AN APPRAISAL OF EULER AND JOHNSON BUCKLING THEORIES UNDER DYNAMIC COMPRESSION BUCKLING LOADING.

Hussain Abdul Aziz,
Asist. Lecturer
University of Technology
Educational Technology Dep.
Mech. Eng. Division

H.J. Mohamed Al-alkawi.
Prof.
University of Technology
Educational Technology Dep.
Mech. Eng. Division

ABSTRACT:

This research presents an appraisal for Euler and Johnson theories based on experiment tests under compression dynamic buckling load.

20 specimens (columns) made from two materials, namely 1020 Hot Rolled and 5052 Aluminum alloy, are tested under compression dynamic buckling load. The following remarks can be concluded from the present work :

1- Euler (for long columns) and Johnson (for short columns) theories can be used to estimate the dynamic critical buckling load with design factor of 3 or more.
2- Initial deflection of column has an important effect on compression dynamic critical buckling load.

Keywords: Buckling ; Design

الخلاصة :

البحث الحالي يعرض تقييم لنظريتي أويلر و جونسون معتمداً على الفحوصات العملية تحت الاحمال الانبعاجية الضغطية الحركية. عشرون عينة (اعمدة) صنعت من مادتين هماالفولاذ 1020 Steel Alloy Hot Rolled وسبيكة الألمنيوم Aluminum 5052 Alloy وفحصت تحت الأحمال الانبعاجية الانضغاطية الحركية. النقاط التي استنتجت من البحث الحالي هي :

1- نظريتي أويلر (للاعمدة الطويلة) و جونسن (للأعمدة القصيرة) يمكن استخدامها في تقييم الحمل الحرج ألانبعاجي ألانضغاطي الحركي مع عامل تصميمي بمقدار 3 أو اكثر.
2- الانحراف الأولي للعمود له تأثير مهم على الحمل الانبعاجي الضغطي الحركي.

1-2 Factors Affecting column buckling phenomena:

The tendency for a column to buckle is dependent on the shape and dimensions of its cross-section, along with its length and the manner of attachment to adjacent members or supports.

*** The Crosssectional properties that are important are as follows:** [3].

a-The cross-sectional area A

b-The moment of inertia of the cross-section, I , with respect to the axis about which the value of I is minimum.

c-The least value of the radius of gyration of the cross-section R. R is computed from:

$$R = \sqrt{\frac{I}{A}} \quad \text{..................(1)}$$

*** The end fixity and effective length**

End fixity refers to the manner in which the ends of a column are supported. The forms of end restraint are pinned, fixed, and free. Fig.(1) shows the types of end fixity. The manner of support of both ends of the column affects the effective length of the column. Effective length (L_e) may be defined as :

$$L_e = KL \quad \text{..................(2)}$$

Where L=actual length of the column between supports.
K= end fixity constant. Fig.1 gives the theoretical and experimental values of K.

