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Atangana- Baleanu Fractional Variational Iteration Method for Solving Fractional Order Burger's Equations

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Abstract

To solve fractional order fractional order Burger's equations, a hybrid technique named Atangana- Baleanu fractional variational iteration method (ABVIM) has been applied. Two challenges are overcome to validate and demonstrate the efficacy of the current process. It is also shown that the results acquired using the suggested technique is extremely like those obtained using other strategies. For a range of science and engineering difficulties, the proposed solution has been shown to be efficient, dependable, and simple to implement.

Keywords: Fractional order Burger's equations, Fractional variational iteration method, Atangana- Baleanu fractional operator.

1. Introduction

In recent years, engineering and applied sciences have shown a great deal of fractional calculation. The principles of fractional calculus are located in [1, 2]. One kind of differential equations are fractional differential equations, or FDEs, which are considered a broad kind of differential equations, involve derivatives of any complex or real order. Fractional partial differential equations can be used to solve a variety of problems in the real world, and they've been discovered to be a tool that's useful for interpreting and modeling all areas of science and mathematical applications concerns [5, 6, 8].

The precise and estimated results for PDEs with fractions have recently received a lot of attention (PDEs). For the solution of fractional PDEs, numerous motivated strategies have been used in this work such as HAM, expansion methods, HATM, FDM, operational method, VIM, HPM, direct approach, Lie symmetry analysis, DTM, reproducing kernel method, EDTM, mesh less methods, SVIM, SDM, LHPM, LVIM, and other methods [3-70]. The goal of this paper is introduce ABFVIM and use it to resolve the fractional order Burger's equations. The rest is separated as, in section 2, some FC definitions are giving. In section 3, the analysis of the ABFVIM is achieved. Examples of ABFVIM are shown in the section 4. Section 5 is where this paper's conclusion is found.

2. Preliminaries

In this section, we'll go over some of the most important fractional calculus definitions and formulas [1, 2, 7].

Definition 1. The ABFD of order α is given as follows [44]:

$$ {}^{AB}_{a}D^{\alpha}_{t}u(t) = \frac{M(\alpha)}{1-\alpha}\int_{a}^{t} E_{\alpha}\left(\frac{-\alpha(t-x)^{\alpha}}{\alpha-1}\right) u'(x)dx \tag{2.1} $$

where $0 < \alpha < 1$ and $M(0) = M(1) = 1$.

Definition 2. The ABFI of order α defined as follows [44]:

$$ {}^{AB}_{a}I^{\alpha}_{t}\, u(t) = \frac{1-\alpha}{M(\alpha)} u(t) + \frac{\alpha}{M(\alpha)} \frac{1}{\Gamma(\alpha)} \int_{a}^{t} (t-x)^{\alpha-1} u(x)\, dx. \qquad (2.2) $$

The properties of ABFI is defined as follows**:**

1. ${}^{AB}_{a}I^{\alpha}_{t}\, {}^{AB}_{a}D^{\alpha}_{t}\ u(t) = u(t) - u(0).$
2. ${}^{AB}_{a}I^{\alpha}_{t}\ c = \frac{c}{M(\alpha)} \left(1-\alpha + \frac{t^{\alpha}}{\Gamma(\alpha)}\right).$
3. ${}^{AB}_{a}I^{\alpha}_{t}\ t^{k} = \frac{t^{k}}{M(\alpha)} \left(1-\alpha + \frac{\alpha\,\Gamma(k+1)\ t^{\alpha}}{\Gamma(\alpha+k+1)}\right).$

3. Analysis of FVIM

Consider the following: partial differential equation with fractions

$$ {}^{AB}D^{\omega}_{t}(g(x,t)) + R\, g(x,t) + Ng(x,t) = h(x,t), \qquad 0 < \omega \le 1 $$

with the initial condition

$$ g(x,0) = F(x) $$

where ${}^{AB}D^{\omega}_{t}$ is ABFD , R is the linear differential operator ,N denotes the nonlinear term,and h(x,t) denotes the source term .

