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DYNAMIC ANALYSIS OF STIFFENED AND UNSTIFFENED COMPOSITE PLATES

Prof. Dr. Muhsin J. Jweeg
Mechanical Engineering Department
Al-Nahrain University
Muhsinjj@yahoo.com

Dr. Muhannad Al-Waily
Mechanical Engineering Department
Al-Kufa University
muhannad_al_waily@yahoo.com

ASTRACT

A suggested solution for dynamic analysis of stiffened and un-stiffened laminated plates is presented in this work. The First order laminated plate theory is used. The equations of motion are solved by using the modal analysis of forced vibration for multi degrees of freedom. The applied load for this study aresine, rectangular, expansion, ramp and triangular pulses. The method of applied these loads is centrally and uniformly distributed across the plate.

The deflection and stresses for each layer are presented for stiffened and un-stiffened, symmetrical and un-symmetrical, cross ply and angle ply, laminated composite plate with respect to the plate side-to-thickness ratio, aspect ratio, material orthotropy, and lamination scheme, number of layer of laminated plate, number of stiffeners, high to width of stiffener ratio, high of stiffener, the width of stiffener, and the stiffener properties.

The results are very close compared with finite elements method using ANSYS program and with**Reddy 1982**.

الخلاصة :-

تم اقتراح حل تحليلي للتصرف الديناميكي للصفائح المتراكبة المقوات والغير المقوات. حيث تم في هذا البحث استخدام نظرية ال(FSDT). تم حل المعادلات العامة للحركة عن طريق استخدام نظرية ال(Modal Analysis) للاهتزاز متعدد درجات الحرية بتأثير احمال متغيرة مع الزمن. حيث تم تسليط حمل ديناميكي متغير مع الزمن من نوع (Transient Load) على شكل موجة جيب، مستطيلة، دالة لوغاريتمية، مثلثة (من قيمة صفر ال قيمة اخرى)، او دالة مثلثة الشكل. حيث تم تسليط هذا الحمل على الصفيحة بطريقتين، متمركز في المنتصف او موزع بصور منتظمة على الصفيحة.

الازاحة والاجهادات لكل طبقة من طبقات الصفيحة تم ايجاد للصفائح المقوات والغير المقوات (المتناظرة والغير متناظرة، للطبقات المتعامدة الالياف والمائلة الالياف) بتاثير نسبة طول الصفيحة للسمك، طول الصفيحة للعرض، نسبة خواص الصفيحة، زاوية الالياف للصفيحة، عدد الطبقات للصفيحة، عدد اجزاء التقوية، نسبة ارتفاع الى عرض اجزاء التقوية، ارتفاع اجزاع التقوية، عرض اجاء التقوية، وخواص اجزاء التقوية.

النتائج تبين تقارب جيد مقارنة مع نتائج طريقة العناصر المحددة المستخدمة عن طريق استخدام برنامج ال(ANSYS) ومقارنة مع الابحاث السابقة (**Reddy 1982**).

KEYWORDS

Dynamic Composite, Stiffened Plate, Modal Analysis.

INTRODUCTION

The composite plates have high stiffness-to-weight ratio, and flexible anisotropic property which can be tailored through variation of the fiber orientation and stacking sequence, Fiber-reinforced laminated composites are finding increasing applications, and therefore, the stress and deformation characteristics of composite plates are receiving greater attention.

With the increased application of composites in high performance aircraft, the studies involving the assessment of the dynamic response of laminated composite-structure designers are also increased.

Much of the previous research in the analysis of composite plates is limited to linear problems, and many of them were based on the classical thin-plate theory, which neglects the transverse shear deformation effects.

Librescu et al. 1990, presented the dynamic loading conditions considered here include sine, rectangular and triangular pulses while spatially, they are considered as sinusoidally distributed. The results obtained as per a higher-order plate theory are compared with their first order transverse shear deformation and classical counter-parts.**Reddy 1982**, presented numerical results for deflections and stresses showing the effect of plate side-to-thickness ratio, aspect ratio, material orthotropy, and lamination scheme.

Cederbaum and Aboudi 1989, applied the first-order as well as two high-order shear deformation theories for the investigation of the laminated plate's response.

Khdeir and Reddy 1991, used the exact solutions of rectangular laminated composite plates with different boundary conditions are studied.The Levy-type solutions of the classical first-order and third-order shear deformation theories are developed using the state-space approach.

Bose and Reddy 1998, presented a unified third-order laminate plate theory that contains classical, first-order and third order theories as special cases are presented. Analytical solutions using the Navier and Levy solution procedures are presented.

Reddy and Chao 1981**, obtained the numerical results of deflections and stresses for rectangular plates for various boundary conditions, loading, staking and orientation of layers, and material properties.**

In this work the analytical solution for displacement and stresses of laminated plates in bending subject to dynamic loading is presented. The investigations deal with the analytical solution of composite plates subjected to time dependent loading, therefore,

EQUIVALENT SINGLE LAYER THEORIES (ESL)

In the "ESL" theories, the displacements or stresses are expanded as a linear combination of the thickness coordinate and undetermined functions of position in the reference surface,

$$\phi_i(x,y,z) = \sum_{j=0}^{N_i} \phi_j^i(x,y) Z^j \text{ , for i=1,2,3.} \quad \textbf{(1)}$$

Where N_i are the number of terms in the expansion. ϕ_i^j can be either displacements or stresses.

Classical Laminated Plates Theory (CLPT)

The displacement filed of laminated plates are, **Rao 1999**,

$$u_1(x,y,z,t) = u(x,y,t) - Z\frac{\partial w}{\partial x}$$
$$u_2(x,y,z,t) = v(x,y,t) - Z\frac{\partial w}{\partial y} \qquad (2)$$
$$u_3(x,y,z,t) = w(x,y,t)$$

Where (u,v,w) are the displacements, along the coordinate lines, of a material point on the xy-plane.

The equations of motion are,

$$N_{x,x} + N_{xy,y} = I_1.u_{,tt} - I_2.W_{,xtt}$$
$$N_{xy,x} + N_{y,y} = I_1.v_{,tt} - I_2.W_{,ytt}$$
$$M_{x,xx} + 2.M_{xy,xy} + M_{y,yy} + q(x,y) = \qquad (3)$$
$$I_1.W_{,tt} + I_2.u_{,xtt} + I_2.v_{,ytt} - I_3.W_{,xxtt} - I_3.W_{,yytt}$$

Where,

$$I_1 = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho^{(k)} dz \qquad (4)$$

For $\rho^{(K)}$ being the material density of K^{th} layer.

The laminate constitutive equations can be expressed in the form,

$$\begin{bmatrix} N_i \\ M_i \end{bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{bmatrix} [\varepsilon] \\ [K] \end{bmatrix} \qquad (5)$$

Where,

$\varepsilon_x = u_{,x}$, $\varepsilon_y = v_{,y}$, $\gamma_{xy} = u_{,y} + v_{,x}$, $K_x = w_{,xx}$, $K_y = -w_{,yy}$, $K_{xy} = -2\, w_{,xy}$ (6)

The A_{ij}, B_{ij}, D_{ij} (i,j =1,2,6) are the respective inplane, bending –inplane coupling, and bending or twisting, respectively,

$$(A,B,D) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \overline{Q}^{(k)} (1, Z, Z^2) dz \qquad (7)$$

Here Z_m denotes the distance from the mid-plane to the lower surface of the K^{th} layer.

Eqs. (3) and (5) can be conveniently expressed in the operator form as,

$$[L][\Delta] + [f] = [M][\ddot{\Delta}] \qquad (8)$$

Where,

$M_{11} = I_1$, $M_{12}=0$, $M_{13}=-I_2\, d_x$, $M_{22}=I_1$, $M_{23}=-I_2\, d_y$, $M_{33}=I_1 - I_3\,(d_{xx}+d_{yy})$.

$[\Delta]=[u \quad v \quad w]^T$,$[f]=[0 \quad 0 \quad q(x,y,t)]^T$.

And,

$L_{11}=A_{11}\, d_{xx} + 2\, A_{16}\, d_{xy} + A_{66}\, d_{yy}$, $L_{12}=A_{12}\, d_{xy} + A_{16}\, d_{xx} + A_{26}\, d_{yy} + A_{66}\, d_{xy}$,

$L_{13}=-B_{11}\, d_{xxx} - B_{12}\, d_{xyy} - 3\, B_{16}\, d_{xxy} - B_{26}\, d_{yyy} - 2\, B_{66}\, d_{xyy}$, $L_{22}=2A_{26}\, d_{xy} + A_{66}\, d_{xx} + A_{22}\, d_{yy}$,

$L_{23}=-B_{16}\, d_{xxx} - 3\, B_{26}\, d_{xyy} - 2B_{66}\, d_{xxy} - B_{12}\, d_{xxy} - B_{22}\, d_{yyy}$,

$L_{33}= -D_{11}\, d_{xxxx} - 2\, D_{12}\, d_{xxyy} - 4\, D_{16}\, d_{xxxy} - 4\, D_{26}\, d_{xyyy} - 4\, D_{66}\, d_{xxyy} - D_{22}\, d_{yyyy}$. (9)

First-OrderShear Deformation Theory (FSDT)

This theory accounts for linear variation of inplane displacements through the thickness,

$u_1(x,y,z,t)=u(x,y,t)+Z\ \psi_x(x,y,t)$
$u_2(x,y,z,t)=v(x,y,t)+Z\ \psi_y(x,y,t)$
$u_3(x,y,z,t)=w(x,y,t)$. **(10)**

Where, t is the time; u_1 , u_2 , u_3 are the displacements in x,y,z directions, respectively; and ψ_x and ψ_x are the slopes in the xy and yz planes due to bending only.

The equations of motion are,

$N_{x,x}+N_{xy,y}=I_1\, u_{,tt} +I_2\psi_{x,tt}$
$N_{xy,x}+N_{y,}y=I_1\, v_{,tt}+ I_2\psi_{y,tt}$
$N_{xz,x}+N_{yz,y}+q(x,y,t)=I_1 w_{,tt}$
$M_{x,x}+M_{xy,y} -N_{xz}=I_2\, u_{,tt} +I_3\psi_{x,tt}$
$M_{xy,x}+M_{y,y}- N_{yz} =I_{2\ v,tt}+ I_3\psi_{y,tt}$ **(11)**

Where,

$$(I_1, I_2, I_3) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho^{(k)} (1, Z, Z^2) dz \quad \textbf{(12)}$$

The laminated constitutive equations can be expressed in the form,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ K \end{bmatrix}, \begin{bmatrix} N_{yz} \\ N_{xz} \end{bmatrix} = \begin{bmatrix} k_{44}^2 A_{44} & k_{45}^2 A_{45} \\ k_{45}^2 A_{45} & k_{55}^2 A_{55} \end{bmatrix} \begin{bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{bmatrix} \quad \textbf{(13)}$$

Where,

$\varepsilon_x=u_{,x}$, $\varepsilon_y=v_{,y}$, $\gamma_{xy}=u_{,y}+v_{,x}$, $\gamma_{yz}=\psi_y+w_{,y}$, $\gamma_{xz}=\psi_x+w_{,x}$, $K_x=\psi_{x,x}$, $K_y=\psi_{y,y}$, $K_{xy}=\psi_{x,y} +\psi_{y,x}$ **(14)**

And,$K_{45,}$ K_{44} and K_{55} are correction factors.

Eqs. (11) and (13) can be conveniently expressed in the operator form as,

$$[L][\Delta]+[f]=[M][\ddot{\Delta}] \quad \textbf{(15)}$$

Where,

$[\Delta]=[u\ \ v\ \ w\ \ \psi_x\psi_y]^T$, $[F]=[0\ \ 0\ \ q(x,y,t)\ ]^T$.

And,$M_{11}=M_{22}=M_{33} =I_1$, $M_{44} =M_{55} =I_3$, $M_{14} =M_{25}=I_2$,and other terms of $M_{ij}=0$ (for í≠j).

And, [L] is given as,

$L_{11}=A_{11}d_{xx}+2A_{16}d_{xy}+A_{66}d_{yy}$, $L_{12}=A_{12}d_{xy}+A_{16}d_{xx}+A_{26}d_{yy}+A_{66}d_{xy}$, $L_{13}=0$

$L_{14}=B_{11}d_{xx}+2B_{16}d_{xy}+B_{66}d_{yy}$, $L_{15}=B_{12}d_{xy}+B_{16}d_{xx}+B_{26}d_{yy}+B_{66}d_{xy}$

$L_{22}=2A_{26}d_{xy}+A_{66}d_{xx}+A_{22}d_{yy}$, $L_{23}=0$, $L_{24}=B_{16}d_{xx}+B_{66}d_{xy}+B_{12}d_{xy}+B_{26}d_{yy}$

$L_{25}=2B_{26}d_{xy}+B_{66}d_{xx}+B_{22}d_{yy}$, $L_{33}=2A_{45}d_{xy}+A_{55}d_{xx}$, $L_{34}=A_{55}d_x+A_{45}d_y$, $L_{35}=A_{45}d_x+A_{44}d_y$
$L_{44}=D_{11}d_{xx}+2D_{16}d_{xy}+D_{66}d_{yy}-A_{55}$, $L_{45}=D_{12}d_{xy}+D_{16}d_{xx}+D_{26}d_{yy}+D_{66}d_{xy}-A_{45}$
$L_{55}=2D_{26}d_{xy}+D_{66}d_{xx}+D_{22}d_{yy}-A_{44}$.

