STUDYING THE EFFECT OF TEMPERATURE ON THE BUCKLING LOAD OF LAMINATED COMPOSITE PLATE UNDER DIFFERENT BOUNDARY CONDITIONS

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ABSTRACT

In this study, effect of temperature, aspect ratio, number of layers and boundary conditions on critical buckling load of composite laminated plate was investigated numerically. Six different boundary conditions: (SSSS, SSFC, SSFS, SSFF, SSCC and CCFF), three temperatures (40C, 60C and 80C) and four aspect ratios (1, 1.3, 1.5 and 2) would consider. These variables were analyzed with different types of laminated plate (unidirectional, cross - ply and angle - ply). The thickness of the plate was changed by increasing the number of layers.

The results showed that in the case of (a/b = 1 and θ = 0), and the temperature changed from (40C) to (80C), the maximum value of critical buckling load dropped by about (53 %) for (CCFF), while the minimum value of critical buckling load dropped by about (63 %) for (SSCC).

Also, it was found that the maximum value of critical buckling load dropped by about (60 %) at the ply orientation (0/0) and (20/-20) at a/b = 1 and CCFF, while the minimum value drops by about (64 %) at the ply orientation (0/0), a/b = 1 and SSCC as the temperature changed from 40C to 80C.

Keywords : Plate, Composite, Finite element, Buckling, Temperature load

دراسة تأثير درجة الحرارة على حمل الانبعاج للصفائح المصنوعة من مواد مركبه تحت ظروف حدية مختلفة

الخلاصة

تم في هذه الدراسة ، دراسة تأثيـر درجـة الحرارة, نسبـة الطول إلى العرض ، عدد الطبقات وحالات التثبيت على حمل الانبعاج في الصفائح المركبـة عدديا.المتغيرات التي درست تضمنت ستة ظروف حدية (SSSS ، SSFC ، SSFS ، SSFF ، SSCC و CCFF)، ثلاث احمال حرارية (^{o}C40 ، ^{o}C60 و ^{o}C80) ونسب مختلفه من الطول الى العرض (1 ، 1.3 ، 1.5 ، 2). ثلاثة انواع من الصفائح تمت دراستها وهي(unidirectional، cross – ply وangle – ply).

أظهرت النتائج أنه في حالة (a/b = 1، θ = 0) ، وتغيير درجة الحرارة من (C40)إلى (C80), القيمه العظمى للانبعاج انخفضت بنحو (53%)لـ(CCFF) ، في حين انخفضت القيمة الصغرى بنحو (63%) لـ(SSCC). واظهرت النتائج ايضا انخفاض القيمة العظمى للانبعاج بحوالي (60 %) لاتجاه الالياف (0/0) (20 and/-20)عند نسبة الطول الى العرض تساوي 1 و تحت الظرف الحدي CCFF في حين انخفضت القيمه الدنيا للانبعاج الى (64 %)لاتجاه الالياف (0/0) عند نسبة الطول الى العرض تساوي 1 و طريقة التثبيتSSCC عند تغير درجة الحرارة من(^{o}C40) الى (^{o}C 80).

كلمات اساسية : صفيحة، مواد مركبة، طريقة العناصر المحددة، الانبعاج، الحمل الحراري

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INTRODUCTION

Laminated composites find wide applications in aero-space structures. In addition to the mechanical loads, they are subjected to a thermal load which is caused by aerodynamic heating. Thermal loads are important factors in composite structures. These composite structural components are subjected to severe thermal loads and are found to buckle at high temperature without the application of mechanical loads Singha [2000]. The external skin of high speed flight vehicles experiences high temperature rise due to aerodynamic heating, which can induce thermal buckling and dynamic instability Ibrahim et. al. [2007].The earliest studies to examine behavior effect of thermal loading on buckling of composite plates were conducted by Lien-Wen and Lei-Yi [1989] who investigated the thermal post-buckling behavior of composite laminated plates subjected to a non uniform temperature by finite element method. The effect of thermal residual stresses on the buckling load of stringer reinforced composite plates was investigated by Muller and Hansen [1997]. These stresses arise the difference in thermal expansion coefficients and elastic behavior of the plate and the stiffeners. It is shown that thermal residual stresses can be tailored to significantly enhance the performance of the structure.

William [1998] studied the mechanical and thermal buckling strengths analysis of the perforated rectangular plates with central circular and square cutouts used finite element structural analysis method. Pandey et. al [2006] presented buckling and post-buckling response of moderately thick laminated composite rectangular plates subjected to in-plane mechanical and uniform temperature loading. The buckling /limiting load and post-buckling strength increase with increase in number of layers but become insignificant if the number of layers is more than 8 to 10. Bojanic [2008] focused on stability analysis of the thin-walled isotropic and composite structures subjected to both mechanical and thermal loading.

From the literature review, it is clear that a substantial amount of work has been done on the buckling analytical solution with using finite element method of the composite laminated plates and a little research on thermal buckling and no work on the effect of temperature on buckling load under various boundary conditions.

In this study, effect of temperature loading on buckling of composite plate is carried out for cross play and angle ply laminated composite plate for different boundary conditions using ANSYS APDL Language. Glass/epoxy composite are used for the composite laminated plates. The buckling behavior of the plates are investigated for different stacking sequences, aspect ratio and temperature load.

