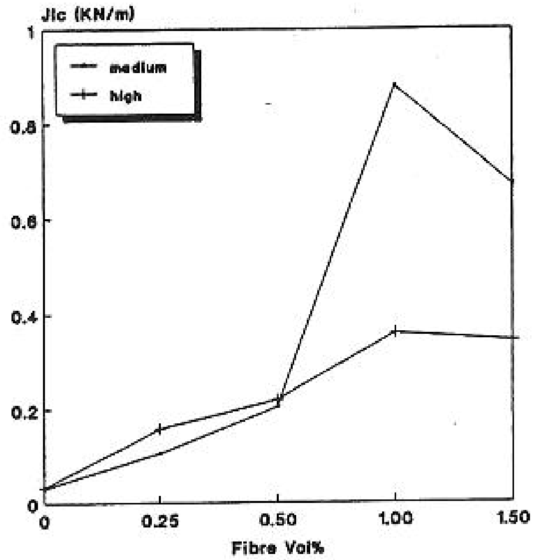

Fig. 3 b Relation between Jic and fibre contents for perpendicular alinged fibres(medim and high strength concrete)

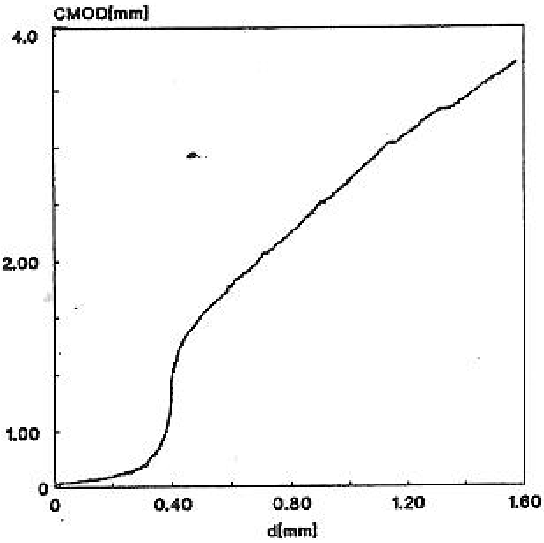

Fig. 3 C Typical CMOD-deflection graph

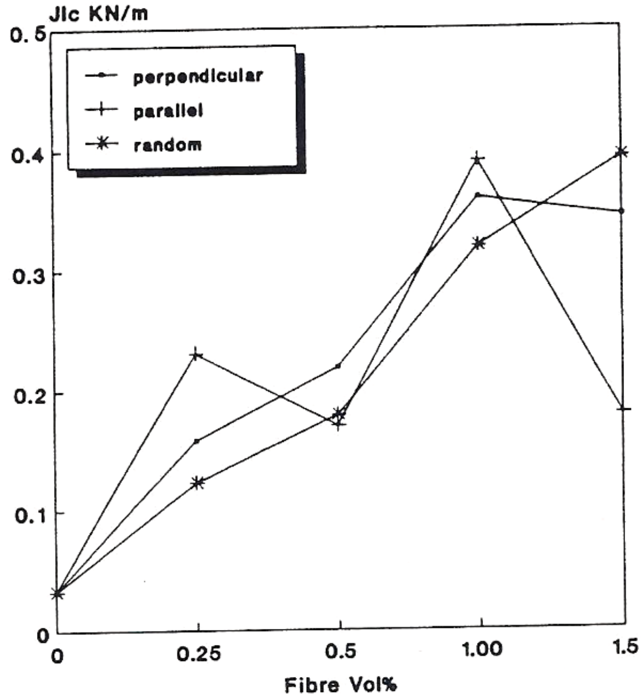

Fig. 5 a The J-integral values versus fibre content(high strength concrete)

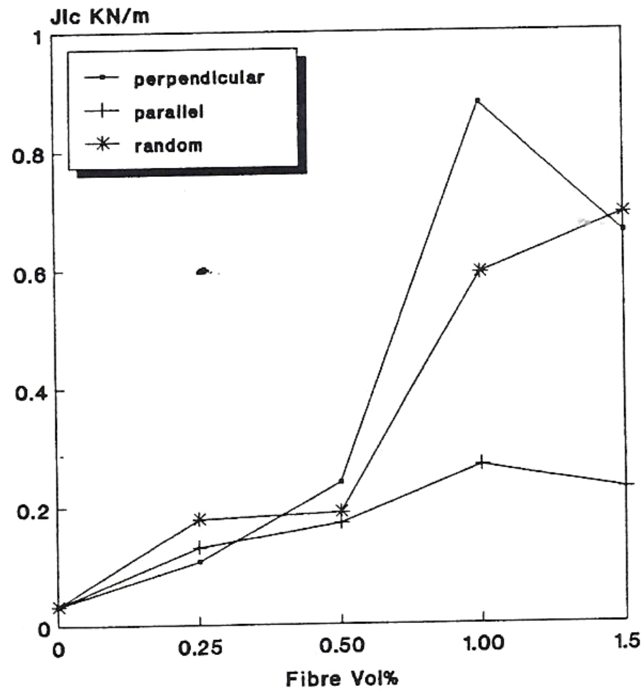

Fig. 5b The J-integral values versus fibre content(medium strength concrete)