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VISCOELASTIC AND VISCOPLASTIC ANALYSIS OF POLYETHYLENE PIPE UNDER PRESSURE LOADING USING SPACE-TIME FINITE ELEMENT METHOD

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Abstract

In this paper, the viscoelastic and viscoplastic behavior of polyethylene (PE) pipe is studied and analyzed using Space-time finite element method (FEM). The (FEM) is achieved for Maxwell model and then the program is developed in three dimensions (two-dimension in space and one dimension in time) and applied to (PE) pipe under pressure loading. The time dependent partial differential equations describing the viscoelastic deformation of (PE) pipe under pressure loading for Maxwell element is solved using variational Galerkin continuous method in an integral process using time slabs. As a result of this study, the FEM in viscoelastic Maxwell model used to predict the relaxation modulus give accurate result compared to analytical which used Prony series. The stress increases to large values at the first stage of applied pressure loading and then decrease with increasing time. The viscoplastic surface density of micro crack is increasing function with increasing plastic strain.

في هذا البحث، تم دراسة وتحليل التصرف المرن–اللزج و اللدن –اللزج لمادة البولي اثيلين المستخدمة في صناعة ألأنابيب باستخدام طريقة العناصر المحددة الفضاء–الزمن. استخدمت طريقة العناصر المحددة الثلاثية الأبعاد (بعدين في الفضاء وبعد واحد للزمن) لنموذج ماكسويل في تطوير البرنامج وتطبيقه على أنابيب البولي اثيلين وتحت شروط أحمال الضغط. المعادلات التفاضلية المعتمدة على الزمن التي تصف التشوه المرن–اللزج لأنابيب البولي اثيلين تحت تأثير أحمال الضغط لعناصر ماكسويل المتوازية تم حلها باستخدام طريقة كلاركن المستمرة بعمليات تكامل لشرائح الزمن. لقد وجد من النتائج أن طريقة العناصر المحددة للتصرف المرن–اللزج والتي استخدمت لحساب معامل الاسترخاء تعطي نتائج جيدة مقارنة بالطرق التحليلية باستخدام متسلسلة بروني. الإجهاد يزداد إلى قيمة عالية خلال المرحلة الأولى من تسليط الضغط ثم يتناقص مع زيادة الزمن. الكثافة السطحية اللزجة اللدنة للشقوة السطحية هي دالة تزايدية مع زيادة الانفعال اللدن.

Introduction

Polyethylene (PE) is a diverse polymer material used in many product and applications. This diversity is due to the material's ability to be produced with a wide range of properties and characteristics such as: chemical and corrosion resistance, excellent hydraulic properties, lightweight, flexible and durable …etc [1].

Polyethylene is commonly used for the fabrications of gas or water pipes, in pressurized and non-pressurized applications. It has been proven effective for underwater, underground, above-ground, surface, as well as floating applications.

Failure mechanism of (PE) pipe can be occurs under pressure loading. The pressure loading increases deformation in (PE) pipe material and over time can often cause problems since it decrease the ability to withstand fracture (the creep rupture, slow or rapid crack propagation and stresses relaxation).

The determination of viscoelastic and viscoplastic behaviors of PE-pipe is complicated work. Oyen M. [2] used Boltzman hereditary integrals to generate displacement–time solution for loading at constant rate and creep following ramp loading in spherical and conical pyramidal viscoelastic material. Kolarik J. [3] studies on creep behavior of high density polyethylene cycloolefin copolymer blends and develop a predictive format appropriate for the creep of binary blends with co-continuous components showing non-linear creep. Orlik J. [4] develops a numerical procedure to study the problems of hereditary viscoelasticity using two-dimension space-time finite element method. Lemaitre [5] develops a good approach for viscoplastic material analysis.

The purpose of this paper is to develop the space-time finite element method (FEM) program in three dimensions and applied to (PE) pipe under pressure loading. It is indicated that the result obtained from FEM are a good agreement with the result obtained from analytical method.

Viscoelasticity of Polyethylene Pipe

Polyethylene pipe is viscoelastic construction material. Due to its molecular nature, its exhibits the characteristic of viscous flow and elastic deformation. Viscoelastic of polyethylene make it display time-dependent material properties and sensitive to the rates of deformation and loading. The time dependent response of PE pipe to pressure loading gives PE pipe unique resilience and resistance to sudden [6].

The viscoelasticity nature of polyethylene results in two unique engineering characteristics that employed in the design of HDPE water or gas piping system. These are creep and stress relaxation.

Creep is the time dependent viscous flow component of deformation. It refers to the response of polyethylene over time to a constant static load. Stress relaxation, when polyethylene pipe is subjected to a constant strain (deformation of specific degree) that is maintained over time, the load or stress generated by the deformation slowly decrease over time. The Stress relaxation reduced modulus of elasticity of PE pipe since it increase the strain [3].