The correctional functional for is approximately expressed as follows :

$$ g_{n+1}(x,t) = g_n(x,t) + {}^{AB}_{a}I^{\omega}_{t} [\lambda(\mu)(AB_{D}{}^{\omega}_{\mu} g_n(x,\mu) + R\tilde{g}(x,\mu) + N\breve{g}(x,t) - $$

$h(x,\mu))]$,

where $\lambda(\mu)$ is general lagrane s multipier. $\tilde{g}$ and h are considered as restricted variations .the relevant adjustment in place and making it functioning and noticing $\delta\tilde{g} = 0$ *and* $h = 0$, we obtain

$$ \delta g_{n+1}(x,t) = \delta g_n(x,t) + {}^{AB}I^{\omega}_{t}\left[\delta\lambda(\mu)\left({}^{AB}D^{\omega}_{\mu} g_n(x,\mu)\right)\right]. $$

Or

$$ \delta g_{n+1}(x,t) = \delta g_n(x,t) + \lambda(\mu)\delta g_n(x,t) - {}^{AB}I^{\omega}_{t}\left[\delta\lambda(\mu)\left({}^{AB}D^{\omega}_{\mu} g_n(x,\mu)\right)\right]. $$

This produces the stationary conditions

$$\lambda'(\mu) = 0$$

$$1 + \lambda(\mu) = 0$$

There for, we identified $\lambda = -1$,

$g_{n+1}(x,t) = g_n$ (x,t) + ${}^{AB}_{a}I^{\omega}_{t}$ [$\lambda(\mu)$(${}^{AB}D^{\omega}_{t} g_n$ (x,μ) +R g_n (x,μ) +Ng_n (x,t) –h(x,μ))]

Finally ,we have

$$g(\mathrm{x},\mathrm{t}) = \lim_{n\to\infty} g_n$$

4. Applications of Applications of Burger's equations

Example 1: Consider the fractional Burger equation

$${}^{\mathrm{ABC}}\mathrm{D}^{\omega}_{\mathrm{t}}\, g + g g_{\mathrm{x}} = g_{xx}\,, \quad 0 < \omega \le 1$$

with initial conditions

$$g(x,0) = x$$

Solution: Applying ABFVIM to gets,

$$g_{n+1}(x,t) = g_n(x,0) - \int_0^t \left(\frac{\partial^{\alpha} g_n(x,\tau)}{\partial \tau^{\alpha}} + g_n (g_n)_x - (g_n)_{xx}\right) d(\tau)^{\omega}$$

$$g_0(x,t) = x$$

$$g_1(x,t) = x - \int_0^t \left(\frac{\partial^{\alpha} g_0(x,\tau)}{\partial \tau^{\alpha}} + g_0 (g_0)_x - (g_0)_{xx}\right) d(\tau)^{\omega}$$

$$= x - x\left(1 - \omega + \frac{\omega t^{\omega}}{\Gamma(\omega+1)}\right)$$

$g_2(x,t) = x - \omega^2\left(1-\omega+\frac{\omega t^{\omega}}{\Gamma(\omega+1)}\right) - 2\left[(1-\omega)\frac{\omega t^{\omega}}{\Gamma(\omega+1)}+\frac{\omega^2 t^{2\omega}}{\Gamma(2\omega+1)}\right] + [(1-$

$\omega)\frac{\omega^2 t^{2\omega}}{\Gamma(2\omega+1)}+\frac{\omega^3 t^{3\omega}}{\Gamma(3\omega+1)}]\;]$

$\vdots$

$g_n(\mathrm{x},\mathrm{t}) = x - \omega^2\left(1-\omega+\frac{\omega t^{\omega}}{\Gamma(\omega+1)}\right) - 2\left[(1-\omega)\frac{\omega t^{\omega}}{\Gamma(\omega+1)}+\frac{\omega^2 t^{2\omega}}{\Gamma(2\omega+1)}\right] + \Big[(1-$

$\omega)\frac{\omega^2 t^{2\omega}}{\Gamma(2\omega+1)}+\frac{\omega^3 t^{3\omega}}{\Gamma(3\omega+1)}\Big] + \cdots$