MATHEMATICAL FORMULATION

The laminated plate of length (a) and width (b) is consist of (N) layers. Each layer is of thickness t_k, so that $h=\sum_{k=1}^{N} t_k$ is the total thickness of the laminate **Fig.1** Yapici [2005].
The difference between the plate and ambient temperatures is T.
The linear stress-strain relation for each layer in the x,y axes has the form

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}_k=\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k\begin{Bmatrix}\varepsilon_x-\alpha_x\Delta T\\ \varepsilon_y-\alpha_y\Delta T\\ \gamma_{xy}-\alpha_{xy}\Delta T\end{Bmatrix}_k$$

(1)

$$\begin{Bmatrix}\tau_{yz}\\ \tau_{xz}\end{Bmatrix}=\begin{bmatrix}\bar{Q}_{44} & \bar{Q}_{45}\\ \bar{Q}_{45} & \bar{Q}_{55}\end{bmatrix}\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}$$

Where $_{x}$, $_{y,}$ $_{xy,}$ $_{yz,}$ and $_{xz}$ are the stress components, $\bar{Q}_{ij}$ are the transformed reduced stiffnesses, which can be expressed in terms of orientation angles and engineering constants of the material Chen et. al. [1991]; α_x and α_y are the coefficients of thermal expansion along x and y axes, respectively, but α_{xy} is the coefficient of thermal shear;

(2)

$$\alpha_x = \alpha_1 \cos^2\theta + \alpha_2 \sin^2\theta,$$
$$\alpha_y = \alpha_2 \cos^2\theta + \alpha_1 \sin^2\theta,$$
$$\alpha_{xy} = 2\left(\alpha_1 - \alpha_2\right)\cos\theta\sin\theta,$$

Where α_1 and α_2 are the coefficients of linear thermal expansion of the lamina along and across the fiber direction, respectively.

In this study, the first-order shear deformation theory (FSDT) is used . The FSDT extends the kinematics of CLPT by including a gross transverse shear deformation in its kinematic assumptions: i.e., the transverse shear strain is assumed to be constant with respect to the thickness coordinate. Inclusion of this rudimentary form of shear deformation allows the normality restriction of the CLPT theory to be relaxed Reddy [2004].The displacements u,v, and w can be expressed as

$$u(x,y,z) = u_o\left(x,y\right) + z\psi_x\left(x,y\right)$$

(3)

$$v(x,y,z) = v_o\left(x,y\right) - z\psi_y\left(x,y\right)$$
$$w(x,y,z) = w_o\left(x,y\right)$$

Where u_o, v_o, and w_o are displacements of midsurface points; $_x$ and $_y$ are the rotation angles about the normal to the y and x axes, respectively. The bending strains $_x$ and $_y$ and the transverse shear strains $_{xy}$, $_{yz}$, and $_{xz}$ at any point of the laminate are

(4)

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{vmatrix} \dfrac{\partial u_o}{\partial_x} \\ \dfrac{\partial v_o}{\partial_y} \\ \dfrac{\partial u_o}{\partial_y} + \dfrac{\partial v_o}{\partial_x} \end{vmatrix} + z \begin{vmatrix} \dfrac{\partial \psi_{xo}}{\partial_x} \\ \dfrac{\partial \psi_y}{\partial_y} \\ \dfrac{\partial \psi_x}{\partial_y} + \dfrac{\partial \psi_y}{\partial_x} \end{vmatrix}, \begin{vmatrix} \gamma_{yz} \\ \gamma_{xz} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial w}{\partial y} - \psi_y \\ \dfrac{\partial w}{\partial x} + \psi_x \end{vmatrix}$$

The resultant forces N_x, N_y, and N_{xy}, the moments M_x, M_y, and M_{xy} and the shearing forces Q_x and Q_y per unit length of the plate are given as

(5)

$$\begin{bmatrix} N_x & M_x \\ N_y & M_y \\ N_{xy} & M_{xy} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} (1, z)\, dz$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz$$

The total potential energy of the laminated plate under the thermal loading is

(6)

$$\Pi = U_b + U_s + V$$

Where U_b is the energy of bending strain, U_s is the energy of shear strain, and V is the potential energy of in plane strains due to the change in temperature Yapici [2005]:

(7)

$$U_b = \frac{1}{2} \int_{-h/2}^{h/2} \left[\iint_R \left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy} \right) dx\, dy \right] dz$$

(8)

$$U_s = \frac{1}{2} \int_{-h/2}^{h/2} \left[\iint_R \left(\tau_{xz} \gamma_{xz} + \tau_{yz} \gamma_{yz} \right) dx\, dy \right] dz$$

(9)

$$V = \frac{1}{2} \iint_R \left[\bar{N}_1 \left(\frac{\partial w}{\partial x} \right)^2 + \bar{N}_2 \left(\frac{\partial w}{\partial y} \right)^2 + 2\bar{N}_{12} \left(\frac{\partial w}{\partial x} \right) \left(\frac{\partial w}{\partial y} \right) \right] dx\, dy - \int_{\partial R} \left(\bar{N}_n^b u_s^o \right) ds$$

$\bar{N}_n^b$ and $\bar{N}_s^b$ the in plane loads applied to the boundary. For equilibrium, the potential energy must be stationary. The equilibrium equations of the cross-ply laminated plate subjected to temperature changes can be derived from the variational principle through use of stress strain displacement relations. These equations can be obtained form the condition $\delta\Pi = 0$. Bathe [1996]

FINITE ELEMENT FORMULATION

In general, it is difficult to obtain a closed- form solution for buckling problems Bathe [1996]. Therefore, numerical methods are usually used to find an approximate solution.

In order to study the buckling of the plate, an eight node Lagrange finite element is employed in this study using ANSYS program. The stiffness matrix of the plate is obtained using the principle of minimum potential energy. The bending stiffness $[K_b]$, and geometric stiffness $[K_g]$ matrices can be expressed as

(10)

$$\left[k_b \right] = \int_A \left[B_b \right]^T \left[D_b \right] \left[B_b \right] dA$$

(11)

$$\left[k_g\right] = \int_A \left[B_g\right]^T \left[D_b\right]\left[B_g\right] dA$$

Where

$$\left[D_b\right] = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix}$$

$$\left[D_g\right] = \begin{bmatrix} \bar{N}_1 & \bar{N}_{12} \\ \bar{N}_{12} & \bar{N}_2 \end{bmatrix}$$

(12)

