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Dispersive Optical Potential Study for Neutron Stripping from Nucleus Using Dispersion Relations.

Marwah Abdulkareem Saud^{*}, Akram Mohammed Ali

ABSTRACT



Department of Physics, College of Science, University of Anbar, Ramadi, Iraq;

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Introduction

structures resulting from these reactions. As this potential during the reaction is subject to dispersion by the nuclei surface as well as the total size of the nucleus, it is very important to calculate the dispersive behaviour. We present an analytical and numerical expressions used to evaluate the dispersive contribution to the real potential of the imaginary part and surface contribution and applied to ${}^{15}C + {}^{9}Be$ at energy 60 MeV. For both imaginary volume and surface potentials, solutions to the nuclear optical model dispersion relation have been obtained. For the volume imaginary term, a typical Brown-Rho shape has been considered with the parameters (n=2,m=2,4), and for the surface contribution, a Brown-Rho shape multiplied by the decreasing exponential. For exponent with any even value appearing in these forms, the analytical solutions are valid. The energy representation of the real and imaginary parts of the OMP is carried out by these approaches.

Usually in nucleon-nucleon reactions, optical potential is used to study the nuclear

A lot of theoretical focus has recently been placed on creating an optical model potential (OMP) that acts as a consistent formulation of the mean-field of nucleons and nuclei in both positive and negative energies [1,6].

This type of potential is defined by two terms, first one is Hartree-Fock which represents the component of the mean-field that varies with energy slowly, i.e. independent of energy. The second one called the correction term which is more complicated and dependent on energy derived from a dispersion relation. Two terms are called dispersive optical potential. It can be describing the bound state properties as probabilities of occupation etc and nucleon-nucleus scattering system. The two parts of optical potential real and imaginary are satisfying the dispersion relation [7]. which is an attempt at this idea in our paper. The analysis of OMP was taken care of in the work of Mahaux and Sartor [8] where their dispersive analysis of potential achieved by an integral relation links real and imaginary parts.

At the Fermi surface region, the depth of real potential differs from the regular behavior because of the dispersion phenomena. So, the main two parts the potential, real and imaginary, will related in dispersion relations. This elation and according to Cauch's theorem have two poles that can be neglected. The potential depth does not depend on particular nuclei and can be considered as an interaction property, and this property be the due heavy nucleus and high incident energy of particle [9]

The analyzes conducted on the optical potentials used in nuclear reactions using dispersion relations proved insufficient in giving us additional information about the high-energy region, as the data were not accurate or comprehensive enough. Therefore, it was necessary to distinguish between real dependence on energy and false dependence in

^{*}Corresponding author at: Department of physics, College of Science , University of Anbar, Ramadi, Iraq; ORCID:https://orcid.org/0000-0001-5754-5800; Tel:+9647505819140

E-mail address: marow.soad@uoanbar.edu.iq

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order to obtain accurate data and thus improve this data. In the present paper, the dispersive optical model analysis of the neutron removed from ¹⁵C nucleus will be done and the results comparing with global parametrization of the optical model potential results at energy range up to 103 MeV by taking the calculation step for energy range whose value (1)MeV and using Mahaux and Sartor and Brown and Rho functions values.

2. Theory:

When the nucleon-nucleus interact and scattering happen, the optical potential U is given as [10-14]

$$U(r,E) = -V_{V}(r,E) - V_{SO}(r,E).\vec{\sigma}.\vec{\ell} + V_{S}(r,E) + V_{C}(r) + i(W_{V}(r,E) - W_{S}(r,E)) + W_{SO}(r,E).\vec{\sigma}.\vec{\ell}) \qquad \dots (1)$$

Where V_C is the Coulomb potential, $V_{V,S}$ are real parts of volume (*V*) and surface (*S*) generated by the Wood-Saxon well, and $W_{V,S,SO}$ are imaginary parts has a combination of volume and surface where the volume term can be considered to have a Wood-Saxon shape. The spin-orbit potential V_{SO} has Thomas form and has an effect on the polarization of the scattering of particles. LAB energy is represented by E (*MeV*) for incident nucleon. The scalar product of operators $\vec{\sigma}.\vec{\ell}$ given as [15]:

$$\vec{\sigma}.\vec{\ell} = \begin{cases} 1 & \text{for } j = 1 + 1/2 \text{ parallel} \\ -(\ell+1) & \text{for } j = 1 - 1/2 \text{ anti-parallel} \end{cases}$$

