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VIBRATION AND STABILITY OF CURVED PIPE STIFFENED BY LINEAR SPRING CONVEYING FLUID

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Abstract

The vibration analysis and stability of curved pipe with linear spring stiffener for fixed-fixed ends conveying fluid consisting from several of straight pipe elements is investigated by using three dimensional finite element model. The characteristic matrices consisting of stiffness, inertia, damping and contradictory terms which derived by finite element method and the effect of the internal flow velocity, axial force, Coriolis force and force of spring stiffener in connected elbow are considered. Some parameters that affect the dynamic characteristic have been studied such as curved pipe angle, location of spring stiffener, diameter ratio and wall pipe thickness. It is found that the location of spring stiffener would influence the critical flow velocity and the natural frequency of the system. Where the natural frequency of curved pipe increased with closed the spring stiffener to center of curved pipe as the flow velocity increase. The results are compared with the numerical approach and gives good agreement of used technique.

Keys: curved pipe, conveying fluid, angle of spring location, straight pipe, critical velocity, coriolis force.

دراسة الاهتزاز والاستقرار لأنبوب مقوس مصلب بنابض خطي ناقل للمائع

الخلاصة

تمت دراسة تحليل الاهتزاز والاستقرارية لانبوب مقوس مصلب بنابض خطي مثبت النهايتين ناقل للمائع المركب من عدد من عناصر الأنابيب الطولية باستخدام نموذج العنصر المحدد ثلاثي الأبعاد.اشتملت خواص المصفوفات المشتقة بطريقة العنصر المحدد على الجساءة والكتلة والتخميد والمتناقضة، واعتبر تأثير كل من سرعة التدفق الداخلية والقوة المحورية وقوة كوريكليس وقوة النابض المصلب في مفصل الربط. تم دراسة بعض المتغيرات التي تؤثر على الخواص الديناميكية مثل زاوية القوس المائل وموقع النابض المصلب ونسبة الأقطار وسمك جدار الأنبوب. وجد من خلال الدراسة إن موقع النابض المصلب يؤثر على السرعة التدفق الحرجة و التردد الطبيعي للنظام حيث إن التردد الطبيعي للأنبوب المقوس يزداد باقتراب النابض من وسط الأنبوب المقوس وكذلك زيادة في سرعة التدفق. قورنت النتائج مع الطرق العددية وأعطت الطريقة المستخدمة توافق جيد من الطرق المقارنة .

NOMENCLATURES

Symbol	Definition	Basic Unit
A_i	Fluid cross-sectional area	m^2
A_p	Pipe cross-sectional area	m^2
C_{oveall}	Damping matrix	-
E	Modulus of elasticity of pipe	N/m^2
Fx	Tension force in the pipe	N
G	Shear modulus of elasticity of pipe	N/m^2
I	Unity matrix	-
I_y, I_z	Pipe second moment of area in y and z directions	m^4
J	Polar second moment of area	m^4
k_1	Stiffness matrix of pipe	-
l	Element length of curved pipe	m
M	Fluid mass per unit length	kg/m
m	Pipe mass per unit length	kg/m
$M_{overall}$	Pipe mass matrix	-
N_i	Shape function	-
OD	Outer diameter of pipe	m
R	Raduis ofdcurved pipe	m
r	Radius of gyration of pipe section	m
t	Time	s
U	Fluid velocity relative to the pipe	m/s
q	Displacement vector	-
$\dot{q}$	Velocity vector	-
$\ddot{q}$	Acceleration vector	-
x,y,z	Cartesian axes	-
β	Curved pipe angle	degree
λ	Transformation matrix	-

INTRODUCTION

The flow induced vibration occurs in many industrial fields including, refrigerators, heat exchangers, air conditions, chemical plants, nuclear reactor components, fuel lines of aircraft and missiles. Flow induced vibration analysis of curved pipes conveying fluid have been one of the attractive subjects in structural dynamics.

For computationally analyzing curved beams or arches, many prefer using straight beam elements based on straight beam theories. This is a simple and good approximation for slender curved beams or flexible curved beams although more elements will be used to get a satisfactory accuracy. Others prefer using curved beam, arch elements to analyze curved beams or arches based on slender beam theories to reduce the number of elements used.

Some studies have investigated vibration of fluid conveying straight pipes, for examples of straight pipe conveying fluid studied can be found in Ref **[Chol [1991], Huang[2010, Ni[2011] and Nawras[2011]]**, but there are significantly fewer studies on curved pipes Among the first to study the hydroelastic vibration of curved pipes was **Svetlitskii [1966].** He investigated the out-of-plane motion of a fluid conveying perfectly flexible hose, treating it as a string, and therefore neglecting the bending rigidity. **Unny et al. [1970]**considered the in-plane divergence of initially circular tubular beams with fixed ends. The equations of motion were derived using Hamilton's principle, and critical flow velocities for instability were obtained for pinned and clamped ends. **Chen[1972,1973]** proposed an early dynamic model for the vibration analysis of a curved pipe conveying fluid in his papers. **Hill and Davis [1974]** studied the effect of initial forces on the vibrating and stability of curved clamped-clamped pipes conveying fluid, shaped as circular arcs, as well as S-shaped, L-shaped and spiral configurations. **Ko and Bert [1984]** considered the first-order nonlinear interaction between the pipe structure and the flowing fluid and formulated the governing equations of motion for the in-plane vibrations of a circular-arc pipe containing flowing fluid. **Kohli and Nakva [1984]** analyzed the straight and curved tubes conveying fluid by means of straight beam finite elements. **Dupuis & Rousselet [1985]** have carried out a study on the dynamics of fluid-conveying planar curved pipes modelled as Timoshenko beams. The extension of the centerline was taken into account. This study used the transfer matrix method. **Fan and Chen [1987]** investigated the vibration and stability analysis of helical pipe conveying fluid based on finite element method analysis. **Misra et al. [1988]** studied the differences of the dynamics between the curved pipes with extensible and inextensible centerlines. **Lee et al. [1996]** presented a transfer matrix formulation for three dimensional vibration analysis of straight and curved piping systems containing fluid flow with small computer core memory usage. **Ni et al. [2005]** investigated a fluid-conveying curved pipe with demisemi-arc shape placed on nonlinear foundations. Based on numerical analysis, three final steady states were detected, and chaotic transients found, as a function of the flow velocity parameter. **Ni et al. [2006]** developed a fluid-conveying curved pipe model subject to motion constraints placed arbitrarily along the pipe axis. It was shown that chaotic motions may occur at sufficiently high flow velocities for such a self-vibration system. **Wang Lin et al.[2007]** investigated the nonlinear dynamics of a fluid-conveying curved pipe subjected to motion constraints and harmonic excitation. **Jung [2008]** analyzed the in-plane and out-of-plane motions of a semi-circular pipe conveying fluid. Assuming that the centerline of the semi-circular pipe was extensible, nonlinear equations of in-plane and out-of-plane motions are derived according to the extended Hamilton's principle. The derived equations of motion were discretized by applying the Galerkin method. Linearized equations around the equilibrium position were obtained from the discretized equations, and then the dynamic characteristics of the pipe were investigated.

