

SPECTRAL ANALYSIS OF A TRAWLER STRUCTURAL MEMBERS SUBJECTED TO HIGH SEA WAVES

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Abstract

The purpose of this study is to clarify the trawler structural members subjected to random sea wave groups with large waves (hydrodynamic forces) to examine the trawler behavior under the above random loading condition to avoid future damages. By adopting the general complex modal theory, identifying the natural frequencies with corresponding mode shapes, the response of the trawler structure analysis with input from the high sea waves is carried out using a kind of direct spectrum analysis method in frequency domain. With the input of power spectral density function given, the explicit expression of the power spectral density function of the output response can be obtained. By taking Fourier inverse transforms, the integrated expressions of the correlation function and of the spectrum are obtained. The mean square values and variety of statistical values can be obtained conveniently leads to the formulation of a mathematical model of a trawler structure model. The detailed model will be used to identify the fundamental vibration mode shapes and corresponding excitation frequencies which will be used to control the safety of the trawler and the crew, to increase the trawler structure optimization, simulating the increased protection of the Trawler.

Keywords spectrum analysis - complex modal - random vibration

التحليل النسقي لهيكل سفينة صيد معرضة لأمواج البحر العالية

الخلاصة

الغرض من هذه الدراسة هو توضيح الهيكلية لسفينة صيد متعرضة لمجموعة موجة البحر العشوائية ذات الموجات الكبيرة (القوى الهيدروديناميكية) لفحص تصرف السفينة تحت ظروف الحمل العشوائي لتجنب الأضرار في المستقبل. من خلال اعتماد نظرية النسق المركبة العامة عن طريق تحديد الترددات الطبيعية والأشكال النسقية المماثلة، كذلك تحليل استجابة هيكل سفينة الصيد مع المدخلات من ارتفاع أمواج البحر حيث تم استخدام نوع مباشر في أسلوب تحليل الطيف في مجال التردد مع مدخلات دالة كثافة طيف القدرة يمكن الحصول على تعبير واضح لاستجابة خارج دالة كثافة طيف القدرة عن طريق اتخاذ تحويلات فوريير العكسية، يتم الحصول على تعبير متكامل لعبارات الترابط والطيف. متوسط مربع القيم ومجموعة من القيم الإحصائية يمكن الحصول عليها بشكل ملائم يؤدي إلى صياغة نموذج رياضي لنموذج هيكل سفينة الصيد. هذا النموذج المفصل سوف يستخدم لتحديد الأشكال النسقية الاهتزازية الأساسية وإثارة الترددات المناظرة التي سيتم استخدامها للسيطرة على سلامة السفينة وطاقمها، لزيادة تحسين هيكل السفينة، ومحاكاة لزيادة حماية السفينة.

كلمات مرشدة تحليل كثافة الطيف-النسق المركب-الاهتزاز العشوائي

Introduction

Data from sea keeping trials of a Scottish trawler are analyzed. The trawler sailed an octagonal course, each leg took over 20 minutes and data recorded twice a second. The natural frequencies of vibration for each of the six rigid body modes are estimated from the heave, surge, sway, pitch, yaw and roll time series. The time series are investigated for evidence of non-linearity. A time domain model is fitted to a roll time series, and second order amplitude response functions are then obtained using autoregressive estimators [1].

The stationary Gaussian random process (SGRP) is a widely used mathematical model of random loading. The statistical description of SGRP is exhausted by the spectral density of power. The principal characteristic used in the calculation of fatigue is the rms value of random load determined from spectral density[2].

The purpose of this study is to clarify the trawler structural members subjected to random sea wave groups with large waves (hydrodynamic forces) to examine safety, economy, and marine environment protection.

Early investigations of the response of trawler structures to High Sea Waves led to the development of spectrum analysis. This is defined as the maximum response of each of a series of simple mechanical systems having different frequencies. The spectrum is usually expressed as the maximum acceleration plotted as a function of natural frequency.

The motion of a ship in a fluid can be described by displacements along orthogonal axes xyz and rotations about these axes as shown in Figure (1), the displacements are surge, sway and heave along the x , y and z -axes respectively. The corresponding rotations are pitch, roll, and yaw.

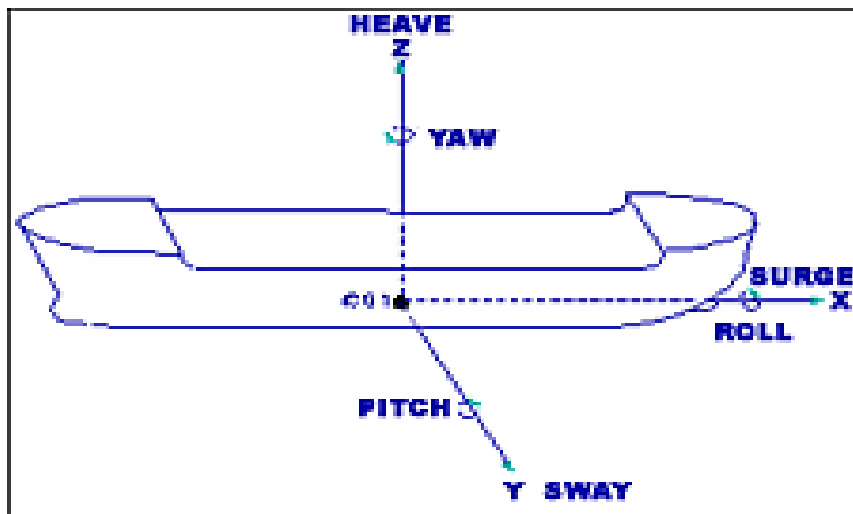


Fig.(1) The six degree of freedom motions of a moving ship are the three translations of surge, sway, heave, and the three rotations of roll, pitch, and yaw.