All these parts in eq.(1) are in well-depths $(V_v, V_s, W_v, W_s, W_{SO})$ with energy dependent and radial parts $f(r, R_i, a_I)$ with energy independent define as the radial geometrical form factor of V_{real} , V_{SO} , V_v , and V_s potentials and take a standard formula of Wood-Saxon shape that given as [16]:

Where the nuclear radius $R_i = r_i A^{\frac{1}{3}}$, with atomic mass number A, the diffuseness parameters a_i , and the functional i = V, SO, S geometry parameters represent the volume, spin-orbit, and surface potentials. By solving the Schrodinger equation using the complex potential we shall get a prediction about some basic observables, such as total cross-section, and elastic angular momentum distribution [17].

The causality principle, which states that a scattered wave cannot be emitted before the incident wave arrives [1], naturally leads to certain dispersion relation. The depth of absorptive part in the dispersive of the potential is determined by the imaginary part can be expressed by dispersion relation as [18]:

Where *E* is projectile incident energy, *E'* is the scattering energy, and the symbol *P* represented the principal value of the integral. Now, by assuming that W(r, E') is symmetric according to Fermi energy (the amount of energy holds between the last occupied shell and final empty shell), i.e $V(r, E_F + E) = -V_{HF}(r, E - E_F)$, the total real-central potential can be written as a sum of Hartree-Fock component V_{HF} (r,E) and total dispersion potential and given as:

$$V(r,E) = V_{HF}(r,E) + \Delta V(r,E) \quad \dots \quad (4)$$

and as the imaginary component has two parts, volume, and surface, the last term (dispersion term) in eq. 4 is determined as:

$$\Delta V(r, E) = \Delta V_V(r, E) + \Delta V_D(r, E) \quad \text{with}$$

$$\Delta V(r, E) = \Delta V_V(E) f(r, R_V, a_V) - 4a_D \Delta V_D(E) \frac{d}{dr} f(r, R_D, a_D) \quad \dots \dots \quad (5)$$

The values of volume and surface correction terms of dispersion are given by:

$$\Delta V_V(E) = \frac{P}{\pi} \int_{-\infty}^{\infty} \frac{W_V(E')}{E' - E} dE' \qquad \text{and}$$

The HF energy dependence potential is given as $V_{HF}(r, E) = A_{HF} \exp[-\lambda_{HF}(E - E_F)]$ being:

$$\mathbf{A}_{HF} = \mathbf{V}_o \left[1 + (-1)^{\varepsilon' + 1} \frac{C_{viso}}{V_o} \frac{N - Z}{A} \right]$$

while V_{o} , λ_{HF} , and C_{viso} are undetermined constants.

2.1 Numerical solution:

The numerical solution is the simplest integral method used to solve the dispersion relation. Since there

is a pole at $E'=E_F$ in eq.3, this equation cannot be used to have a numerical solution.

There is a difficulty represented as the potentials required to cover the whole energy range $(-\infty \rightarrow +\infty)$. i.e. this integral must be analytic for absorption potential energy dependence. To make the analytical solution available for dispersive correction calculation, some conditions must be taken as the symmetric of imaginary potential (either volume or surface) with respect to Fermi surface energy E_F for the target, i.e: $W(E_F - E) = W(E + E_F)$, is used for energies below Fermi, so the eq. 3 be [19]:

$$\Delta V(E) = \frac{2}{\pi} (E - E_F) \int_{E_F}^{\infty} \frac{W(r, E')}{(E' - E_F)^2 (E - E_F)^2} dE' \qquad \dots \dots \dots (7)$$

Note that the term P is dropped because the integral does not have a singular point at E' = E.