In this paper, the out of plane curved pipe conveying fluid with linear spring location composed from several straight pipes are constructed in three-dimensional space and analyzed by finite element method. A fixed- fixed end conditions with change location of spring stiffener will be adopted in this study. The matrices such as mass, stiffness, damping (Carioles) are derived and the pipe natural frequency relates with inlet flowing velocity and the velocity caused the frequency is zero (critical velocity) are then performed. The forces occur due to momentum change and pressure when the fluid pass by the elbow (joint point of two adjacent pipe elements) part is also considered.

EQUATION OF MOTION

The differential equation of motion in three dimensional coordinate's vibration of a pipe conveying a moving fluid was used to take into account the presence of motion constraints. The system consists of a several of straight pipe of a length (*l*), where ($l = R * \phi$), R is the radius of curved pipe and (ϕ) is the element front angle measured from curved pipe center point. E and G are the pipe axial and shear moduli of elasticity respectively, I_y and I_z are moment of inertia of the pipe in y and z directions respectively, J is the pipe polar moment of inertia, (*m) is the* mass of the pipe per unit length, conveying fluid of mass per unit length (*M*), U is the steady mean flow velocity of fluid with respect to pipe, *Ft* is the tension force in the pipe, A_p, A_i, r are the cross section pipe area, internal pipe area (fluid area), and pipe radius of gyration respectively. Fig.(1) shows a simple representation of the problem within hand which is consist of several pipes joint at their junction by an elbow. The equation of motion of whole system is given by (Païdoussis, 2004)

$$
\begin{aligned}
EI_y \frac{\partial^4 w}{\partial x^4} + EI_z \frac{\partial^4 v}{\partial x^4} + \{MU^2 + (pA_i - Ft)\}[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 v}{\partial x^2}] + 2MU[\frac{\partial^2 w}{\partial x \partial t} + \frac{\partial^2 v}{\partial x \partial t}] \\
+ (m+M)[\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2 v}{\partial t^2}] + EA_p \frac{\partial^2 u}{\partial x^2} + GJ \frac{\partial^2 \theta}{\partial x^2} + (m+M)[\frac{\partial^2 u}{\partial t^2} + r^2 \frac{\partial^2 \theta}{\partial t^2}] = 0
\end{aligned}
\tag{1}
$$

Where u, w, and v are the coordinate axes in the directions of x, y, and z respectively, θ is the pipe torsional angular displacement.

FINITE ELEMENT DISCRETIZATION

Fig.(2) Shown the location of spring stiffener and represent the node points(i,j) of finite element of length (l). Each node point should have six degrees of freedom to describe their motion which consist of three linear displacements u, w, v and three rotational displacements θ_x , θ_y θ_z. Therefore the finite element has the total twelve degrees of freedom. According to this the displacement vector for a pipe element in space may be represent by [Rao 2004]

$$
\{q\}^T = \{u_1 \quad w_1 \quad v_1 \quad \theta_{x1} \quad \theta_{y1} \quad \theta_{z1} \quad u_2 \quad w_2 \quad v_2 \quad \theta_{x2} \quad \theta_{y2} \quad \theta_{z2}\}
\tag{2}
$$

The equation of motion of finite element may be derived by using the extended Hamilton's Principle [Rao 2004]

$$
\int_{t1}^{t2} (P.E_{pipe} - K.E_{pipe+fluid} + D.E_{fluid} + P.E_{pressure+force})dt = 0
\tag{3}
$$

The Eq.(1) represent the general form of motion of the whole system. Another form containing Kinetic, Potential and Dissipation energies were driven in more specific form as ($P.E_{pipe}$) is potential energy of the pipe, ($K.E_{pipe+fluid}$) a kinetic energy of both fluid and pipe structure, ($P.E_{pressure+force}$) a potential energy in pipe which done by internal pressure and forces produced by fluid flow, ($D.E_{fluid}$) is the dissipation energy which comes from Coriolis acceleration due to fluid flow with velocity *(U)* relative to the pipe and (t_1 and t_2) time at any two instants.