ACTUAL DISPLACEMENTS FOR SIMPLY-SUPPORTED LAMINATED PLATE

Cross-Ply Laminated Plate

The general actual displacements for cross-ply laminated plate are, **Bose and Reddy 1998**,

$$
\begin{aligned}
u(x,y,t) &= \cos\alpha x.\sin\beta y.u(t) = \bar{u}(x,y).u(t) \\
v(x,y,t) &= \sin\alpha x.\cos\beta y.v(t) = \bar{v}(x,y).v(t) \\
w(x,y,t) &= \sin\alpha x.\sin\beta y.w(t) = \bar{w}(x,y).w(t) \\
\psi_x(x,y,t) &= \cos\alpha x.\sin\beta y.\psi_x = \bar{\psi}_x(x,y).\psi(t) \\
\psi_y(x,y,t) &= \sin\alpha x.\cos\beta y.\psi_y(t) = \bar{\psi}_y(x,y).\psi_y(t)
\end{aligned}
\qquad \textbf{(16)}
$$

Angle-Ply Laminated Plate

The general actual displacements for Angle-ply laminated plate are,

$$
\begin{aligned}
u(x,y,t) &= \sin\alpha x.\cos\beta y.u(t) = \bar{u}(x,y).u(t) \\
v(x,y,t) &= \cos\alpha x.\sin\beta y.v(t) = \bar{v}(x,y).v(t) \\
w(x,y,t) &= \sin\alpha x.\sin\beta y.w(t) = \bar{w}(x,y).w(t) \\
\psi_x(x,y,t) &= \cos\alpha x.\sin\beta y.\psi_x = \bar{\psi}_x(x,y).\psi(t) \\
\psi_y(x,y,t) &= \sin\alpha x.\cos\beta y.\psi_y(t) = \bar{\psi}_y(x,y).\psi_y(t)
\end{aligned}
\qquad \textbf{(17)}
$$

GENERAL SOLUATION FOR EQUATIONS OF MOTION

The general equations of motion are,

$$[L][\Delta] + [f] = [M][\ddot{\Delta}] \qquad \textbf{(18)}$$

By substituting the actual displacements, eq. (16) or (17), into eq. (18), than by premultiplying the result by [$\bar{\Delta}$ (x,y)]^T and integral of xy, we get,

$$[M][\ddot{\Delta}] + [K][\Delta] = [F] \qquad \textbf{(19)}$$

Where,

$$[\bar{\Delta}(x,y)] = [\bar{u}(x,y) \quad \bar{v}(x,y) \quad ....]^T$$

And [M] and [K] are mass and stiffness matrices, respectively; [Δ(t)] and [F] are displacement of time and load vector, respectively.

Cross-Ply Laminated Plate

Depending on the used theory, substitution eq. (16) into eq. (18) , get,

$$[M][\ddot{\Delta}] + [K][\Delta] = [F] \qquad \textbf{(20)}$$

Where,[M], [K], [Δ(t)] and [F] as, for (FSDT),
$K_{11}=\alpha^2 A_{11}+\beta^2 A_{66}$, $K_{12}=\alpha\beta(A_{12}+A_{66})$, $K_{13}=0$, $K_{14}=\alpha^2 B_{11}$, $K_{15}=0$, $K_{22}=\alpha^2 A_{66}+\beta^2 A_{22}$, $K_{23}=0$
$K_{24}=0$, $K_{25}=\beta^2 B_{22}$, $K_{33}=\alpha^2 A_{55}+\beta^2 A_{44}$, $K_{34}=\alpha A_{55}$, $K_{35}=\beta A_{44}$, $K_{44}=\alpha^2 D_{11}+\beta^2 D_{66}+A_{55}$

$K_{45}=\alpha\beta(D_{12}+D_{66})$, $K_{55}=\alpha^2 D_{66}+\beta^2 D_{22}+A_{44}$
And ,[M] as in eq. (15)
$[\Delta(t)]=[u(t) \;\; v(t) \;\; w(t) \;\; \psi_x(t) \;\; \psi_y(t)]^T$, $F(t)=[0 \;\; 0 \;\; \overline{q}(t) \;\; 0 \;\; 0]^T$.
Where,

$$\overline{q}(t)=\frac{4}{ab}\int_0^b\int_0^a \sin\alpha x.\sin\beta y.q(x,y).dx.dy.f(t)$$

The [M] and [K] matrix for symmetric cross-ply are as for antisymmetric cross-ply for subjected ($B_{ij}= E_{ij}=G_{ij}=0$).

Angle-ply Laminated Plate

Depending on the used theory, substitutioneq. (17) in to eq. (18), get,

$$[M][\ddot{\Delta}]+[K][\Delta]=[F] \qquad \textbf{(21)}$$

Where, [M], [K], [Δ(t)] and [F] as, for (FSDT), for anti-symmetric angle-ply laminated plates,
$K_{11}=\alpha^2 A_{11}+\beta^2 A_{16}$, $K_{12}=\alpha\beta(A_{12}+A_{66})$, $K_{13}=0$, $K_{14}=2\alpha\beta B_{16}$, $K_{15}=\alpha^2 B_{16}+\beta^2 B_{26}$
$K_{22}=\alpha^2 A_{66}+\beta^2 A_{22}$, $K_{23}=0$, $K_{24}=\alpha^2 B_{16}+\beta^2 B_{26}$, $K_{25}=2\alpha\beta B_{26}$, $K_{33}=\alpha^2 A_{55}+\beta^2 A_{44}$, $K_{34}=\alpha A_{55}$
$K_{35}=\beta A_{44}$, $K_{44}=\alpha^2 D_{11}+\beta^2 D_{66}+A_{55}$, $K_{45}=\alpha\beta(D_{12}+D_{66})$, $K_{55}=\alpha^2 D_{66}+\beta^2 D_{22}+A_{44}$.
And, [F] and [Δ(t)] as in equation (20).
And, $M_{11}=M_{22}=M_{33}=I_1$,$M_{44}=M_{55}=I_3$,$M_{ij}=0$ for $i\neq j$.

STIFFENED LAMINATED PLATES

To achieve a uniform distribution over the entire section consisting of plate and ribs, the spacing of the ribs must be small in comparison with whole span.A typical orthotropic element stiffened eccentrically with open ribs in x and y-directions is shown in **Fig. 1**.

Classical Plate Theory (CLPT)

The refined analysis of such a plate, the governing differential equations are expressed in terms of the displacements, u, v, and w, of the middle surface of the plate in the directions x, y, and z, respectively.

The force and moment relations for stiffened laminated plate are, **Troitsky (1976)**,

$$\begin{bmatrix} N \\ M \end{bmatrix}=\begin{bmatrix} N \\ M \end{bmatrix}_{un.st.pl.}+\begin{bmatrix} N \\ M \end{bmatrix}_{st.pl.} \qquad \textbf{(22)}$$

Where, $\begin{bmatrix} N \\ M \end{bmatrix}_{un.st.pl.}$ are force and moment relations for un stiffened laminated plate, and $\begin{bmatrix} N \\ M \end{bmatrix}_{st.pl.}$ are force and moment relations for stiffened plate as,

$$\begin{bmatrix} N_x \\ N_y \\ M_x \\ M_y \end{bmatrix} = \begin{bmatrix} A_{11}^{st.} & 0 & B_{11}^{st} & 0 \\ 0 & A_{22}^{st} & 0 & B_{22}^{st} \\ B_{11}^{st} & 0 & D_{11}^{st} & 0 \\ 0 & B_{22}^{st} & 0 & D_{22}^{st} \end{bmatrix} \begin{bmatrix} \varepsilon_x^{(0)} \\ \varepsilon_y^{(0)} \\ \mathrm{K}_x \\ \mathrm{K}_y \end{bmatrix} \qquad \textbf{(23)}$$

For,

$$\left(A_{11}^{st}, B_{11}^{st}, D_{11}^{st}\right) = \int_{Z_{i-1)x}^{st}}^{Z_{i)x}^{st}} \frac{t_x}{b_x} E_x^{st}\left(1, Z, Z^2\right)\, dz \ , \ \left(A_{22}^{st}, B_{22}^{st}, D_{221}^{st}\right) = \int_{Z_{i-1)y}^{st}}^{Z_{i)y}^{st}} \frac{t_y}{b_y} E_y^{st}\left(1, Z, Z^2\right)\, dz$$

Where, E_x^{st} *and* E_y^{st} are modules of elasticity of stiffeners in x and y-directions, respectively.

Then, by substituting equation (22) in to equations of motion, we get [M] and [K] matrices as, $[M]=[M]_{st.}+[M]_{un.st.}$ and [K]=-[L], for $[L]=[L]_{st} + [L]_{un.st}$.

Where, $[M]_{un.st.}$,$[L]_{un.st.}$ are mass and stiffness matrices for un stiffened laminated plate, and $[M]_{st.}$,$[L]_{st.}$ are mass and stiffness matrices for stiffeners determined as,

$$L_{11}^{st} = A_{11}^{st} d_{xx} \ , L_{12}^{st} = 0, L_{13}^{st} = -B_{11}^{st} d_{xxx} \ , L_{22}^{st} = A_{22}^{st} d_{yy} \ , \ L_{23}^{st} = -B_{22}^{st} d_{yy} \ ,$$
$$L_{33}^{st} = -D_{11}^{st} d_{xxxx} - D_{22}^{st} d_{yyyy} \qquad \textbf{(24)}$$
$$M_{11}^{st} = I_1^{st)x}, M_{12}^{st} = 0, M_{13}^{st} = -I_2^{st)x} d_x, \ M_{22}^{st} = I_1^{st)y} \ , \ M_{23}^{st} = -I_2^{st)y} dy,$$
$$M_{33}^{st} = I_1^{st)x} + I_1^{st)y} - (d_{xx} + d_{yy})(I_3^{st)x} + I_3^{st)y})$$

Where,

$$(I_1^{st)x}, I_2^{st)x}, I_3^{st)x}) = \int_{Z_{i-1)x}^{st}}^{Z_{i)x}^{st}} \rho_x^{st}(1, Z, Z^2)\ dz \ \ , and \ \ , (I_1^{st)y}, I_2^{st)y}, I_3^{st)y}) = \int_{Z_{i-1)y}^{st}}^{Z_{i)y}^{st}} \rho_y^{st}(1, Z, Z^2)\ dz$$

(25)

For ρ_x^{st} *and* ρ_y^{st} are the density of stiffeners in x and y-directions respectively.

First-Order Shear Deformation Theory (FSDT)

The force and moment relations for stiffened laminated plate are,

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} N \\ M \end{bmatrix}_{un.st.pl.} + \begin{bmatrix} N \\ M \end{bmatrix}_{st.pl.} \ , \text{and,} \begin{bmatrix} Q_y \\ Q_x \end{bmatrix} = \begin{bmatrix} Q_y \\ Q_x \end{bmatrix}_{st.pl.} + \begin{bmatrix} Q_y \\ Q_x \end{bmatrix}_{un.st.pl.} \qquad \textbf{(26)}$$

Where, [N], [M] as in equations (22) , $\begin{bmatrix} Q_y \\ Q_x \end{bmatrix}_{un.st.pl.}$ are shear force for un stiffened laminated plate and $\begin{bmatrix} Q_y \\ Q_x \end{bmatrix}_{st.pl.}$ are shear force for stiffeners as,

$$\begin{bmatrix} Q_y \\ Q_x \end{bmatrix}_{st.pl.} = \begin{bmatrix} A_{44}^{st.} & 0 \\ 0 & A_{55}^{st.} \end{bmatrix} \begin{bmatrix} \gamma_{yz} \\ xz \end{bmatrix} \quad \textbf{(27)}$$

For,

$$A_{44}^{st.} = \int_{Z_{i-1)x}^{st.}}^{Z_{i)x}^{st.}} G^{st)x} \frac{t_x}{b_x} \, dz \ , \ A_{55}^{st.} = \int_{Z_{i-1)y}^{st.}}^{Z_{i)y}^{st.}} G^{st)y} \frac{t_y}{b_y} \, dz$$

Where, $G^{st)x}$ and $G^{st)y}$ are shear modules of elasticity of stiffeners in x and y-directions, respectively.

Then, by substituting equation (26) into equations of motion, Eq. (11), we get [M] and [K] matrices as,

$[M]=[M]_{st.}+[M]_{un.st.}$ and $[K]=-[L]$, for $[L]=[L]_{st}+[L]_{un.st}$.

Where, $[M]_{un.st.}$,$[L]_{un.st.}$ are mass and stiffness matrices for un stiffened laminated plate, and $[M]_{st.}$,$[L]_{st.}$ are mass and stiffness matrices for stiffeners determined as,

$$L_{11}^{st} = A_{11}^{st} d_{xx} \ , L_{14}^{st} = B_{11}^{st} d_{xx} \ L_{22}^{st} = A_{22}^{st} d_{yy} \ , L_{25}^{st} = B_{22}^{st} d_{yy} \ , L_{33}^{st} = A_{55}^{st} d_{xx} + A_{44}^{st} d_{yy} \ ,$$
$$L_{34}^{st} = A_{55}^{st} d_x L_{35}^{st} = A_{44}^{st} d_y \ , L_{44}^{st} = D_{11}^{st} d_{xx} - A_{55}^{st} \ , L_{55}^{st} = D_{22}^{st} d_{yy} - A_{44}^{st} \ , \textit{and other term of } L_{ij} = 0$$
$$\textit{and} \ , M_{11}^{st} = I_{1)x}^{st} , M_{14}^{st} = I_{2)x}^{st} \ , M_{22}^{st} = I_{1)y}^{st} \ , M_{25}^{st} = I_{2)y}^{st} \ , M_{33}^{st} = I_{1)x}^{st} + I_{1)y}^{st} \ , M_{44}^{st} = I_{3)x}^{st} \ ,$$
$$M_{55}^{st} = I_{3)y}^{st} \ , \textit{and other term of } M_{ij} = 0.$$

MODAL ANALYSIS

For a system with (n) coordinates or degrees of freedom, the governing equations of motion are a set of (n) coupled ordinary differential equations of second order. The solution of these equations becomes more complex when the degree of freedom of the system (n) is large and/or when the forcing functions are non-periodic. In such cases, a more convenient method known "Modal analysis" can be used to solve the problem.

