

ANALYTICAL STUDY OF TORSIONAL VIBRATION EFFECT ON TURNING SHAFTS IN SUDDEN STOP CASE

Hadi Rahim Almaamuri Hadialmaamuri@yahoo.com

ABSTRACT :-

The sudden stop of rotating shafts for any external & arbitrary reason due to develop of circumference distortions because of dynamic angular displacement & by dividing on static angular displacement produce ratio called (SSRSR) to undesired elevations. The response according to "Bernoli – Euler " theory in undamped torsional vibration, one – dimension bars & homogenous, continuous material of bars, hence, derived response for time & pointed position (furrier functions), the result depend on shaft engineering directory & modulus rigidity of shaft material & which is get convergence for theory & practical results.

KEY WORDS : Rotating shaft , Torsional vibration , sudden stop .

دراسة تحليلية لتأثير اهتزازات الفتل على الأعمدة الدوارة فى حالة التوقف المفاجئ هادى رحيم المعموري

الخلاصة :

أن التوقف المفاجيء للأعمدة الدوارة لأي سبب خارجي أعتباطي يؤدي الى نمو تشوهات محيطية نتيجة الأزا حات الزاوية الديناميكية وبقسمتها على الأزاحات الزاوية السكونية تكون نسبة سميت (SSRSR) الى مستويات غير مرغوب فيها ، حيث أشتقت رياضيا استجابة العمود للإجهاد المتولد وفق نظرية (بيرنولي –أويلر) في أهتزازات الفتل غير المخمدة لأعمدة أحادية البعد منتظمة متصلة المادة أذ أستتبطت الأستجابة بالنسبة للزمن والموقع النقطي (دوال فورير) وتعتمد النتائج على الصفات الهندسية للعمود ومواصفات الجساءة لمعدن العمود ،

1-INTRODUCTION :

All the problems of torsional vibration which occur in helicopter propeller or in secondary end of connecting rod in engines ,and can be that loadings of moment of inertia for the propellers are connecting with gear box ,all there are effect with normal frequency For torsional vibration of connecting rod and after that due to failure , and the results clearing that the shaft density is big there for it has low twisting , there for to avoid this statIn design by increasing shaft material density .[(Schlaifer,1950),(Satori,2012)]

It can be using lesser measurement base for torsional vibration and facility of readings by Using techniques of modern devices .[Gatzwiller,2005]

In case of to decrease the sudden acceleration and deceleration using modern methods to design ideal case for vibration translate as well as controlling on moment reference to produce the vibration ,it can be used system of mechanical simulation , hence can be create stat of translate path for vibration of three dimensions , and by simulation method can be testing of vibration damped .[Takeuch . T .2003]

There are untraditional methods to continuous test and warning about small variables in cracks propagation of it in feedback shaft for air conditioning compressor ,and this method Index for the variable properties in natural frequencies for torsional stresses which related with crack propagation in shaft .[Mitchell .S.L.2004]

2 - FREE UNDAMPED TORSIONAL VIBRATION OF RODS :

This type of vibrations are logical simulate for rotating shafts case in cutting stateBy turning processes for **free** shafts supported from one end as cantilever and this can be considered as assuming case study, perhaps can be determine angular displacement and stresses values due to twisting in all point along of the rod with its main case in figure (1),

Hence show cross section part of rod equal (dx) .The molecules which cross section rod contain will moving on rod cross section circumference, the expansions and distortions which transverse and angular, this is very important for research, while longitudinal displacements are ignore because of small value .[Aggurwal,2008]

From figure (1) show that the rod, cutting from it section far of distance (x) from fixed end The twisting angle in any point through section (dx) from rod produce due to torque (T):[Timoshenko,1974]

Where :

I_p=Polar moment of Inertia

G= Shear modulus of elasticity

The torque on two section of rod [$T + (\partial T/\partial x)dx$] as shown in figure (1):

The torque is consulate from equation (1):

$$\frac{\partial T}{\partial x \partial} dx = I_p * G * \frac{\partial^2 \theta}{\partial x^2} dx \dots \dots \dots \dots (2)$$

And by equaling the turning torque with mass moment of inertia $[(\rho/g).I_p.dx]$ for the element in angular acceleration $(\partial^2 \Theta/\partial x^2)$, where :