$$
K.E_{pipe+fluid} = \underbrace{\tfrac{1}{2}\int_0^l (m+M)(\frac{\partial w}{\partial t})^2 dx}_{transverse\ \ y-dir.} + \underbrace{\tfrac{1}{2}\int_0^l (m+M)(\frac{\partial v}{\partial t})^2 dx}_{transverse\ \ z-dir.} + \underbrace{\tfrac{1}{2}\int_0^l (m+M)(\frac{\partial u}{\partial t})^2 dx}_{axial} + \underbrace{\tfrac{1}{2}\int_0^l (m+M) r^2 (\frac{\partial \theta_x}{\partial t})^2 dx}_{torsional}
\tag{4-a}
$$

$$
P.E_{pipe} = \underbrace{\tfrac{1}{2}\int_0^l EI_y (\frac{\partial^2 w}{\partial x^2})^2 dx}_{transverse,\ \ y-dir.} + \underbrace{\tfrac{1}{2}\int_0^l EI_z (\frac{\partial^2 v}{\partial x^2})^2 dx}_{transverse,\ \ z-dir.} + \underbrace{\tfrac{1}{2}\int_0^l EA_p (\frac{\partial u}{\partial x})^2 dx}_{axial} + \underbrace{\tfrac{1}{2}\int_0^l GJ (\frac{\partial \theta_x}{\partial x})^2 dx}_{torsional}
\tag{4-b}
$$

$$P.E_{pressure+force} = \tfrac{1}{2}\int_0^l \{MU^2 + (pA_i - Fx)\}(\frac{\partial w}{\partial x})(\frac{\partial w}{\partial x})dx + \tfrac{1}{2}\int_0^l \{MU^2 + (pA_i - Fx)\}(\frac{\partial v}{\partial x})(\frac{\partial v}{\partial x})dx \quad (4\text{-c})$$

$$D.E_{pipe} = \tfrac{1}{2}\int_0^l 2MU(\frac{\partial w}{\partial x})(\frac{\partial w}{\partial t})dx + \tfrac{1}{2}\int_0^l 2MU(\frac{\partial v}{\partial x})(\frac{\partial v}{\partial t})dx \quad (4\text{-d})$$

The displacement models can be expressed as [Meirovitch, 2002]

-for transverse displacements

$$w(x) = v(x) = \sum_{k=1}^{4} N_k(x) q_k \quad (5)$$

-for axial and torsional displacements

$$u(x) = \theta_x(x) = \sum_{k=5}^{6} N_k(x) q_k$$

And the shape functions $N_i(x), (i = 1.....6)$ for transverse (flexural) pipe vibration are

$$\left.\begin{aligned} N_1 &= \frac{1}{l^3}(2x^3 - 3lx^2 + l^3) \\ N_2 &= \frac{1}{l^2}(x^3 - 2lx^2 + l^2 x) \\ N_3 &= \frac{1}{l^3}(3lx^2 - 2x^3) \\ N_4 &= \frac{1}{l^2}(x^3 - lx^2) \end{aligned}\right\} \quad (6)$$

While for axial and torsional vibrations the shape functions are

$$\left.\begin{aligned} N_5 &= (1 - \frac{x}{l}) \\ N_6 &= (\frac{x}{l}) \end{aligned}\right\} \quad (7)$$

By inserting eqs (6, 7) in to eqs (4), can be obtain the element matrices defined in the following equation of motion

$$[M_{overall}]\{\ddot{q}\} + [C_{overall}]\{\dot{q}\} + [K_{overall}]\{q\} = \{0\} \quad (8)$$

Where ($M_{overall}$) the overall mass matrix of pipe and fluid from eq(4-a) the element mass matrix has the form

$$
M_{overall}=\frac{(m+M)l}{420}\begin{bmatrix}
140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\
0 & 156 & 0 & 0 & 0 & 22l & 0 & 54 & 0 & 0 & 0 & -13l \\
0 & 0 & 156 & 0 & -22l^2 & 0 & 0 & 0 & 54 & 0 & 13l & 0 \\
0 & 0 & 0 & 140r^2 & 0 & 0 & 0 & 0 & 0 & 70r^2 & 0 & 0 \\
0 & 0 & -22l^2 & 0 & 4l^2 & 0 & 0 & 0 & -13l & 0 & -3l^2 & 0 \\
0 & 22l & 0 & 0 & 0 & 4l^2 & 0 & 13l & 0 & 0 & 0 & -3l^2 \\
70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\
0 & 54 & 0 & 0 & 0 & 13l & 0 & 156 & 0 & 0 & 0 & -22l \\
0 & 0 & 54 & 0 & -13l & 0 & 0 & 0 & 156 & 0 & 22l & 0 \\
0 & 0 & 0 & 70r^2 & 0 & 0 & 0 & 0 & 0 & 140r^2 & 0 & 0 \\
0 & 0 & 13l & 0 & -3l^2 & 0 & 0 & 0 & 22l & 0 & 4l^2 & 0 \\
0 & -13l & 0 & 0 & 0 & -3l^2 & 0 & -22l & 0 & 0 & 0 & 4l^2
\end{bmatrix} \quad (9)
$$

($C_{overall}$) is the overall damping matrix of fluid. From eq (4-d) the element damping matrix can be written as,

$$
[C]=\frac{MU}{30}\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -30 & 0 & 0 & 0 & -6l & 0 & -30 & 0 & 0 & 0 & 6l \\
0 & 0 & -30 & 0 & -6l & 0 & 0 & 0 & -30 & 0 & 6l & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 6l & 0 & 0 & 0 & 0 & 0 & -6l & 0 & l^2 & 0 \\
0 & 6l & 0 & 0 & 0 & 0 & 0 & -6l & 0 & 0 & 0 & l^2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 30 & 0 & 0 & 0 & 6l & 0 & 30 & 0 & 0 & 0 & -6l \\
0 & 0 & 30 & 0 & 6l & 0 & 0 & 0 & 30 & 0 & -6l & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -6l & 0 & -l^2 & 0 & 0 & 0 & 6l & 0 & 0 & 0 \\
0 & -6l & 0 & 0 & 0 & -l^2 & 0 & 6l & 0 & 0 & 0 & 0
\end{bmatrix} \quad (10)
$$

And ($[K_{overall}]=[K_1]_e-[K_2]_e$), the overall stiffness matrices, where ($[K_1]_e$) the element stiffness matrix of pipe. From eq (4-b) can be written as

$$[K_1]_e = \begin{bmatrix} \frac{EA_p}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA_p}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} & 0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} \\ 0 & 0 & \frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & -\frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 \\ 0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 \\ 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} \\ -\frac{EA_p}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA_p}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} & 0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} \\ 0 & 0 & -\frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 \\ 0 & 0 & 0 & -\frac{GJ}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GJ}{l} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 \\ 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} \end{bmatrix} \quad (11)$$