The sum of volume absorption potential, W_V (*E*), and surface energy, $W_S(E)$ give imaginary optical potentials, W(E), that give birth to the terms $\Delta V_V(E)$ and $\Delta V_S(E)$, which are subdivided into the dispersive term $\Delta V(E)$. The core components of the dispersive OMP's actual volume $V_V(E)$ and surface $V_S(E)$, while the $V_S(E)$ is taken as a correction term:

$$\begin{cases} V_V(E) = V_{HF}(E) + \Delta V_V(E) \\ V_S(E) = \Delta V_S(E) \end{cases}$$
(8)

It is beneficial express the fluctuation of the surface $W_s(E)$ and volume $W_v(E)$ with energy in functional forms appropriate for the dispersive optical model analysis, as was mentioned in Refs. [9,13]. The energy dependence of the volume imaginary portion is defined in this study as:

$$W_{V}(E) = \begin{cases} 0 & E_{F} \prec E \prec E_{P} \\ A_{V} & \frac{(E - E_{P})^{n}}{(E' - E_{P})^{n} - (B_{V})^{n}} & E \ge E_{P} \end{cases}$$
(9)

where to define such region there is an average energy called single-particle state energy E_p [20]. Likewise, the energy dependence of imaginary-surface potential was given by [21]:

$$W_{S}(E) = \begin{cases} 0 & E_{F} \prec E \prec E_{P} \\ A_{S}e^{-C_{S}|E-E_{P}|} \frac{(E-E_{P})^{m}}{(E'-E_{P})^{m} - (B_{S})^{m}} & E \ge E_{P} \end{cases} \dots \dots (10)$$

Where A_V , B_V , A_S , B_S , and C_S are undetermined constants [8]. The constants (*n*) and (*m*) must have values with even numbers for integral (1) to exist, where Mahaux and Sartor [8] was suggested n=4, while Brown and Rho [22] suggested n=2, while m = 2,4. Such a region needs to define the is the average energy of the single-particle states E_P , so the energy shift (offset) for the particle-hole region is $E_{of}=E_p - E_F$. According to eq's 5 and 6, W(E) will be zero at $E=E_F$ and with any value everywhere else,

2.2 Analytical solution:

Recent research [23,24] has derived the analytical solutions given by Eq. (7) for the frequently utilized forms of W(E), used in DOM investigations (Eq's. (5)–(7)):

$$\Delta V_{V}(E) = -\frac{A}{\pi} \left\{ \sum_{i=1}^{n} Z_{j}^{n} \ln(-p_{j}^{n}) + R_{+}^{n} \ln E_{+} + R_{-}^{n} \ln |E_{-}| \right\}$$

.....(11)
$$\Delta V_{S}(E) = \frac{A}{\pi} \left\{ \sum_{i=1}^{m} Z_{j}^{m} e^{-p_{j}^{n}C} E_{1}(-p_{j}^{m}C) - R_{+}^{m} e^{CE_{+}} E_{i}(-CE_{+}) - R_{-}^{m} e^{-CE_{-}} E_{i}(CE_{-}) \right\} \dots \dots (12)$$

the p_{j}^{n} are the *n*-zeroes of $(U^{n} + B^{n})$, that is:

The dispersion relation that emerges from the asymmetric form of W_V (E) (Eq's. (8) and (9)) have to derived directly from Eq. (2) then separated into extra three parts [8]. As a result, the dispersive correction for the imaginary part in the form can be written as:

Where $\Delta V_{<}(E)$ and $\Delta V_{>}(E)$ are the dispersive corrections produced from the asymmetric terms of Eq's. (9-10), and ΔV_{V} (E) is the dispersive correction as a result of the symmetric imaginary part of Eq(5). The dispersive correction for the surface imaginary potential can be expressed as follows:

$$\Delta V_{s}(E) = \Delta V_{s}(E) + \alpha_{s} \Delta V_{\succ}(E) \dots \dots \dots (15)$$

$$\Delta V_{\prec}(E) = \frac{AE_{x}}{\pi} \left\{ \sum_{i} \operatorname{Res} \left[f(z), z_{j} \right] \ln(E_{a} - E_{F} - z_{j}) \right\} \dots \dots (16)$$

where the function f(z) has a residue given as Res[f(z), zj] at the pole $z = z_j$. The total in Eq. (16) is over (n+ 4) terms because two real poles (z=-E, z=-E_F).