 ρ =Rod density (N/unite volume), there for the partial equation for turning motion was :

$$\frac{\rho}{g}I_p.\,dx\frac{\partial^2\theta}{\partial t^2} = I_p.\,G.\frac{\partial^2\theta}{\partial x^2}.\,dx\,\dots\,\dots\,\dots\,(3)$$

And by delete the similar formulas from equation two sides ,there for the result was :

From previous equation consulate that the angular displacement (Θ) is the time function ,and for position (x) along the length of rotating rod while the formula ($\left(\frac{c_g}{c}\right)$) that is

constants for rod material properties .[(Timoshenko,1974),(Betta machinery,2010)] Can be putting result dividing of rigidity modulus (G) to rotating rod material density ,its called "speared velocity of stress wave " or angular velocity for shaft and represented function of physical and mechanical properties for shaft material .[Betta machinery,2010]

3 - THEORY REVIEW OF FREE SUDDEN STOP FOR ROTATING SHAFT :

The sudden stop for rotating shaft is occur often in engines due to high friction in sleeve bearing or any cause such as electrical default or sudden stop for shaft in turning processes That is lead to vibration case with all probable and without any force effecting .In case of stopping for rotating shaft movement ,there for the time equal zero (t=0) , and angular displacement was position function (x) , and maximum angular displacement for rotating shaft tip ($\Theta \ell = \chi_0 \ell$),while angular velocity was derivative of angular displacement (θ) And was constant along axial of rotating shaft , where the symbols as follows :[Sabah ,2005] M_o =Rotating rod mass (Kg)

 $N_o = Rod rotating velocity (r.p.m)$

 $\gamma_o =$ Free rod shear strain

The general solution for the angular displacement and strain to any point from the rotating shaft points ,for correct solution might be separate of motion variables [(Selim,2009),(PISKUNOV,1974)], with another meaning the function (Θ)take of arbitrary vibration mode ,there for the general solution become all the probable solutions for free vibration mode and with another meaning was :

Where: i= Integer number for function.

Hadi R.

$$\begin{split} &\Theta_i(x) = \text{angular displacement function relative to only position} .\\ &T_i(t) = \text{angular displacement function relative to only time.}\\ &The function [T_i(t)] \text{was triangle functions relative to time and it is cyclic} .\\ &[(Sabah, 2005), (SZENASI, 1974)] \end{split}$$

Where : A_i , B_i ; represented function capacity constants ... , w_i : angular frequency And when substitution produce the following equation :

And by substitution in moving equation produce :

$$\sum_{i=0,1,2}^{\infty} \theta_i^{"}(x) [A_i \cos w_i t + B_i \sin w_i t] = \sum_{i=0,1,2}^{\infty} \theta_i(x) (\frac{-w_i^2}{a^2}) [A_i \cos w_i t + B_i \sin w_i t] \dots \dots (9)$$

And by corporation the similar limits following produce :

The general solution for differential equation get the following form :[5]

and by substitution in equations produce : \sum_{∞}^{∞}

And can be putting conditions related with angular displacement and time for every end from rod ends, after that application these conditions in previous equation to get the equation constants with new expressions ...

At clamed – end and in case any time (t), the applied condition was as following : At x=0, $\Theta(x,t)=0$(13) While at free end for shaft and in case at any time (t) ,the applied condition was :....

At x=L
$$\frac{\partial \theta(x,t)}{\partial x} = 0$$

ω

Then was :

 $\therefore C_i = 0$

The angular displacement (Θ)in equation (12) become as following :

Where :

At substitution in equation (16) produce :

In case of shaft rotating stopping ,where (t = 0) and the conditions of angular displacement and rotating speed as following :

Was angular displacement for time.) $\boldsymbol{\theta}^{*}$ (Where

From fig. (2-3) produce following: [5]

And by substitution eq.(23) in eq.(20) produce the following :

At multiplying equation two sides by formula $[\sin(j\frac{\pi x}{2\ell})]$

With integral relative to position (x=0 to $x=\ell$) and produce :

Where ,j=integer No. and applied integration with used two conjugations as follows :

By substitution eq.(22) in eq. (18) produce :

By multiplied two sides of equation by expression($\sin \frac{j\pi x}{2\ell}$) and integrate two sides:

$$\int_{0}^{\ell} w_{o} \sin\left(\frac{j\pi x}{2\ell}\right) dx$$
$$= \sum_{i=1,3,5}^{\infty} \left(\frac{i\pi a}{2\ell}\right) B_{i}^{-} \int_{0}^{\ell} \sin\left(\frac{i\pi x}{2\ell}\right) \cdot \sin\left(\frac{j\pi x}{2\ell}\right) dx \dots \dots \dots \dots (30)$$