Influence of Metacentric Height

If the metacentric height (GM) Figures (2-4) of a ship is large, the righting arms that develop, at small angles of heel, will be large. Such a ship is “stiff” and will resist roll. However, if the metacentric height of a ship is small, the righting arms that develop will be small. Such a ship is tender and will roll slowly. In ships, large GM and large righting arms are desirable for resistance to damage. However, a smaller GM is sometimes desirable for a slow, easy roll that allows for more accurate gunfire; therefore, the GM value for a naval ship is the result of compromise [3].

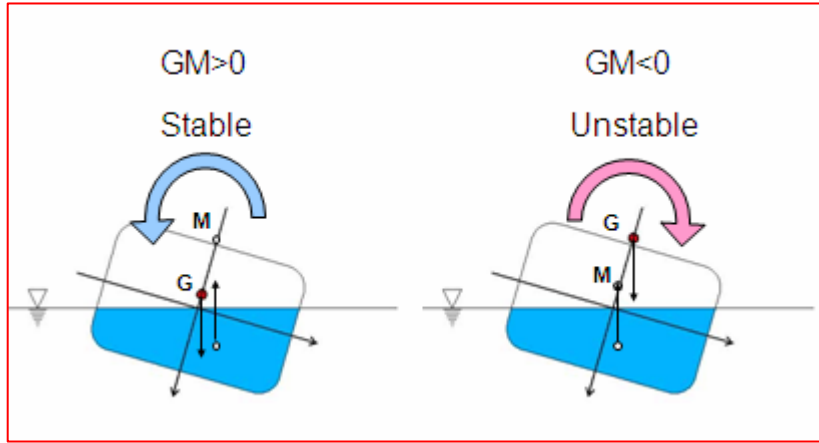


Fig.(2) Ship motion in waves –dynamic stability

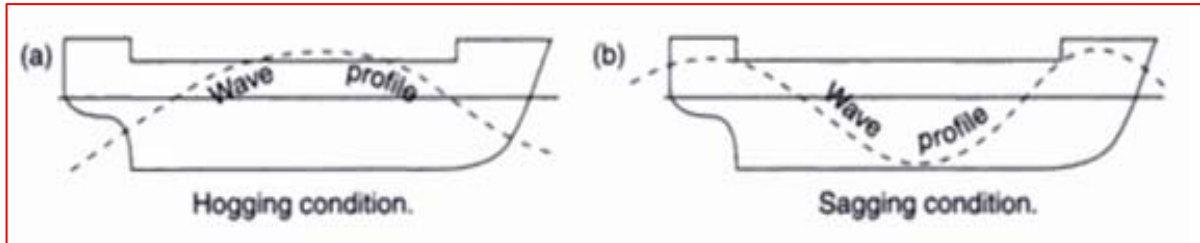


Fig.(3) Ship motion in waves a-Hogging condition (Crest condition)
b- Sagging condition (Trough condition)

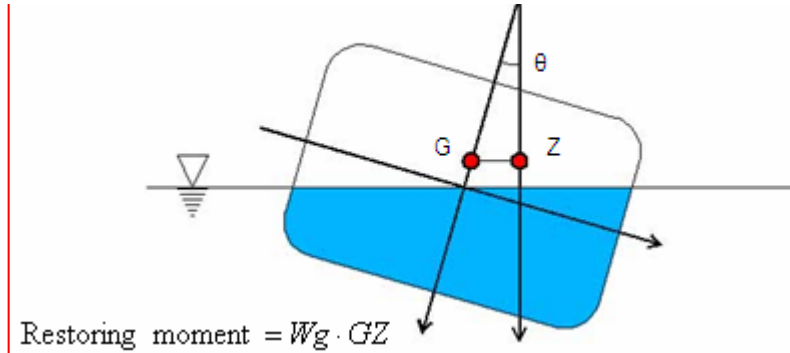


Fig.(4) Restoring arm GZ

Direct spectral density (spectra) Procedure

The equations of motion [4] of the trawler can be written in the form

$$\sum_{j=1}^6 \left\{ (M_{kj} + m_{kj}) \ddot{X}_j(t) + C_{kj} \dot{X}_j(t) + \left[\int_{-\infty}^t K_{kj}(\tau) d\tau \right] X_j \right\} = \Omega(t) \quad (1)$$

