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ROBUST ACTIVE DAMPING DESIGN FOR A SUSPENSION SYSTEM

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ABSTRACT
This paper proposes a control of robust active damping for a quarter car model with three degrees of freedom by using an optimal control approach. The design takes into consideration the uncertainty of system parameters. The solution to the robust damping problem is obtained by translating it into an optimal control problem, and the subsequent design of appropriate feedback controllers for the active suspension system can be obtained by solving the LQR (linear quadratic regulator) problem. The results show that the proposed robust design provides superior kinematic and dynamic performance compared to those of the passive system.

KEYWORDS: - robust control, active suspension, quarter car, uncertainty, lqr methodology

الخلاصة :-
يقترح هذا البحث السيطرة على التخميد النشيط القوي لنموذج ربع سيارة مع ثلاثة درجات من الحرية من خلال استخدام اسلوب السيطرة الأمثل. يأخذ هذا التصميم بعين الاعتبار حالة عدم التأكد لمعلمات النظام .وتحل مشكلة التخميد القوية بتحويلها الى مشكلة سيطرة مثلى ومن ثم يمكن الحصول على تصميم لاحق لمسيطرات التغذية العكسية لنظام التعليق النشيط من خلال حل مشكلة LQR . وأظهرت النتائج أن التصميم المقترح يوفر أداءً حركياً وديناميكياً متميز مقارنة مع تلك التابعة للمنظومة السلبية

List of symbols

A_0 Stiffness matrix of dimension $n \times n$

B_0 Matrix of dimension $n \times m$

C_0 Matrix of dimension $n \times p$

D_1 Damping coefficient of passenger seat, N.s/m

D_2 Damping coefficient of car body, N.s/m

$f_0(x, \dot{x})$ Uncertainty

F Uncertainty matrix

K_1 Stiffness of passenger seat, N/m

K_2 Stiffness of the car body spring, N/m

K_3 Stiffness of the car wheel, N/m

H Weight matrix

m_p Mass for passenger seat, Kg

m_b Mass for car body, Kg

m_w Mass for car wheel, Kg

M_0 Mass matrix of dimension $n \times n$

r Road disturbances input, m

u Control force, N

x_1, x_2, x_3 Displacement of passenger seat, car body, and car wheel, respectively, m

1. INTRODUCTION

In recent years, a considerable attention has been paid by many automotive manufacturers and researchers to the possibility of reducing the vibration effects in vehicles by means of active control techniques. This is due to the increasing interest and awareness of the potential of the electronically controlled suspensions to improve vehicle performance [W.Sie, R.Lian and B. Lin 2006].

The development of active suspensions has become necessary in order to improve simultaneously the ride comfort and road holding ability since the conventional passive suspensions tuned at a particular speed or at a certain road conditions [Emanuele G. and Tudor S. 2008].

A robust controller is meant to provide a reasonable level of performance in systems with uncertain parameters, no matter how fast they vary, but it usually requires the knowledge (or an estimate) of the bounds of the uncertainty. The typical structure of a robust control law is composed of a nominal part (such as a state feedback or an inverse model control law), plus a term which deals with model uncertainty. Finally, some systems may be subject to deterioration or other changes during their life time. All these facts indicate the necessity of having robust controller for such systems [Feng Lint 2000].

For active control of a vehicle suspension system various approaches have been proposed in the literature. [M. Senthil Kumar and S.Vijayarangon 2006] used the linear quadratic optimal control theory, based on two different control approaches [conventional method (CM), and acceleration dependent method (ADM)].It was shown that active suspension system had a better potential to improve both the ride comfort and road holding, since the acceleration has been reduced for active CM system (19.58%) and for active ADM system (34.08%) compared to passive one.

[Mouleeswaran S. 2008] studied the active suspension system for the quarter car model by used a proportional integral derivative (PID) controller. The controller design deals with the selection of proportional, derivative gain and integral gain parameters. The results show that the active suspension system has reduced the peak overshoot of spring mass displacement, spring mass acceleration, suspension travel and tire deflection compared to passive suspension system.

