On α -Replete Semi–Groups on Semi–Topological Groups

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Abstract :

The aim of this paper is to introduce α-replete semigroup, deals in some type of topological groups investigation some properties their images under homomorphism using the work of W. Gottschalk(1955)[5],Coven(1969)[4],Al-Kutaibi (1996)[1],Al-Kutaibi (1997)[2].

Introduction:

Let X be topological space and A be any subset of X ,let $\overset{\circ}{A}$, $\overline{A}$ and A^c denote the interior set , closure and complement of A respectively . A is called α - open set (α - open) in X if and only if $A \subset \overset{\circ-\circ}{A}$ [6] , the complement of an α -set is called α -closed set .

Definition:-

Let X be topology space and $A \subset X$ then A called α -compact (αC) if and only if for every $\alpha-$open cover for A has finite subcover.

Remark:-

Through out this paper we mean by AL(G) to be the set of all topological ,semi- topological ,feebly topological , irresolute topological , feebly irresolute topological group [1] .

Definition:-

Let A be a subset of $G \in$ AL(G) ,then A called α -replete (αR) if and only if for every α -compact (αC) K subset of G there exist $g_1, g_2 \in G$ such that $g_1 K g_2 \subset A$.

Remark:-

1- A semigroup $A \subset G \in$AL(G) is said to be (αR) if for every (αC) K subset of G there exists $g \in G$ such that $gK \subset A$ or $Kg \subset A$[2].

2- If $G \in$AL(G) then G is (αR) semigroup in G .

Proposition:-

Let A be αR subset of $G \in$ AL(G) then gA and Ag are αR subset of G for every $g \in G$.

Proof :- Let $g \in G$, since A is αR subset of G for every αC subset K of G there exists $g_1, g_2 \in G$ such that $g_1 K g_2 \subset A$ then $g.g_1 K g_2 \subset gA$.

Let $m = g.g_1$, $m \in G$ hence $mKg_2 \subset gA$ and so gA is αR in G .

by the same way we prove Ag is αR in G .

Corollary:-

If A is αR subset of $G \in$ AL(G) then every bilateral translate [3] of A $(g_1 A g_2)$ is αR subset of G $\forall\, g_1, g_2 \in G$.

Proposition:-

If A and B are semi-group of $G \in$ AL(G) such that $A \subset B$ and A is αR then B is αR .

Proof :- since A is αR semigroup ,for every αC subset K of G there exist $g \in G$ such that $gK \subset A$ since $A \subset B$, $gK \subset B$ then B is αR .

Corollary:-

If A and B are αR semigroup of $G \in$ AL(G) then $A \cup B$ is αR semigroup of G .

Remark:-

If A, B are αR subset of $G \in$ AL(G) such that $A \cap B \neq \phi$ then $A \cap B$ is not αR subset of G in general the following example shows that .

Example:-

Let $G = R$ and $(-\infty, 0]$, $[0, \infty)$ are αR subset of G but $(-\infty,0] \cap [0,\infty) = \{0\}$ and $\{0\}$ is not αR in G .

Proposition:-

If A, B are αR semigroup of $G \in$ AL(G) then $A \times B$ is αR semigroup of $G \times G$ with the direct product .

Proof :- since A, B are semigroup of G then $A \times B$ is semigroup of $G \times G$ since A and B are αR in G . for every K_1 , K_2 αC subset of G there exist $g_1, g_2 \in G$ such that $g_1 K_1 \subset A$ and $g_2 K_2 \subset B$.

Let K be αC subset of $G \times G$ such that $K \subset K_1 \times K_2$ then

$$\begin{aligned}(g_1 \,.\, g_2)\,.\,K &\subset (g_1 \,.\, g_2)\, K_1 \times K_2 \\ &= g_1 K_1 \times g_2 K_2 \\ &\subset A \times B\end{aligned}$$

and $(g_1 , g_2) \in G \times G$ thus $A \times B$ is αR subset of $G \times G$.

Proposition:-

If G and H are irresolute (feebly irresolute) topological group and $f : G \to H$ is an onto irresolute (feebly irresolute) homomorphism [1] and A is αR semigroup of H then $f^{-1}(A)$ is αR semigroup of G .

Proof :- since A is semigroup in H , $f^{-1}(A)$ is semigroup in G [5] let K be αC subset of G , $f(K)$ is αC subset of H ,since A is αR in H there exist $h \in H$ such that $hf(K) \subset A$, since f is onto there exist $g \in G$ such that $f(g) = h$,then $f(g)f(K) \subset A$ and since f is homomorphism $f(gK) \subset A$ thus $gK \subset f^{-1}(A)$ and hence $f^{-1}(A)$ is αR in G .

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في شبه الزمرة α – المفعمة في الزمرة شبه تبولوجية

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الملخص:

يهدف البحث إلى تقديم تعريف شبه الزمرة α – المفعمة ودراستها في بعض أنواع الزمر التبولوجية ، واستقصاء بعض العمليات عليها وصورها تحت تأثير الهومومورفزم ، مستعملين بعض المفاهيم التي قدمها كوستجوك (١٩٥٥)[5] ،وكوفن (١٩٦٩)[4] ، والكتبي (١٩٩٦)[1] ، والكتبي(١٩٩٧)[2] .