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \bar{Q}_{ij}\left(1, z, z^2\right) dz = \sum_{k=1}^{N} \int \bar{Q}_{ij}^{(k)}\left(1, z, z^2\right) dz \quad (i, j = 1, 2, 6)$$

$$(A_{44}, A_{55}) = \int_{-h/2}^{h/2} \left(\bar{Q}_{44}, \bar{Q}_{55}\right) dz$$

or

$$A_{ij} = \sum_{k=1}^{N} \bar{Q}_{ij}^{(k)}\left(z_{k+1} - z_k\right)$$

$$B_{ij} = \frac{1}{2}\sum_{k=1}^{N} \bar{Q}_{ij}^{(k)}\left({z^2}_{k+1} - {z^2}_k\right)$$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{N} \bar{Q}_{ij}^{(k)}\left({z^3}_{k+1} - {z^3}_k\right)$$

Where [A] Extensional stiffness matrix.
[B] Bending – extension coupling matrix.
[D] Bending stiffness matrix.

k_1 and k_2 are the shear correction factors; for a rectangular cross section $k_1^2 = k_2^2 = 5/6$ Whitney [1973].
The application of the principle of total potential energy to the plate leads to the eigenvalue problem

(13)

$$\left[\left[K_o\right] - \lambda\left[K_{og}\right]\right]\begin{Bmatrix} u_i \\ v_i \\ w_i \end{Bmatrix} = 0,$$

(14)

$$\left[K_o\right] = \left[K_b\right], \qquad -\lambda_b\left[K_{og}\right] = \left[K_g\right]$$

ELEMENT GEOMETRY

ANSYS provides a variety of element types ranging from one dimensional element to three dimensional elements. For the purpose of the present work, an element called (SHELL281 - 8-Node Finite Strain Shell) is used.

SHELL281 is suitable for analyzing thin to moderately-thick shell structures. It is an 8-node element I, J, K, L, M, N, O and P with six degrees of freedom at each node; translations in the x, y, and z axes, and rotations about the x, y, and z-axes as shown in **Fig. 2**.

SHELL281 may be used for layered applications for modeling laminated composite shells or sandwich construction. The accuracy in modeling composite shells is governed by the first order shear deformation theory.

FINITE ELEMENT MODEL

The physical structure that used in this work is a fiber reinforced composite plate, shown in **Fig. 3**. The model was developed in ANSYS 12.1, using the 100 elements. The global x coordinate is directed along the length of the plate, while the global y coordinate is directed along the width and the global z direction is taken to be the outward normal of the plate surface. There are 10 elements in the axial direction and 10 along the width. Reasons for choosing the particular mesh used in this study will be described later in the discussion on convergence study. A linear buckling analysis was performed on the model to calculate the minimum buckling load of the structure.

The plates were analyzed under six different boundary conditions: SSSS, SSFS, SSFF, SSFC, SSCC and CCFF. **Fig.4** to **Fig. 9** visually show the boundary conditions and the applied load as they were entered into ANSYS.

MATERIAL PROPERTIES

The material properties used throughout this study are shown in **Table 1**. These properties are obtained by using mechanics of material approach Jones [1999] except the property G_{23} at which the semiempirical stress partitioning parameter (SSP) technique by Barbero [1998] is used to calculate its value. It is assumed that $E_2=E_3$, $G_{12}=G_{13}$, $_{12}=_{13}$.

The fiber glass (*f*) and the epoxy matrix (*m*) have the following mechanical properties:

(15)

E_f = 72.4GPa ; G_f = 29.67 GPa ; $_f$ = 0.22

E_m= 3.4 GPa ; G_m = 1.27 GPa ; $_m$ = 0.34

Table 1 Material Properties (Glass – Epoxy)

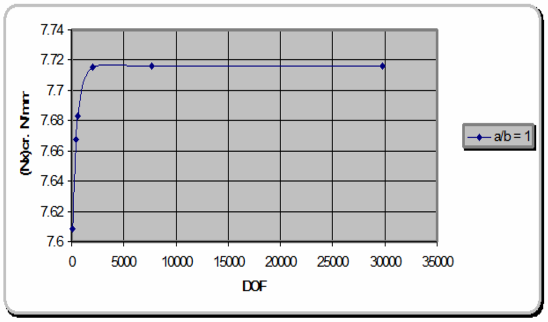
13	23	12	G_{13} (MPa)	G_{23} (MPa)	G_{12} (MPa)	E_3 (MPa)	E_2 (MPa)	E_1 (MPa)
0.26	0.34	0.26	2312	2345	2312	5235	5235	28522

Mesh Convergence

The convergence results of ANSYS computer program must be achieved in order to decide the optimum number of degrees of freedom (DOF) for different lamination schemes.

The size and number of elements influence directly the accuracy of the results. However, if the number of elements is increased beyond a certain limit, the accuracy will not be improved by any significant amount.

A Plot of the convergence study for one of the suggested cases is given in **Fig. 10**.



PROGRAM VERIFICATION CASE STUDY

In order to confirm the reliability of the ANSYS program three verification cases are considered. The first case is verify effect of layer numbers and boundary conditions on critical buckling loads of cross - ply square plates while the second case is built to devoted to test the ANSYS program for effect of both aspect ratio and degree of orthotropy on the critical buckling load of rectangular laminate plate,. Finally, the third case is concerned with the effect of thermal load on deflection of laminated composite plate with various boundary conditions. **Tables** from **2** to **4** shown the verification results.

Table (2) Comparison of Results of Critical Buckling Load obtained from ANSYS and Ready [1989]

No. of Layer	Theory	SSSS	SSFC	SSFS	SSFF	SSSC
Layer-2	**Ready**	11.3530	6.1660	5.3510	4.8510	16.437
	ANSYS	11.4708	6.0504	5.3385	4.6281	16.1740
	% Diff	1.0372	1.8742-	0.2327-	4.5954-	1.6260 -
Layer-10	**Ready**	25.4500	14.3580	12.5240	12.0920	32.614
	ANSYS	24.9763	14.1693	12.3451	11.9682	32.9151
	% Diff	1.8611-	1.3145-	1.4281 -	1.0235 -	0.91477

Table (3) Comparison of Results of Critical Buckling Load obtained from ANSYS and Reddy [2004]

a/b	Theory	E_1/E_2				
		5	**10**	**20**	**30**	**40**
0.5	**Reddy**	13.9000	18.1260	21.8780	22.8740	24.5900
	ANSYS	13.7651	17.9084	21.4444	22.3243	23.6843
	% Diff	0.9703 -	1.2004 -	1.9820 -	2.4031 -	3.6832 -
	Reddy	5.6500	6.3470	6.9610	7.1240	7.4040
	ANSYS	5.6009	6.3016	6.9101	7.0674	7.3276
	% Diff	0.8692 -	0.7152 -	0.7307 -	0.7939 -	1.0322 -
1.5	**Reddy**	5.2330	5.2770	5.3100	5.3180	5.3320
	ANSYS	5.1952	5.2456	5.2826	5.2910	5.3020
	% Diff	0.7219 -	0.5952 -	0.5152 -	0.5082 -	0.5619 -

Table (4) Comparison of Results] Effect of Thermal Load on Deflection obtained from ANSYS and Khder et .al[1992] .