Can be produce the following :

 ∞

$$B_i^- = w_o \, \frac{8\ell}{\pi^2 i^2 a} \dots \dots \dots \dots \dots \dots \dots \dots \dots (31)$$

And by substitution two equation (28)and (31) in equation (19) produce :

$$\theta(x,t) = \sum_{i=1,3,5}^{n} \sin\left(\frac{i\pi x}{2\ell}\right) \cdot \frac{8\ell}{\pi^2 i^2} [\gamma_o(-1)^{\frac{i-1}{2}} \cos\left(\frac{i\pi a}{2\ell}\right) t + \frac{W_o}{a} \sin\left(\frac{i\pi a}{2\ell}\right) t] \dots \dots \dots (32)$$

Can be produce the following form :

$$\theta(x,t) = 8\gamma_o \ell \sum_{\substack{i=1,3,5\\ \nu_o a}}^{\infty} \sin\left(\frac{i\pi x}{2\ell}\right) \cdot \frac{8\ell}{\pi^2 x^2} \left[(-1)^{\frac{i-1}{2}} \cos\left(\frac{i\pi a}{2\ell}\right) t + \frac{W_o}{\gamma_o a} \sin\left(\frac{i\pi a}{2\ell}\right) t \right] \dots \dots (33)$$

Where the static displacement value (θ_0) from shear strain concept can be take the following form :

$$\theta_o = \gamma_o.\,\ell\,\dots\,\dots\,(34)$$

Where ; θ_o =Static tip angular displacement for shaft material .

 $\gamma_o = static$ total shear strain for shaft .

and by dividing equation(33) in position $(x=\ell)$ on the static shear strain produce :

$$\frac{\theta(\ell,t)}{\theta_o} = 8 \sum_{i=1,3.5}^{\infty} (-1)^{\frac{i-1}{2}} \frac{1}{\pi^2 i^2} \left[(-1)^{\frac{i-1}{2}} \cos\left(\frac{i\pi a}{2\ell}\right) t + \frac{W_o}{\gamma_o a} \sin\left(\frac{i\pi a}{2\ell}\right) t \dots \dots \dots (35) \right]$$

Where; $\theta(\ell, t)$:Dynamic angular displacement at shaft tip.

$$\frac{\theta(l,t)}{\theta_0} = Angular \ distorsion \ ratio \ (ADR)$$

And to determine stress function ,hence ,the application of shear strain by the expression

$$\left(\gamma(x,t)=\frac{\partial\theta(x,t)}{\partial\theta}\right)$$
 and applied it in equation (33)produce :

Can be used (shear stress-shear strain)law according to following formula :

 $\tau(x,t) = G.\gamma(x,t) \dots \dots \dots \dots (3-39)$ Where; T(x,t) = stress function of displacement & time .(N/m²)

G = Shaft modulus of rigidity. (GPa)

And by substitution above equation in equation (38) produce the following :

$$\tau(x,t) = 4(G\gamma_o) \sum_{\substack{i=1,3,5\\ \nu=1,3,5}}^{\infty} \cos(\frac{i\pi x}{2\ell}) \cdot \frac{1}{i\pi} [(-1)^{\frac{i-1}{2}} \cos\left(\frac{i\pi a}{2\ell}\right) t + \frac{w_o}{\gamma_o \cdot a} \sin(\frac{i\pi a}{2\ell}) t \dots \dots (40)$$

The value of static shear stress as the following definition :[1]

And by definition dynamic stress at shaft fixed area to static shear stress produce [surface shear stress (SSSR)] or according to the following formula :

$$SSSR = \frac{\tau(o,t)}{\tau}$$

$$\therefore SSSR = 4 \sum_{i=1,3,5}^{\infty} \frac{1}{i\pi} \left[(-1)^{\frac{i-1}{2}} \cos\left(\frac{i\pi a}{2\ell}\right) t + \frac{W_o}{\gamma_o a} \sin\left(\frac{i\pi a}{2\ell}\right) t \dots \dots (42)$$

In order to formulating the present equation form (35,42) to view more application to be compatible with engineering knowing's to case study ,& according to angular frequency which represent main frequency by the following formula :