Where M, C, K are the mass, damping, and stiffness matrices of the Trawler, respectively, m_{ij} is the Hydrodynamic mass matrix in kgm^2 . $\Omega(t)$ Is the forced high sea waves (random loading). Define $H_{jk}(\omega)$ as the complex frequency response function for the (χ_{ij}) output due to a harmonically varying imposed displacement of unit amplitude at the (Ω_j) input

Let [4]

$$\Omega(t) = \int_{-\infty}^{\infty} \Omega(i\omega) \text{Exp}(i\omega t) d\omega$$

$$\chi(t) = \int_{-\infty}^{\infty} \chi(i\omega) \text{Exp}(i\omega t) d\omega$$

$$\dot{\chi}(t) = \int_{-\infty}^{\infty} i\omega \chi(i\omega) \text{Exp}(i\omega t) d\omega$$

$$\dot{\chi}(t) = \int_{-\infty}^{\infty} -i\omega \chi(i\omega) \text{Exp}(i\omega t) d\omega$$

By substitution in Eq.(1) and canceling $\text{Exp}(i\omega t)$ terms yields

$H(i\omega) = \text{Response/Excitation}$

$$H(i\omega) = [k - \omega^2 (M_{kj} + m_{kj}) + i\omega C]^{-1} [\Omega(i\omega)] \quad (2)$$

Then

$$\chi(i\omega) = H(i\omega) \Omega(i\omega) \quad (3)$$

The power spectral density of the generalized displacement response (χ) can be expressed in terms of the input spectral densities and the system response function as follows

Using Fourier transformations [5] yields,

$$S_{\chi_j}(\omega) = |H(i\omega)|^2 S_{\Omega_j}(\omega) \quad (4)$$

or

$$S_{\chi_j}(\omega) = H(\omega) S_{\Omega_j}(\omega) H^*(\omega) \quad (5)$$

Where (*) denotes matrix conjugate transposition. $S_{\chi_j}(\omega)$ is 6×6 matrix containing the spectral densities of the (χ) vector along the diagonal and the cross spectral densities on off-diagonal elements.

From the definition of autocorrelation [6]

From the impulse response function

$$R\chi(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \chi_j(t) \chi_j(t + \tau) dt \quad (6)$$

$$\chi_j(t) = \int_0^{\infty} \Omega_j(t - \tau_1) h_{jk}(\tau_1) d\tau_1 \quad (7)$$

The integer j can take values between 1 and m, where m is the number of degree of freedom, and h_{jk} is the response function for a unit impulse.

Similarly

$$\chi_j(t + \tau) = \int_0^{\infty} \Omega_j(t + \tau - \tau_2) h_{jk}(\tau_2) d\tau_2 \quad (8)$$

Substituting from Eq.(8) and (7) in Eq.(6) and rearranging the order of integration

$$R_{\text{But}}\chi(\tau) = \int_0^\infty h_{ik}(\tau_1) \int_0^\infty h_{jk}(\tau_2) \lim_{t \rightarrow \infty} \frac{1}{2T} \times \int_{-T}^T \Omega_j(t - \tau_1) \Omega_j(t + \tau - \tau_2) dt d\tau_2 d\tau_1 \quad (9)$$

$$R_{\text{Then}}\Omega(t + \tau - \tau_2) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T \Omega_j(t - \tau_1) \Omega_j(t + \tau - \tau_2) dt \quad (10)$$

$$R\chi(\tau) = \int_0^\infty h_{jk}(\tau_1) \int_0^\infty h_{jk}(\tau_2) R\Omega(\tau - \tau_2 + \tau_1) d\tau_2 d\tau_1 \quad (11)$$

From Winer-Khinchine relation [7] or the Fourier pair

$$R\chi(\tau) = \frac{1}{4\pi} \int_{-\infty}^\infty \chi(\omega) \text{Exp}(i\omega\tau) d\omega \quad (a) \quad (12)$$

$$S\chi(\omega) = 2 \int_{-\infty}^\infty R\chi(\tau) \text{Exp}(i\omega\tau) d\tau \quad (b)$$

Where $S\chi(\omega)$ is the spectral density of the responses. Substituting from Eq. (11) in the second term of fourier pair relation

$$S\chi(\omega) = \int_{-\infty}^\infty \text{Exp}(-i\omega\tau) \int_0^\infty h_{jk}(\tau_1) \int_{-\infty}^\infty h_{jk}(\tau_2) R_{\Omega}(\tau - \tau_2 - \tau_1) d\tau d\tau_2 d\tau_1 \quad (13)$$