Numerical Analysis By Using Generalized Maxwell Model

The behavior of viscous material is usually assumed to be restricted to devatoric strains and stresses (the volumetric components are purely elastic). The generalized visco-elastic Maxwell model shown in Fig.1. This model consists of an arbitrary number of Maxwell elements (spring and dashpot arranged in parallel). The stress in the spring is proportional to the strain, while the stress in the dashpot is proportional to the strain rate. The material parameters are shear moduli μ_p, and viscosity parameter η_p, where p = 1, 2, … M as shown in Fig.1, where the spring and dashpot symbols are used for deviatoric quantities. [6].

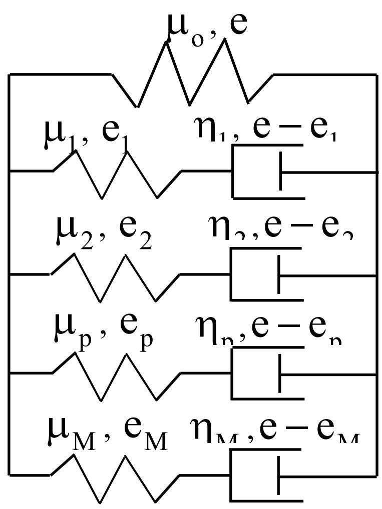

Fig.1 Maxwell Model

In the three dimensions, space-time FEM formulations for viscoelastic PE pipe can be summarizing as follows:

The total deviatoric stresses and strain are [7]:

$$S = \sum_{p=0}^{M} S_p = \sum_{p=0}^{M} 2\mu_p e_p \qquad \dots (1)$$

$$S_p = 2\eta_p e_p = 2\eta_p (\dot{e} - \dot{e}_p) \qquad \dots (2)$$

Where,

P = 1, 2, 3… , M

The Deviatoric strain e_p of the P-the serial spring can be considered as an internal variable of the constitutive equations.

Eq.2 can be written in the following form:

$$\dot{e}_p + \beta_p e_p - \dot{e} = 0 \qquad \dots (3)$$

where,

$$\beta_p = \frac{\mu_p}{\eta_p}$$

The total tensors of volumetric strain (ε):

$$\dot{\varepsilon}_p + \beta_p \varepsilon_p - \dot{\varepsilon} = 0 \qquad \dots (4)$$

The displacement is defined as:

$$\dot{u}_p + \beta_p u_p - \dot{u} = 0 \qquad \dots (5)$$

The strains are defined as:

$$\varepsilon = \nabla^5 u \qquad \dots (6)$$

$$\varepsilon_p = \nabla^5 u_p \qquad \dots (7)$$

The total stress (σ) is the sum of viscoelastic deviatoric (eq.1) and elastic mean ($\sigma_o I$) parts and given as the following:

$$\sigma = S + \sigma_o I$$

$$\sigma = 2\mu_o \mathrm{dev}\varepsilon + k(\mathrm{tr}\varepsilon)I + \sum_{p=1}^{M} 2\mu_p \mathrm{dev}\varepsilon_p$$

$$\sigma = \frac{E}{\varepsilon} + \sum_{p=1}^{M} \frac{E_p}{\varepsilon_p} \quad \ldots (8)$$

For plain strain state, the tensors E and Ep can be write in the following matrix:

$$E = \begin{bmatrix} K+\frac{4}{3}\mu_o & K-\frac{3}{3}\mu_o & 0 \\ K-\frac{2}{3}\mu_o & K+\frac{4}{3}\mu_o & 0 \\ 0 & 0 & \mu_o \end{bmatrix}$$

And,

$$E_p = \begin{bmatrix} \frac{4}{3}\mu_o & -\frac{3}{3}\mu_o & 0 \\ -\frac{2}{3}\mu_o & \frac{4}{3}\mu_o & 0 \\ 0 & 0 & \mu_p \end{bmatrix}$$

The following boundary conditions are applied:

$u = \bar{u}(r,t)$ For $x \in \Gamma_u(t)$, $t \in [0,T]$

$\sigma.n = \bar{t}(r,t)$ For $x \in \Gamma_u(t)$, $t \in [0,T]$

$\Gamma = \Gamma_u \text{ U } \Gamma_\sigma$, $\phi = \Gamma_u \cap \Gamma_\sigma$