While ($[K_2]_e$) the element stiffness matrix of pressure and forces. From eq(4-c) can be written as

$$[K_2]_e = \frac{MU^2 + pA_i - Ft}{30l} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 36 & 0 & 0 & 0 & 3l & 0 & -36 & 0 & 0 & 0 & 3l \\ 0 & 0 & 36 & 0 & 3l & 0 & 0 & 0 & -36 & 0 & 3l & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3l & 0 & 4l^2 & 0 & 0 & 0 & -3l & 0 & -l^2 & 0 \\ 0 & 3l & 0 & 0 & 0 & 4l^2 & 0 & -3l & 0 & 0 & 0 & -l^2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -36 & 0 & 0 & 0 & -3l & 0 & 36 & 0 & 0 & 0 & -3l \\ 0 & 0 & -36 & 0 & -3l & 0 & 0 & 0 & 36 & 0 & -3l & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3l & 0 & -l^2 & 0 & 0 & 0 & -3l & 0 & 4l^2 & 0 \\ 0 & 3l & 0 & 0 & 0 & -l^2 & 0 & -3l & 0 & 0 & 0 & 4l^2 \end{bmatrix} \quad (12)$$

Eq (4-c) which contains the force per unit length (stiffness unit) that conforms the fluid to the pipe (weakening effect) besides the axial tension force (stiffening effect). The stiffness spring $K_s(i,j)$ will added in element matrix (k_1), where i,j represent the position of k_s in overall matrix. Here, we will call the above matrix as a contradictory matrix because it contains two opposite component effects. Where *Ft* is an axial tension force (tangential) that caused by the change in fluid's momentum and pressure in a pipe bend (elbow). Fig (3**)** shows the induced axial tension forces in the pipe bend.

From Fig.(3**)**, the axial tension forces in pipe bend are equal to (Munson *et al.*, 2002):

$$Ft_1 = pA[\cos(\alpha) - 1] + \rho_{fluid} QU [1 + \cos(\alpha)]$$
$$Fn_1 = pA\sin(\alpha) + \rho_{fluid} QU \sin(\alpha)$$

$R = [Ft_1{}^2 + Fn_1{}^2]^{\frac{1}{2}}$

and

$$Ft_2 = R\{\frac{\sin(\psi)}{\sin(\alpha)}\} \quad \ldots\ldots\ldots\ldots \quad (13)$$

Where

$$\psi = \tan^{-1}\{\frac{\sin(\alpha)}{1+\cos(\alpha)}\}$$

Where (Ft_1) tension force (tangential) and (Fn_1) normal force on pipe element.

From the mathematical formulation presented above, it is clear that the overall stiffness is composed of two parts, namely the contradictory and pipe structural stiffness matrices.

It can be seen that the 12 * 12 element matrices ($[M]_e, \{C\}_e, [K_1]_e \, and [K_2]_e$) are with respect to the local *xyz* coordinate system. Since the nodal displacements for the angled pipe are in different local coordinates, thus it must transform the local coordinate to global coordinate system. The transformation matrix, λ, can be identified as (Rao, 2004).

$$\lambda = \begin{bmatrix} l_{ox} & m_{ox} & n_{ox} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_{oy} & m_{oy} & n_{oy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_{oz} & m_{oz} & n_{oz} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{ox} & m_{ox} & n_{ox} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{oy} & m_{oy} & n_{oy} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & l_{oz} & m_{oz} & n_{oz} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & l_{ox} & m_{ox} & n_{ox} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & l_{oy} & m_{oy} & n_{oy} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & l_{oz} & m_{oz} & n_{oz} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_{ox} & m_{ox} & n_{ox} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_{oy} & m_{oy} & n_{oy} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & l_{oz} & m_{oz} & n_{oz} \end{bmatrix} \quad (14)$$

Here, l_{ox}, m_{ox}, and n_{ox} denote the direction cosines of the *x*-axis; l_{oy}, m_{oy} , and n_{oy} represent the direction cosines of the *y*-axis; and l_{oz}, m_{oz}, and n_{oz} indicate the direction cosines of the *z*-axis with respect to the global axes. This leads to the following global element matrices:

$$[M_{overall}]_{Global} = [\lambda]^T [M_{overall}][\lambda] \quad (15)$$

$$[K_{overall}]_{Global} = [\lambda]^T [K_{overall}][\lambda] \quad (16)$$

$$[C_{overall}]_{Global} = [\lambda]^T [C_{overall}][\lambda] \quad (17)$$

DYNAMIC ANALYSIS

The solution of eigenvalues problem should be executed to the characteristic matrix [Ω], [Meirovitch, 2002], which is equal to

$$[\Omega] = \begin{bmatrix} [0] & [I] \\ -[M_{overall}]_{Global}^{-1}[K_{overall}] & -[M_{overall}]_{Global}^{-1}[C_{overall}] \end{bmatrix}_{Global} \tag{18}$$

The solution of eigenvalues problem yields complex roots. The imaginary part of these roots represents the natural frequencies of damped system. The real part indicates the rate of decay of the free vibration.

RESULTS AND DISCUSSION

In this section, the out of plane results of curved pipe conveying fluid were presented. Table (1) presents the number of elements, its represents the major parameter for accuracy of the results
and the consumed time to solve the problem. Some types of error are increase or decrease with increasing or decreasing the number of elements and it is noted that, 256 elements gave a convergence in the results and then they will be used to discretize the curved pipe system.

To verify the results a simple comparison with another researcher was achieved. In 2008 Jung, used Galerkin Method to list a different values for velocity of fluid and correspond of frequency values without adding a spring stiffener. Table (2) presents the compassion.