Laminate	b/h	Theory	SS	SC	CC	FS
0	5	Khder et al	1	0	0	1
		ANSYS	1	0	0	1
		% Diff	1.9	0.2	1.0	1.3
	10	Khder et al	1	0	0	1
		ANSYS	1	0	0	1
		% Diff	1.6	0.0	0.2	1.5
0/90/0	5	Khder et al	1	0	0	1
		ANSYS	1	0.7691	0	1
		% Diff	2.0	5.	7.5	1.6
	10	Khder et al	1	0	0	1
		ANSYS	1	0	0	1
		% Diff	2.2	4.1	4.9	2.0
10 ....90/0 Layers	5	Khder et al	1	0	0	1
		ANSYS	1	0	0	1
		% Diff	1.6	1.9	1.8	2.7
	10	Khder et al	1	0	0	1
		ANSYS	1	0	0	1
		% Diff	1.6	0.9	0.9	2.7

RESULTS AND DISCUSSION

In this study, the critical buckling load of the laminated composite plate subjected to uniform thermal loading with different temperatures (40C, 60C and 80C) is considered. Fig. (11) shows the relation between the temperature and the critical buckling load for orthotropic laminated plate with different aspect ratios and boundary conditions. It can be noted that in case of (a/b = 1 and θ = 0), and the temperature change from (40C) to (80C), the maximum value of critical buckling load drops by about (53 %) for (CCFF), while the minimum value of critical buckling load drops by about (63 %) for (SSCC).

Figs.(12) and (13) present that in the case of boundary condition (SSFS) and (a/b=2) the behavior of buckling load with temperature is different, at which the curve is concave for unsymmetric laminate $0^o/90^o$ and is convex for symmetric laminate $0^o/90^o/0^o$ (the same behavior at 0^o laminate) and that can be explain as the effect of boundary condition and aspect ratio may be differ for symmetric and unsymmetric laminate. when the aspect ratio is constant (a/b = 1), and the temperature changes from (40C) to (80C), the maximum value of critical buckling load drops by about (43 %) for ply orientation (0/90) and by about (58 %) for ply orientation (0/90/0) and both are subjected to (CCFF) boundary conditions while the minimum value of critical buckling load drops by about (65 %) for both ply orientation of (0/90) and (0/90/0) and both are subjected to (SSCC) boundary conditions. Also it is noted from tables 5, 6, 7 that in the case of boundary condition(SSSS, SSFF and SSCC) for (a/b=1 and T=40^oC) the buckling load decrease with increasing the value of angle ply orientation (i.e. For $20^o/-20^o$ to $60^o/-60$) while the behavior is inverse in the case of boundary conditions(SSFC and SSFS) in which the buckling load increase with increase in angle ply orientation and one can explain this contacted behavior as the sever effect of boundary conditions on buckling load of heated laminated plates.

It can be noted that the general behavior of critical buckling load is the same for increasing aspect ratios and with increasing numbers of layers to 3 or 4- layers.

Also, it is clear from these tables that in the case of boundary conditions (SSSS, SSSC, SSFS, SSCC and CCFF) the buckling load increase with increase of aspect ratio for all (/-, /-/ and /-//- where =20°, 40 °, 60°) while the same behavior can be noted for boundary condition (SSFF) and angle ply =20° in contract to the boundary condition (SSFF) and ply angle =60° in which the buckling load nearly remain constant for different aspect ratio and this also can be explain as the effect of boundary conditions on buckling load of angle ply laminate .

CONCLUSIONS

The present work deals with determination of the effect of temperature, plate's aspect ratio, number of layers and various plate boundary conditions on critical buckling loads.

Based on the results obtained from this study for laminated composite plate, it can be concluded that:-

1. The temperature applied to the specimen affects the critical buckling load. When temperature value increases, the critical buckling load decreases. It can be noted that in the case (a/b = 1, θ = 0), as the temperature changes from 40C to 80C, the maximum value of critical buckling load drops by about 53 % for (CCFF), while the minimum value drops by about (63 %) for (SSCC).
2. The length to width ratio affects the critical buckling load. The critical buckling load increases as a/b ratio increases. It can be noted that the maximum value of critical buckling load is about (449.88 N/mm) for (0/90/0) at (a/b = 2, T = 40C and CCFF), while the minimum value is about (2.33 N/mm) for (0/90) at (a/b = 1, T = 80C and SSCC).
3. The change in fiber orientation affects the critical buckling load. It can be noted that in case of changing the fiber orientation [(0/0), (0/90), (20/-20), (40/-40), (60/-60)], as the temperature changes from 40C to 80C, the maximum value of critical buckling load drops by about (60 %) at the ply orientation (0/0) and (20/-20) at a/b = 1 and CCFF, while the minimum value drops by about (64 %) at the ply orientation (0/0), a/b = 1 and SSCC.