Can be extracting time of one cycle :

The formula related with material property of rotating shaft can be describe it by symbol (AVS) as following :

$$AVS = \frac{w}{a} \cdot \frac{1}{\gamma_o} = Angular \, Velocity \, ratio \, to \, shear \, strain \dots \dots \dots (45)$$

And application tow relationships(44,45)in function(TADR&RSSR) product as following :

$$TADR = 8 \sum_{i=1,3,5}^{\infty} \frac{(-1)^{\frac{i-1}{2}}}{\pi^2 i^2} [(-1)^{\frac{i-1}{2}} \cos 2\pi i (\frac{t}{T}) + AVS \sin 2\pi i (\frac{t}{T}) \dots \dots \dots \dots (46)$$

$$SSSR = 4 \sum_{i=1,3,5}^{\infty} \frac{1}{i\pi} [(-1)^{\frac{i-1}{2}} \cos 2\pi i (\frac{t}{T}) + AVS \sin 2\pi i (\frac{t}{T}) \dots \dots \dots \dots (47)$$

5 - PRACTICAL WORK :

1 - Selection perform mean carbon steel C_{15} according to properties (DIN) and it chemical compounding illustrated in table (1) as following :[12]

2- Associating perform testing shaft in triple decoder vise for normal turning machine multi velocities as in fig .(9)and bracing accessory of shear stress test equipment torsion result, And there readings represent twist angle values ,molest install on shaft by andiron contain Thinly bush with thickness (0.5mm) to reduce the lost in practicality readings for different velocities ,and by knowing metal properties for shaft and rotating torque for electrical motor to turning machine can be calculate static twisting angle values by application of the following equation .[13]

Where:

T=Rotation moment (Power=T. ω)(ω =2 π N)

 ℓ =Rotating shaft length(100mm)

G=Rigidity modulus for shaft material(Pa)

3 -By dividing dynamic angular twist values $[\theta(x,t)]$ on static angular twist values (θ_0) can be

Get important practical ratio $\left[\frac{\theta(x,t)}{\theta_0}\right]$ to compare with theoretical ratio from equation (35)which represent shear strain ratio in equation (46)and shear stress in equation (47) that

can be calculate it theoretical by using suitable computer program to get maximum

Values curve for angular displacement ratios with different values for mechanical

Properties variable to material shaft(AVS).

4 -Enable putting table for variable values (AVS)according to number of providing machine Turning revolutions in gear box table according to the following relationships :

$$AVS = \frac{w}{a} \cdot \frac{1}{\gamma_o}$$

$$\tau_o = G \cdot \gamma_o \qquad \quad w = \frac{2\pi N}{60} \quad \quad a = \sqrt{\frac{G}{\rho}}$$

According to shaft metal properties which using as previously mention its metallic construct T=275 MPa $\,$, G=76 GPa $\,$, ρ =7700 Kg/m^3

The relationship between number of revolutions and variable mechanical properties was (AVS=0.0092*N)and the following table illustrate practical and theoretical using values..

6 - RESULTS & DISCUSSION :

- 1- According to equation(46)represent dynamic angular displacement ratio to static angular Displacement by using computer program (Visual Fortran program under window)can be Drawing of curves (3,4,5,6,7,8,9,10)for variable (AVS).[13]
- 2 From practical angular displacement ratio which symmetry in its values shear stress and Maximum values for theoretical shear stress ratio ,can be putting table (3),and by using Static relationship can be determine standard deviation and correction factor for practical And theoretical results .[14]

N = No. of statements.

$$\left(\frac{\theta_d}{\theta_o}\right)mean = \frac{\sum \left(\frac{\theta_d}{\theta_o}\right)exp.}{N}$$

3 - to achieving the probative twisting case that is angular displacement at shaft limb and the produce shaft at root or attaching area for steel shaft which fixing from one limb by using simulation program and according to model partition to finite element and by using (auto disc inventor program)and using practical information's

With turning machine power (1500w) and rotating velocity (120rpm), hence, can be:

Calculate twisting torque value (P=T. $(2\pi N/60)$ =1193 N.m) that is illustrate in Figures (11,12,13).[15,16]

The reading produce practically was dynamic angular displacement readings during rotational moving of shaft produces torsional vibration different rotational velocities and by dividing on static angular displacement produces equally ratio for shear

Stress ratio at shaft fixing area , and by compare practical and theoretical values as in Table (2) ,molest illustrate data gradient magnitude and correction factor (R^2),and can be illustrate the following :

a - The variable (AVS) ascribe for it direct influence on displacement ratio or strain ratio or stress ratio and its role depend on shaft physical properties.

b - The stress proportionally with shaft angular velocity and inversely with static shear strain and spread velocity of stress(a) and the result is effect on shaft fixing ,while in cutting case by turning machine therefor will be effect on cutting quality .