Fig.(4) present the effect of the curved pipe angle on natural frequency without additional spring stiffener. The frequency of the system depends on the relationship between the stiffness and inertia. With decreasing the curved pipe angle the stiffness leads to weaken the structure stiffness of the curved pipe angle. This behavior was same as investigated (Jung, and Chung,2008).

Fig.(5) present the effect of inlet fluid velocity on the natural frequency, it's clear that any increase velocity will lead to decreasing in frequency and static instability buckling phenomena occurs at critical velocity. This figure was confirmed previously by Lee (1996) used Transfer Matrix Method, for clamped-clamped curved pipe for curved angle (180^0) and the number of element is (128). It was note that, there was as good agreement between the results. The different was result for using Transfer Matrix Method

Fig.(6) present the effect of the fluid velocity on natural frequency for different values of curved pipe angle (60^0,90^0,120^0,150^0,180^0). The general sight is the frequency was decreased with increasing the flow velocity. Because the frequency depended on the structure stiffness of curved pipe and when the velocity of fluid increased leads to weaken the structure stiffness according to the overall stiffness matrix [$K_{overall}$].

Fig.(7) present effect the ratio of spring location to angle of curved pipe on the critical velocity of curved pipe conveying fluid with different values of curved pipe angle. Which is the critical velocity of the fluid caused the corresponding frequency become zero for whole system. As a general view, it was noted that the critical velocity is increase with increasing the ratio of spring location to angle of curved pipe. For the same value of ratio, the critical velocity is seemed to be increased with increasing ratio for different values of curved pipe

angle. This behavior was dominated for the specific value of ratio stared from fixed end to center of curved pipe.
From this figure, two interested were observed. The first, there was the critical velocity value in which the response is started from change of ratio. The second, the spring location which attain its critical location at center of curved pipe because the symmetric.

Fig.(8) present the effect of fluid velocity on the natural frequency for different angles of spring location values for two types of curved pipe (90^0 and 180^0) . As a general view is the frequency was decreased with increasing the fluid velocity. The frequency of the system depend on the stiffness of curved pipe and the flow velocity of fluid as shown in eq.(4-b and 4-c).The increasing or decreasing in the stiffness of system and velocity of fluid effect on the hydrodynamic results of system. the increasing velocity leads to increasing in hydrodynamic and thereby decreasing in frequency of the curved pipe.

The effect of the diameter ratio on the natural frequency present in **Fig.(9)** for different values of spring stiffener location for two types of curved pipe angle and at flow velocity (100m/s). It is clear,
The increasing in the diameter ratio caused increasing in structure stiffness and inertia of curved pipe due to the direct relationship between them. The diameter ratio leads to strong the structure stiffness.

Fig.(10) indicated that effect of the thickness pipe on critical velocity with different values of spring location for two types of curved pipe angle. Where at the same value of thickness the critical velocity increasing with change in spring location, and it has high value of critical velocity when the location of spring closed to center of curved pipe(at angle 90^0 for curved pipe 180^0 and 45^0 for curved pipe 90^0).

CONCLUSIONS

The vibration of three dimensional curved piping systems consisting from several of straight pipes is analyzed by the finite element method. As a result, the following conclusions are obtained:

1- The location of spring stiffener has great effect on the critical velocity of fluid and natural frequency of system.
2- The stability of curved pipe dose not loses at a high fluid velocity and when changes in location of spring stiffener.
3- For each location of spring stiffener, there are pipe thickness and diameter ratio that gives the best dynamic characteristics.
4- The increase in ratio of spring location to curved pipe angle leads to decreasing in natural frequency for different values of curved pipe angle..
5- The Finite Element Method appears good agreement when comparison with numerical approach.

Table [1]: Effect number of elements on length of curved and frequency (v=0m/s, t=1mm, OD=30mm, $\beta=180^0$, and p=100kPa), Exact length of curved is 3.1415 m

Number of Elements	Length of Curved (m)	Frequency (rad/s)
8	3.1214	2.7045
16	3.1365	2.6700
32	3.1403	2.6612
64	3.1413	2.6590
128	3.1415	2.6585
256	3.1415	2.6584

Table [2]: Comparison the natural frequencies (Hz) for a curved pipe (t=1.5875mm OD=22.375mm, $\beta=180^0$, $m_f=0.3874$kg/m, $m_p=0.1415$kg/m, $\rho_f=1000$kg/m^3, $\rho_p=7800$kg/m^3 and R=0.511m.

Velocity of fluid m/s	Jung Hz	Present work Hz	%Error
20	20.8423	19.9451	4.3
80	11.0412	10.8327	1.9