Table (5) Effect of Aspect Ratio, Number of Layers and Boundary Conditions on Buckling Loads

Pcr [N/mm]of Angle-Ply θ = 20Plates

Boundary Conditions	a/b	20-/20			20/20-/20			20-/20/20-/20		
		C^0 40	C^0 60	C^0 80	C^0 40	C^0 60	C^0 80	C^0 40	C^0 60	C^0 80
SSSS	1	28.3469	16.4781	11.2009	80.7254	44.4017	30.4863	146.0200	70.6059	44.9823
	1.3	32.2264	18.1521	12.5940	88.2182	47.5669	32.5018	158.8150	83.9099	56.9672
	1.5	34.5003	18.8419	12.9448	91.9416	48.7525	33.1462	162.5871	84.7794	57.3195
	2	39.6833	20.5538	13.8668	100.0185	51.2188	34.4215	172.4394	87.6350	58.7430
SSFC	1	20.4417	11.6393	8.1096	55.9639	30.7317	21.1509	96.0998	50.9526	34.6426
	1.3	35.3422	20.3551	14.2230	96.8704	53.2700	36.6639	163.6393	87.0358	39.8755
	1.5	47.1187	27.2415	19.0497	128.2155	70.2082	48.2489	215.1328	104.2347	94.2083
	2	60.9569	46.8641	32.7711	212.2152	115.3860	79.1064	350.5159	185.9991	126.4563
SSFS	1	14.1567	7.9771	5.5400	39.8232	21.7264	14.9221	67.3832	35.5642	24.1447
	1.3	22.1847	12.7664	8.9209	63.0002	34.6870	23.8878	105.9810	56.3525	38.3471
	1.5	28.7720	16.7536	11.7391	81.9045	45.0786	31.0270	137.1186	73.1042	49.7816
	2	47.3528	28.1000	19.7455	134.2758	73.2249	50.2266	220.8940	117.8806	80.2699
SSFF	1	17.5610	12.3135	9.2865	75.2063	34.7470	24.7876	113.4793	66.5169	46.8056
	1.3	20.8902	17.2223	13.8917	78.0025	51.9036	38.0128	160.2755	100.7024	72.1380
	1.5	21.6713	19.6382	16.8348	88.2973	63.8393	47.8356	187.0473	125.3316	91.0185
	2	21.6897	21.6041	21.0608	97.0166	88.8636	74.3033	218.9048	185.6405	143.6947

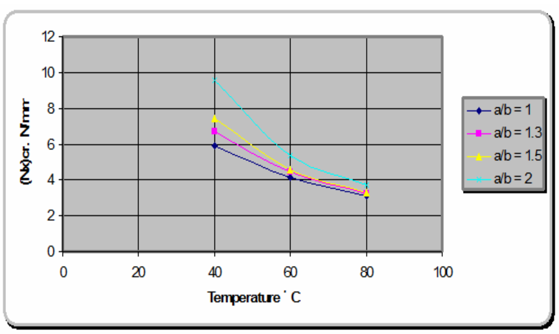
SSCC	1	7.0664	3.6325	2.4445	17.0574	8.6831	5.8238	31.5638	15.9994	10.7154
	1.3	8.6533	4.3876	2.9388	20.6084	10.3986	6.9536	36.8993	18.5775	12.4137
	1.5	10.3114	5.2039	3.4801	24.3647	12.2568	8.1879	42.8583	21.5283	14.3744
	2	16.0615	8.0599	5.3797	37.2064	18.6482	12.4422	63.3105	31.7133	21.1552
CCFF	1	29.4078	16.5656	11.5076	85.4049	45.9539	31.4080	149.5285	79.0220	53.6729
	1.3	42.0952	24.9977	17.6906	127.3699	71.0411	49.1595	224.8668	121.7695	83.3922
	1.5	53.6033	33.8574	24.4807	168.1948	97.4981	68.3218	301.1436	166.9475	115.1670
	2	77.8560	62.2126	48.4808	282.3337	188.3514	134.9039	514.2574	287.0888	197.0832

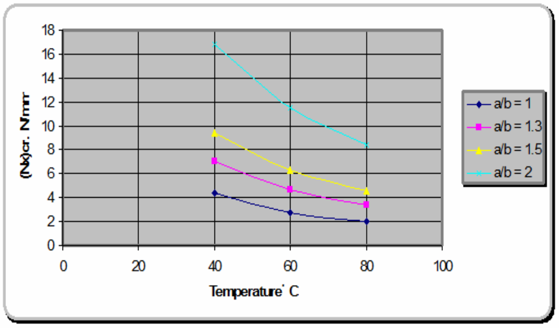
Table (6) Effect of Aspect Ratio, Number of Layers and Boundary Conditions on Buckling Loads Pcr [N/mm]of Angle-Ply θ= 40 Plates