$u(r_i,0) = 0$, $\varepsilon(r_i,0) = 0$

$\sigma(r_i,0) = -Pr.$, and $0 \le t \le T$

The system of differential equations can be transformed into the following one with respect to the variables (u) and (u_p) as followers:

$$LZ = 0 \quad \ldots (9)$$

Where,

$$z^T = \left(u_1 \;\; u_2 \;\; \ldots \;\; u_p \;\; \ldots \;\; u_m \;\; u\right)$$

$$L = \begin{bmatrix} \phi_1 & \phi_2 & \cdots & \phi_p & \cdots & \phi_M & \phi \\ \beta_1 + \frac{\partial}{\partial t} & 0 & \cdots & 0 & \cdots & 0 & \frac{\partial}{\partial t} \\ 0 & \beta_2 + \frac{\partial}{\partial t} & \cdots & 0 & \cdots & 0 & \frac{\partial}{\partial t} \\ 0 & 0 & \cdots & \beta_p + \frac{\partial}{\partial t} & \cdots & 0 & \frac{\partial}{\partial t} \\ 0 & 0 & \cdots & 0 & \cdots & \beta_M + \frac{\partial}{\partial t} & \frac{\partial}{\partial t} \end{bmatrix}$$

Where,

$$\phi_1 = \nabla.\left(\frac{E_1}{\nabla^5}\right), \qquad \phi_2 = \nabla.\left(\frac{E_2}{\nabla^5}\right)$$

$$\phi_P = \nabla.\left(\frac{E_P}{\nabla^5}\right), \qquad \phi_M = \nabla.\left(\frac{E_M}{\nabla^5}\right)$$

$$\phi = \nabla.\left(\frac{E}{\nabla^5}\right)$$

The deviatoric components e_p found from integrate the differential eq.3 with the initial condition given:

$$e_p(r,t) = \int_0^t \dot{e}\ (r,\zeta) e^{\beta_p(\zeta - t)} d\zeta \qquad \ldots (10)$$

And obtained the following integral differential form of constitutive eq.3 for viscoelastic materials:

$$s = 2\mu_o e + \sum_{p=1}^{M} \int_0^t \dot{e}(r,\zeta) e^{\beta_p(\zeta - t)} d\zeta \qquad \ldots (11)$$

The displacement is given by:

$$u(r,t) = e^{-\beta t} \int_0^t \dot{u}(x,\zeta) e^{\beta_p \zeta} d\zeta \qquad \ldots (12)$$

Non - Symmetric Continuous Galerkin Method in the three Dimensional Case

The Galerkin method allows to transform differential to the simplest form and to get approximated solutions. In three-dimension case by multiplying Eq.9 with a test function $\delta\lambda$ and integrating over the space-time domain [8]:

$$\int_{\Omega \times [0,T]} \delta\lambda L Z d\Omega dt = 0 \qquad \ldots (13)$$

Where, $z \in \Omega\,(0, T)$ and $\delta\lambda = 0$ on Γ_u.

Then, for the choice of $\delta\lambda$:

$$\delta\lambda = \varphi_1 * \varphi_2 \qquad \ldots (14)$$

Where,

$\varphi_1 =$

$$\begin{bmatrix} -c_1 & -c_2 & \ldots & -c_p & \ldots & -c_M & -c_o \\ \beta_1+\frac{\partial}{\partial t} & 0 & \ldots & 0 & \ldots & 0 & -\frac{\partial}{\partial t} \\ 0 & \beta_2+\frac{\partial}{\partial t} & \ldots & 0 & \ldots & 0 & -\frac{\partial}{\partial t} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \beta_p+\frac{\partial}{\partial t} & \ldots & 0 & -\frac{\partial}{\partial t} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & \ldots & \beta_M+\frac{\partial}{\partial t} & -\frac{\partial}{\partial t} \end{bmatrix}$$

$$\varphi_2 = \begin{bmatrix} \delta u_1 \\ \delta u_2 \\ \vdots \\ \delta u_p \\ \vdots \\ \delta u_M \\ \delta u \end{bmatrix}$$

From eq.9, eq.13 and eq.15 it follows:

$$- \int_{\Omega\times[0,T]} \left(\sum_{p=0}^{M} c_p \delta u_p^T \right) (\nabla.\sigma) d\Omega dt \qquad \ldots (15)$$

$$+ \int_{\Omega\times[0,T]} \delta Z^T L_1^T L_1 Z d\Omega dt = 0$$

Where,

$$L_1 = \begin{bmatrix} \beta_1+\frac{\partial}{\partial t} & 0 & \ldots & 0 & \ldots & 0 & -\frac{\partial}{\partial t} \\ 0 & \beta_1+\frac{\partial}{\partial t} & \ldots & 0 & \ldots & 0 & -\frac{\partial}{\partial t} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \beta_1+\frac{\partial}{\partial t} & \ldots & 0 & -\frac{\partial}{\partial t} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & \ldots & \beta_1+\frac{\partial}{\partial t} & -\frac{\partial}{\partial t} \end{bmatrix}$$