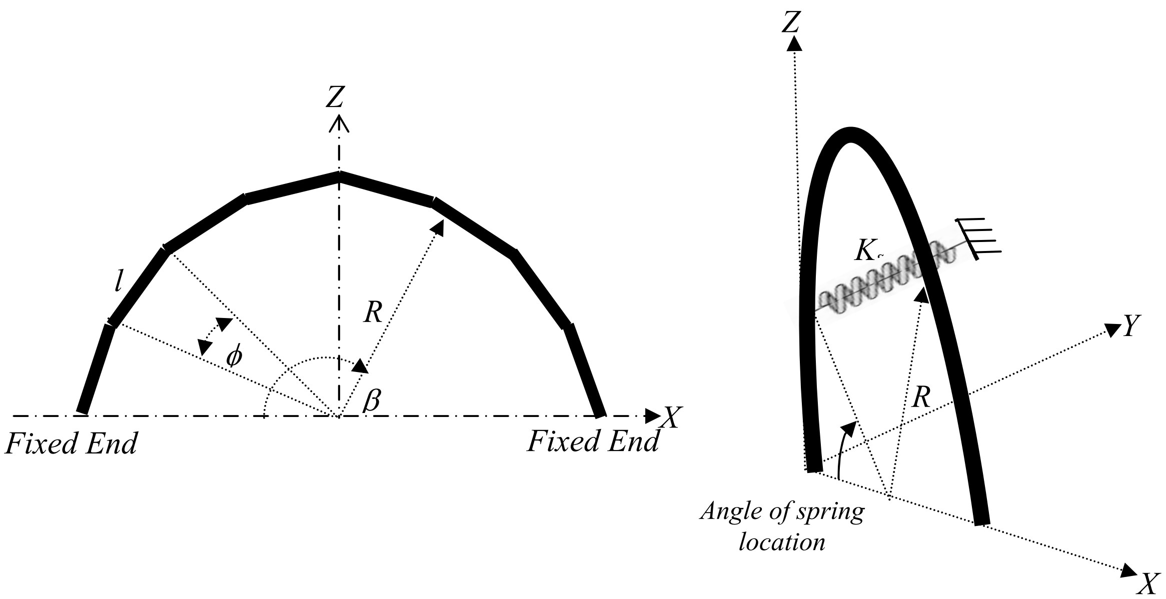


Fig.(1): Curved Pipe Model and Angle of Spring Location.

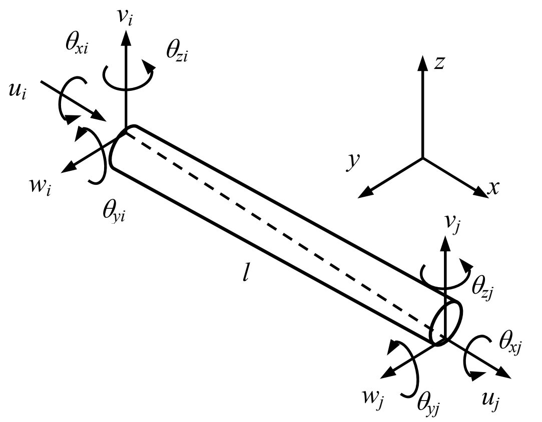


Fig.(2): Degree of freedom of pipe element.

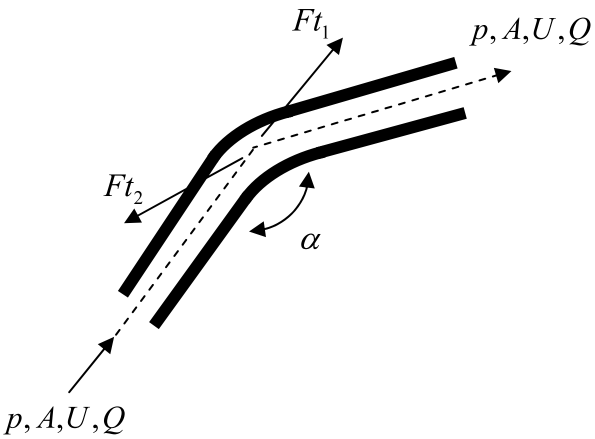


Fig.(3): Tension forces in curved pipe bend.

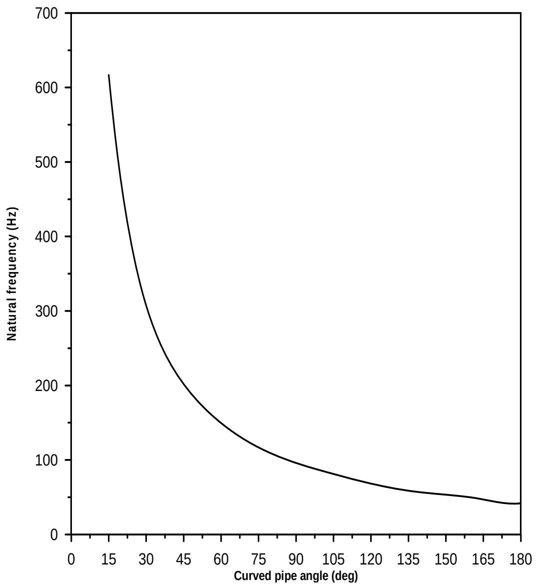


Fig.(4): Effect the curved pipe angle on the natural frequency

Young's modules(200Gpa), Density of pipe(8000Kg/m^3), Density of fluid(1000Kg/m^3), pipe outside diameter(30mm), pipe wall thickness(3mm),pressure(100kpa), and pipe radius curved (3m)

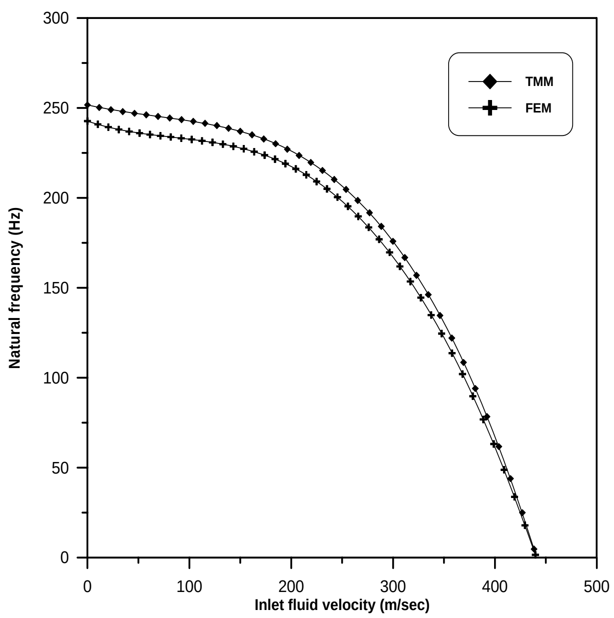


Fig.(5): Effect the inlet fluid velocity on the natural frequency

Young's modules(207Gpa), Density of pipe(8000Kg/m^3), Density of fluid(1000Kg/m^3), pipe outside diameter(9.54mm), pipe wall thickness(1mm),pressure(130Mpa), curved angle (180^0) and pipe radius curved (0.125m)