When $\omega = 1$, we have

$g\,(x,t) = x[1-t+t^2-\cdots..]$

$= \frac{x}{1-t}$

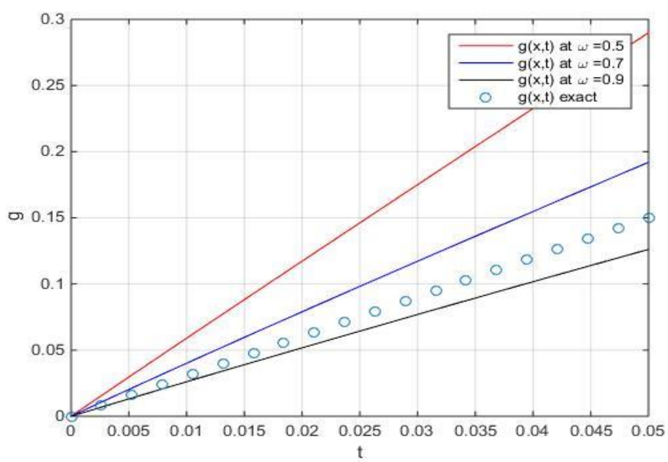

Figure 1. Plot of the exact and approximate solutions g(x ,t) for different values of ω with fixed values x=1

Example (2): consider the fractional Burger equation

$^{AB}D_t^{\omega} g(x,t) - g_{xx} - 2gg_x + (gw)_x = 0 \qquad 0 < \omega \le 1$

$^{AB}D_t^{\beta} w(x,t) - w_{xx} - 2ww_x + (gw)_x = 0 \quad 0 < \omega \le 1$

The condition $g(x,t) = sinx$, $and\ w(x,0) = sinx$

Solution : Aplying ABFVIM, we get

$$g_{n+1}(x,t) = g_n(x,t) - \int_0^t \left(\frac{\partial^\omega g_n(x,\tau)}{\partial\tau^\alpha} - (g_n)_{xx} - 2g_n(g_n)_x + (g_n w_n)_x\right) d(\tau)^\omega$$

$$w_{n+1}(x,t) = w_n(x,t) - \int_0^t \left(\frac{\partial^\beta w_n(x,\tau)}{\partial\tau^\alpha} - (w_n)_{xx} - 2w_n(w_n)_x + (g_n w_n)_x\right) d(\tau)^\beta$$

$$g_0(x,t) = sinx \qquad w_0(x,t) = sinx$$

$$g_1(x,t) = sinx - \int_0^t \left(\frac{\partial^\alpha g_0(x,\tau)}{\partial\tau^\alpha} - (g_0)_{xx} - 2g_0(g_0)_x + (g_0 w_0)_x\right) d(\tau)^\omega$$

$$= sinx - sinx(1 - \omega + \frac{\omega t^\omega}{\Gamma(\omega+1)})$$

$$w_1(x,t) = sinx - \int_0^t \left(\frac{\partial^\beta w_0(x,\tau)}{\partial\tau^\alpha} - (w_0)_{xx} - 2w_0(w_0)_x + (g_0 w_0)_x\right) d(\tau)^\beta$$

$$= sinx - sinx(1 - \beta + \frac{t^\beta}{\Gamma(\beta+1)})$$

$$g_2(x,t) = sinx + sinx\left[-\omega\left(1 - \omega + \frac{\omega t^\omega}{\Gamma(\omega+1)}\right) + \frac{\omega t^\omega}{\Gamma(\omega+1)}\left(1 - \omega + \frac{\omega t^\omega}{\Gamma(\omega+1)}\right)\right] - 2sinx\ cosx\left((1-\omega)^2 + 2(1-\omega)\frac{\omega t^\omega}{\Gamma(\omega+1)} + \frac{\omega^2 t^{2\omega}}{\Gamma(2\omega+1)}\right) + sinx\ cosx(1 - \omega + \frac{\omega t^\omega}{\Gamma(\alpha+1)})(1-\omega)^2 + 2(1-\omega)\frac{\omega t^\omega}{\Gamma(\omega+1)} + \frac{\omega^2 t^{2\omega}}{\Gamma(2\omega+1)}) + 2sinx\ cosx[(1-\omega)(1-\beta) + (1 - \omega - \beta + \beta\omega) + \frac{t^{\beta+\omega}}{\Gamma(\omega+\beta+1)}] - sinx\ cosx[\left(1 - \beta + \frac{t^\beta}{\Gamma(\beta+1)}\right)\left((1-\omega)^2 + 2(1-\omega)\frac{\omega t^\omega}{\Gamma(\omega+1)} + \frac{\omega^2 t^{2\omega}}{\Gamma(2\omega+1)}\right]$$