[R.K.PeKgokgoz 2010] investigated the fuzzy logic to control (FLC), the active suspension and the membership functions are optimized by using genetic algorithm operations. The model has been applied to a sample one quarter car model. The results of proposed model were

compared with those of PID controller and the efficiency of the FLC controller model has been assessed. It has been shown that the fuzzy-logic controller displays better performance than the PID controller for both the minimization of the maximum body deflection and the efficiency of the actuator force of the controller.
[Supavut Chantrauwathana and Huei Peng 2000] proposed a method for adaptive robust controllers for force tracking application in a quarter-car active suspension system. The overall active suspension system was decomposed into two loops. At the main-loop, the desired force signal is calculated while the sub-loop force tracking controller tries to keep the actual force close to this desired force. It was found that force-tracking of up to 5Hz can be reliably achieved. It is, however, found to be unreliable in experiments, especially when high frequency disturbances are present.

[Mohammad S. F. and W. Fang 2009] used H_∞ controller for control the active suspension system with measured car body velocity for feedback. The system parameter variations are treated with multiplicative uncertainty model. The simulation results are shown to verify the effectiveness of the H_∞ controller comparing to the passive system.
[RAJU JAITWAR 2011] studied the sliding mode control technique. A nonlinear surface is used to ensure fast convergence of vehicle's vertical velocity. The nonlinear surface changes the system's damping. The effect of sliding surface selection in the proposed controller is also presented. Extensive simulations are performed and the results obtained shows that the proposed controller perform well in improving the ride comfort and road handling for the quarter car model using the hydraulically actuated suspension system.
[A. Mat Amir 2006] developed an robust control technique that based on backstepping theory is presented and controls the nonlinear active suspension system. The performance of the systems used backstopping control approach is compared to linear quadratic regulator (LQR) control approach.

The main objective in this paper, is to reduce motions of the spring mass (vehicle body), and to minimize the vertical forces transmitted to the passenger. The approach to be presented in this work is to translate the robust control problem into an optimal control problem. The robust control problem is guaranteed to be solving by using LQR technique and the solution is verified with MATLAB.

2. DYNAMIC MODEL OF THE VEHICLE SUSPENSION

In this work, quarter car model with three degrees of freedom has been considered for the analysis as shown in Fig.1, which leads to simplified analysis. The model consists of passenger seat, sprung mass, which refer to the part of the car that is supported on spring and unspring mass, which refers to the mass of tire and axel assembly. The tire has been replaced with its equivalent stiffness and tire damping is neglected. The suspension, tire and passenger seat are modeled by linear springs in parallel with dampers [Padraig Dowds and Aidan O. Dwyer 2005]. The uncertainty of the system appears in the dashpot constant as we do not know its exact value; nevertheless we assume that we know its bounds, that is, $\mathrm{D} \in [\mathrm{D}_{\mathrm{min.}}, \mathrm{D}_{\mathrm{max.}}]$ [Joha Wiley and Sons Ltd 2007].

For active suspension system, the following equations of motion are derived for the quarter - car model using Newton's laws of motion:

$$m_p\ddot{x}_1 = -K_1(x_1 - x_2) - D_1(\dot{x}_1 - \dot{x}_2) \tag{1}$$

$$m_b\ddot{x}_2 = u + K_1(x_1 - x_2) - K_2(x_2 - x_3) + D_1(\dot{x}_1 - \dot{x}_2) - D_2(\dot{x}_2 - \dot{x}_3) \tag{2}$$

$$m_w\ddot{x}_3 = -u + K_2(x_2 - x_3) + D_2(\dot{x}_2 - \dot{x}_3) - K_3(x_3 \ \ - r) \tag{3}$$

The system of equations can be represented in the robust damping problem with uncertainties:

$$M_0\ddot{x} + A_0 x = B_0 u + C_0 f_0(x,\dot{x}) \tag{4}$$

In order to simplify the notation used in this paper, we introduce the following variables and matrices:

$$y = M_0^{\frac{1}{2}}x, \quad A = M_0^{-\frac{1}{2}}A_0 M_0^{-\frac{1}{2}}, \quad B = M_0^{-\frac{1}{2}}B_0, \quad C = M_0^{-\frac{1}{2}}C_0 ,$$

$$f(y,\dot{y}\,) = f_0(M_0^{-1/2}y, M_0^{-1/2}\dot{y})$$

Then equation (4) can be rewritten as:

$$\ddot{y} + Ay = Bu + Cf\,(y,\dot{y}) \tag{5}$$

We stack the displacement and its velocity to obtain the following first-order model.

$$\frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -A & 0 \end{bmatrix}\begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix}u + \begin{bmatrix} 0 \\ C \end{bmatrix}f(y,\dot{y}) \tag{6}$$