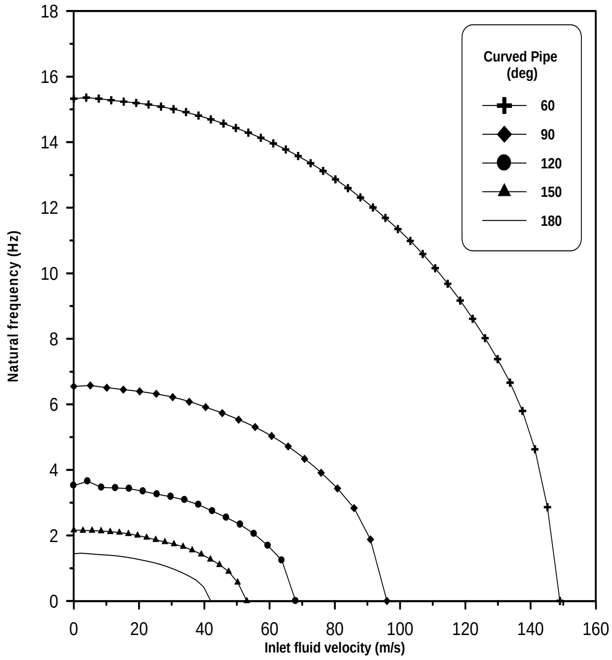


Fig.(6): Effect the inlet fluid velocity on the natural frequency with different Curved angle

Young's modules(200Gpa), Density of pipe(8000Kg/m^3), Density of fluid(1000Kg/m^3), pipe outside diameter(30mm), pipe wall thickness(3mm),pressure(100kpa) and pipe radius curved (3m)

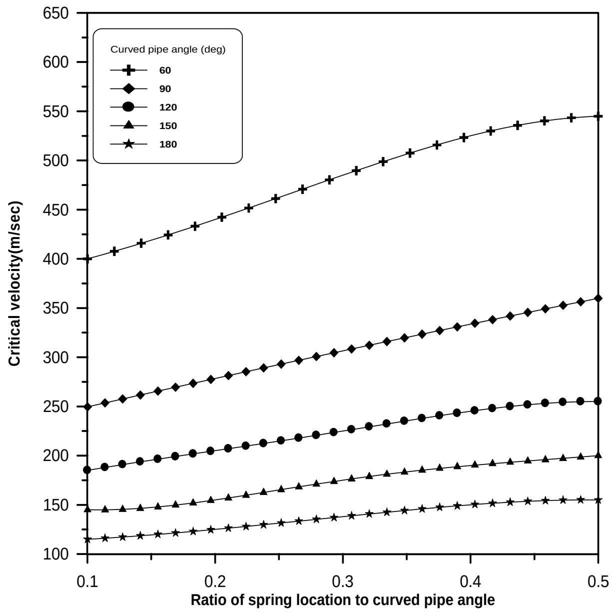
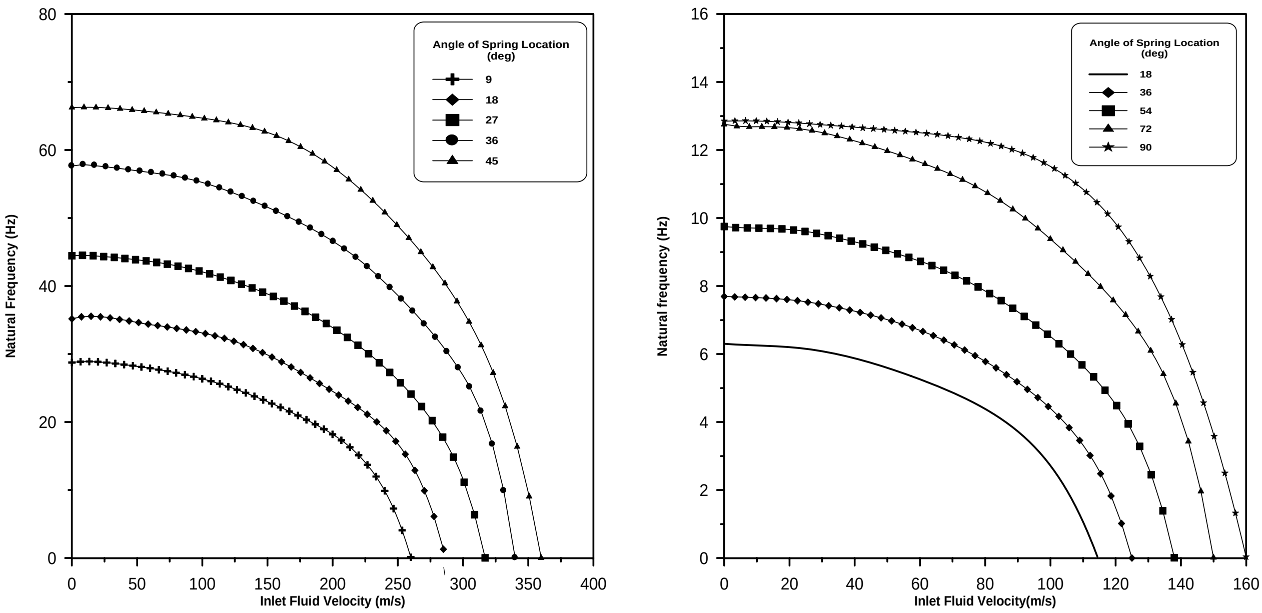
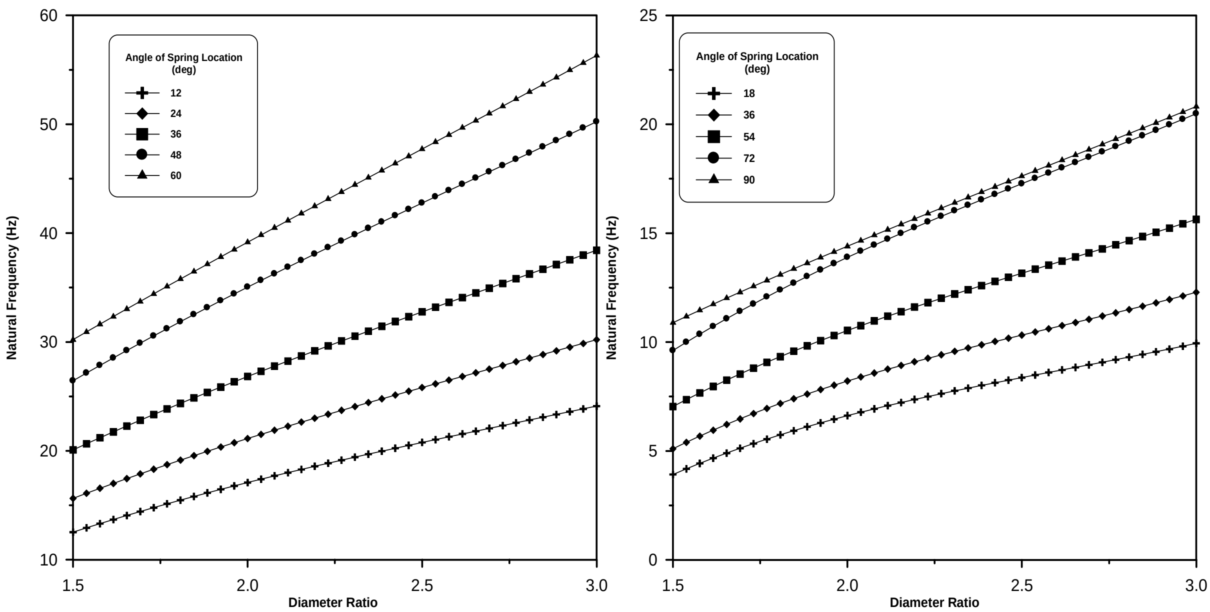