For solving Eq.15 by finite element method, the standard finite element approximation for displacement used as:

$$u(x,,y,t) = \sum_{i=1}^{ne} N_i(x,y,t) a_{ix}$$

$$v(x,y,t)=\sum_{i=1}^{ne} N_i(x,y,t)a_{iy}$$

And the general systems of equations can be written in the form:

$$\sum_{e=1}^{m}\left[K_{staf.}+K_{sym.}^{e}\right]*a_{staf.}=f_{staf.} \qquad \text{… (16)}$$

Where,

$$K_{staf.}=\begin{bmatrix} [k_{11}^e] & [k_{11}^e] & \cdots & [k_{1p}^e] & \cdots & [k_{1M}^e] & [k_{10}^e] \\ [k_{21}^e] & [k_{22}^e] & \cdots & [k_{2p}^e] & \cdots & [k_{2M}^e] & [k_{20}^e] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ [k_{q1}^e] & [k_{q2}^e] & \cdots & [k_{qp}^e] & \cdots & [k_{qM}^e] & [k_{q0}^e] \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ [k_{M1}^e] & [k_{M2}^e] & \cdots & [k_{Mp}^e] & \cdots & [k_{MM}^e] & [k_{M0}^e] \\ [k_{01}^e] & [k_{01}^e] & \cdots & [k_{0p}^e] & \cdots & [k_{0M}^e] & [k_{00}^e] \end{bmatrix}$$

$$a_{staf.}=\begin{bmatrix} a_1^e \\ a_2^e \\ \cdot \\ \cdot \\ \cdot \\ a_p^e \\ \cdot \\ \cdot \\ \cdot \\ a_M^e \\ a^e \end{bmatrix}, \quad f_{staf.}=\begin{bmatrix} f_1^e \\ f_2^e \\ \cdot \\ \cdot \\ \cdot \\ f_p^e \\ \cdot \\ \cdot \\ \cdot \\ f_M^e \\ f^e \end{bmatrix}$$

$$k_{qp}=c_q\int_{(\Omega\times[0,T])_e} B^T E_p \, d\Omega dt$$

$$k_{sym}^e=\int_{(\Omega\times[0,T])_e} B_1^T B_1 \, d\Omega dt$$

$$f_p^e=c_p\int_{(\Gamma\times\{0,T\})_e} N\bar{t} \, d\Gamma dt$$

The Eq.16 is solving use iterative solver within parallel calculation to determine the displacement. After calculation displacement we use eq.6 and eq.8 to obtain the strain and stresses.

The appendix A contains analytical method to calculating strains and stresses in HDPE pipe, which is depending on experimental results.

Viscoplastic of the Polyethylene Pipe

If the viscoplastic is localize in PE- pipe then it can be extended originally plasticity constitutive relation for time-independent plasticity to viscoplastic. It gives the von mises equivalent stress which is sufficient to estimate the damage if the stresses is known. The damage law in viscoplastic material [5] used scalar isotropic damage variable (D) to define the surface of micro defects in any plane of representative volume element and this gave as follows:

$$D_{n+1} = D_n + \left(\frac{Y_{n+1}}{S}\right)^n \Delta\dot{P} \qquad \ldots (17)$$

$$Y_{n+1} = \frac{1+\nu}{2E}\left[\frac{\sigma_{n+1}}{\sigma_{n+1} + [h(\frac{1-D_n}{1-hD_n})^2]}\right]$$

$$-\frac{\nu}{2E}(\mathrm{tr}\sigma_{n+1})^2 + (h(\frac{1-D_n}{1-hD_n})^2 * \qquad \text{Where,}$$

$$(-\mathrm{tr}\sigma_{n+1})^2$$

$$\sigma_{n+1} = (1-D_{n+1})\overline{\sigma}_n$$

$$\Delta\dot{P} = \left(\frac{2}{3}\right)^{0.5} \varepsilon_{ij}^p$$

$$\varepsilon_{ij}^p = \frac{3}{2}\frac{\sigma_{ij}^d - X_{ij}}{2(\overline{\sigma} - X)_{eq}} \dot{M}$$

$$\sigma_{eq.} = \left(\frac{3}{2}\right)^{0.5} \sigma_{ij}^d$$

$$\sigma_{ij}^d = \sigma_{ij} - \sigma_H$$

$$\sigma_H = \frac{1}{3}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right)$$

$$\dot{M} = \left(\frac{F}{K}\right)^N$$

$$F = (\overline{\sigma} - X)_{eq.} - R - \sigma_y$$

$$R = R_\infty\left(1 - e^{b\dot{M}(1-D)}\right)$$

$$\overline{\sigma}_{n+1} = \frac{\overline{\sigma}_n}{1 - D_n h} \quad \text{For compression.}$$

$$\overline{\sigma}_{n+1} = \frac{\overline{\sigma}_n}{1-D_n} \quad \text{For tension.}$$

$0 \le h \le 1$, often $h = 0.2$

For plain strain conditions, $\dot{\varepsilon}_{22} = \dot{\varepsilon}^e_{22} = 0$ and $\varepsilon^p_{22} = 0$ the elastic strain matrix, plastic strain matrix and stress matrix are:

$$[\underline{\varepsilon}]^e = \begin{bmatrix} \varepsilon^e_1 & 0 & 0 \\ 0 & \varepsilon^e_2 & 0 \\ 0 & 0 & \varepsilon^e_3 \end{bmatrix}, \quad [\underline{\varepsilon}]^p = \begin{bmatrix} \varepsilon^p_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\varepsilon^p_{11} \end{bmatrix}$$

$$\underline{\sigma} = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \frac{\sigma}{2} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For comparison a result, we us program developed by Lemaitre [5] which is called damage 2000.