Boundary Conditions	a/b	40-/40			40/40-/40			40-/40/40-/40		
		C^0 40	C^0 60	C^0 80	C^0 40	C^0 60	C^0 80	C^0 40	C^0 60	C^0 80
SSSS	1	25.0963	17.6306	12.4899	73.5828	48.3407	33.1865	174.8219	94.1370	64.3022
	1.3	34.9790	18.7561	12.8055	106.4863	56.3448	38.2883	181.0855	93.4199	62.9359
	1.5	37.3646	19.1822	12.9007	111.0302	56.8256	38.3786	182.4821	92.1423	61.6282
	2	47.7764	23.4105	15.4988	135.4227	66.8066	44.3333	215.0378	106.1722	70.4829
SSFC	1	28.3728	17.5890	12.9987	94.1240	55.7394	39.3775	169.1531	94.4791	65.3752
	1.3	46.1871	30.0682	21.8422	150.9370	87.9169	61.7344	274.0044	153.6960	106.4425
	1.5	59.2811	38.5527	27.9545	184.7879	107.2914	75.1911	342.0677	191.0539	132.1084
	2	92.6677	63.3754	45.7896	184.6410	135.4328	103.7435	521.7179	288.4948	198.8306
SSFS	1	18.7564	11.8075	8.5182	62.1236	36.7507	25.9650	113.5936	63.3422	43.8128
	1.3	28.4222	18.7976	13.7076	96.4913	56.7106	39.8924	177.2930	99.9833	69.3464
	1.5	34.5909	23.3701	17.0768	118.4379	68.6587	48.0797	216.6182	122.1797	84.6980
	2	52.6587	37.2550	27.2109	152.9657	102.6537	72.5449	328.1615	184.0282	127.2680
SSFF	1	11.7724	11.2173	10.1622	52.2212	42.0048	33.0067	115.9216	89.4219	68.6294
	1.3	11.5049	11.5841	11.4668	52.1250	51.0074	45.5689	119.3316	113.4281	97.3596
	1.5	11.2456	11.3363	11.3586	49.8623	51.0565	50.0890	115.2179	116.4954	110.9411
	2	10.7832	10.7627	10.7302	45.2252	46.0638	46.7027	105.6856	107.1653	107.9112
SSCC	1	5.8774	2.9750	1.9915	15.7133	7.9190	5.2932	29.6256	14.9020	9.9545
	1.3	8.1178	4.0819	2.7264	21.8522	10.9646	7.3183	39.5060	19.8077	13.2173
	1.5	10.0426	5.0392	3.3635	27.0904	13.5741	9.0558	47.7608	23.9221	15.9573
	2	16.1475	8.0836	5.3912	43.9390	21.7106	14.4768	72.5364	36.2899	24.1981
CCFF	1	27.9728	17.7171	12.7539	87.0822	50.3778	35.3314	176.9822	99.5155	69.0651
	1.3	41.1194	31.4948	24.6889	142.1259	93.5659	68.5295	308.9704	191.9796	137.4959
	1.5	45.6112	40.7727	34.8971	171.1762	129.4395	98.7969	399.3768	270.6555	195.8977
	2	45.4845	44.7875	43.4233	175.9390	165.1833	146.5714	453.1184	387.4972	276.7569

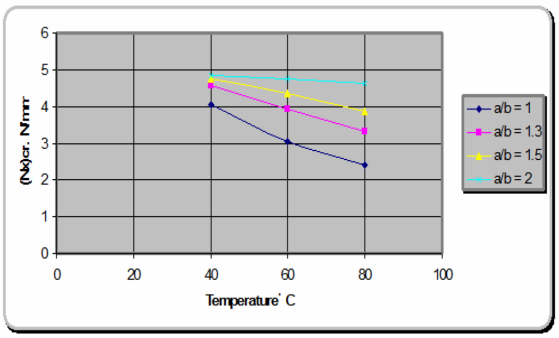
Table(7) Effect of Aspect Ratio, Boundary Conditions on Buckling and Number of Layers θ= 60Plates Loads Pcr [N/mm] of Angle-Ply

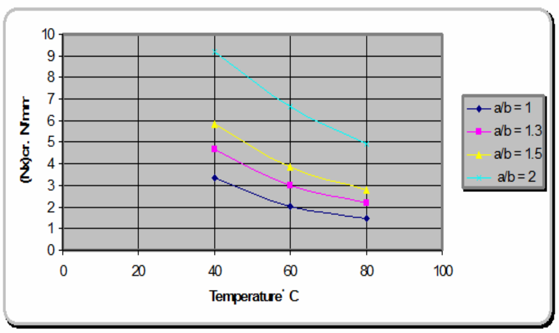
Boundary Conditions	a/b	60-/60			60/60-/60			60-/60/60-/60		
		C^0 40	C^0 60	C^0 80	C^0 40	C^0 60	C^0 80	C^0 40	C^0 60	C^0 80
SSSS	1	17.8021	9.4880	6.4641	53.5212	28.1248	19.0682	88.0391	45.1963	30.3989
	1.3	21.4352	10.7821	7.2013	61.9280	31.1455	20.8017	98.4439	49.1934	32.7880
	1.5	25.2782	12.5257	8.3232	71.8730	35.7585	23.7954	113.2721	56.2289	37.3940
	2	40.6632	20.1265	13.3652	111.0087	55.2859	36.8003	175.6056	87.2082	58.0025
SSFC	1	30.9561	23.5079	17.7546	114.1471	71.0763	50.9684	216.9859	129.6763	91.5690
	1.3	47.8546	37.2798	27.9283	133.7203	102.3568	74.8019	332.0605	195.3510	137.2251
	1.5	56.5276	47.6790	35.6553	134.1570	109.5821	88.1496	413.9701	242.0991	169.6404
	2	76.1731	74.2220	57.8018	183.1467	141.4030	114.5548	623.0495	363.7701	256.4563
SSFS	1	19.4071	15.0530	11.5040	67.9023	44.0550	31.7963	138.5025	84.3140	59.8149
	1.3	25.9317	22.6973	17.5762	99.1975	66.3559	47.4591	204.8925	125.6811	89.0235
	1.5	30.4840	28.9990	22.5488	99.6029	83.3257	60.3159	258.3430	158.1525	111.7877
	2	43.3240	49.3505	38.0439	115.8119	98.8733	83.6137	420.1944	251.3047	176.5677
SSFF	1	7.8662	7.9411	7.9421	28.3578	28.8812	28.6546	66.9479	67.4138	65.2963
	1.3	7.6597	7.7186	7.7637	26.9036	27.3566	27.6855	63.8258	64.8755	65.4669
	1.5	7.5640	7.6014	7.6333	26.2756	26.5638	26.8027	62.3721	63.0871	63.6221
	2	7.4220	7.4325	7.4421	25.4292	25.5212	25.6042	60.3327	60.5673	60.7687
SSCC	1	6.1095	3.0796	2.0586	17.8113	8.9484	5.9751	31.0585	15.5853	10.4026
	1.3	9.0521	4.5422	3.0317	27.0447	13.5463	9.0362	45.1448	22.6035	15.0759
	1.5	11.3699	5.6972	3.8008	34.2526	17.1448	11.4339	55.7702	27.9056	18.6083
	2	18.0289	9.0203	6.0148	54.4407	27.2361	18.1608	84.1717	42.0922	28.0629
CCFF	1	31.1476	27.5322	23.7231	104.9990	83.3180	66.3900	241.6334	183.0593	141.8386
	1.3	32.1567	31.9808	31.3727	111.3921	105.3847	96.1761	270.2437	258.1202	229.7416
	1.5	31.8303	31.8337	31.6580	109.7926	106.1031	100.3255	266.3570	261.7924	248.6971
	2	31.2691	31.1884	31.0253	107.1214	104.9048	101.5290	258.2398	254.7650	247.8197