 \mathbf{c} - Illustrate that the variable (AVS)effect on moving function form and curve quality .

d - The sudden stop case of shaft will be effect on its working life with continuous increasing to stress ratio with increasing of variable (AVS).

 \mathbf{e} - The old turning machine will be produce high vibration ,also that connection case of angular strains reading equipment on the rotating shaft by a few thick bush will effect surely on practical readings for angular displacement .

Elem	C%	Si%	Mn	P%	S%	Ni%	Cr%	Mo%	V%	Cu%	Zn%
ent			%								
Actu	0.163	0.201	0.50	0.016	0.023	0.113	0.089	0.01	0.002	0.031	0.002
al			3								
ratios											

Table(1) : chemical compounding for mean carbon steel C₁₅

AVS	N(rpm)
0.1	15
0.4	45
1.2	140
1.4	160

Table(2) illustrate (AVS)values with revolution number

Tabe (3)

Ν	AVS	(RSSR) _{exp.}	(RSSR) _{th.}	$\Delta(RSSR)$			
15	0.1	1.29	1.12	0.17			
45	0.4	1.58	1.38	0.2			
140	1.2	2.65	2.2	0.45			
160	1.4	2.8	2.5	0.3			
Standard Deviation (S.D.)=0.6							
Correction Factor (R ²) =66%							





Figure (1)



Fig.(2): illustrate progressive of angular velocity from fixed area to shaft free tip



Fig.(3) illustrate one torsional vibration cycle for rotate shaft with Velocity(15rpm)&(AVS=0.1)to shear strain ratio



Fig.(4) illustrate one torsional vibration cycle for rotate shaft with Velocity(15rpm)&(AVS=0.1)to shear stress ratio



Fig.(5): illustrate one torsional vibration cycle for rotate shaft Velocity(45rpm)&(AVS=0.4)to shear strain ratio







Fig.(7): illustrate one torsional vibration cycle for rotate shaft with Velocity(145rpm)&(AVS=1.2)to shear strain ratio





Velocity(160rpm)&(AVS=1.4)to shear strain ratio



Fig.(10) : illustrate one torsional vibration cycle for rotate shaft with Velocity(160rpm)&(AVS=1.4)to shear stress ratio



Fig. (11): illustrated max. displacement value at fixed right shaft tip when subjected torsional force on simulation model by auto disc inventor



Fig. (12): illustrate details of using equipments in practical work

REFERENCES :-

- * Aggurwal .Er.R.K."Metal fatigue due to excess vibration &dynamic stresses on hydro Power plant",2008.Member(Power)BBMB.
- * DOYLE . L . E ,"Manfacturing Processes & Material for Engineers",1973
- *- Gatzwiller.K."Measurment of Torsional vibration", 2005
- * Palazzolo"Simulation Software for Torsional vibration Prediction of Machinery Trains with Variable Frequency Drive Motors",2011-2012,a-palazzolo@tamu.
- * PISKUNOV . N, "Differential & Integral Calculus ", 2nd ed.1974.
- * Presz.W & Andersen .B. "Analysis & modeling", 2006.
- *-Satori .D.R."What is Torsional Vibration?".Follow@wise GEEK.2012
- * Schlaifer .R."Torsional Vibration",1950

- * Selim.M.M."Torsional Vibration of carbon nanotubes under initial compression Stress",2009.brazilian Journal of physics.
- *- Shigly.J. "Machine Design",1983
- * Szenasi.F.R.&Blodgett.l.e."Isolation OF Torsional Vibration In Rotating Machinery", 1975.
- * Takeuch.T. "Simlution of Torsional Vibration",2003
- * Timoshenko .B .S . ,Yong .D.H & Weaner .w "Vibration Problems in Engineering",4th ed.1974

*عبيد ، صباح حاتم "التقييم العملي والنظري لأجهاد الصدم المؤثر على عدة الثقب "2005 ، رسالة ماجستير