Pipe Specimen Properties and FEM Meshes

Most polyethylene pipe now made from high density polyethylene material (HDPE) since the smooth inside surface maintains its exceptional flow characteristics [9]. The (HDPE) pipe specimen and meshes are shown in Fig.2 and Fig.3 with the following dimensions:

$D_i = 10.8^{//}$ (27.43 cm)

$D_o = 12^{//}$ (30.48 cm)

$t = 0.6^{//}$ (1.525 cm)

Pr. = 5 MPa.

The elastic and voscoelastic properties:

Elastic spring modulus = 10^8 Pa.

Viscosity modulus $\eta = 10^{10}$ Pa..s

Bulk modulus = $1.8 * 10^9$ Pa.

Density = 955 Kg/ m^3.

Length of pipe = $236.2^{//}$ (600 cm).

X = 116 MPa.

Fig.2 Polyethylene pipe specimen

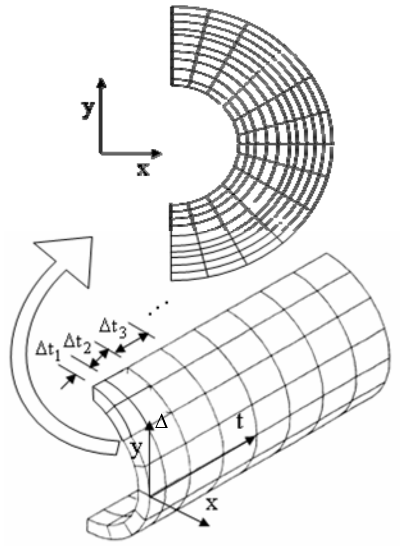

Fig.3 Space time finite element meshes for polyethylene pipe.

Due to symmetry only half pipe will be meshed using 8-node hexahedral linear element. The element shape is shown in Fig.4. The total numbers of element used for analysis are 300 elements. The boundary conditions for the time interval $0 \le t \le 10^n$ where n = 0, 1, 2 …8. Appendix B contains shape function for this element [11].

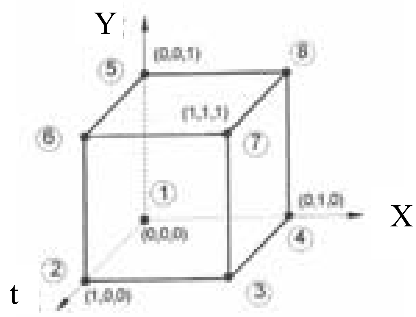

Fig.4 Eight- node hexahedral element

Results and Discussion

Using the presented formulation based on continuous Galerkin method, the space-time finite element method discretization can be used to calculate displacements, stresses and strains. The stresses are divided by constant rate of applied strain to convert it to the relaxation modulus. Fig.5 shown logarithmic relaxation modulus vs. logarithmic time results from FEM and analytical method. As indicated there is very good agreement between them, also it can be seen the relaxation modulus decrease as time increase and the relation of decrease relaxation modulus is not linearly.

Fig.6 plot stresses vs. time dependent, as indicated there is a very good agreement between the two methods used for analysis. It can be seen that the stresses reach maximum values after pressure loading is applied and then decrease with increasing time.

Fig.7 radial stresses were plotted against radial strain for PE pipe. It is apparent from figure there are very good agreement between FEM results and analytical result. It can be seen that the stresses dependence on strain as well as time and there is larger stresses during primary stage of the loading and then the stresses increase with small values with increasing strain.

A numerical solution for the displacements in the x-axis and y-axis is shown in Fig.8. For an evaluation of the solution accuracy of space-time finite element, numerical values of displacements is compared with the analytical. As indicated there is good closeness between results obtained from FEM and analytical method.