The equation of motion of a multi-degree of freedom system under external forces are given by,**Singiresu 1995**,

$$[M][\ddot{\Delta}] + [K][\Delta] = [F] \quad \textbf{(28)}$$

To solve equation (28) by modal analysis, it is necessary first to solve the eigenvalue problem and find the natural frequencies ω_1,ω_2, ……, ω_n and the corresponding normal weighted modal $[\tilde{P}]$.

The solution vector of equation (28) can be expressed by a linear combination of the normal weighted modal,

$$\overline{\Delta}(t) = \left[\tilde{P}\right]\vec{q}_p(t) \qquad \textbf{(29)}$$

$$\text{Where, } \vec{q}_p(t) = \begin{bmatrix} q_{p_1}(t) \\ q_{p_2}(t) \\ \vdots \\ q_{p_N}(t) \end{bmatrix} \qquad \textbf{(30)}$$

Where $q_{p_1}(t)$, $q_{p_2}(t)$,….., $q_{p_n}(t)$ are time-dependent generalized coordinates, also known as the "principal coordinates or modal participation coefficients".
By substituting equation (29) into equation (28) , than, premultiplying throughout by $\left[\tilde{P}\right]^T$, get,

$$\ddot{q}_p(t) + \left[\ddots\omega^2\ddots\right]q_p(t) = \vec{Q}_p(t) \qquad \textbf{(31)}$$

Where,

$$[I] = \left[\tilde{P}\right]^T [M]\left[\tilde{P}\right], \left[\ddots\omega^2\ddots\right] = \left[\tilde{P}\right]^T [K]\left[\tilde{P}\right] \text{ and } \overline{Q}_p(t) = \left[\tilde{P}\right]^T \vec{F}(t)$$

Equation (31) denotes a set of (n) uncoupled differential equations of second order,

$$\ddot{q}_{p_i}(t) + \omega_i^2 q_{p_i}(t) = Q_{p_i}(t) \qquad \textbf{(32)}$$

COMPUTER PROGRAMMING

The computer programs designed in this work are concerned with solving the dynamic problems for composite laminated plates using any theory for laminated plates. The computer programs constructed herein are coded in "Fortran Power Station 4.0" language, the following flow chart of the dynamic program ,as shown in **Fig. 2**.

RESULTS AND DISCUSSION

Un-Stiffened Plates

The case study discussed here is a un-stiffened laminated simple supported plate **Fig. 3.** with dimensions and material properties give below using the first-order shear deformation theory (FSDT) and applying the suggested analytical solution and finite element method.

Fig. 4.shows a comparison of the present work solutions by Analytical and finite elements method with the numerical solution of **Reddy, J. N. (1982)**they are given for two layer simply supported cross-ply laminated plate subjected to sinusoidal Pulse loading (q(x,y,t)=P(x,y), for P(x,y)=q_o sin(π x/a)sin(π y/b) ,qo=10 N/cm^2) and the properties of plate, $E_2=2.1*10^6$ N/cm^2, $E_1/E_2=25$, $G_{12}=G_{13}=G_{23}=0.5E_2$, ρ=800 Kg/m^3, ν=0.25, a=b=25 cm , h=5 cm.

Fig. 5.shows a comparison of the present work with the numerical solution of **Reddy, J. N. (1982)**they are given for simply supported two layer cross-ply laminated plate subjected to sinusoidal Pulse loading (q(x,y,t)=P(x,y), for
P(x,y)=q_o sin(π x/a)sin(π y/b) ,qo=10 N/cm^2) ,for properties of plate:-

E_2=2.1*10^6 N/cm^2, E_1/E_2=25, $G_{12}=G_{13}=G_{23}=0.5E_2$, ρ=800 Kg/$m^3$, ν=0.25, a=b=25 cm , h=1 cm.

The following properties were using for simply supported Laminated Plates ,in **Figs**. 6, 7, and 8, for qo=10 N/cm^2 ,t_o=0.0005 sec, simply supported laminated plates,
E_2=2.1*10^6 N/cm^2, E_1/E_2=25, $G_{12}=G_{13}=G_{23}=0.5E_2$, ρ=1500 Kg/$m^3$, ν=0.25.
a=b=25 cm , h=5 cm .

Fig. 6. represents the variation of central transverse deflection with time for antisymmetric cross-ply (0/90/0/…) laminated plates under sinusoidal variation loading (plus q(x,y,t)=P(x,y) ,Ramp loading q(x,y,t)=P(x,y) t/t_o and sine loading q(x,y,t)=P(x,y) sinπt/t_o) for qo=10 N/cm^2 ,t_o=0.0005 sec) solutions by analytical and (F.E.M).The deflection due to pulse loading higher in magnitude than the other loading because the pulse load subjected suddenly with constant value with time.

Fig. 7. represents the variation of central transverse deflection with time for Antisymmetric cross-ply (0/90/0/…) laminated plates under sinusoidal (P(x,y)=q_o sin(πx/a)sin(πy/b)) and uniform (P(x,y)=q_o) plus loading solutions by analytical and (F.E.M). The deflection due to uniform load higher in magnitude than the deflection due to sinusoidal loading.

Fig. 8. represents the variation of central transverse deflection with time for angle-ply and cross-ply laminated under sinusoidal Ramp loading solution by analytical and (F.E.M). The (0/90/…) laminated higher in magnitude than the (45/-45/…) laminated because at (θ=45^0/-45^0/…) the extension and bending stiffnesses A_{16}, A_{26}, D_{16} and D_{26} appear to have a significant effect while at (θ=0^0/90^0/…)the extension and bending stiffnesses A_{16}, A_{26}, D_{16} and D_{26} are zero.

The following properties were used for simply supported Laminated Plates, **Figs.** (9 to 14),
E_1=130.8 Gpa, E_2=10.6 Gpa, $G_{13}=G_{23}$=6 Gpa ,G_{23}=3.4 Gpa , ρ=1580 Kg/m^3, ν=0.25.
a=b=1 m h=0.02 m, and q_o=10 kn/m^2 t_o=0.05 sec.

Fig. 9. represents the effect of the degree of othotropy (E_1/E_2) (E_2=10.6 Gpa) on the deflection with time of simply supported antisymmetric cross-ply laminated plates subjected to sinusoidal plus loading solution by analytical and (F.E.M). From the figure, increasing the material orthotropy ratio (E_1/E_2) will decreases the deflection. **Fig. 10.** shows the effect of the aspect ratio (a/b) on the deflection of the simply supported antisymmetric cross-ply laminated plates(a=1 m) subjected to sinusoidal Ramp loading solution by analytical and (F.E.M). From the results, the increase of (a/b) ratio increases the deflection.

Fig. 11. shows the effect of the (a/h) ratio on the deflection of the simply supported antisymmetric cross-ply laminated plates (a=1 m)subjected to sinusoidal sine loading solution by analytical and (F.E.M). From the results, the increase of (a/h) ratio increases the deflection of laminated plates.**Fig. 12.** shows the effect of the number of layer of simply supported antisymmetric cross-ply laminated plates on the deflection of plate subjected to sinusoidal Pulse loading solution by analytical and (F.E.M) . The central deflection of laminated plates decreases with increasing number of layers.

Fig. 13. shows the effect of the lamination angle ($θ^0$) on the deflection of simply supported antisymmetric angle-ply laminated plates under sinusoidal ramp loading solution by analytical and (F.E.M). It is apparent from the results that the deflection decreases with increasing the angle of laminated.**Fig. 14.** shows the effect of the number of layer of simply supported antisymmetric angle-ply laminated plates on the deflection of plate subjected to sinusoidal sine loading solution by analytical and (F.E.M). The central deflection of laminated plates decreases with increasing number of layers.

The following properties were used for simply supported laminated plates, for analytical solutions, in figs. 15 to 20, qo=10 kn/m^2 ,t_o=0.05 sec, simply supported**,**
E_1=130.8 Gpa ,E_2=10.6 Gpa ,$G_{12}=G_{13}$=6 Gpa, G_{23}=3.4 Gpa ,ρ=1580 Kg/m^3, ν=0.28.

a=b=1 m ,h=0.02 m.

Fig. 15. represents the stress-x in each layer, at the middle of layers, with time for four layers Antisymmetric cross-ply (0/90/0/…) laminated plates under uniformly ramp loading $q(x,y,t)=q_o\ t/t_o$ for qo=1 N/cm^2 ,t_o=0.05 sec), at x=a/2, y=b/2. The maximum value of σ_x is at layer-1 and the stress-x are antisymmetric about the middle plane. **Fig. 16.** represents the stress-x in layer-1, at the middle of layer, with time for different number of layer for Antisymmetric cross-ply (0/90/0/…) laminated plates under uniformly ramp loading $q(x,y,t)=q_o\ t/t_o$ for qo=1 N/cm^2 ,t_o=0.05 sec), at x=a/2, y=b/2. The value of σ_x at layer-1 increase with increase the number of layers

Fig. 17. represents the effect of the lamination angle(θ^0) on theσ_x at layer-1 for four layers antisymmetric angle-ply laminated plates under uniformly ramp loading, at x=a/2, y=b/2. From the results the σ_x decreases with the increase of the angle of laminated to the 45^0 ,the minimum value at 45^0 and the maximum value at 0^0.**Fig. 18.** represents the comparison of stress-x with stress-y at layer-1 for four layers antisymmetric cross-ply laminated plates for difference E_1/E_2 under uniformly pulse loading, at x=a/2, y=b/2. From the results, stresses-x are more than stresses-y at $E_1/E_2\neq1$ and Stress-x equal stress-y for $E_1/E_2=1$.

Fig. 19. represents the comparison stress-x with stress-y at layer-1 for four layers antisymmetric cross-ply laminated plates for difference aspect ratio under uniformly ramp loading, at x=a/2, y=b/2. From the results, stresses-x are more than stresses-y.

Fig. 20. represents the stress-y in layer-1, at the middle of layer, with time for different number of layer for Antisymmetric cross-ply (0/90/0/…) laminated plates under uniformly sine loading $q(x,y,t)=q_o\sin(\pi t/t_o)$ for qo=1 N/cm^2 ,t_o=0.05 sec), at x=a/2, y=b/2. The value of σ_y at layer-1 decreases with the increase of the number of layers.

Stiffened Laminated Plates

The case study discussed here is a stiffened laminated simple supported plate **Fig. 1**. with dimensions and material properties give below using the first-order shear deformation theory (FSDT) and applying the suggested analytical solution and finite element method.

The following properties were used for simply supported stiffened laminated plates, in **Figs**. 21. to 27, qo=10 kn/m^2 ,t_o=0.05 sec, dynamic numerical and analytical solution:
E_1=130.8 Gpa ,E_2=10.6 Gpa ,$G_{12}=G_{13}$=6 Gpa, G_{23}=3.4 Gpa ,ρ=1580 Kg/m^3, ν=0.28.
a=b=1 m ,h=0.02 m.
And, for stiffeners:
$E_{st')x}=E_{st)y}=E_1$,$G_{st.)x}=G_{st)y}=G_{12}$,$h_x=h_y$=0.025 m, $t_x=t_y$=0.0025 m,$\rho_{st)x}=\rho_{st)y}=\rho$.

Comparison of the stiffened laminated plates with the un stiffened laminated plate are shown in **Fig.** 21 , for cross-ply two and four layer for un stiffened laminated and two and four layer, and three stiffeners in x and y-directions for stiffened laminated plates, subjected to sinusoidal pulse loading.

Fig. 22. shows the effect of the number of stiffeners of the four layer antisymmetric cross-ply stiffened laminated plates subjected to sinusoidal pulse loading. The figure shows that the increase of the numbers of stiffeners decreases the deflection of stiffened laminated plates.

Fig. 23. shows the effect of (E_{st}/E_1) ,(for $E_{st)x}=E_{st)y}$ and E_1=130.8 Gpa) for four layer and four stiffeners of stiffened laminated plates subjected to sinusoidal pulse loading. From the figure, the deflection of stiffened laminated plates decreases with increase ($E_{st.}/E_1$) ratio, decreases with increase $E_{st.}$.

Fig. 24. shows the effect of the higher to width (h_s/t_s), ratio of stiffeners ,(for $h_s=h_x=h_y$=.025 m and $t_s=t_x=t_y$) for four layer and four stiffeners cross-ply stiffened laminated plates subjected to sinusoidal Ramp loading. From the figure, the deflection of stiffened plates increase with increases (h_s/t_s) ratio, increase with decreases t_s .

Fig. 25. represents the effect of the distance of stiffeners in x and y directions on the central deflection for four layer antisymmetric cross-ply stiffened laminated plates under sinusoidal pulse loading. From the results, the deflection decrease with decrease the distances of stiffeners.

Fig. 26. represents the effect of the aspect ratio for different distance of stiffeners in x and y directions on the central deflection for four layer antisymmetric cross-ply stiffened laminated plates under sinusoidal pulse loading.

Fig. 27. shows the effect of the higher of stiffeners to the thickness of Laminate (h_s/h_p) ratio, ($h_s=h_x=h_y$ and $h_p=h=0.02$ m), for four layer and three stiffeners cross-ply stiffened laminated plates subjected to sinusoidal sine loading. The figure showed that the deflection or stiffened laminated plates decreases with increase (h_s/h_p) ratio, decreases with increase h_s .