After some arrangements and multiplying by $\text{Exp}(i\omega\tau_1)$, $\text{Exp}(i\omega\tau_2)$, $\text{Exp}[-i\omega(\tau + \tau_1 - \tau_2)]$ where their product is unity

$$\begin{aligned} S\chi(\omega) &= 2 \int_{-\infty}^\infty h_{jk}(\tau_1) \text{Exp}(i\omega\tau_1) \int_{-\infty}^\infty h_{jk}(\tau_2) \text{Exp}(-i\omega\tau_2) \\ &\quad \times \int_{-\infty}^\infty R_{\Omega}(\tau - \tau_2 + \tau_1) \text{Exp}[-i\omega(\tau + \tau_1 - \tau_2)] d\tau d\tau_2 d\tau_1 \\ &= \int_{-\infty}^\infty h_{jk}(\tau_1) \text{Exp}(i\omega\tau_1) \int_{-\infty}^\infty h_{jk}(\tau_2) \text{Exp}(-i\omega\tau_1) \times 2 \int_{-\infty}^\infty R_{\Omega}(\tau_3) \text{Exp}(-i\omega\tau_3) (d\tau d\tau_2 d\tau_1 \end{aligned} \quad (14)$$

Where $\tau_3 = \tau - \tau_2 + \tau_1$

$$S\chi(\omega) = D1 + D2 + D3$$

$$D1 = H_{jk}(\omega), D2 = H_{jk}(-\omega), D3 = S\Omega_j(\omega)$$

Where $H_{jk}(-\omega)$ is the matrix conjugate transposition

Thus

From the first term of Fourier pair yields

$$S\chi_j(\omega) = |H_{jk}(\omega)|^2 S\Omega_j(\omega) \quad (15)$$

$$R(0) = E\left[\chi_j^2(t)\right] = \frac{1}{4\pi} \int_{-\infty}^\infty S\chi_j(\omega) d\omega = \frac{1}{2\pi} \int_0^\infty S\chi_j(\omega) d\omega \quad (16)$$

Substituting equation (15) in equation (16) gives

$$E\left[\chi_j^2(t)\right] = \frac{1}{2\pi} \int_{-\infty}^\infty |H_{jk}(\omega)|^2 S\Omega_j(\omega) d\omega \quad (17)$$

Random Loading Spectral densities $Sp(\omega)$

For white noise Gaussian random loading $Sp(\omega) = S_0$ [8] shown in Fig.5(a), the mean response (displacement) is found from equation (2) and equation (17) as

$$E\left[\chi_j^2(t)\right] = \frac{1}{2\pi} \int_0^\infty \frac{S_0}{k^2 [(1 - \omega^2 / \omega_n^2)^2 + (2\xi\omega / \omega_n)^2]} d\omega \quad (18)$$

Integrating equation (18) by the calculus of residues gives

$$E\left[\chi_j^2(t)\right] = \frac{S_o \omega_n}{8\xi k^2} \quad (19)$$

For Band-Limited Gaussian random loading in Fig.5(b), the spectral density $S_p(\omega)$ is uniform up to a cut-off frequency, which is well above the natural frequency

The mean value is found as

$$E\left[\chi_j^2(t)\right] = \frac{1}{2\pi} \int_0^{\omega_c} \frac{S_o}{k^2[(1 - \omega^2/\omega_n^2)^2 + (2\xi\omega)^2]} d\omega \quad (20)$$

$$E\left[\chi_j^2(t)\right] \approx \frac{S_o \omega_n}{8\xi k^2} \quad (21)$$

For slow Gaussian random loading in Fig.5(c), the spectral density $S\Omega(\omega)$ varies fairly slowly with ω according to the curve $S\Omega(\omega) = S_o - S_2(\omega/\omega_n)$.

Then the mean value is,

$$E\left[\chi_j^2(t)\right] = \frac{S\Omega(\omega) \omega_n}{8\xi k^2}$$

$$E\left[\chi_j^2(t)\right] = \frac{(S_o - S_2) \omega_n}{8\xi k^2} \quad (22)$$

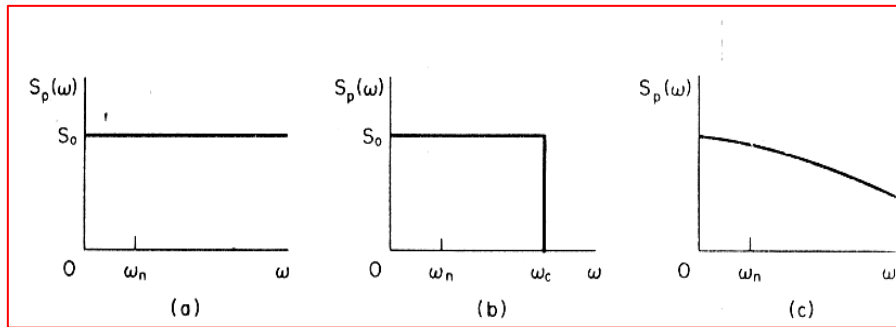


Fig. 5 Example of spectral density $S_p(\omega)$ for an applied force

Standard deviation of random stress

The root mean square value (RMS) [9] of the random stress and strain for the given spectral density (see Figs. 8 and 9) can be obtained as follows:

The variance of the random stress

$$\sigma_s^2 = E[S^2] \quad (23)$$

$$\sigma_s^2 = \frac{K^2}{A^2} E\left[\chi_j^2(t)\right] \quad (24)$$

The standard deviation (RMS) [10] of the stress is,

$$\sigma_s = \frac{K}{A} E \left[\chi_j^2(t) \right] \quad (25)$$

Where A, k is the material geometrical.