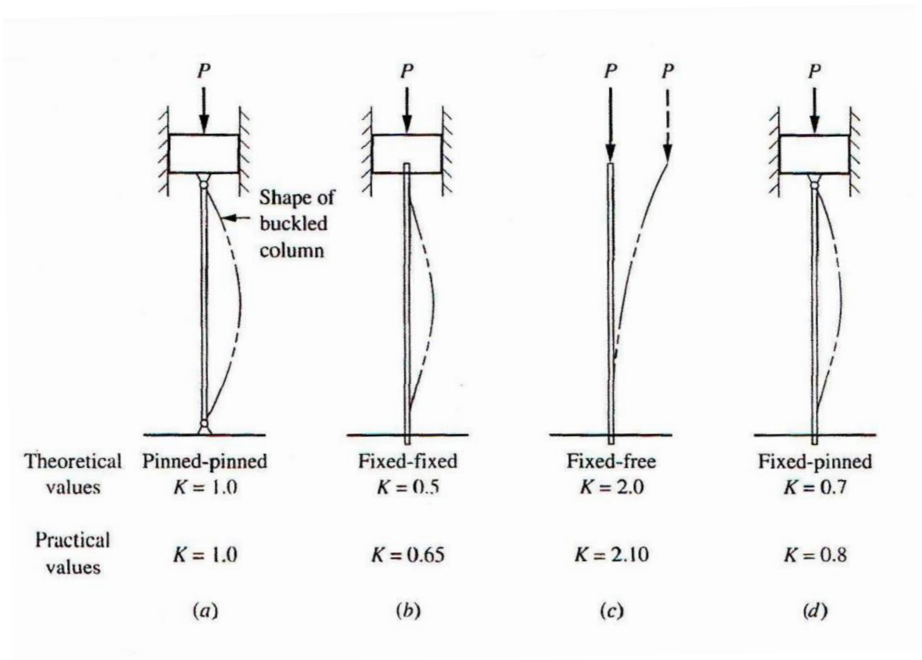

Fig. 1: The types of end fixity.[3]

*** The Slenderness Ratio (S. R.).**[3]

S. R. is the ratio of the effective length to its least radius of gyration. That is

$$S.R.= \frac{L_e}{R_{\min}} = \frac{KL}{R_{\min}} \quad \text{……………..(3)}$$

*** The Column constant (C_c).**[3]

C_c may be defined as

$$C_c = \sqrt{\frac{2\pi^2 E}{\sigma_y}} \quad \text{…………….(4)}$$

Where E = modulus of elasticity of column material.

σ_y = yield stress of the material.

It is clear that the column constant depends on the mechanical properties of material used.

Column are divided into three categories, i.e short column, long column and columns of intermediate length. Table 1 gives the three types of column based on slenderness ratio (S.R.) for different material [4].

Table 1: Slenderness ratio of column for different materials [5].

Material	Short column	Intermediate column	Long column
Structural steel	S.R. $\leq$ 40	40 $\leq$ S.R. $\leq$ 150	S.R. $\geq$ 150
Aluminum alloy 6061-T_6	S.R. $\leq$ 9.5	9.5 $\leq$ S.R. $\leq$ 66	S.R. $\geq$ 66
Aluminum alloy 2024-T_4	S.R. $\leq$ 12	12 $\leq$ S.R. $\leq$ 55	S.R. $\geq$ 55
Wood	S.R. $\leq$ 11 $\leq$ (18 – 30)	11 $\leq$ S.R. $\leq$ (18 – 30)	(18 – 30)$\leq$ S.R. $\leq$ 50

1-3 The Euler and Johnson formulas.

Analysis of a long column employs the Euler formula.[3].

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad \text{………………(5)}$$

More details of deriving and assumption for determining the above equation can be seen elsewhere[1].

It is clear that the buckling load (P_{cr}) is dependent only on the geometry (length and cross section) of the column and the stiffness of the material represented by the modulus of elasticity. The strength of material is not involved at all. For these reasons, it is often of no benefit to specify a high-strength material in a long column application [3].

When the actual (S.R.) for a column $\frac{KL}{R}$ is less than the column constant (C_c) then the column is short and Johnson formula should be used. The Johnson formula is written as follows [5].

$$P_{cr} = A\ \sigma_y \left[1 - \frac{\sigma_y [KL/R]^2}{4\pi^2 E} \right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

The critical load (P_{cr}) in Johnson formula, equation (5), is affected by the strength of the material in addition to its stiffness, E. while strength is not a factor for a long column when the Euler formula is used.

The aim of this work is to appraisal the Euler formula and Johnson formula for long and short columns respectively. This appraisal is dependent on the experimental work, using two alloys under dynamic compression loading.

2-Experimental Work.

This section outlines the details of specimens used and the mechanical properties, two types of materials were used, namely 1020 steel alloy Hot Rolled alloy and 5052 Aluminum alloy. The mechanical properties are given in Table 2.

Table 2 : Mechanical properties for the materials used

1020 Steel alloy Hot Rolled	σ_y MPa	σ_u MPa	Elong. %	Hardness		G GPa	E GPa	ν
				Brinell	Vicker			
Experimental	196	384	22	124	-	79	203	0.265
AISI Standard	207	379	25	111	-	80	207	0.27
5052 Aluminum Alloy	σ_y MPa	σ_u MPa	Elong. %	Hardness		G GPa	E GPa	ν
				Brinell	Vicker			
Experimental	101	198	27.5	45	63	27	70	0.29
AISI Standard	90	193	30	43	62	30	71	0.29

For specimen Design, Table 3, shows the dimensions of the specimen used for 1020 Hot Rolled.