The following properties were using for simply supported laminated plates, in figs. 28 to 32, qo=10 kn/m^2 ,t_o=0.05 sec, dynamic analytical solution:
E_1=130.8 Gpa ,E_2=10.6 Gpa ,G_{12}=G_{13}=6 Gpa, G_{23}=3.4 Gpa ,ρ=1580 Kg/m^3, ν=0.28.
a=b=1 m ,h=0.02 m.
And, for stiffeners:- $E_{st')x}=E_{st)y}=E_1$,$G_{st.)x}=G_{st)y}=G_{12}$,$h_x=h_y$=0.025 m, $t_x=t_y$=0.0025 m,$\rho_{st)x}=\rho_{st)y}$= ρ.

Fig. 28. represents the effect of the distance of stiffeners in x and y directions on the stress-x in layer-1 for four layer antisymmetric cross-ply stiffened laminated plates under uniformly ramp loading, at x=a/2, y=b/2. From the results, the stress-x decrease with decrease the distances of stiffeners.

Fig. 29. represents the stress-x in layer-1, at the middle of layers, with time for four layers Antisymmetric cross-ply (0/90/0/…) stiffened laminated plates for difference number of stiffeners in x and y directions under uniformly sine loading, at x=a/2, y=b/2. From the results, the stress-x decrease with increase the number of stiffeners.

Fig. 30. represents the stress-x in layer-4, at the middle of layers, with time for four layers Antisymmetric cross-ply (0/90/0/…) stiffened laminated plates for difference number of stiffeners in x and y directions under uniformly sine loading, at x=a/2, y=b/2. From the results, the stress-x decrease with increase the number of stiffeners.

Fig. 31. represents the comparison σ_x for stiffened with unstiffened laminated plates in each layer for four layer antisymmetric cross-ply for stiffeners in x and y directions under uniformly sine loading, at x=a/2, y=b/2. The stresses-x for stiffened plates are less than that corresponding of unstiffened plate.

Fig. 32. represents the comparison σ_y for stiffened with unstiffened laminated plates in each layer for four layer antisymmetric cross-ply for stiffeners in x and y directions under uniformly sine loading, at x=a/2, y=b/2. The stresses-y for stiffened plates are less than that corresponding of unstiffened plates.

CONCLUSIONS

1. The suggested analytical solution is a powerful tool for un-stiffened laminated plated subjected to time depended loading, by solution the general differential equations of motion of (FSDT) for laminated plated by using separation method for differential equation and model analysis method foe forced vibration.
2. The presented work showed that the increasing the numbers of layers for laminated, the angle of fibers, the modules of elasticity E_1 more than E_2, the aspect ratio, the thickness of laminated decreases the deflection of laminated plates.
3. The presented work shows that the increasing the aspect ratio or angle of fibers decreases the stress-x, and the increase of number of layer or the E_1/E_2 ratio increases the stress-x. And the increase of the number of layers or the E_1/E_2 ratio decreases the stress-y.

4. The presented work shows that increasing the number of stiffeners, the higher, the width, and the modules of elasticity of stiffeners decreases the deflection of stiffened laminated plates. And the increasing the numbers of stiffeners decreases the stress in each layer for stiffened laminated plates.

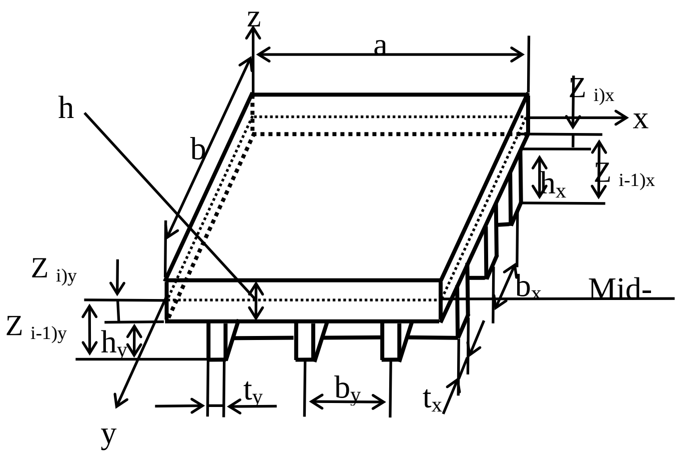

Fig. 1. Dimensions and Directions of Stiffened Laminated Plates.

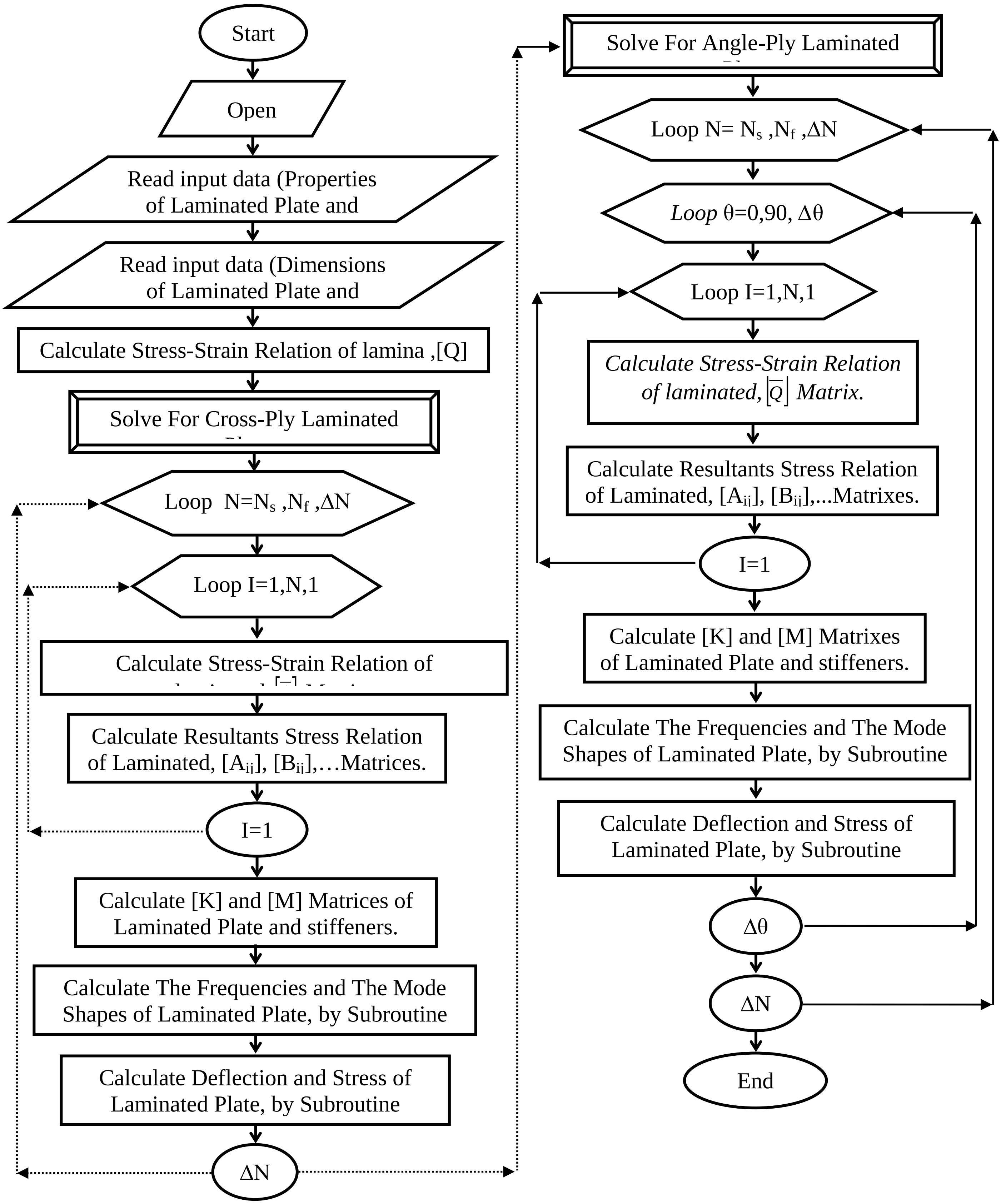

Fig. 2. Flow Chart for Computer Program of Dynamic Solution for Laminated Plates.

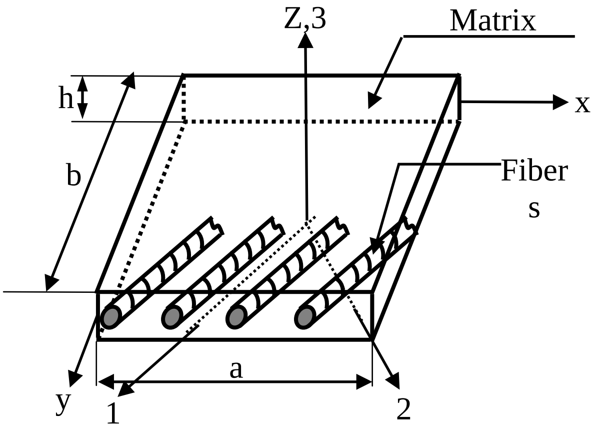

Fig. 3. Dimensions and Directions of Un-Stiffened Laminated Plate.

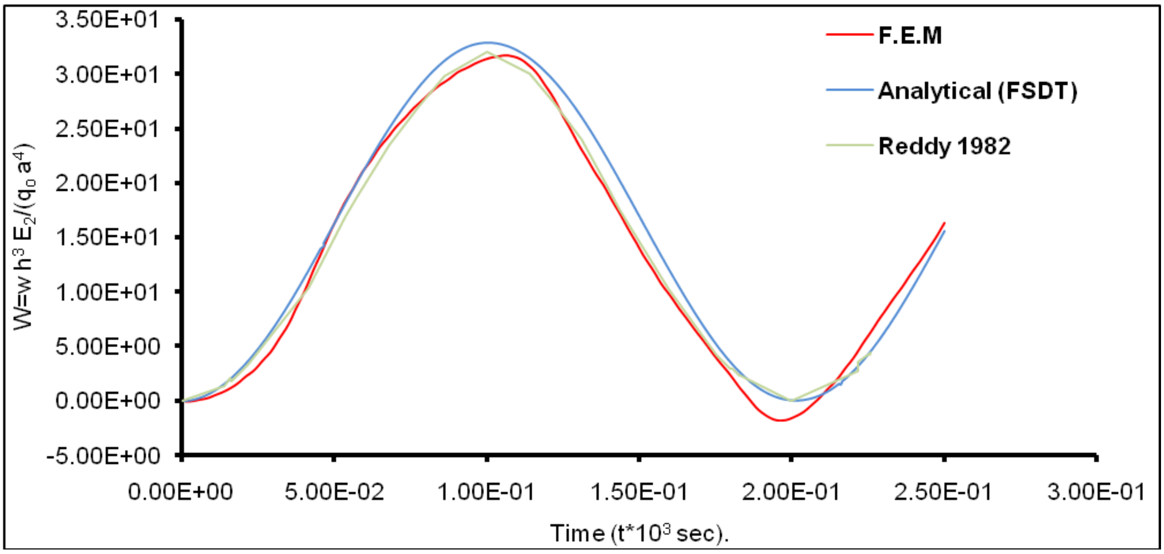

Fig. 4. Central deflection due to Sinusoidal Pulse loading for two Layer.

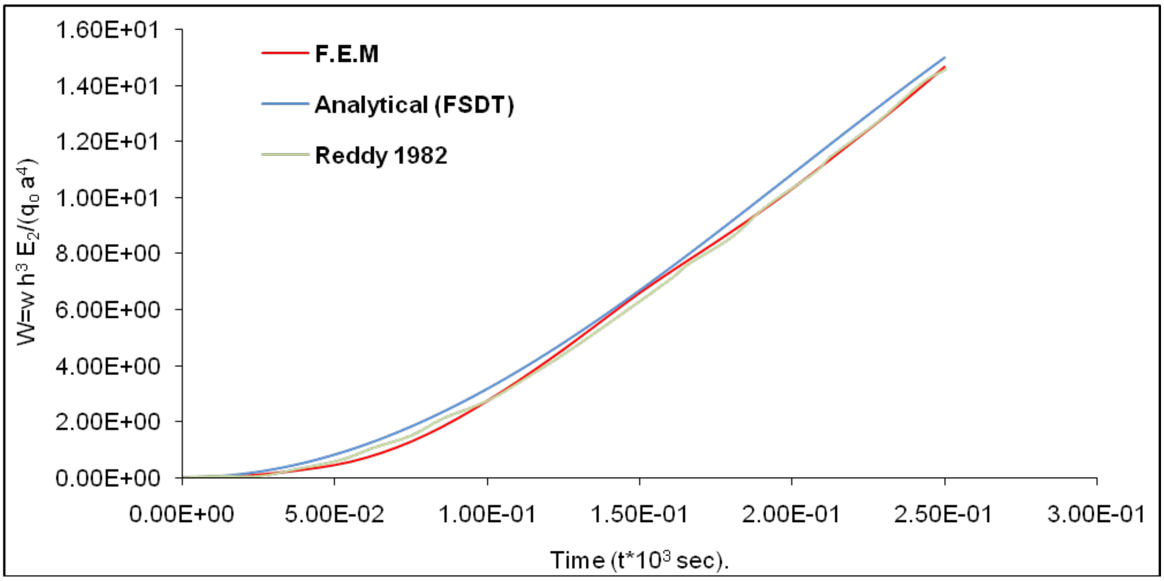

Fig. 5. Central deflection due to Sinusoidal Pulse loading for two Layer.