Generalized response model

For general n-degree-of-freedom lumped trawler with mass matrix [mi], stiffness matrix [kij], damping matrix [ci], and column matrix of external random forces [Ωj], the total damage is the sum of that in each mode according to the linear damage criteria as follows:

$$E[\chi_j^2(t)] = \sum_{i=1}^m \frac{S\Omega(\omega_i) \omega_i \psi_{ij}^2 \psi_{is}^2}{8 \xi_i k_i^2} \quad (26)$$

The symbols ψ_{ij}, ψ_{is} represents the eigen vectors for modal matrix of the structure [11], and s is the location where the random load is applied, ω_i is the eigenvalues of the structure

Result and discussion

In order to investigate the spectral analyses, the test case corresponds to a typical trawler vessel vibrating due to high sea waves [12] shown in fig.(6)

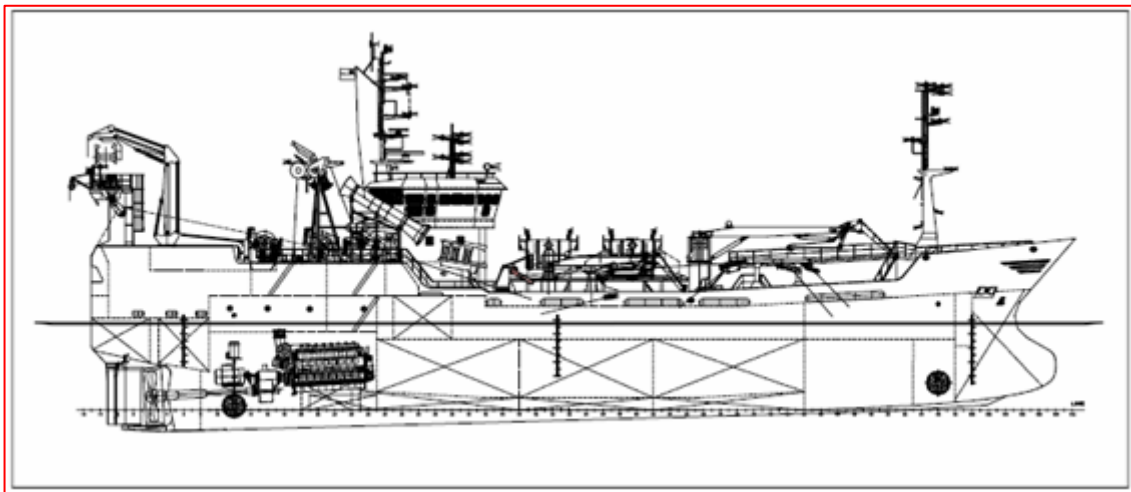


Fig.(6) Trawler Vessel

Trawler Vessel Main Particulars

Length between perpendiculars (L): 70.50 m
tonnes

Maximum beam (B): 12.00 m

Depth to main deck (D): 7.45 m

Design draught (T): 6.20 m

Light displacement (Δ): 1587.00

Fish hold capacity: 1800.00 m³

Service speed (U): 16.00 knots

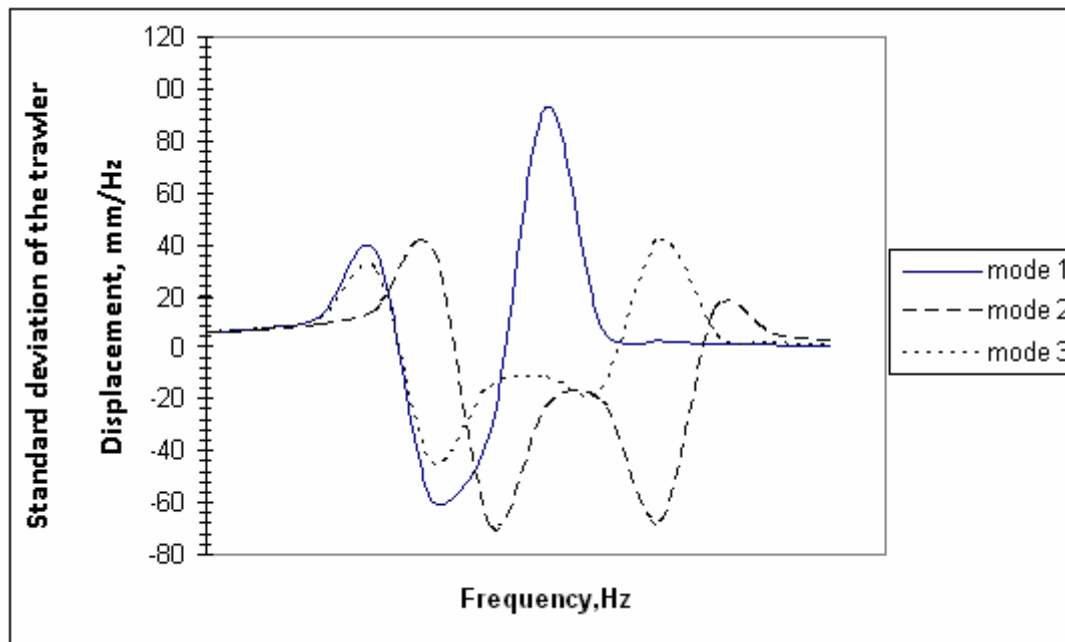


Fig.(7) standard deviation of the Trawler modal shapes

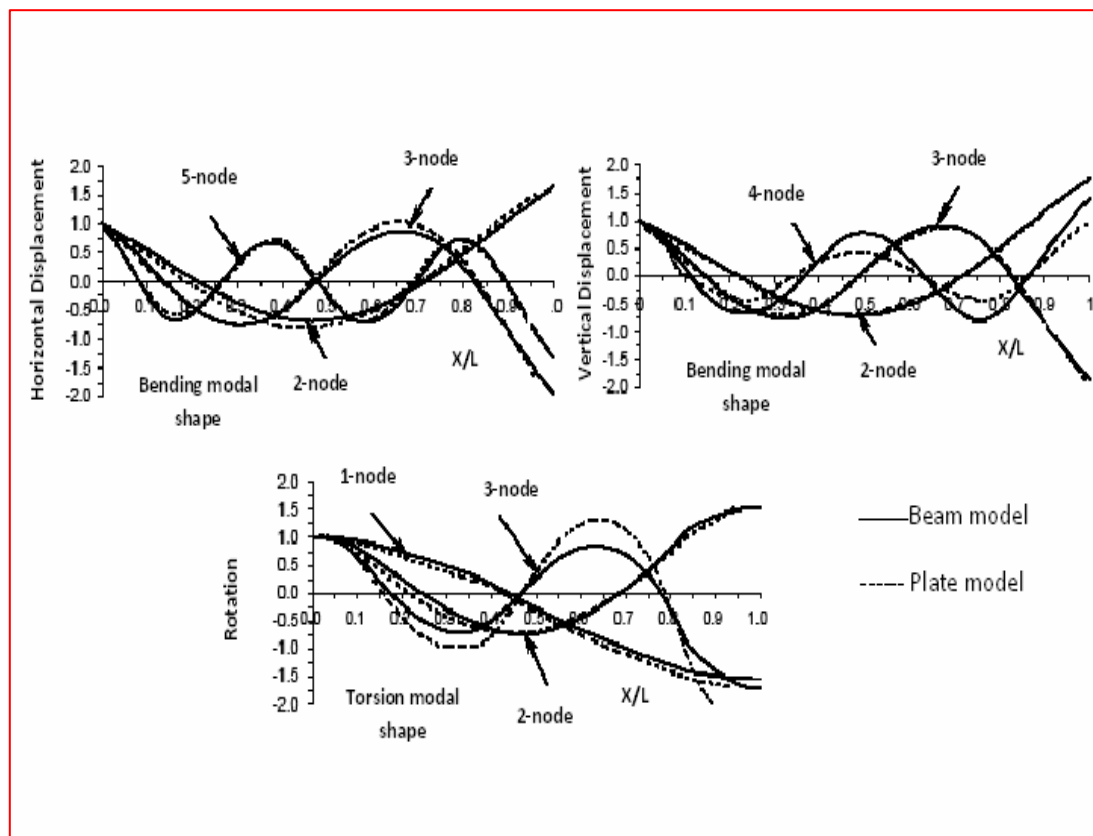


Fig.(8)Trawler Modal Shapes

MSC/NASTRAN solution

The dynamic analysis procedure is to calculate the free vibration frequencies and mode shapes for the structure. This requires solving the free vibration eigenvalues problem:

$$[K]\{\phi\} - \omega^2[M]\{\phi\} = 0 \quad \text{Where;}$$

$[K]$ =stiffness, $\{\phi\}$ =mode shape, $[M]$ =mass, ω = natural frequency

For a system with n mass degrees of freedom, there will be n free vibration mode shapes. For multiple degree of freedom trawlers, the eigenvalues problem can be solved by rewriting the equation in the form:

$$[K] - \omega^2[M]\{\phi\} = 0$$

MSC/NASTRAN solution can be used to determine the eigenvalues and eigenvectors for multi-degree of freedom models.

Once the mode shapes and frequencies have been determined, the modal mass can be computed. The stationary Gaussian random process (SGRP) is a widely used mathematical model of random loading. The statistical description of SGRP is exhausted by the spectral density of power $S(\omega)$. The principal characteristic used in the calculation the rms value of random load determined from spectral density

Modal analysis of a trawler Vessel

Natural Frequencies of Trawler Modes shown in table (1) and the Modal loads for trawler model are readily available in the form of bending or torsion modes which can be observed in figures (9, 10).