$$w_2(x,t) = sinx + sinx\left[-\beta\left(1 - \beta + \frac{\beta t^\beta}{\Gamma(\beta+1)}\right) + \frac{\alpha t^\beta}{\Gamma(\beta+1)}\left(1 - \beta + \frac{\beta t^\beta}{\Gamma(\beta+1)}\right)\right] - 2sinxcosx((1-\beta)^2 + 2(1-\beta)\frac{\beta t^\beta}{\Gamma(\beta+1)} + \frac{\beta^2 t^{2\beta}}{\Gamma(2\beta+1)}) + sinxcosx\left(1 - \beta + \frac{\beta t^\beta}{\Gamma(\beta+1)}\right)[(1-\beta)^2 + 2(1-\beta)\frac{\beta t^\beta}{\Gamma(\beta+1)} + \frac{\beta^2 t^{2\beta}}{\Gamma(2\beta+1)}] + 2sinxcosx\left[(1-\omega)(1-\beta) + \right.$$

$(1-\omega-\beta+\beta\omega)+\frac{t^{\beta+\omega}}{\Gamma(\omega+\beta+1)}\Big]-sinxcosx\left(\left(1-\omega+\frac{\omega t^{\omega}}{\Gamma(\omega+1)}\right)(1-\beta)^2+\right.$

$\left.2(1-\beta)\frac{\beta t^{\beta}}{\Gamma(\beta+1)}+\frac{\beta^2t^{2\beta}}{\Gamma(2\beta+1)}\right)\Big]$

$\vdots$

$$g_n(x,t)=$$

$sinx+sinx\left[-\omega\left(1-\omega+\frac{\omega t^{\omega}}{\Gamma(\omega+1)}\right)+\frac{\omega t^{\omega}}{\Gamma(\omega+1)}\left(1-\omega+\frac{\omega t^{\omega}}{\Gamma(\omega+1)}\right)+\cdots\right]-$

$2sinx\,cosx\left((1-\omega)^2+2(1-\omega)\frac{\omega t^{\omega}}{\Gamma(\omega+1)}+\frac{\omega^2t^{2\omega}}{\Gamma(2\omega+1)}\right)+sinx\,cosx(1-\omega+$

$\frac{\omega t^{\omega}}{\Gamma(\alpha+1)})(1-\omega)^2+2(1-\omega)\frac{\omega t^{\omega}}{\Gamma(\omega+1)}+\frac{\omega^2t^{2\omega}}{\Gamma(2\omega+1)})+2sinx\,cosx[(1-\omega)(1-\beta)+$

$(1-\omega-\beta+\beta\omega)+\frac{t^{\beta+\omega}}{\Gamma(\omega+\beta+1)}]-sinx\,cosx[\left(1-\beta+\frac{t^{\beta}}{\Gamma(\beta+1)}\right)\left((1-\omega)^2+\right.$

$\left.2(1-\omega)\frac{\omega t^{\omega}}{\Gamma(\omega+1)}+\frac{\omega^2t^{2\omega}}{\Gamma(2\omega+1)}\right]+\cdots$

$$w_n(x,t)=$$

$sinx+sinx\left[-\beta\left(1-\beta+\frac{\beta t^{\beta}}{\Gamma(\beta+1)}\right)+\frac{\alpha t^{\beta}}{\Gamma(\beta+1)}\left(1-\beta+\frac{\beta t^{\beta}}{\Gamma(\beta+1)}\right)+\cdots\right]-2sinx\,cosx((1-$