Fig.(7): Effect the location of spring at curved pipe on the critical velocity with varying curved pipe angle
Young's modules (200Gpa), Density of pipe (8000Kg/m^3), Density of fluid (1000Kg/m^3), pressure (100kpa) and pipe radius curved (2m), thickness of pipe (t=5mm) ,out diameter (OD=50mm) and stiffener spring (10^7 N/m)



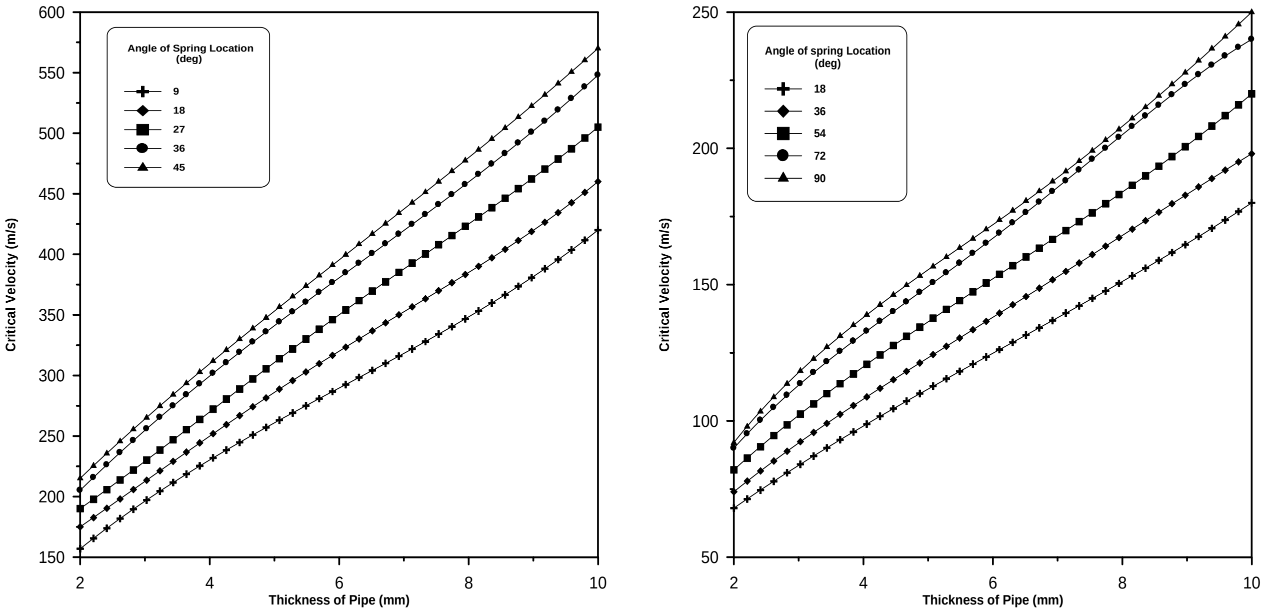
a b

Fig.(8): Effect the inlet fluid velocity on the natural frequency with varying angle of spring location
curved pipe angle (a=90^0 and b=180^0)
Young's modules (200Gpa), Density of pipe (8000Kg/m^3), Density of fluid (1000Kg/m^3), pressure (100kpa) and pipe radius curved (2m), thickness of pipe (t=5mm) ,out diameter (OD=50mm) and stiffener spring (10^7 N/m)



a b

Fig. (9): Effect the diameter ratio on the natural frequency with varying angle of Spring Location with inlet diameter (ID=3mm)
curved pipe angle (a=120^0 and b=180^0)
Young's modules (200Gpa), Density of pipe (8000Kg/m^3), Density of fluid (1000Kg/m^3), pressure (100kpa) , pipe radius curved (2m), thickness of pipe (t=0.5*(OD-ID)) ,inlet diameter (ID=30mm) and stiffener spring (10^7 N/m)



a b

Fig.(10): Effect the curved pipe thickness on the critical velocity with varying angle of spring location curved pipe angle (a=90^0 and b=180^0) Young's modules (200Gpa), Density of pipe (8000Kg/m^3), Density of fluid (1000Kg/m^3), pressure (100kpa) and pipe radius curved (2m) ,out diameter (OD=50mm) and stiffener spring (10^7 N/m)

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