3. UNMATCHED UNCERTAINTY

The uncertainty $f(y,\dot{y})$ depends on y and $\dot{y}$. If the uncertainty satisfies the matching condition, then $B = C$. Otherwise, $B \neq C$, the matching condition is not satisfied. For all uncertainties $f(y,\dot{y})$ satisfying the following assumptions [Feng Lint 2000]:

1- There exists a non-negative function $g_{max}(y,\dot{y})$ such that,

$$\|f(y,\dot{y})\| \leq g_{max}(y,\dot{y})$$

2- There exists a non-negative function $f_{max.}(y,\dot{y}\,)$ such that,

$$\|\, B^+C\; f(y,\dot{y})\|^2 \leq f_{max}\;(y,\dot{y})^2$$

Obviously, the first assumption implies the second. However, by introducing both $g_{max}(y,\dot{y})$ and $f_{max}(y,\dot{y}$), the least restrictive bound can be used. Decompose the uncertainty $Cf\,(y,\dot{y})$ into the sum of a matched component and an unmatched component by projecting $Cf\,(y,\dot{y})$ into the range of B, thus take,

$$Cf\,(y,\dot{y}) = B\; B^+ Cf\,(y,\dot{y}) + \left(I - BB^+\, Cf\,(y,\dot{y})\right) \tag{7}$$

Therefore, the orthogonal decomposition becomes,

$$\begin{bmatrix} 0 \\ C \end{bmatrix} = f(y,\dot{y}) = \begin{bmatrix} 0 \\ BB^+C \end{bmatrix}f(y,\dot{y}) + \begin{bmatrix} 0 \\ (I - BB^+C) \end{bmatrix}f(y,\dot{y}) \tag{8}$$

4. OPTIMAL CONTROL PROPBLEM

The robust active damping will be designed by translating the robust control problem into an optimal control problem, such that the closed-loop system is globally asymptotically stable for all uncertainties $f(y,\dot{y})$ satisfying the conditions [Hrovat D. 2000]. With the above decomposition of the uncertainties, the dynamic equation of the optimal control problem is given by

$$\frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -A & 0 \end{bmatrix}\begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u + \begin{bmatrix} 0 \\ (I - BB^{+}C) \end{bmatrix} v \tag{9}$$

We will find a feedback control law (u_0,v_0) that minimizes the cost functional

$$\int_0^{\infty} f_{max}(y,\dot{y})\ ^2 + \rho^2 g_{max}(y,\dot{y})^2 + \beta^2\|y,\dot{y}\|^2 + \|u\|^2 + \rho^2\|v\|^2)\,dt \tag{10}$$

The relationship between the robust control problem and the optimal control problem is given by the following theorems [Joha Wiley and Sons Ltd 2007].

Theorem (1): If the solution to the optimal problem exists, then it is a solution to the robust control problem. By this theorem, we can solve the robust control problem by solving the optimal control problem. Methods to solve optimal control problems can be found in references [Supavut Chantranuwathana and Huei Peng 2000].

Theorem (2): If one can choose ρ and β such that the solution to the optimal control problem, denoted by (u, v), exists and the following conditions is satisfied,

$$\mathbf{2\rho^2\|v_0\|^2 \le \beta'^2\|y,\dot{y}\|^2}, \qquad \forall(\mathbf{y},\dot{\mathbf{y}}) \in \mathbf{R^n}$$

For some β' such that $|\beta'| < |\beta|$, then $u_0(y,\dot{y})$, which gives that the u-component of the solution to the optimal control problem, is a solution to the robust control problem.

LQR problem, v is an augmented control that is used to deal with the unmatched uncertainty. Note that if the matching condition is satisfied, in this case, the design parameters will be selected so that a sufficient condition in the following theorem is satisfied.

In order to obtain a numerical solution of the LQR problem, we introduce the following matrices:

$$\tilde{A} = \begin{bmatrix} 0 & \mathrm{I} \\ -\mathrm{A} & 0 \end{bmatrix}, \quad \tilde{R} = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & \rho^2\mathrm{I} \end{bmatrix}, \quad \tilde{P} = \begin{bmatrix} \mathrm{A} & 0 \\ 0 & \mathrm{I} \end{bmatrix} P,$$

$$\tilde{Q} = \begin{bmatrix} \mathrm{A} & 0 \\ 0 & \mathrm{I} \end{bmatrix} Q, \qquad Q = F + \rho^2 H + \beta^2 I$$