Given that the W_V (E) for E (E_F, Ea) grows in magnitude by less than 10% of its max. magnitude at E = E_F E_a, a highly accurate estimation of the exact contribution of $\Delta V_{\prec}(E)$ may calculated. Eq. (8) therefore can be approximately be:

$$\tilde{W}_{V}(E) = W_{V}(E) - W_{V}(E_{F} - E_{a}) \frac{(E_{F} - E - E_{a})^{2}}{(E_{F} - E - E_{a})^{2} + E_{a}^{2}} \quad \text{for } E \prec E_{F} - E_{a} \quad ..(17)$$

3. Results and discussion:

The volume and surface terms are major terms for imaginary potential. Analytical and numerical solutions of the dispersion relation of these terms of optical potential will be considered. Different proton energy used to investigate the reaction ${}^{15}C+{}^{9}Be$ at different well depths, as shown in figure (1), with n=2 and m=2,4 that using in eq.'s (9,10). A Fermi-energy describes the depth energy dependence of the volume and surface components of the imaginary part.

To solve the eq.9 for the numerical solution by slow Simpson integration we shall need to reduce the iterations number by making 1000 steps with (50 keV) for each step up to 100 MeV. Based on the Gauss-Legendre integration method (GLIM) [25], the fast convergence was achieved of the integral of dispersion relation and got an accurate numerical solution with finite interval (-1,1) and can be approximated by the formula:

Where W_i^n and X_i^n are the wights suitable for npoint Gauss-Legendre integration. The value of n was taken as n=10 for each interval used in this work. The main parameters used in this work, to describe the proton reaction with incident energy 60 MeV, were taken from the RIPL-3 library [26], and Fermi energy is E_F = -10.1136 MeV. Figure (1) describe the energy dependence that was made from zero to 100 MeV of dispersion correction when get the numerical results with n=2 and m=2.



Figure 1: Energy dependence of volume and surface components with n=2 and m=2,4. Calculations are made

by analytical and numerical.

The event of the surface imaginary part, it is anticipated that the gradual decrease in surface absorption with increasing energy will be indicated by the slow decline of the volume potential at energies above the Fermi energy. The radial form and depth of the real potential are influenced and contributed to by the dispersive of the surface terms and imaginary volume. Although the actual potential fills up as neutron energy rises, the imaginary potential fluctuates parabolical with respect to Fermi surface energy.

From this figure, due to the non-local optical potential, the true volume central potential depth W(E) decreases with the increase in energy. This contrasts with the linear energy dependence of this part of the potential, which is assumed to occur mostly in OMP analysis. The lack of dependence here is due to the fact that the linear dependence is reasonable only in a narrow energy period limited to 10-40 MeV, while depending on the OMP dispersion analysis within the range of high energies, the volume real central part behaves as an exponential function, $\exp(-E)$, to an energy limit of 140 MeV, which is noticeable in the figure above when we

adopt the representation of the exponential function for this real potential of less than 100 MeV.



Figure 2: Components of OPM dependence ON Energy with n=2 and m=4 get by analytical and numerical ways.

Similarly, absorption imaginary potential compounds, surface W_S and volume W_V , depend mainly on E- E_F . At low energies, the absorption obtained from the surface compound Ws dominates, and after about 10 million electron volts, there will be an appearance of the volumetric compound W_v, and it cannot be ignored, but at high energies, the volumetric compound will completely dominate the absorption. In general, at low energies the Brown-Rho function is modest, then it becomes more noticeable till it eventually reaches a fixed amount.. In dispersion analyses, Brown-Rho function may in general be equal to any low integer (n=2,4,6), which is what was found in our work when using the exponent of 2 and 4 in the two relations of the imaginary surface and volume, and it gave a good description of our result as shown in the figure above

Figure (1) illustrates the typical energy dependency of the $W_{\nu}(E)$ and the W_s (E). The surface term, which indicates coupling to long-range correlations-LRC, predominates at energies close to the

Fermi energy. The number of ways a particle-hole can couple to 2h-1p (2p-1h) states rises as energies become more negative (positive), which is why the absorption around the Fermi energy increases. The $\Delta V_{<}(E)$ and $\Delta V_{>}(E)$ were shown in figures (1 and 2) as they the dispersive corrections results of the asymmetric terms.