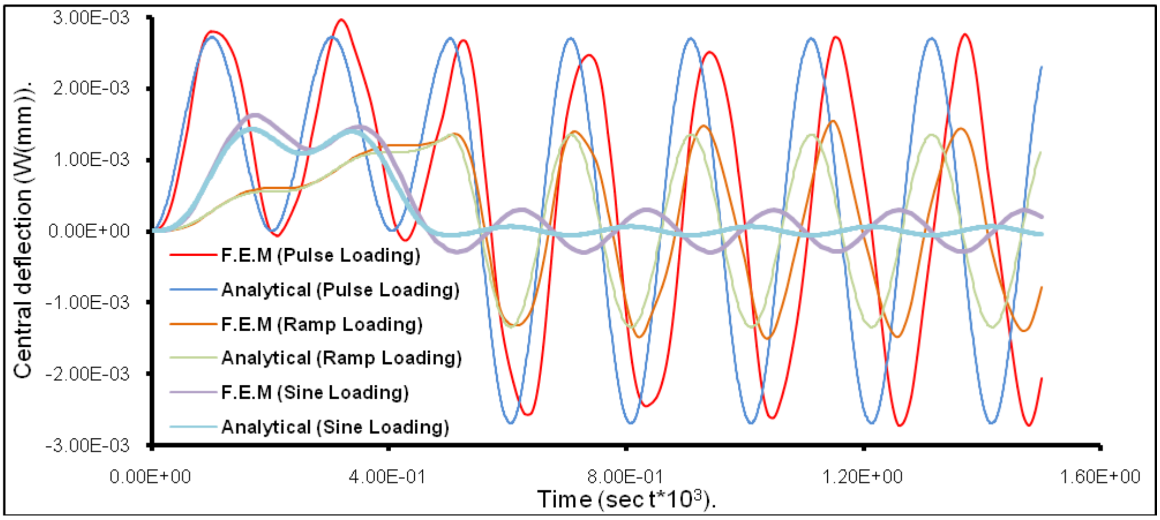

Fig. 6. Central Deflection for Variant dynamic Load for sinusoidal Load (n=4).

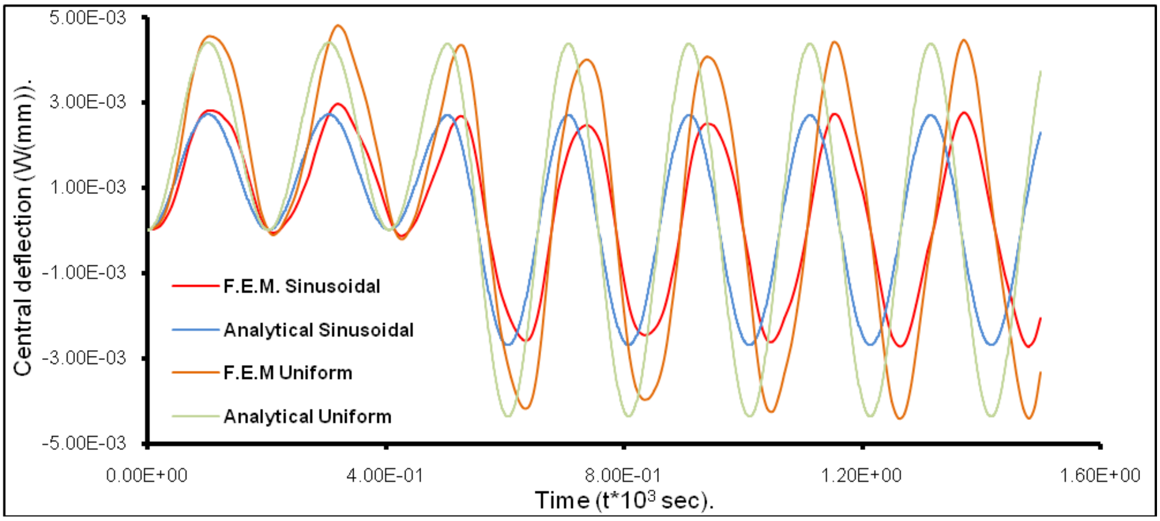

Fig. 7. Central deflection due to Variant d Pulse loading (n=4).

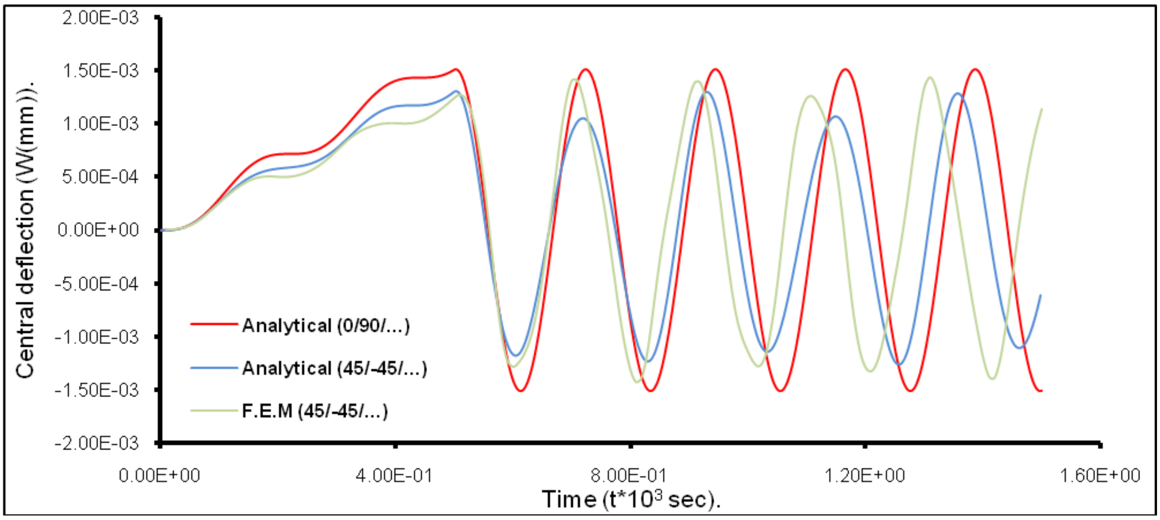

Fig. 8. Central deflection due to Sinusoidal Ramp Loading (n=4).

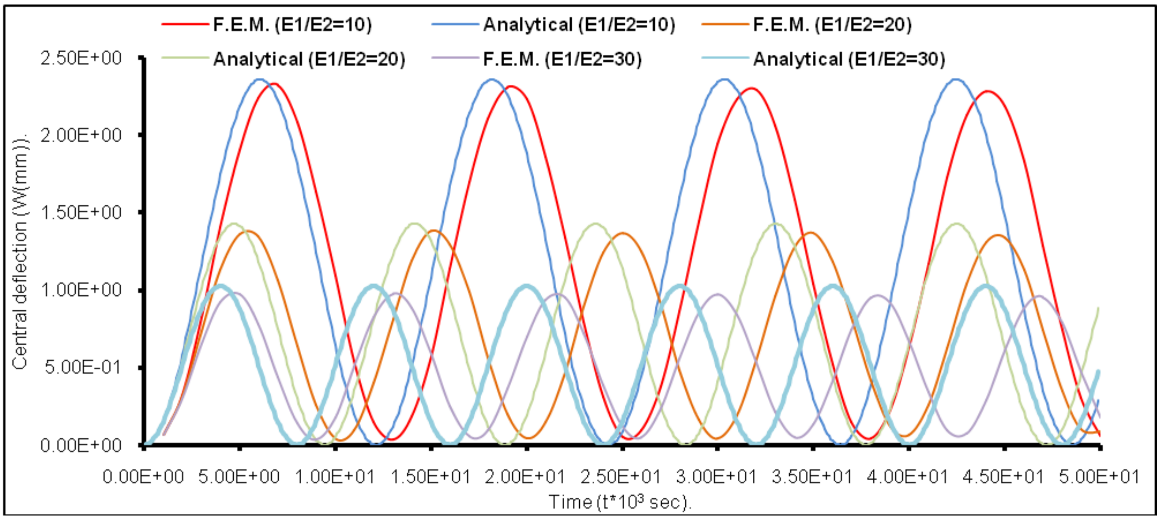

Fig. 9. Central deflection due to Sinusoidal Pulse loading for N=4.

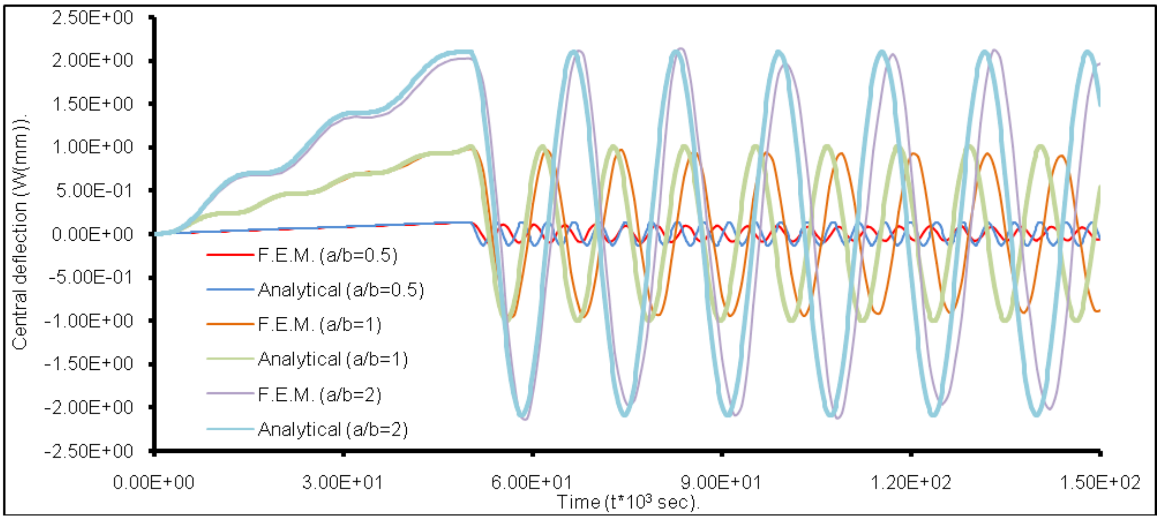

Fig. 10. Central deflection due to Sinusoidal Ramp Loading for (N=4).

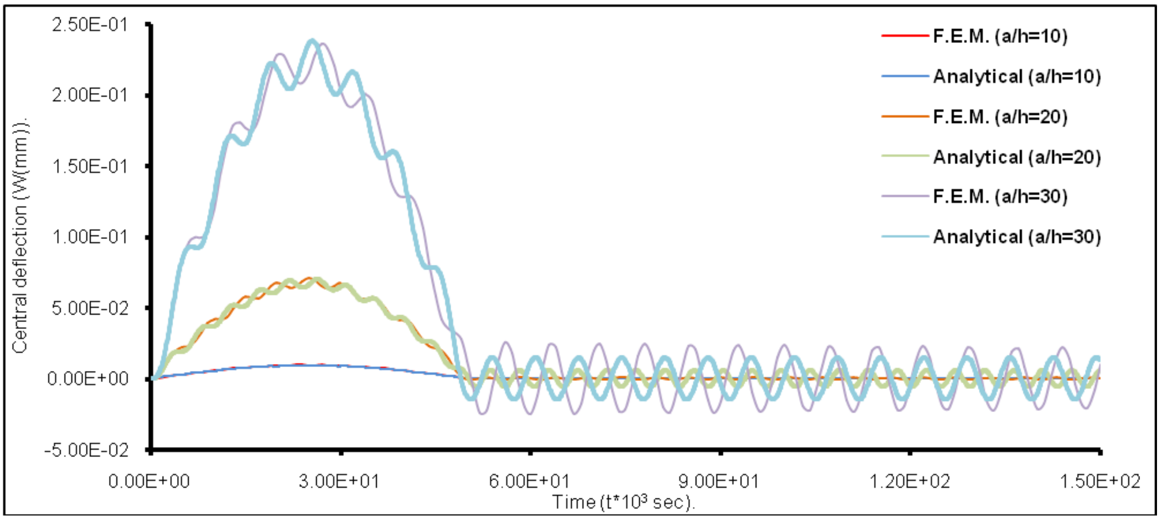

Fig. 11. Central deflection due to Sinusoidal Sine Loading for (N=4)

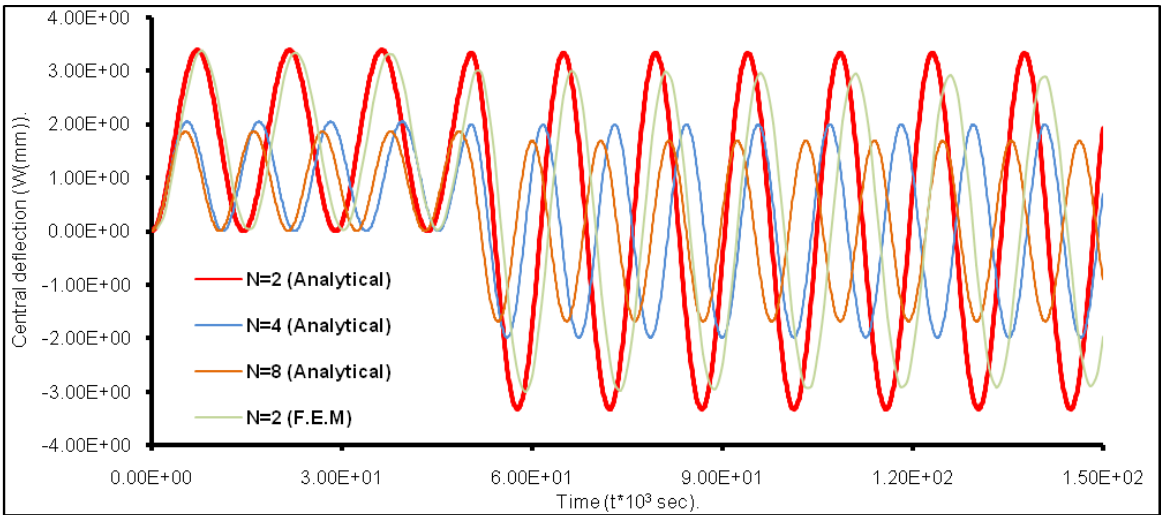

Fig. 12. Central deflection due to Sinusoidal Pulse loading for (0/90/…) plates.