The solution to the LQR problem is given by:

$$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = \tilde{R}^{-1}\tilde{B}'\tilde{P}\begin{bmatrix} y \\ \dot{y} \end{bmatrix} \tag{11}$$

Where, $\tilde{P}$ is the unique positive definite solution to the following Algebraic Riccati equation

$$\tilde{P}\tilde{A} + \tilde{A}'\tilde{P} - \tilde{P}\tilde{B}R^{-1}\tilde{B}'\tilde{P} + \tilde{Q} = 0 \qquad (12)$$

The corresponding optimal control can be written as:

$$\begin{bmatrix} u_0 \\ v_0 \end{bmatrix} = -\begin{bmatrix} B' \\ \rho^{-2}C'(I - BB^{+}) \end{bmatrix}[\tilde{P}_{21} \quad \tilde{P}_{22}]\begin{bmatrix} y \\ \dot{y} \end{bmatrix} \qquad (13)$$

If the condition of unmatched uncertainty is satisfied,

$$\beta^2 I - 2\rho^{-2}[\tilde{P}_{21} \quad \tilde{P}_{22}]'(I - BB^{+})CC'[\tilde{P}_{21} \quad \tilde{P}_{22}](I - BB^{+}) > 0$$

And we can take the design parameters to be ρ = 0, β = 1. Then the robust control is given by:

$$u_0 = -B_0'M_0^{-\frac{1}{2}}(\tilde{P}_{21}y + \tilde{P}_{22}\dot{y}) \qquad (14)$$

5. SIMULATION AND RESULTS

In this section, demonstrate the validity of the proposed controller designed of the active suspension system for the quarter of a car model. The optimal parameters of the controllers should be obtained first. The state feedback is designed by using nominal values of parameters and the matrices Q and R are chosen in such a way that both the state variable response of displacements and velocities performance improvements. For simulation purposes, we choose a step disturbance at initial time. The following parameters are taken from reference [Hrovat D. 1990], and Tabulated in Table 1.

The logarithmic decrement $(\delta = ln\frac{X_1}{X_2})$, represent the rate at which the amplitude of a free damped vibration decreases .The logarithmic decrement is dimensionless and is actually another form of the dimensionless damping ratio [Singires S. Rao, 1986].The result of the logarithmic decrement of system is tabulated in **Table 2**.

We see from the Table 2, the logarithmic decrement is increase for the active suspension system compared with passive system, due to increase the amplitude ratio $(\frac{X_1}{X_2})$ of the system at the same value of damping coefficients $(D_1 \text{ and } D_2)$.The results show that the oscillation of the suspension travel is very much reduced as compared to the passive suspension system. From Fig.2, peak overshoot of passenger displacement for passive system is 0.08m, where as for active suspension system it is 0.04m.Fig.6; maximum wheel deflection for passive system is 0.09m where as for the active suspension system it is 0.0395m. It is clear from these figures, that the control law achieves a good tracking performance and drives successfully the disturbances overshoot to the desired zero value with a fast rate of convergence using a bounded controller applied between the wheel and car body.

For the active suspension system, can effectively absorb the car vibration in comparison to the passive system, it is concluded that the active system retains both better ride comfort and road handling characteristics compared to the passive system ,as shown in Figs.(2,3,4,5,6, and 7).

The suspension design which may have emerged from the use of optimal state- feedback control theory proved to be effective in controlling car vibrations and achieving better performance than conventional passive suspension.

6. CONCLUSION

In this work, the robust controller is used to control the active suspension system for a quarter car model. The system parameter's variations are treated with uncertainty. It is shown that the active suspension system significantly improved regulator response when compared to the passive suspension. It is expected that the active suspension system with robust controller can be applied in car industry as an objective to increase the quality of cars.

Table 1 parameters of the quarter- car model

Parameter	symbol	value	unit
Mass of passenger seat	m_p	60	kg
Mass of car body	m_b	290	kg
Mass of car wheel	m_w	59	kg
Stiffness of passenger seat	K_1	10507	N/m
Stiffness of car body spring	K_2	16812	N/m
Stiffness of car wheel	K_3	190000	N/m
Damping coefficient of car body	D_1	500-1000	N.s/m
Damping coefficient of passenger seat	D_2	250-500	N.s/m
Road disturbance	r	0.1	m

Table 2 Parameters of the logarithmic decrement

States	Logarithmic decrement ($\delta = ln\frac{X_1}{X_2}$)		
	Passive system	LQR with $D_1 = 250$ and $D_2 = 500$	LQR with $D_1 = 250$ and $D_2 = 500$
Passenger	0.346	0.7	1.194
Body	0.291	0.674	1.129
Wheel	0.124	0.765	1.085