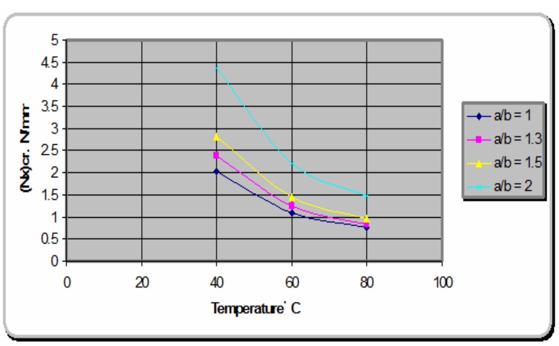


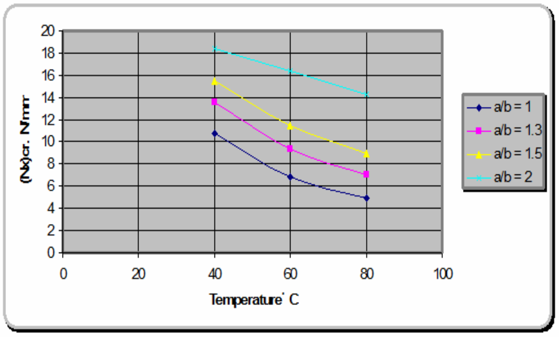
(b) Simple-Simple-Free-Clamp
(a) Simple-Simple-Simple-Simple
(b) Simply*-Simply-*Free-Clam
(a) Simply-Simply-Simply-Simply





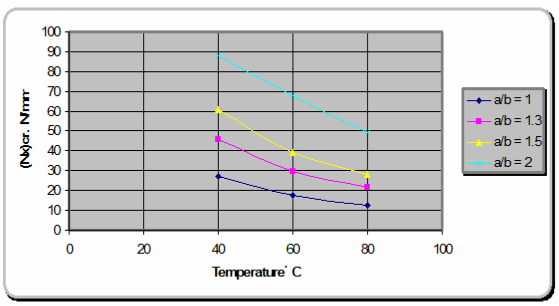
(d) Simple-Simple-Free-Free
(c) Simple-Simple-Free-Simple
(d) Simply-Simply-Free-Free
(c) Simply-Simply-Free-Simply

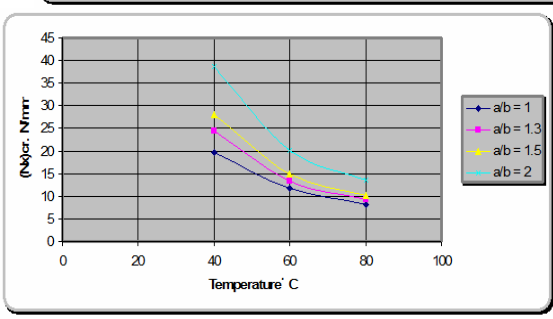




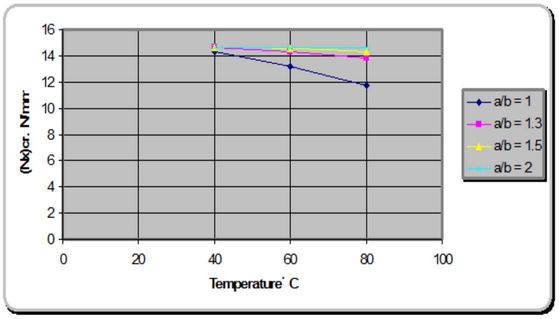
(e) Simple-Simple-Clamp-Clamp
(f) Clamp-Clamp-Free-Free
(f) Clamp-Clamp-Free-Free
(e) Simply-Simply-Clamp-Clamp

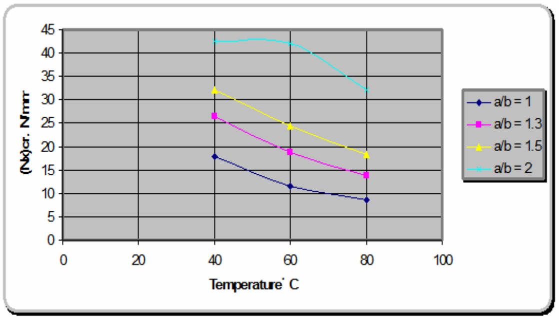
Figure (11) Critical Buckling Load versus Temperature with Aspect Ratio as Parametric for Orthotropic Plates (0) with various Boundary Conditions



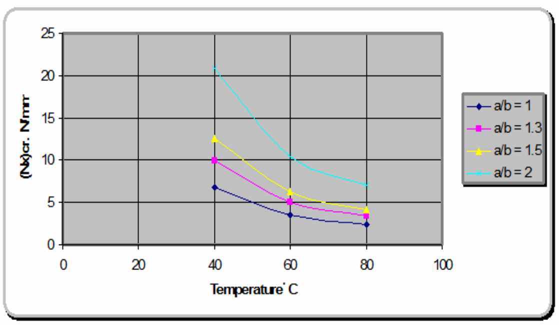


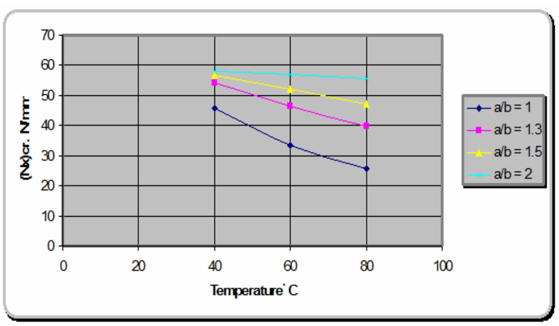
(b) Simple-Simple-Free-Clamp
(a) Simple-Simple-Simple-Simple