Table 3: Solid specimen the dimensions for 1020 Hot Rolled Steel alloy.

Short column							Long column						
L mm	L_e mm	D mm	A mm^2	I mm^4	R mm	S.R.	L mm	L_e mm	D mm	A mm^2	I mm^4	R mm	S.R.
140	98	10	78.5	490.625	2.5	39.2	450	315	8	50.24	200.96	2	157.5

And for 5052 Aluminum alloy the Table 4 illustrates the dimensions of the specimen used.

Table 4: Gives the dimensions of solid specimen used for 5052 Aluminum alloy

Short column							Long column						
L mm	L_e mm	D mm	A mm^2	I mm^4	R mm	S.R.	L mm	L_e mm	D mm	A mm^2	I mm^4	R mm	S.R.
25	17.5	8	50.24	200.96	2	8.75	200	140	8	50.24	200.96	2	70

For the above tables K is taken to be 0.7 (Fixed – Pinned) see Fig.1.

The Failure Definition is defined to be the instance when the specimen buckles to about (1%) of specimen length [6] [7].
The details of the test rig and the experimental procedures are explained elsewhere [5] [7].

3-Experimental Results.

All the specimen are tested under different compressive loads and speed of revolution is 17 RPM. The shear stress which is applied was considered as a constant value (small value was neglected) [7].

All the specimens have an initial deflection created due to manufacturing.
Table 5 shows the experimental results which recorded directly from the testing.

Table 5: Illustrates the data of dynamic bucking results for 1020 steel alloy.

Specimen No.	Column type	N_f (cycles)	(mm) $\delta_{initial}$	(mm) δ_{cr}	P_{cr} (KN)
1	Short	5	0.2	2	7.25
2		6	0.3	2.5	6.97
3		8	0.3	2.4	8.07
4		4	0.2	2.7	6.77
5		5	0.2	2.2	7.45
6	Long	8	0.6	6	2.35
7		4	0.8	8	2.4
8		9	1	10	2.52
9		6	1.1	9	2.39
10		7	0.8	10	2.5

While the experimental results of 5052 Aluminum alloy are presented in Table 6 .

Table 6: Give the compression dynamic buckling results of 5052 Aluminum alloy

Specimen No.	Column type	N_f (cycles)	(mm) $\delta_{initial}$	(mm) δ_{cr}	P_{cr} (KN)
11	Short	11	0.2	1	2.02

12		9	0.3	1.2	2.94
13		8	0.2	1	2.92
14		12	0.4	1.1	2.02
15		7	0.3	1.2	2.17
16	Long	4	0.5	4	2.87
17		6	0.6	6	2.92
18		8	0.4	6	2.84
19		9	0.5	5	2.79
20		7	0.6	3.5	2.89

4- Discussion:

In applying equation (5), Eluer theory to predicate the critical load (P_{cr}) for long columns and Johnson theory, equation (6) for short columns to the data of the above Tables 5, 6. We get the results as in Table 7.

Table 7: Comparison between (P_{cr}) using Euler and Johnson theories with the experimental results.

Spece No.	P_{cr} (KN) Johnson	P_{cr} (exp.)KN
1	14.818	7.25
2	14.818	6.97
3	14.818	8.07
4	14.818	6.77
5	14.818	7.45

Spece No.	P_{cr} (KN) Euler	P_{cr} (exp.)KN
6	4.123	2.35
7	4.123	2.4
8	4.123	2.52
9	4.123	2.39
10	4.123	2.5

Spece No.	P_{cr} (KN) Johnson	P_{cr} (exp.)KN
11	5.06	2.02
12	5.06	2.94
13	5.06	2.92
14	5.06	2.02
15	5.06	2.17

Spece No.	P_{cr} (KN) Euler	P_{cr} (exp.)KN
16	7.076	2.89
17	7.076	2.92
18	7.076	2.84
19	7.076	2.79

20	7.076	2.89

From the test results shown in Table 7 , it is obvious that Euler and Johnson formulas overestimate the critical buckling loads for both material used. This overestimation may be for the following reasons:

1- Euler and Johnson theories considered the column is ideal (no initial deflection) while experimentally this parameter mainly effects on the critical dynamic buckling load [7].
2- Euler and Johnson formulas applied to static buckling columns while the critical load is decreased under dynamic buckling loads [8].