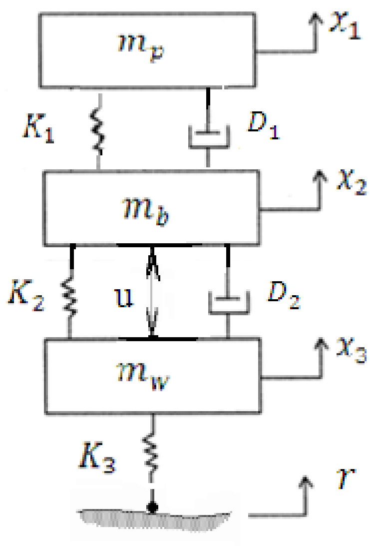

Fig. 1 Quarter- car model

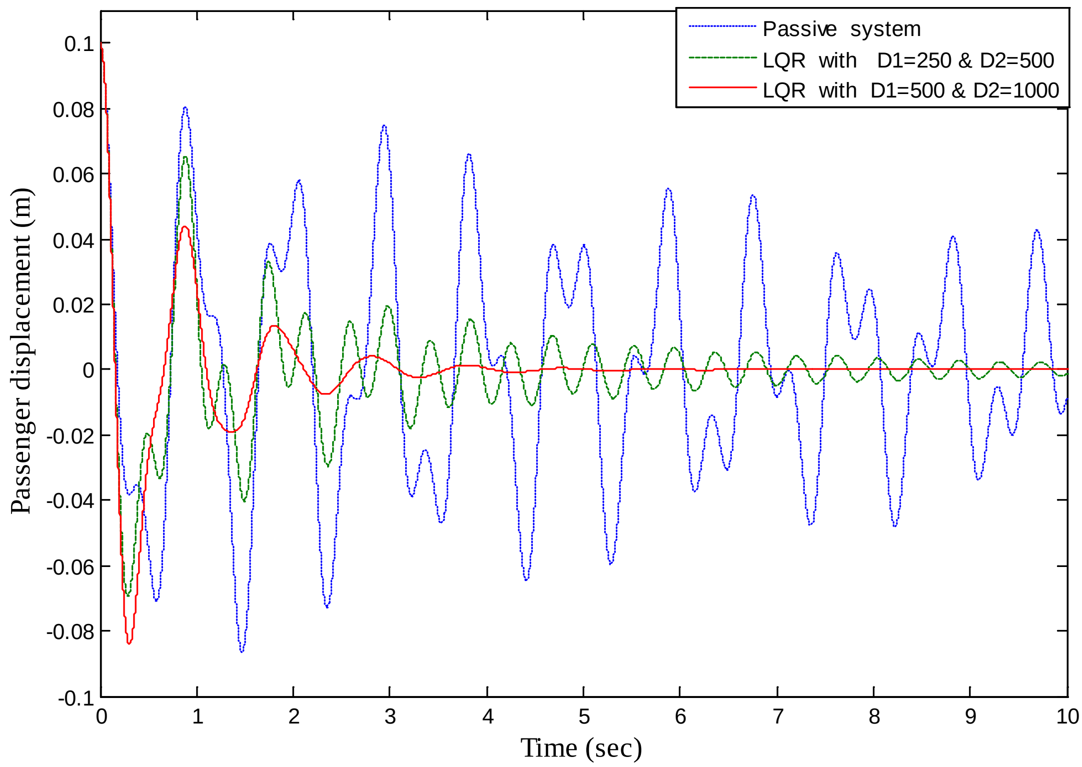

Fig.2 Histories of the passenger displacement for passive and active suspension