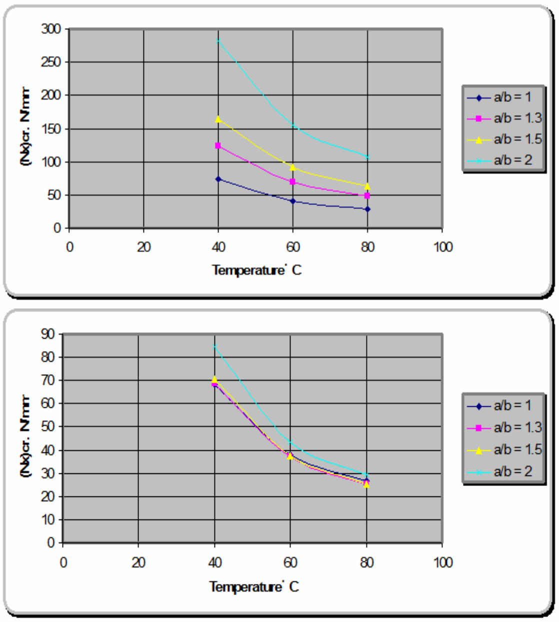
(d) Simple-Simple-Free-Free
(c) Simple-Simple-Free-Simple



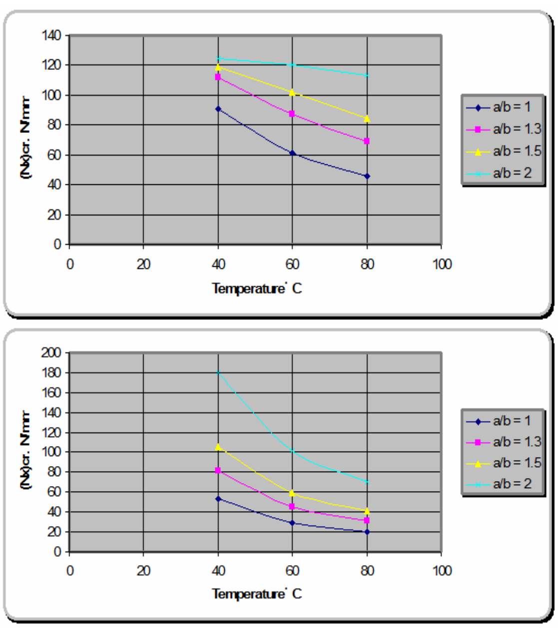


(f) Clamp-Clamp-Free-Free
(e) Simple-Simple-Clamp-Clamp

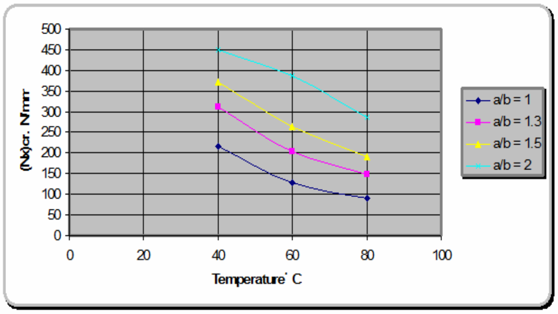
Figure (12) Critical Buckling Load versus Temperature with Aspect Ratio as Parametric for Cross - Ply (0/90) with various Boundary Conditions

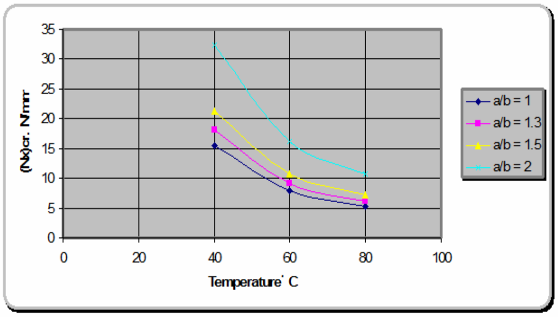


(b) Simple-Simple-Free-Clam
(a) Simple-Simple-Simple-Simple



(d) Simple-Simple-Free-Free
(c) Simple-Simple-Free-Simple





(e) Simple-Simple-Clamp-Clamp

(f) Clamp-Clamp-Free-Free

Figure (13) Critical Buckling Load versus Temperature with Aspect Ratio as Parametric for Cross - Ply (0/90/0) with various Boundary Conditions.

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LIST OF ABBREVIATIONS

Analysis System	ANSYS
Clamped-Clamped-Free-Free	CCFF
Classical laminated plate theory	CLPT
Degree of freedom	DOF
Finite Element	FE
First order shear deformation theory	FSDT
Simple-Simple-Clamped-Clamped	SSCC
Simple-Simple-Free-Clamped	SSFC
Simple-Simple-Free-Free	SSFF
Simple-Simple-Free-Simple	SSFS
Simple-Simple-Simple-Simple	SSSS

LIST OF SYMBOLS

Transformed reduced stiffness matrix	$\overline{Q}_{ij}$
In plane loads	$\overline{N}_n^b$ and $\overline{N}_s^b$
Longitudinal thermal expansion coefficient	$_1$
Transverse thermal expansion coefficient	$_2$
Extensional, bending-extension coupling and bending stiffness	A_{ij}, B_{ij}, D_{ij}

Young's modulus in 1-axis direction	E_1
Young's modulus in 2-axis direction	E_2
Young's modulus in 3-axis direction	E_3
Shear modulus in 1-2 plane	G_{12}
Shear Modulus in 1- 3 plane	G_{13}
Shear modules in 2-3 plan	G_{23}
Total thickness of laminate	h
Moments per unit width	M_x, M_y, M_x
Forces per unit width of the cross section of the laminate	N_x, N_y, N_x
Reduced stiffness matrix	Q_{ij}
Thickness of layer k	t_k
Displacements in the x y, and z direction	u, v, w
The displacement from the undeformed middle surface to the deformed middle surface	u_0, v_0, w_0
Buckling eignvalue	λ
Poisson ratio in 1-3 plan	υ_{13}
Poisson ratio in 2-3 plan	υ_{23}
Poisson ratio in 1-2 plan	υ_{12}