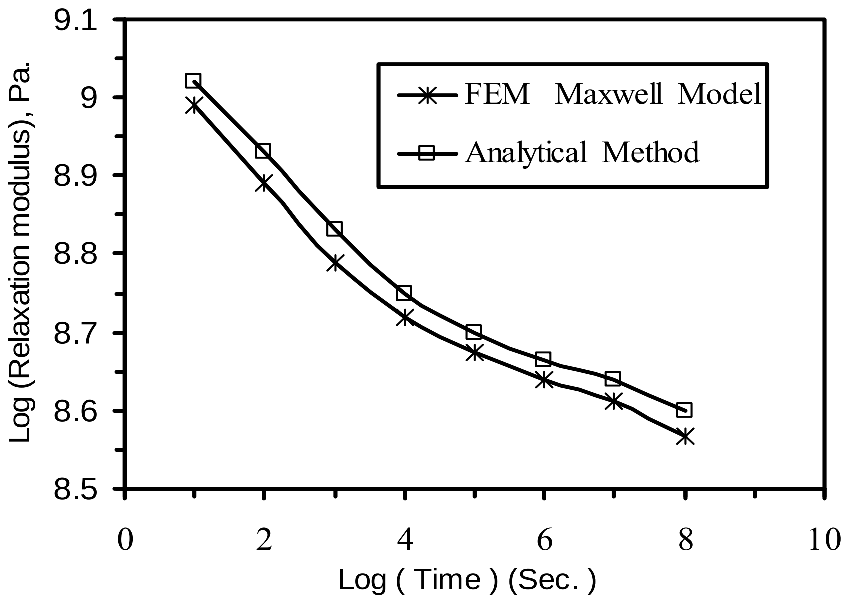

Fig.5 Relaxation modulus verses time for (PE) pipe.

A viscoplastic structure calculation by a finite element analysis gives the histories of the plastic strain ε_{ij}^{p}. The damage evolution is obtained by simple time integration of the damage law this shown in Fig.9. As shown, the surface density of micro cracks ($\dot{D}$) is increasing function with increasing plastic strain rate. This, the damage of polyethylene pipe can be caused by exceeding the critical strain state i.e. when ε_{ij}^{p} = 0.1, ($\dot{D}$) reach maximum value and crack begin to initiate and propagation in polyethylene pipe thickness.

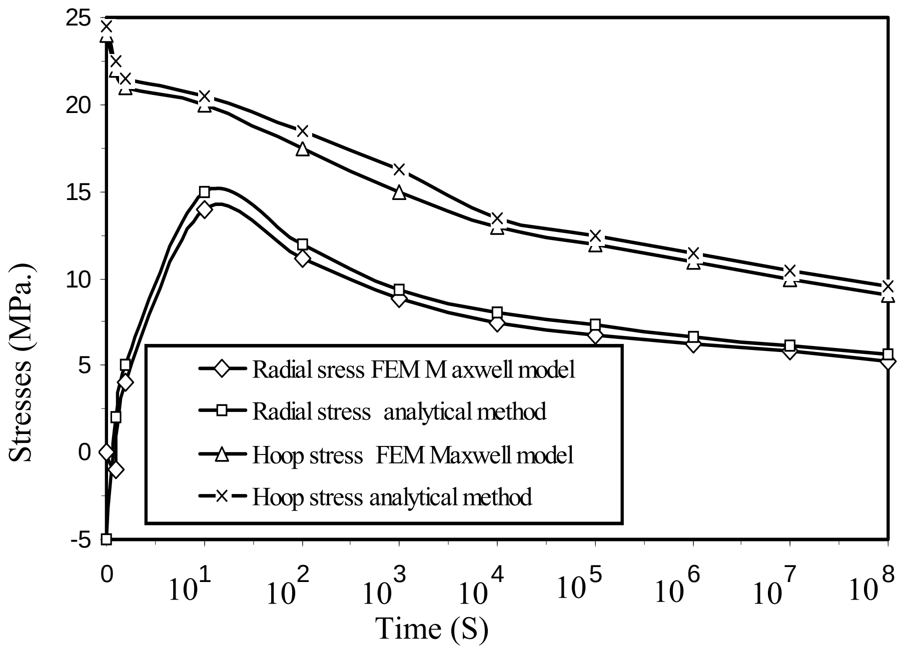

Fig.6 Stresses verses time for polyethylene pipe.