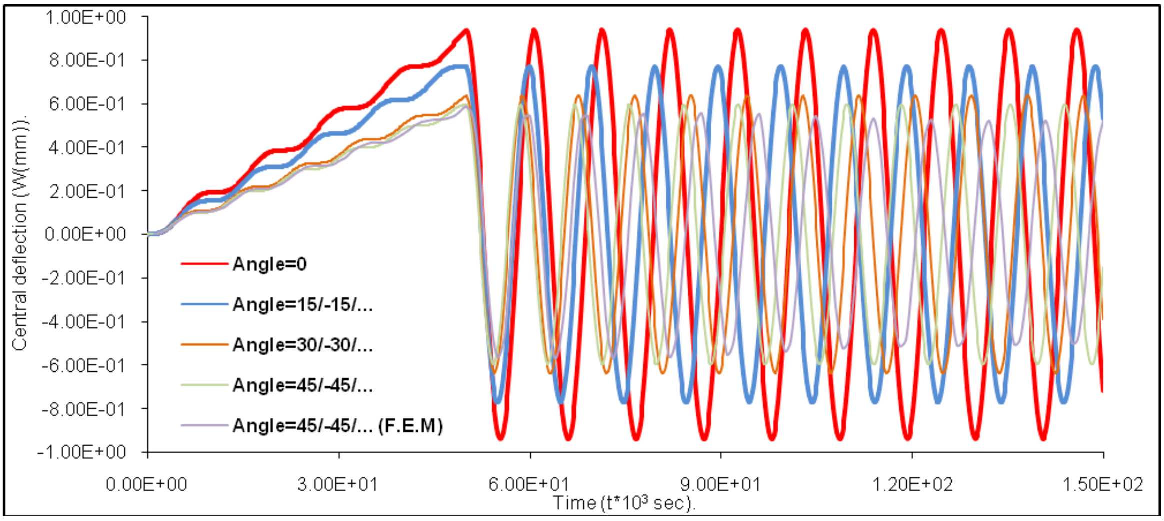

Fig. 13. Central deflection due to Sinusoidal Ramp Loading for (N=6).

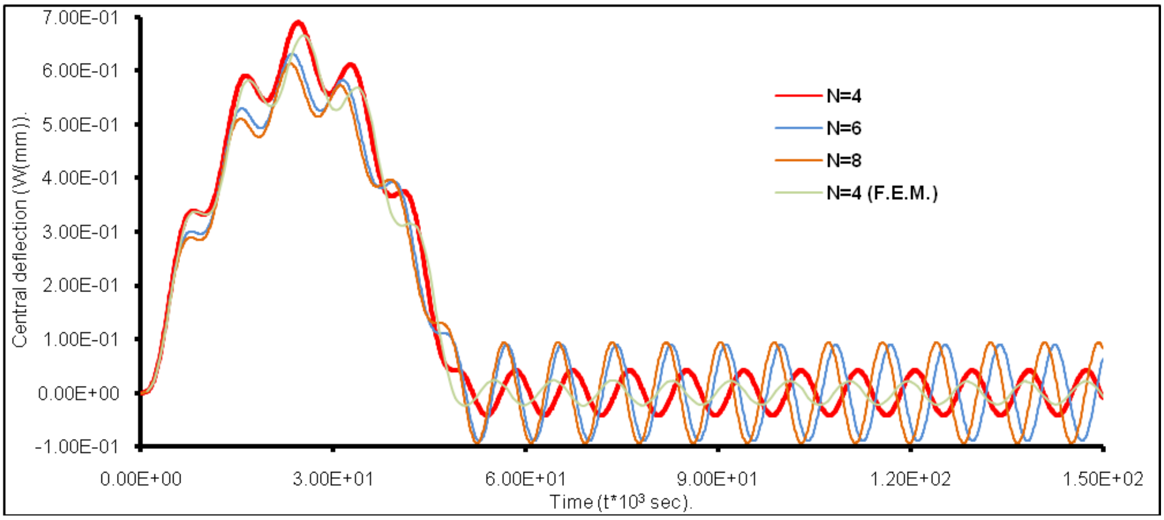

Fig. 14. Central deflection due to Sinusoidal Sine Loading for (θ=45/-45/…).

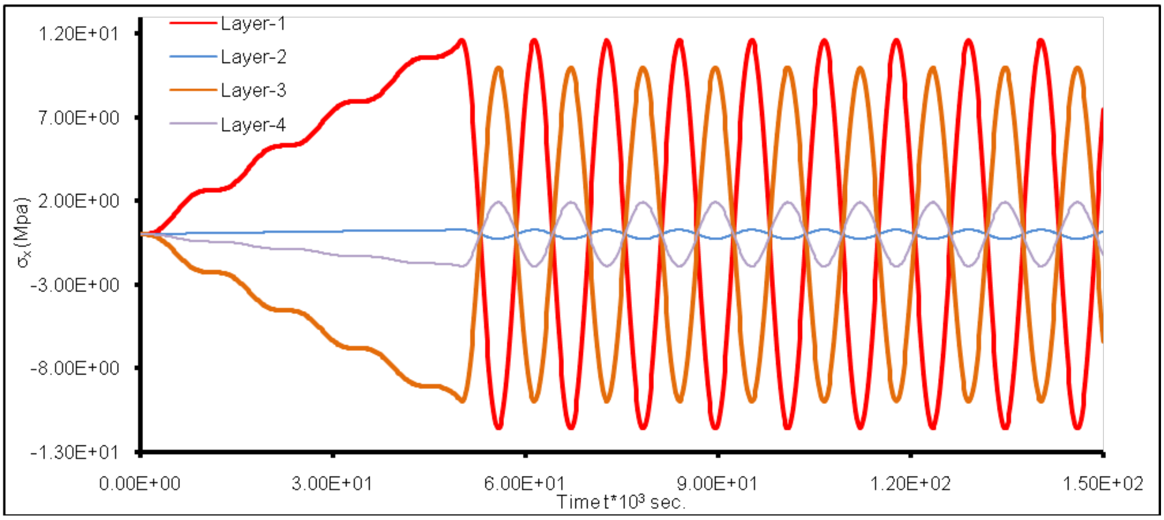

Fig. 15. Stress-x in each Layer due to Uniform Ramp Loading for (N=4).

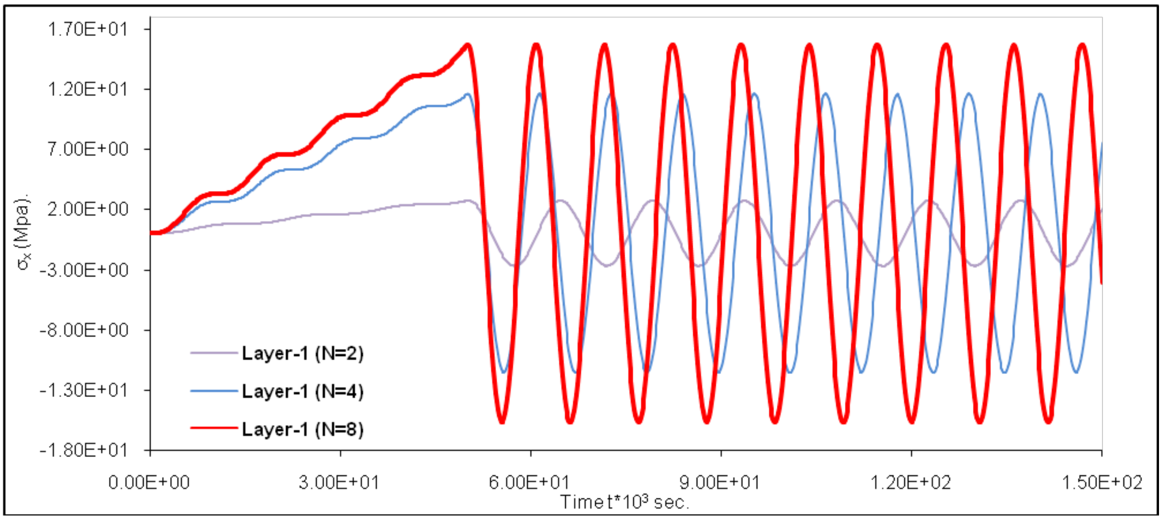

Fig. 16. Stress-x in Layer-1 due to Uniform ramp Loading.

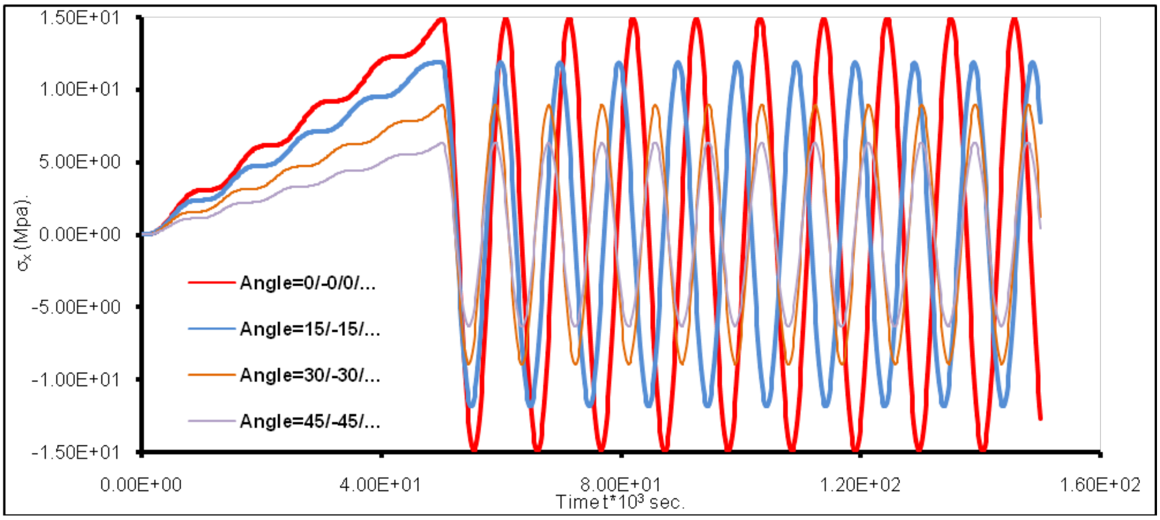

Fig. 17. Stress-x at Layer-1 due to Uniform Ramp Loading for N=4.

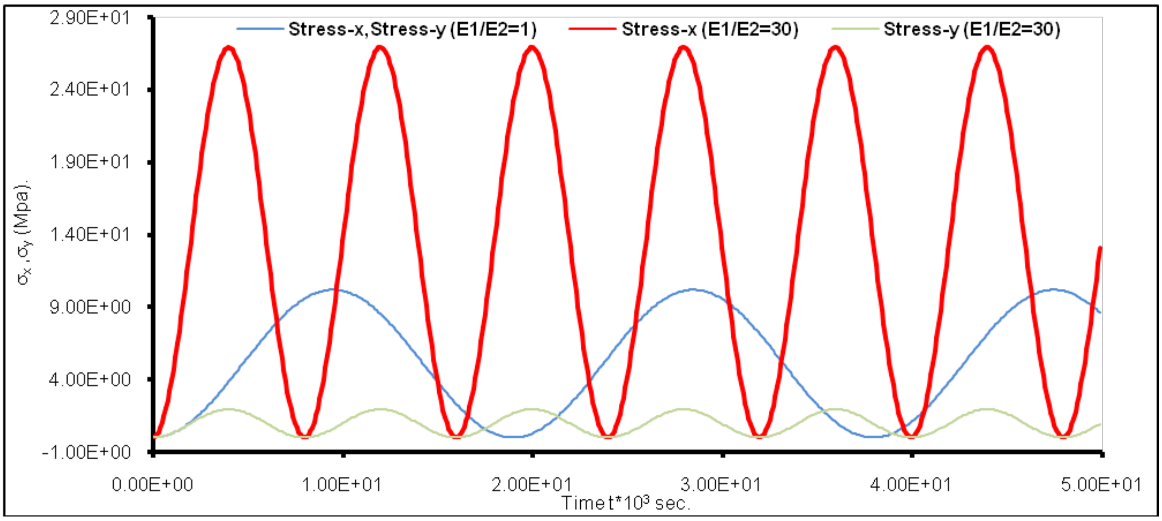

Fig. 18. Stress-x and Stress-y in Layer-1 due to Uniform Pulse loading for N=4.