The critical load (P_{cr}) for a short column is affected by the strength of the material in addition to its stiffness(E), and the strength is not a factor for a long column when the Euler formula is used.

For typical machine design applications ,a design factor of 3 is used. For stationary columns with well-known load and end fixity, a lower factor can be used, such as 2.0. The objective of column analysis and design is to ensure that the load applied to a column is safe, well below the critical buckling load. [3] [9].

Then $$P_a = \frac{P_{cr}}{N} \quad \text{………………..(7)}$$

Where P_a = allowable load.
P_{cr} = critical buckling load.
N = Design Factor

The actual applied load (P), must be less than P_a .
Applying equation (7) to the data in Table 7 . The results are given in Table 8 .

Table 8: comparison between safe load (P_a)obtained theoretically with (P_{cr}) obtained experimentally.

Spece No.	P_a (KN) Johnson (Theory) 1020 Steel alloy Hot Rolled	P_{cr} (exp.)KN
1	4.94	7.25
2	4.94	6.97
3	4.94	8.07
4	4.94	6.77
5	4.94	7.45

Spece No.	P_a (KN) Euler (Theory) 1020 Steel alloy Hot Rolled	P_{cr} (exp.)KN
6	1.377	2.35
7	1.377	2.4
8	1.377	2.52
9	1.377	2.39

10	1.377	2.5

Spece No.	P_a (KN) Johnson (Theory) 5052 Aluminum alloy	P_{cr} (exp.)KN
11	1.686	2.02
12	1.686	2.94
13	1.686	2.92
14	1.686	2.02
15	1.686	2.17

Spece No.	P_a (KN) Euler (Theory) 5052 Aluminum alloy	P_{cr} (exp.)KN
16	2.358	2.89
17	2.358	2.92
18	2.358	2.84
19	2.358	2.79
20	2.358	2.89

5- Conclusions:

1- Euler and Johnson formulas can be used to predict the safe design load under dynamic compression loading using design factor of 3 or more.
2- The initial deflection of columns ($\sigma_{initial}$) has an important effect on the dynamic critical load.

6-References:

[1] Shigley J.E, and C.R. Mischke (2001), Mechanical Engineering Design. 6th ed. McGraw-Hill New York.

[2] Spotts M.F. and T.E.Shoup (1998), Design of Machine elements. 7th ed. Upper saddle River,NJ:prentice Hall.

[3] Mott.P.E.(2004), Machine elements in Mechanical design, Pearson Prentice Hall.

[4] Therib Jameel Hussein (2004), An investigation of elastic buckling for perforated plates in Aircraft structures. Msc Thesis, University of Technology, Iraq.

[5] Kifah Hameed Al-Jubori (2005), Columns lateral buckling under combined dynamic loading. Ph D Thesis, University of Technology, Iraq.

[6] Al-Jubori K.H., and Al-alkawi H.J. (2004), Experimental and Theoretical study for dynamic plastic buckling. J. of Technical Institutes Foundation, Vol. 17, No. 6.

[7] كفاح حميد هاشم الجبوري ، حسين جاسم محمد العلكاوي و انعام الصادق (2005)، تصميم جهاز قياس الانبعاج تحت تأثير أحمال دورية مركبة. مجلة الهندسة والتكنولوجيا ، المجلد 24 العدد 4 .

[8] Young W.C., and R.G. Budynas (2002) , Rook's formulas for stress and strain. 7th ed. McGraw-Hill New York.

[9] Corona Edmundo (1990), Buckling of structural members under cyclic loading. Ph D Thesis , University of Texas at Austn USA.

Nomenclature

σ_y	Yield stress
σ_u	Ultimate stress
ν	Poisson's ratio
$\sigma_{initial}$	Initial deflection
σ_{cr}	Critical deflection
C_c	Column constant
E	Modulus of elasticity
${}_eL$	Effective length
G	Modulus of rigidity
I	Moment of inertia
K	End fixity constant
L	Actual length of column
N	Design factor
N_f	No. of cycles at failure
P	Actual load
P_a	Allowable load
P_{cr}	Critical buckling load
R	Radius of gyration of cross-section
S.R.	Slenderness ratio