Table 1: Natural Frequencies of Trawler Modes

Plate Model	Beam Model
5.472	5.750
7.393	7.378
10.531	11.280
12.352	11.791
15.354	15.147
18.408	20.396
18.982	18.025
19.506	19.148
23.903	23.587
25.419	28.244
29.614	29.416
29.788	30.142
33.763	33.317

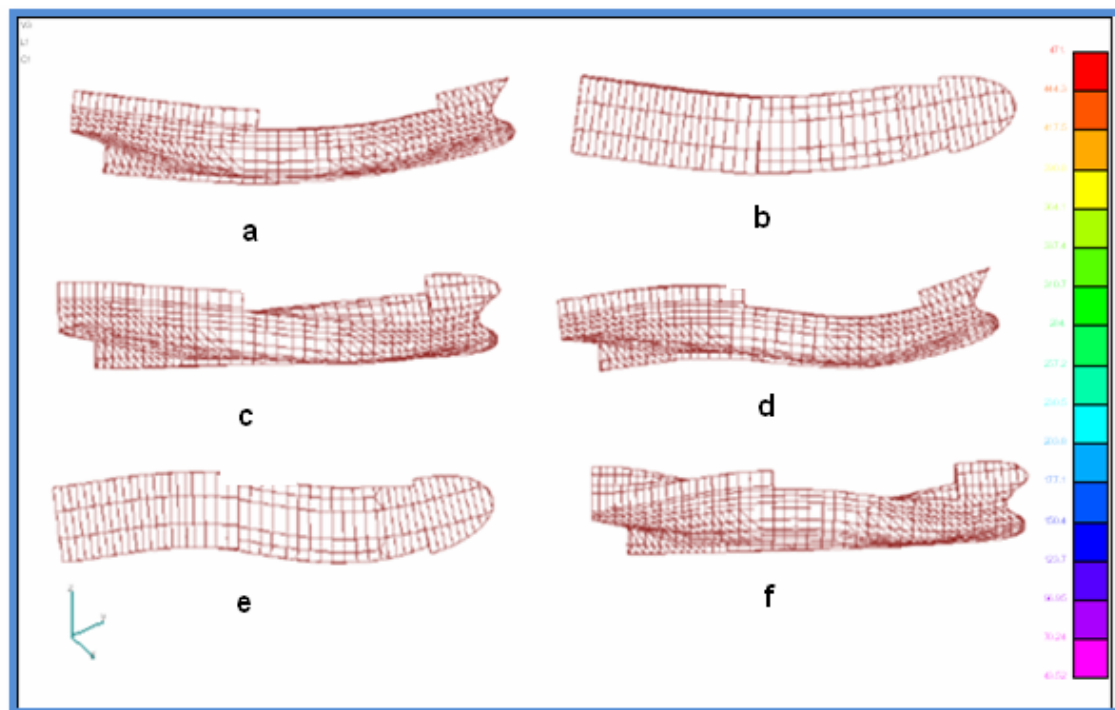
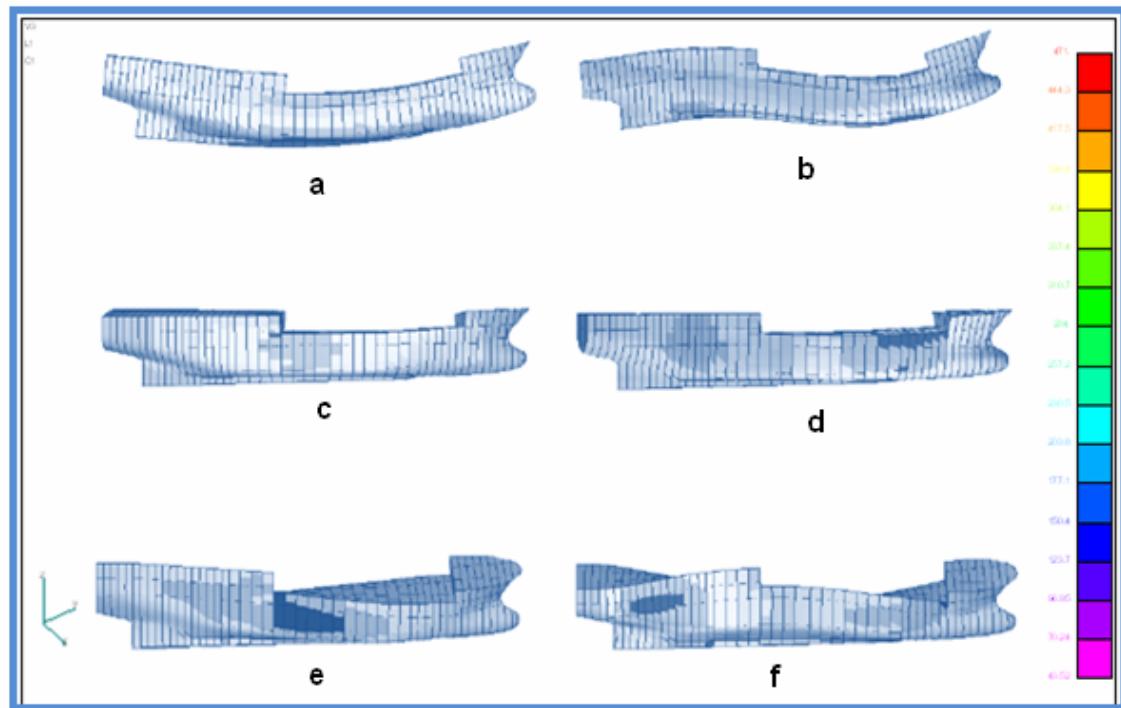