The volume term, which stands for coupling to short-range correlations SRC, dominates at larger and more minuscule energies. The volume term keeps rising as the positive energies increase. This result is caused by the fact that when energy rises, the phase space for 2p1h states rises. The nucleon-nucleon interaction's repulsive core strength determines when the 2p1h states ultimately approach zero and at what energy. Experimentally, the coupling does not reach zero, but theoretical calculations indicate that it does at very high energies [27]. Large negative energies have a quicker volume term that becomes zero. This behavior results from the dominance of high-momentum components in the area of large negative energies and the difficulty of coupling a hole to a high-momentum state as momentum increases. To compare the numerical calculation for ΔV_{v} and ΔV_{s} with method of Gauss-Legendre, table (3) show the results when integration have been don using different values of n and m.

Table 1: The numerical integration calculations of volume and surface correction terms of dispersion with different values of n and m. The error represent the values calculation by Gauss-Legendre integration.

	n,m=2,2		n,m=2,4		n,m=2,2		n,m=2,4	
E	$\Lambda\Lambda abla$	error	$\Lambda \Lambda V$	error	∇VS	error	$\Delta V V$	erro
2	0.91848	0.005	0.70308	0.0051	3.3003	0.003	4.736I I	3x10-5
4	1.08662	0.0005	0.83194	0.006	1.96099	0.0025	2.42243	2.5x10-5
6	1.25288	0.0067	0.96357	0.0068	0.81734	0.0021	0.75125	2x10-5

input values, by analytic and numerical methods. A description of angular momentum distribution for a nucleon-target reaction can be done by analyzing the DOM potential. Also, overlap functions of the reaction can produce an understanding of the nuclear structure. So, DOM analysis introduces an extrapolating for potentials of isotopes.

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4. Conclusions:

Using dispersion relation helped for drawn the dispersive optical potential for ${}^{15}C + {}^{9}Be$ for constant

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دراسة تشتت الجهد البصري في تفاعل تجريد نيوترون من النواة باستخدام علاقات التشتت. مروة عبد الكريم سعود* ، اكرم محمد علي

> قسم الفيزياء، كلية العلوم، جامعة الانبار، الرمادي ، العراق. Email: <u>dr.akram@uoanbar.edu.iq</u>

الخلاصة

عادة في تفاعلات نوكليون-نوكليون، يتم استخدام الجهود الضوئية لدراسة التراكيب النووية الناتجة عن هذه التفاعلات. وبما أن هذه الجهود البصرية الناتجة أثناء التفاعل تخضع للتشتت بواسطة سطح النواة بالإضافة إلى الحجم الكلي للنواة، لذلك من المهم جدًا حساب سلوك التشتت لهذا التفاعل. ثم نبدأ بتقديم تعبيرات تحليلية وعددية تستخدم لتقييم المساهمة المشتتة في الجهود الحقيقية للجزء التخيلي حيث ان المساهمة السطحية ويتم تطبيقها على B+¹⁴C9 عند طاقة 60 ميجا فولت.

إن استخدام علاقة التشتت ساعد في رسم جهود التشتت الضوئية لـ ⁹B + ¹⁴C لقيم المدخلات الثابتة، وذلك بالطرق التحليلية والعددية. بالنسبة لكل من الحجم التخيلي والجهود السطحية، تم الحصول على حلول لعلاقة تشتت النموذج البصري النووي. بالنسبة للمصطلح التخيلي الحجمي، تم اعتبار شكل brown-Rho النموذجي مع المعلمات (n=2,m=2,4)، وبالنسبة للجهود السطحية، تم ضرب شكل Brown-Rho في الأسي المتناقص. بالنسبة للأس مع أي قيمة زوجية تظهر في هذه النماذج، فإن الحلول التحليلية صالحة. يتم تمثيل الطاقة للأجزاء الحقيقية والخيالية لنموذج الجهد البصري من خلال هذه الأساليب.

الكلمات المفتاحية: علاقة التشتت، الإمكانات التخيلية، الحجم والسطح، دالة بر اون-رو.