$\beta)^2+2(1-\beta)\frac{\beta t^{\beta}}{\Gamma(\beta+1)}+\frac{\beta^2t^{2\beta}}{\Gamma(2\beta+1)})+sinx\,cosx\left(1-\beta+\frac{\beta t^{\beta}}{\Gamma(\beta+1)}\right)[(1-\beta)^2+$

$2(1-\beta)\frac{\beta t^{\beta}}{\Gamma(\beta+1)}+\frac{\beta^2t^{2\beta}}{\Gamma(2\beta+1)}]+2sinx\,cosx\left[(1-\omega)(1-\beta)+(1-\omega-\beta+\beta\omega)+\right.$

$\left.\frac{t^{\beta+\omega}}{\Gamma(\omega+\beta+1)}\right]-sinx\,cosx\left(\left(1-\omega+\frac{\omega t^{\omega}}{\Gamma(\omega+1)}\right)(1-\beta)^2+2(1-\beta)\frac{\beta t^{\beta}}{\Gamma(\beta+1)}+\frac{\beta^2t^{2\beta}}{\Gamma(2\beta+1)}\right)\Big]+$

$\cdots\}$

Where $\omega=\beta=1$, we obtain

$g_n(x,t)=sinx\left[1-t+\frac{t^2}{2}+\cdots\right]+2sinxcosx\left[\frac{t^2}{2}-\frac{t^2}{2}\right]=sinx\,e^{-t}$

$w_n(x,t)=sinx\left[1-t+\frac{t^2}{2}+\cdots\right]+2sinxcosx\left[\frac{t^2}{2}-\frac{t^2}{2}\right]=sinxe^{-t}$

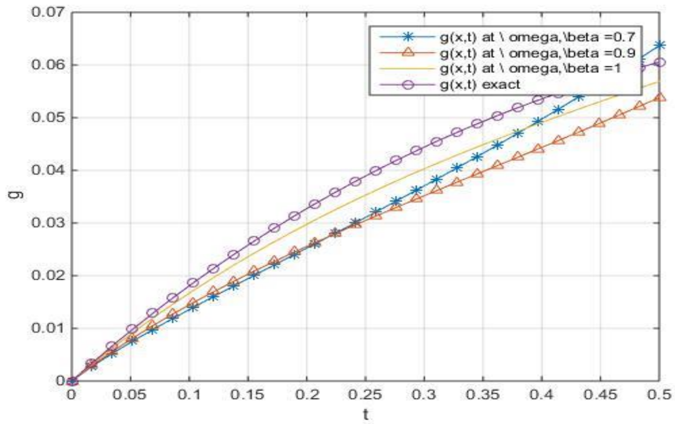

Figure 2. Plot of the exact and approximate solutions W(x ,t) for different values of β, ω with fixed values x=1

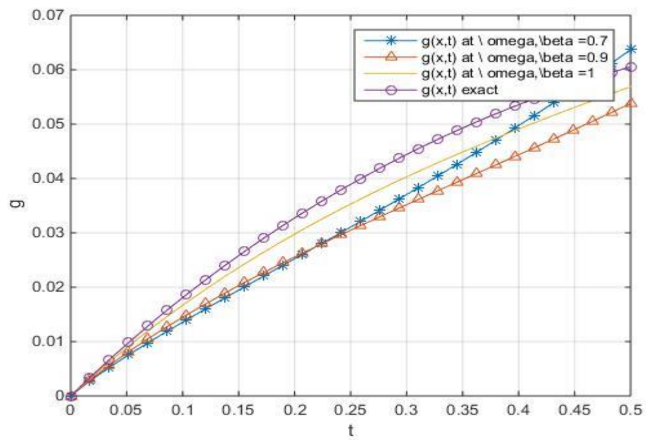

Figure 3. Plot of the exact and approximate solutions g(x ,t) for different values of ω, β with fixed values x=1

5. Conclusions

In the idea of the ABFVIM were both shown to be extremely successful in solving FPDEs. The solution is provided in a series form by the suggested algorithm, if there is an exact solution, it converges quickly. It is obvious from the findings that the ABFVIM produces solutions that are extremely precise with only a few iterates. Because of the efficacy and versatility shown in the examples given, ABFVIM can be operational to higher order FPDEs, according to the findings of this study.

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