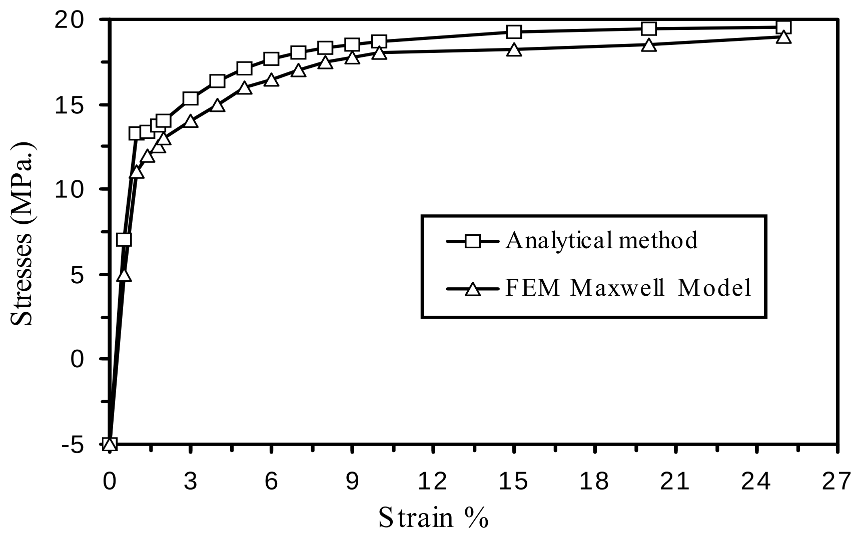

Fig.7 Radial stresses versus radial strain for polyethylene pipe

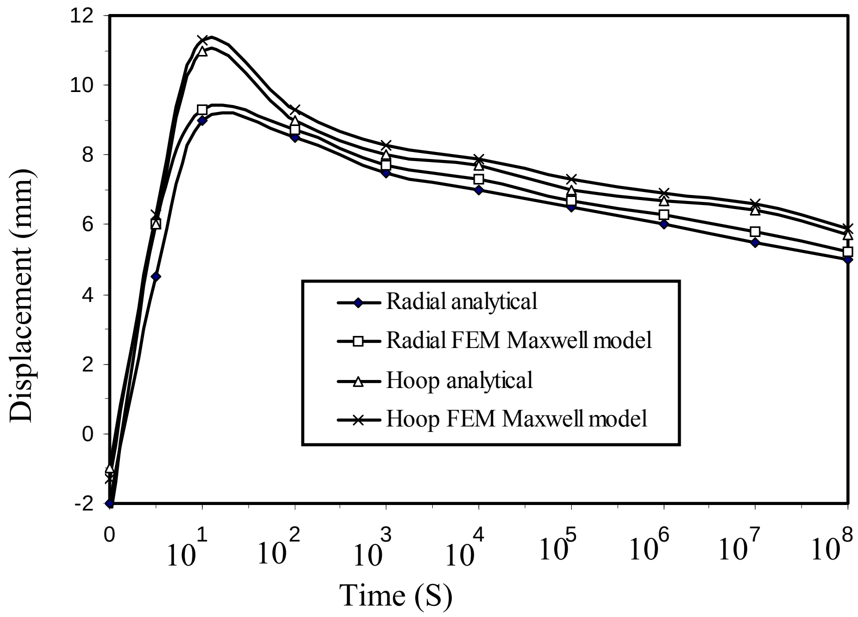

Fig.8 Displacement verses time for polyethylene pipe for 5% strain rate.

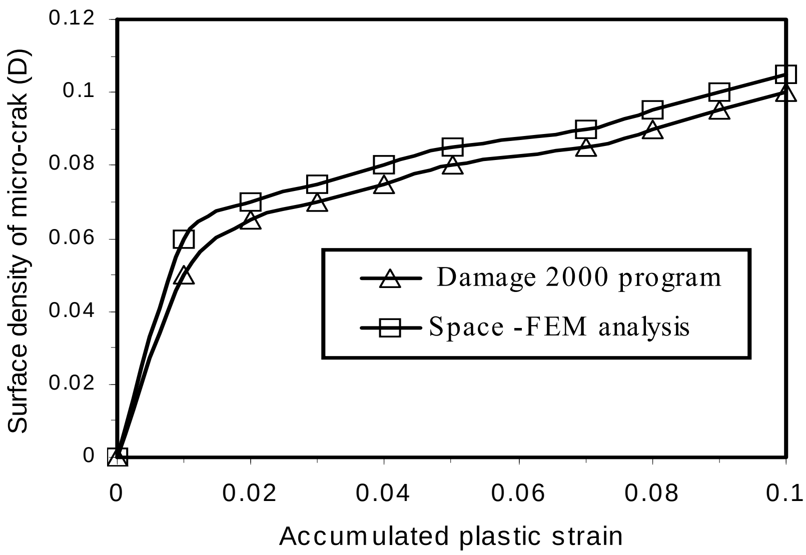

Fig.9 Surface density of micro-cracks against plastic strain for vicoplastic case.

Conclusions

Based on the preceding studies, the following conclusions are reaches:

1- The finite element program included pressure loading is developed and applied for viscoelastic polyethylene pipe.

2- The FEM program gives accurate results for relaxation modulus, stresses strain and displacement compared to analytical results.

3- The results indicated that the relaxation is not linear with the applied stresses and the stresses suddenly increase to maximum values at the primary stage of loading.

4- For viscoplastic case, the relation between surface density of micro-cracks (D) and plastic strain rate is non-linear interaction curve.