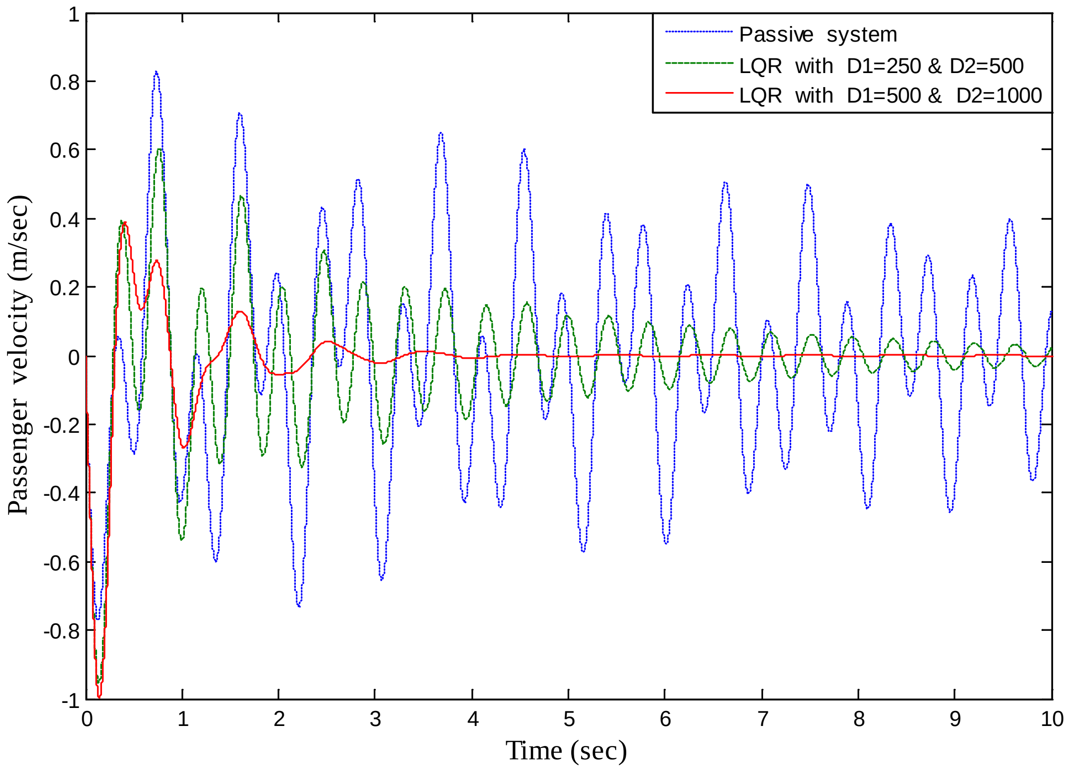

Fig.3 Passenger seat velocity for passive and active suspension vs. time

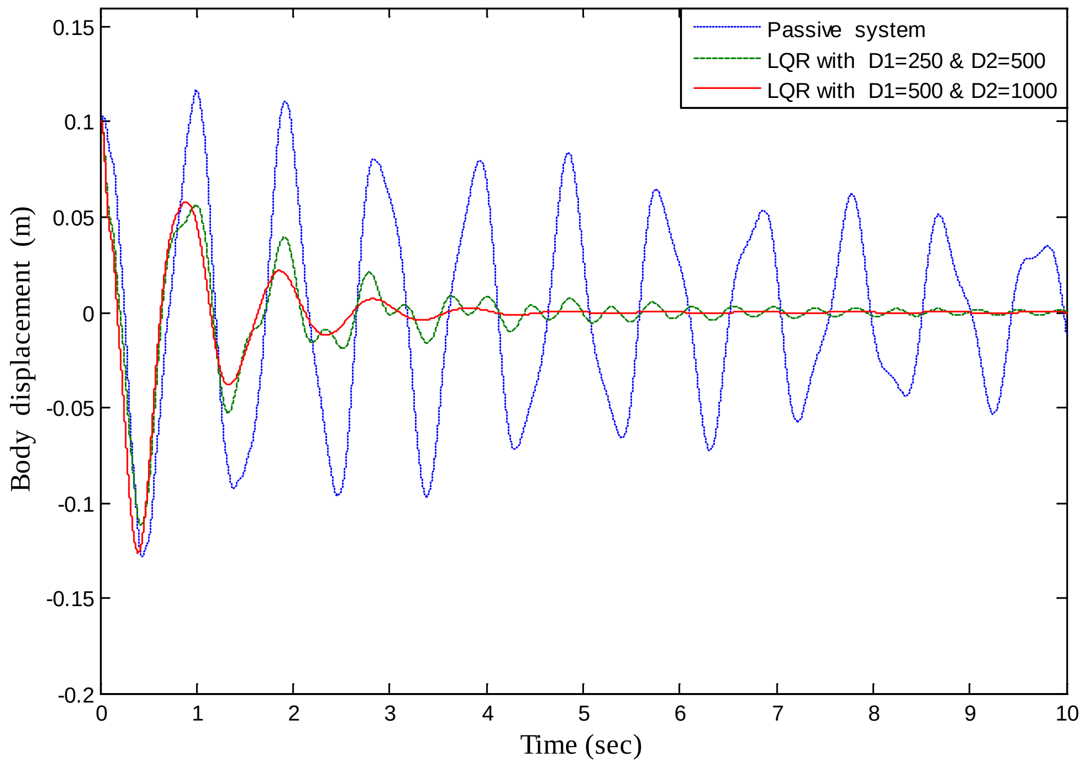

Fig.4 Histories of the body displacement for passive and active suspension