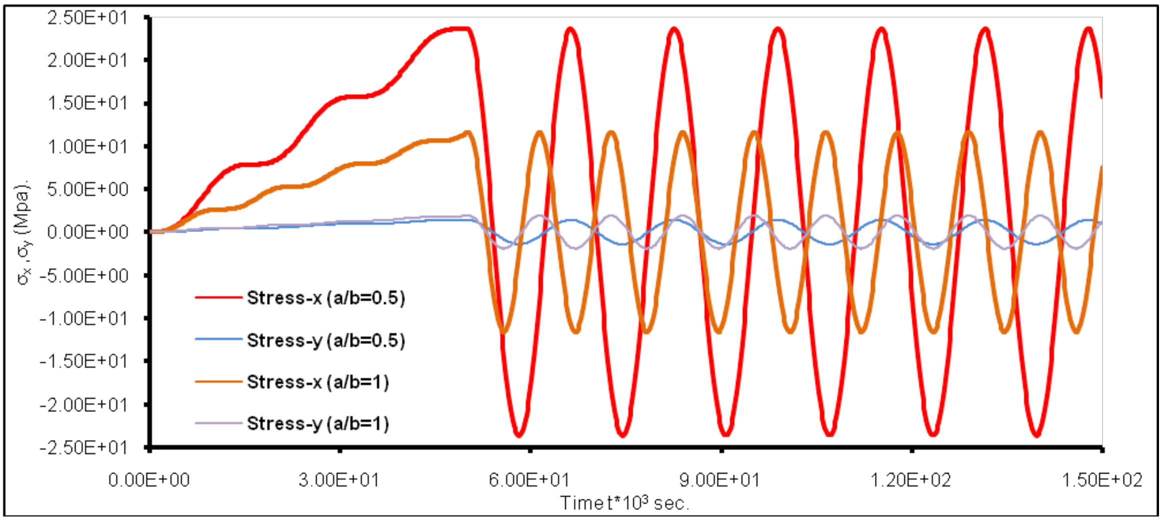

Fig. 19. Stress-x and Stress-y in Layer-1 due to Uniform Ramp loading for N=4.

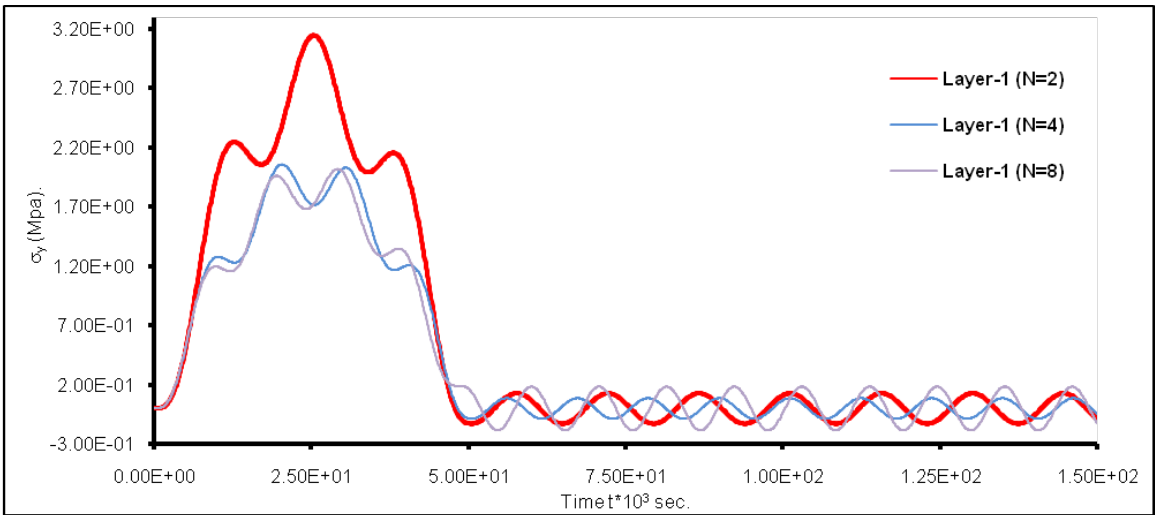

Fig. 20. Stress-y in Layer-1 due to Uniform sine loading.

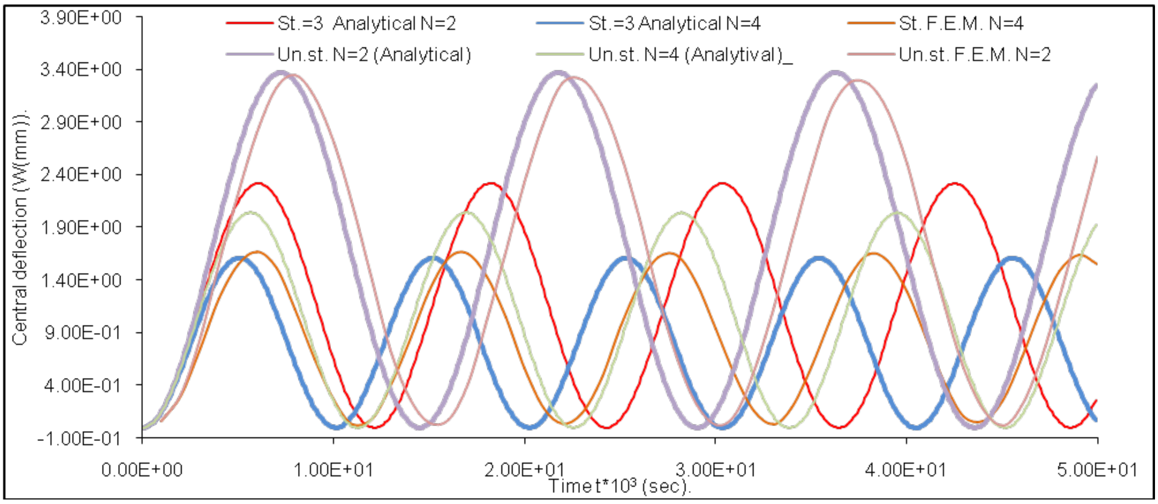

Fig. 21. Comparison of Central Deflection for Stiffened and Un-stiffened Cross-Ply Laminated plates

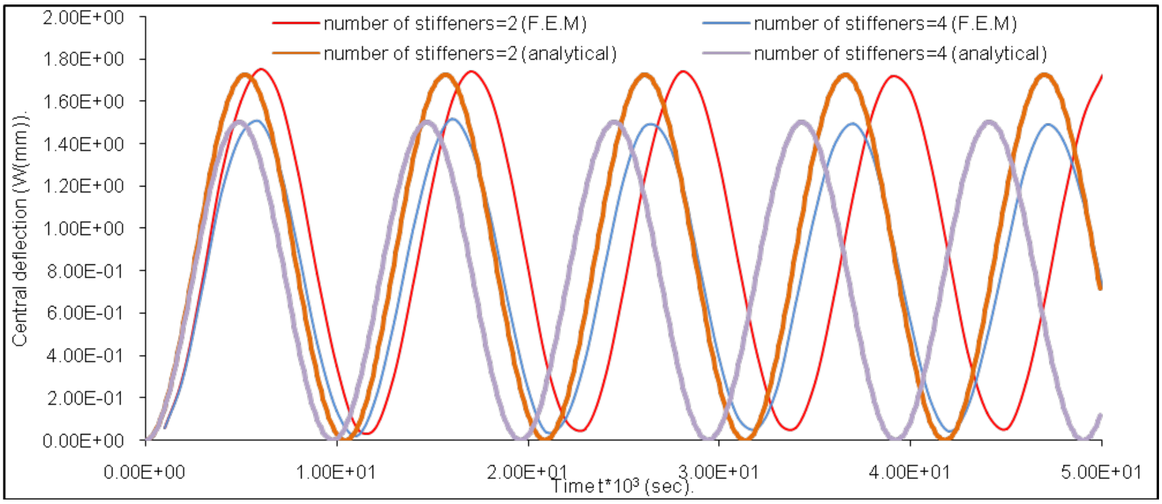

Fig. 22. Central deflection for stiffened Laminated Plates due to sinusoidal pulse loading for cross-ply laminated plates.

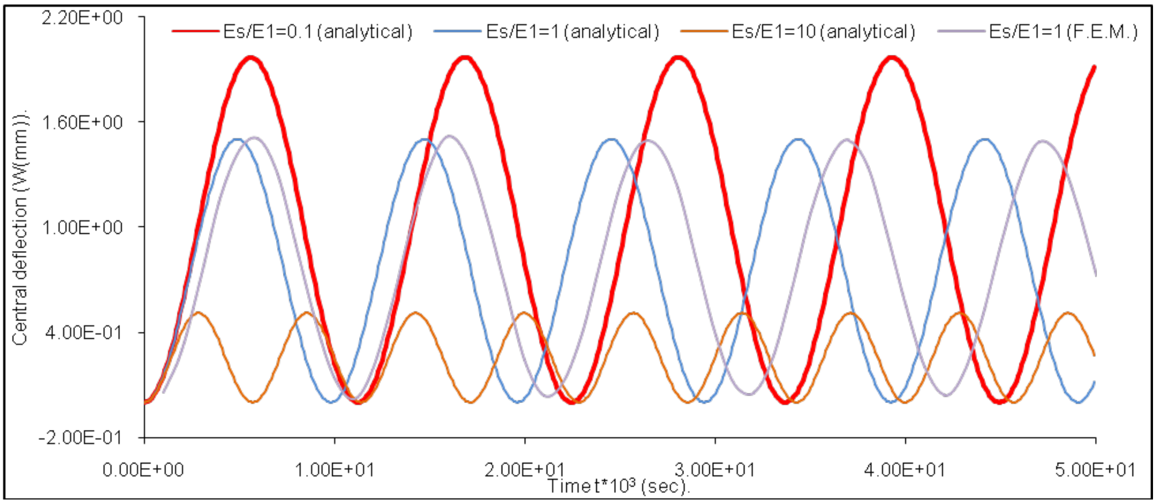

Fig. 23. Central deflection for stiffened Laminated plates due to sinusoidal pulse loading for cross-ply laminated plates.

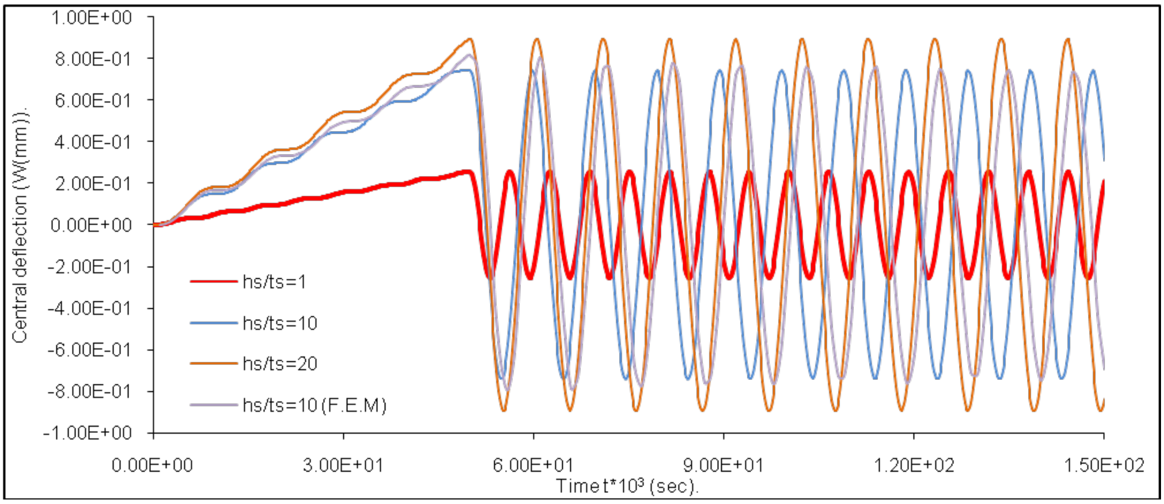

Fig. 24. Central deflection for stiffened plates due to sinusoidal Ramp loading for cross-ply laminated plates.

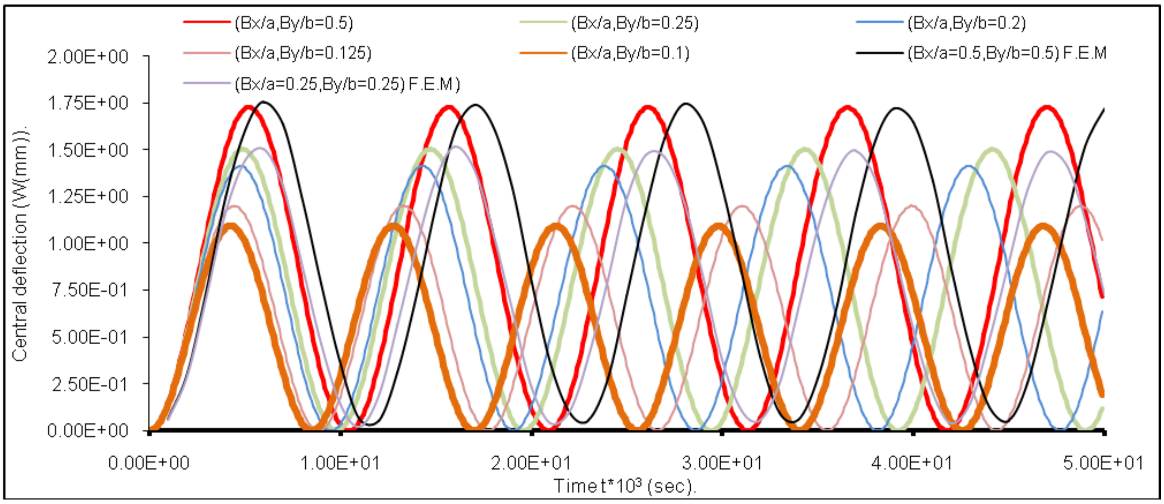

Fig. 25. Central deflection due to Sinusoidal Pulse Loading for stiffened cross-ply laminated plates for N=4 (a/b=1).