Fig.(10) Trawler beam stress mode shapes

a-Vertical bending mode .f=6.34 Hz

c- Torsion mode .f= 12. 67

e- Horizontal bending mode .f=8.45 Hz

b- Horizontal bending mode .f=17.21 Hz

d- vertical bending mode .f=13.44 Hz

f- Torsion mode .f=21.56

Conclusion

The behavior of a Trawler Structure to time-varying excitation is computed. Frequency response analysis computes the structural response to steady-state oscillatory excitation. In addition, it is also possible to conduct a random analysis with frequency response. Modal Analysis is a valuable tool in performing the dry analysis of a marine structure. Plate models are necessary for the investigation of stress distributions; this is a particularly important advantage when a study of highly sea wave areas. A Beam model may be created in a fraction of the time required for a Plate model. Also the computing time employed to perform the modal analysis is significantly lower. This suggests that a Beam model should be preferred when non-detailed calculations are required. From the analysis of highly stressed regions of the trawler structure it is evident that all kind of flexible modes can contribute to the stresses at a particular position of the structure. A structural dynamic analysis of marine structures should allow for all kind of modes to be considered such as vertical bending, elongation-shrinking (symmetric), horizontal bending and torsion (antisymmetric).

References

- [1] Modal analysis of a small ship sea keeping trial, A. Metcalfe, I. Maurits , T. Svenson G. E. Hearnt , R. Thach, ANZIAM J. 47 (EMAC2005) pp.C915-C933, 2007
- [2] EFFECT OF THE SHAPE OF THE SPECTRAL DENSITY OF RANDOM LOADING ON FATIGUE LIFE, B. Z. Kruk, Institute of Strength Problems, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Problemy Prochnosti, No. 10, pp. 36-41, October, 1981. Original article submitted January 5, 1980.
- [3] Toshio Iseki, Ship motion in waves — prediction of dynamic stability, Tokyo University of Marine Science and Technology, international maritime organization, 2007
- [4] S.A. Miedema, J.M.J. Journée and S. Schuurmans, On the Motions of a Seagoing Cutter Dredge, a Study in Continuity, 13th World Dredging Congress, 1992, Bombay, India.
- [5] Thomson, W. T., Theory of Vibration with Applications (4E) Stanley Thorne, 1993. C917
- [6] Nieslony, A., Macha, E., Spectral Method. Multiracial Random Fatigue of Machine Elements and Structures, Part V, Studies and Monographs 160, TU Opole, 2004, 168 ps, (in Polish).
- [7] Müller, A.: Zum Festigkeitsverhalten von mehrachsigen stochastisch beanspruchten Gusseisen mit Kugelgraphit und Temperguss, LBF-Bericht Nr. FB-203, 1994 (in German).
- [8] Crandall, H.ystem, New York, P112, (1963).
- [9] Soong, T.T. and M. Grigoriu (1993), Random Vibrations of Structural and Mechanical Systems, New Jersey, Prentice Hall.
- [10] Sobczyk, K. and J. Trebicki, (1999), "Stochastic response of degrading elastic systems", Proc. of IUTAM Symposium on Rheology of Defected Bodies, R.Wang (ed), Kluwer Acad. Publ., 99-108.
- [11] Hearn, G. E., Metcalfe, A. V. & Lamb, D., All at Sea with Spectral Analysis, Proceedings of the Industrial Statistics in Action 2000 International Conference, University of Newcastle upon Tyne, 8th--10th September 2000, ed. Coleman, S., Stewardson, D. and Fairbairn, L., Volume II, 217--235, University of Newcastle upon Tyne

[12] A. Metcalfe ,L. Maurits ,T. Svenson ,R. Thach ,G. E. Hearn , Modal analysis of a small ship sea keeping trial, July 30, 2007. <http://www.maths.adelaide.edu.au/stats/staff/ametcalfe.html>

NOMENCLATURE

M	Trawler mass matrix in kgm^2
m_{ij}	Hydrodynamic mass matrix in kgm^2
K	Stiffness matrix, [kN/m]
E	Expectation operator
$H_{jk}(\omega)$	Complex frequency response
$I_{s,It}$	Trawler moments of inertia, [kg m^2]
i	$\sqrt{-1}$
R_{xj}	Correlation function
$S_{\Omega}(\omega)$	Sea waves input spectral density matrix, [m ² .sec]
$S_{\chi ij}(\omega)$	Output spectral density matrix, [m ² .sec]
$S_o(\omega)$	Spatial spectral density function, [m ² .sec]
t	Time, [sec]
τ	Time delay, [sec]
χ_{ij}	Vector of generalized coordinate, [m]
ω	Angular frequency, [rad/s]
Ω	Sea wave loads in N or Nm
σ	Standard deviation of the random input, [m]
σ^2	Variance of the random input, [m ²]