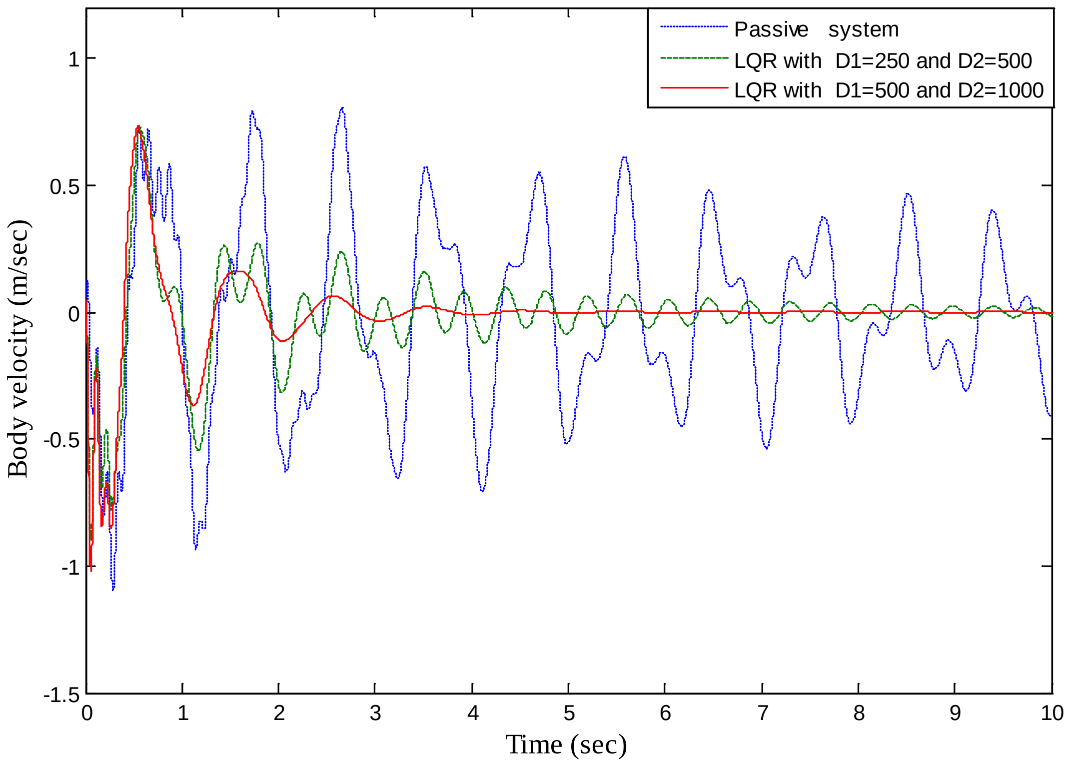

Fig.5 Body velocity for passive and active suspension vs. time

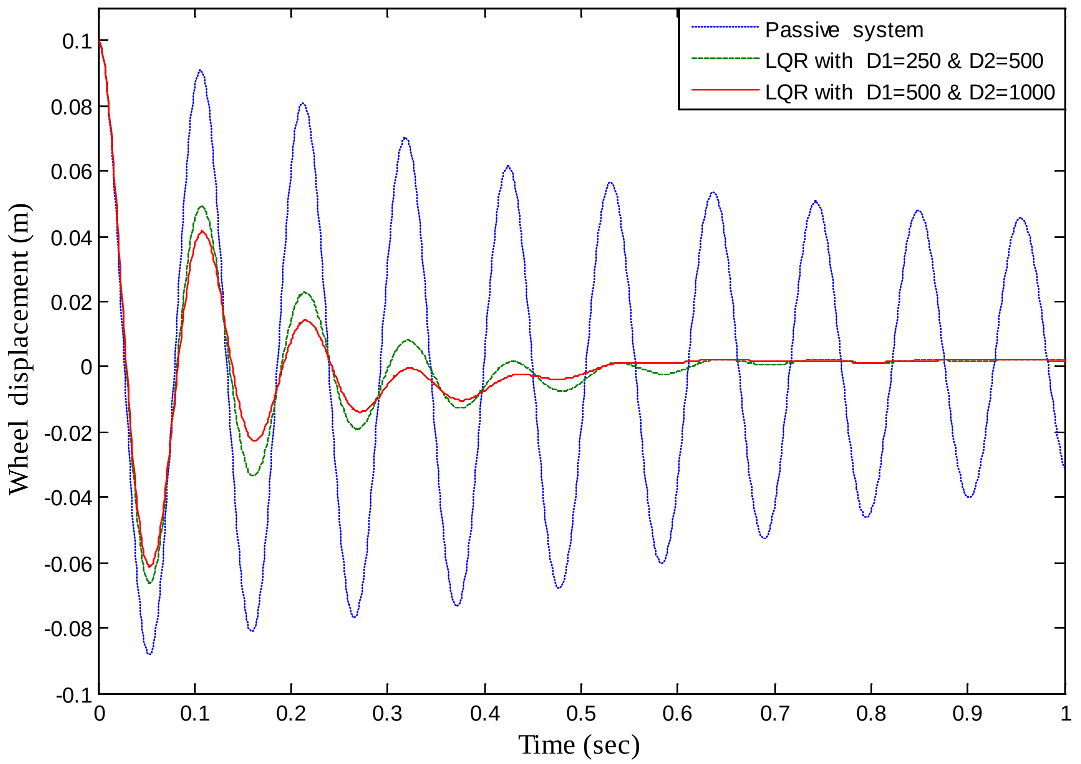

Fig.6 Histories of the wheel