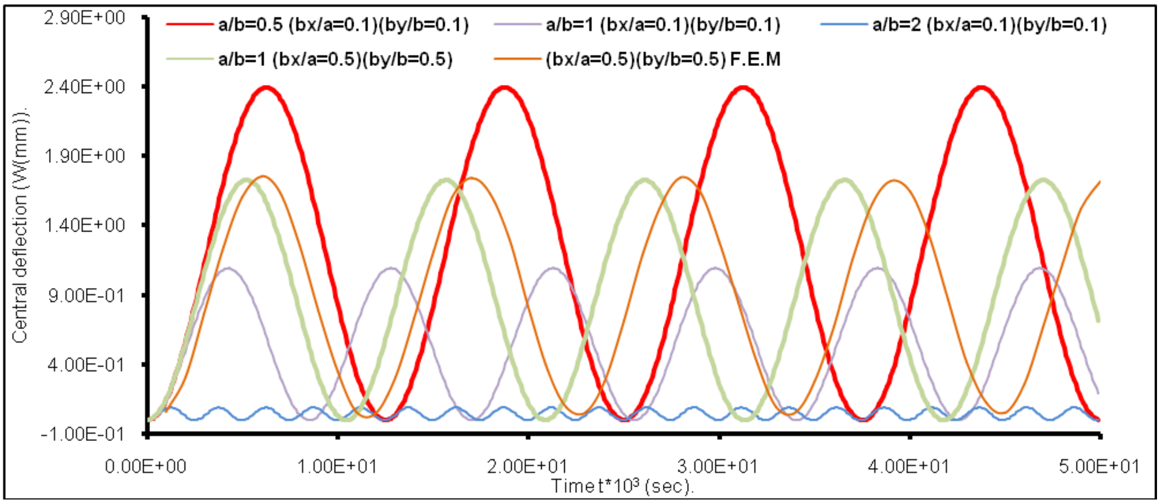

Fig. 26. Central deflection due to Sinusoidal Pulse Loading for stiffened laminated plates for N=4.

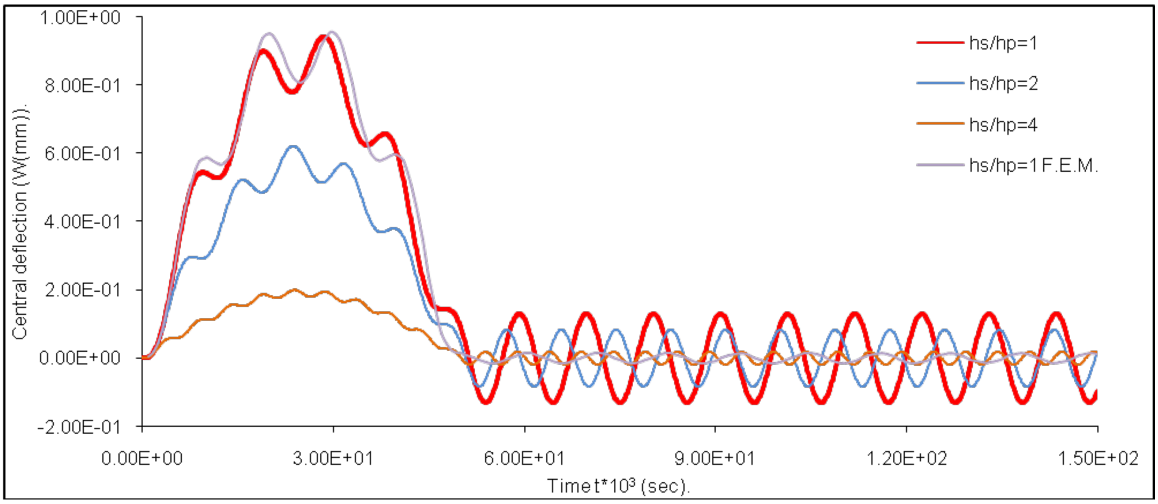

Fig. 27. Central deflection for stiffened plates due to sinusoidal sine loading for cross-ply laminated plates.

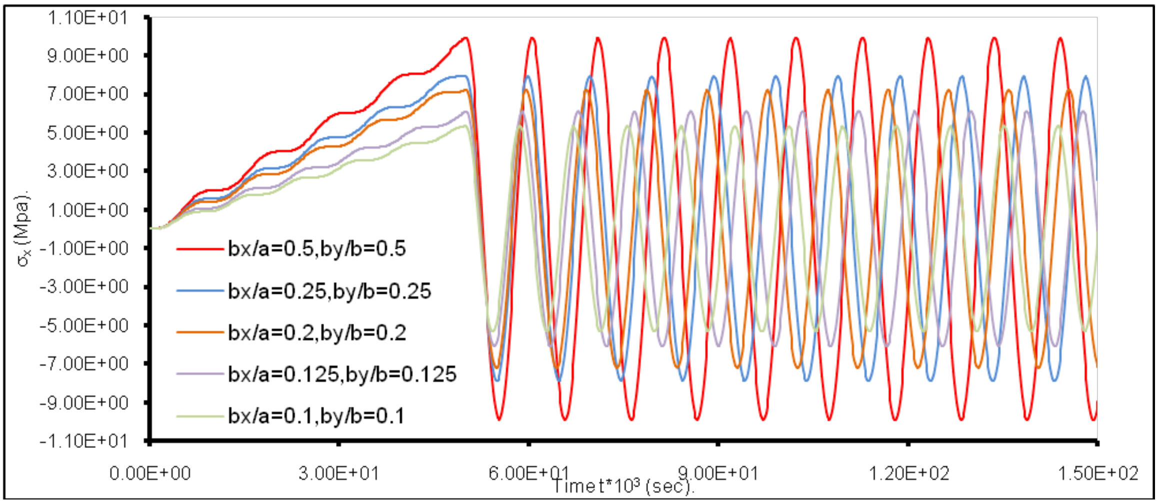

Fig. 28. Stress-x in Layer-1 due to Uniform Ramp loading for stiffened laminated plates for N=4.

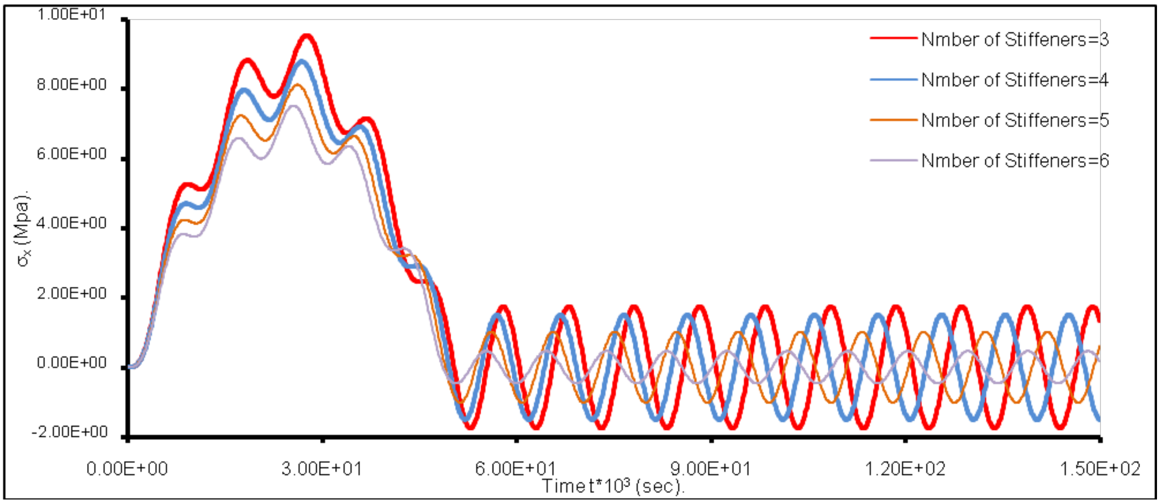

Fig. 29. Stress-x in Layer-1 due to Uniform sine loading for stiffened laminated plates for N=4.

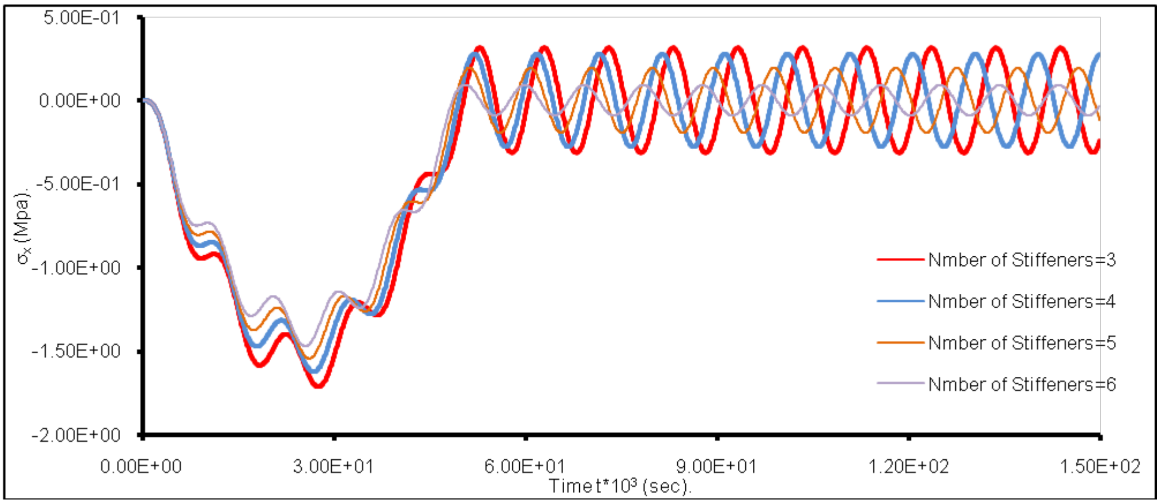

Fig. 30. Stress-x in Layer-4 due to Uniform sine loading for stiffened laminated plates for N=4.

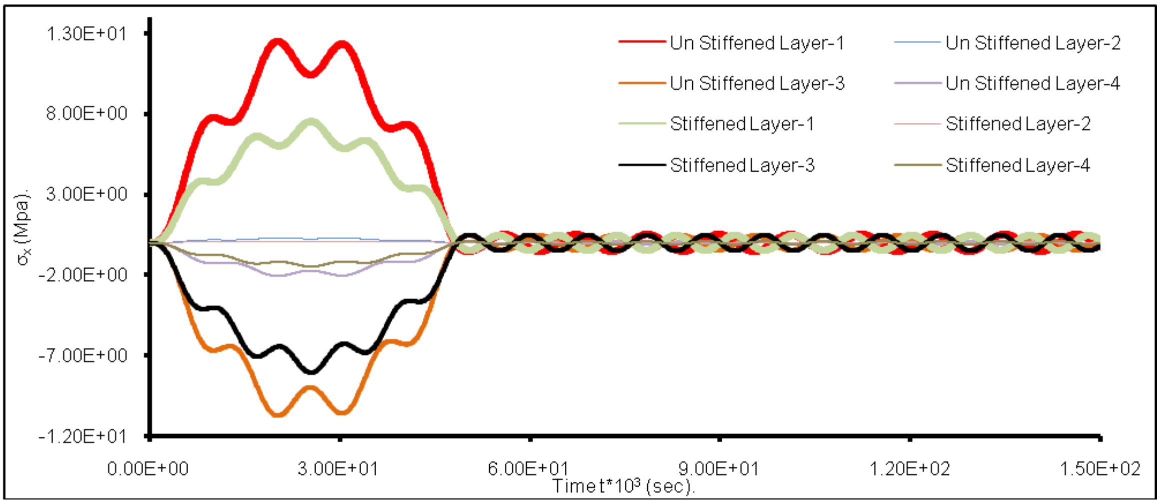

Fig. 31. Stress-x in each Layer due to Uniform sine loading for stiffened laminated plates for Stiffeners=6.

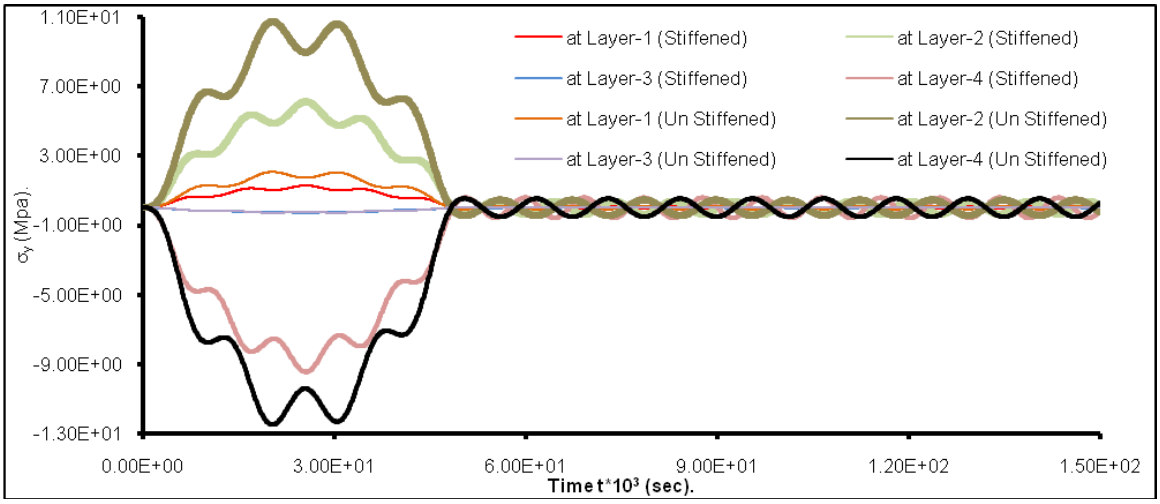

Fig. 32. Stress-y in each Layer due to Uniform sine loading for stiffened laminated plates for Stiffeners=6.

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