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Appendix

A- Analytical Methods for calculate Strain and Stresses

The time dependent strain history can be represented by [12]:

$$\varepsilon = \varepsilon_o t H_1 - 2\varepsilon_o (t - t_o) H_2 \quad \ldots (18)$$

Where,

$H_1 = H(t - 0^+)$: Heavside step function.

$H_2 = H(t - t_o)$: Heavside step function.

The stresses depend on time only is giving by:

$$\sigma(t) = E_o h(\varepsilon_o)\varepsilon_o + \sum_{i=1}^{10} E_i [1 - e^{-\frac{t}{\tau_i}}] h(\varepsilon_o)\varepsilon_o \quad \dots (19)$$

Where,

$$h(\varepsilon_o) = \frac{1}{1 + a\varepsilon_o}$$

$a = 50.1429$

The stresses dependent on time and strain can be given by:

$$\sigma(\varepsilon, t) = E_o h(\varepsilon)\varepsilon + \frac{1}{a}\sum_{i=1}^{10} [1 - e^{-\frac{\varepsilon}{\varepsilon_o \tau_i}} - F_i(\varepsilon, t)] \quad \dots (20)$$

Where,

$$h(\varepsilon) = \frac{1}{1 + a\varepsilon}$$

$$F_i(\varepsilon, t) = \int_0^t \frac{e^{\frac{(z-1)\varepsilon}{\varepsilon_o \tau_i}}}{a\varepsilon z + 1} dz$$

$\tau_i = 10^{i-2}$, $i = 1, 2, 3, \dots 10.$

The relaxation modulus is define by Prony series [11]:

$$E(t) = E_o + \sum_{i=1}^{10} E_i (1 - e^{-\frac{t}{\tau_i}}) \quad \dots (21)$$

where,

The Prony constants can be finding from stress relaxation test. This gives in Ref. [13] in detail. For PE100 pipe are gives as followers:

$E_1 = -57114$ Psi.

$E_2 = -39574$ Psi.

$E_3 = -41012$ Psi.

$E_4 = -25700$ Psi.

$E_5 = -24596$ Psi.

$E_6 = -8580$ Psi.

$E_7 = -7076$ Psi.

$E_8 = -3620$ Psi.

$E_9 = -4191$ Psi.

$E_{10} = -7938$ Psi.

$E_o = 269685$ Psi.

B- Shape Function for 8-node Element

The shape function for 8–node hexahedral element in the intrinsic coordinates (ξ, η, ζ) are given by [11]:

$$N_1(\xi,\eta,\zeta) = (1-\xi)(1-\eta)(1-\zeta)$$
$$N_2(\xi,\eta,\zeta) = \xi(1-\eta)(1-\zeta)$$
$$N_3(\xi,\eta,\zeta) = \xi\eta(1-\zeta)$$
$$N_4(\xi,\eta,\zeta) = (1-\xi)\eta(1-\zeta)$$
$$N_5(\xi,\eta,\zeta) = (1-\xi)(1-\eta)\zeta$$
$$N_6(\xi,\eta,\zeta) = \xi(1-\eta)\zeta$$
$$N_7(\xi,\eta,\zeta) = \xi\eta\zeta$$
$$N_8(\xi,\eta,\zeta) = (1-\xi)\eta\zeta$$

Where,

$$\xi = \frac{x}{L_{ex}}$$
$$\eta = \frac{y}{L_{ey}}$$
$$\zeta = \frac{t}{L_{ez}}$$

Nomenclature

μ_p : Shear modulus.

η_p : Viscous parameter.

M: Number of the simple Maxwell element.

S: Total deviatoric stresses.

e_o : Total deviatoric strain.

e_p : Deviatoric strain of the P-the serial spring.

u : Total spring displacement.

u_p : Total serial spring displacement.

K: Bulk modulus.

E and E_p: Forth order tensor of the elastic module.

Pr. : Pressure loading in MPa.

r : Any radius of pipe (m).

N_i : Matrix shape function.

a_{ix} and a_{iy} : Vector of the nodal displacement in the x and y directions.

ne : Number of element
c_p : Coefficients of scale parameter.
$k^e_{staf.}$ and $k^e_{sym.}$: Stiffness matrix.
B: Matrix contains the derivative of the element shape function.
ε_o : Rate of strain during loading.
t_o : Unloading time in (s).
t : Time of loading in (s).
E_o and E_i : Instantaneous modulus of material and Prony constants.
D: Scalar isotropic damage.
$\dot{P}$: Accumulated plastic strain rate.
Y: Strain energy density release rate.
h: Crack closure parameter.
S: Damage strength.
n : Damage exponent.
$\sigma_{eq.}$: Von-misses equivalent stress.
X_{ij} : Back stress.
R_∞ and b : Norton's parameter.
L_{ex} , L_{ey} and L_{ez} : Element length side in the X, Y and t directions.