displacement for passive and active suspension

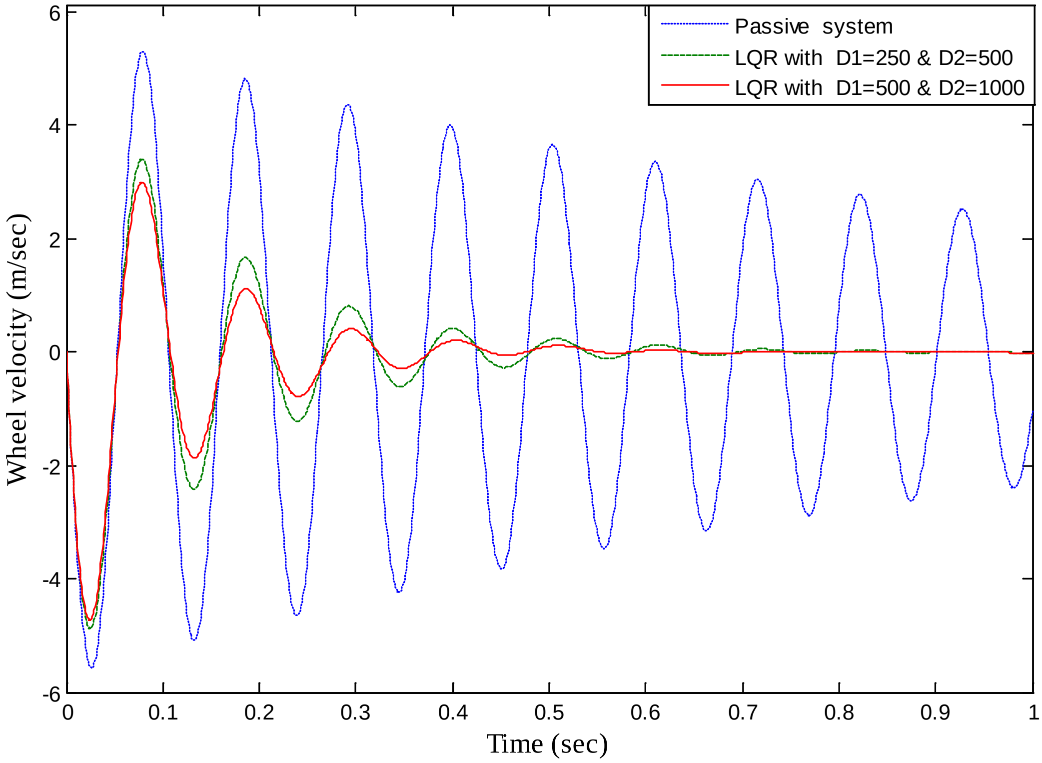

Fig.7 Wheel velocity for passive and active suspension vs. time

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