Pre- Tichonov And Pre- Hausdorff Separation Axioms In Intuitonistic Fuzzy Special Topological Spaces

Fatima M. Mohammad
Department of Mathmatics, College of Education, University of Tikrit, Tikrit, Iraq

Abstract:

Our goal in this paper is to give definitions of pre-Tichonov and pre-Hausdorff intuitionistic special topological spaces, and study relationships between these spaces with intuitionistic special topological spaces (X, To,1) and (X, To,2) and topological spaces (X, T1) and (X, T2)

Introduction :

After the introduction of the concept of fuzzy sets by Zadeh [5], several researches were conducted on the generalizations of the notion of fuzzy set. An Intuitionistic fuzzy special set (IFSS for short) A is an object having the form <x, A1, A2> where A1 and A2 are sub set of a non empty fixed set X, satisfying the following $A1 \cap A2 = Ø$. The set A1 is called the set of member of A, while A2 is called the set of non- member of A ss coker [2]. Every sub set A of a non empty set X is IFSS having the form $<x, A1\ A^c>$. Some Boolean algebra operations on IFSS is defined by Coker [2] as follows :

Let X be anonempty set and IFSS's A,B where A =<x,A1,A2 , B=<x, B1 , B2> respectively. Furthermore, let $\{A\acute{\imath} : \acute{\imath} \in J\}$ be an arbitrary family of IFSS's in X. Then

(í) $A \subseteq B \leftrightarrow A1 \subseteq B1 \quad \Lambda \quad A2 \supseteq B2$;

(íí) $A = B \leftrightarrow \quad A \subseteq B \quad \Lambda \quad A \supseteq B$;

(ííí) $\bar{A} = < x, A2, A1 >$;

(iv) $FA = < x, A1, A^c1 >$, $SA = < x, A^c2, A2 >$;

(v) $UA\acute{\imath} = < x, UA\ \acute{\imath}^{(1)}, \cap A\ \acute{\imath}^{(2)} >$, $\cap A\ \acute{\imath} = < x, \cap A\ \acute{\imath}^{(1)}, UA\acute{\imath}^{(2)} >$

(ví) $Ø^* = < x, Ø, X>$, $X^* = < x, X, Ø >$.

An intuitionsitic fuzzy special topology (IFST for short) on anon empty set X is a family T of IFSS's in X containing Ø* , X* and closed under finite intersection and arbitrary union [2]. In this case the pair (X,T) is called an Intuitiunistic fuzzy special topological space (IFSTS for short) , any IFSS in T is known as Intiotionistic fuzzy special open set (IFSOS for short) in X. Now we construct two IFSTS's when (X,T) be IFSTS, in the following way $To,1 = \{ FG : G \in T\}$ and $To,2 = \{ SG : G \in T\}$, and two topological spaces $T1 = \{G1 : G = < x, G1, G2> \in T\}$ and $T2 = \{ G^c2 : G = < x ,G1, G2 > \in T \}$.

Now let (X,T) be IFSTS, then (X,T) is said to be satisfy Tichonov separation axiom if

$\forall\ x ,\ \forall\ y ,\ x \neq y \in X, x \neq y ,\ \exists\ A \in T,\ \exists\ B \in T$

s.t $x \in A\ \Lambda\ y \in B, x \notin B\ \Lambda\ y \notin A$.;

and it's satisfy Hausdorff separation axioms if

$\forall x,\ \forall y,\ x,y \in X,\ x \neq y\ \exists\ A, B \in T$ s,t

$x \in A\ \ \Lambda\ y \in B,\ A \bigcap B = Ø^*$

where an element x in a IFSSA means by

$(x \in A = < x, A1, A2 > \Leftrightarrow x \in A1 \text{ and } x \notin A2)$

Pre Intuitionistic fuzzy special Topological Spaces

Definition 2.1 :[3]

Let A be any IFS'S in (X,T), when A = < x, A1, A2 > , then A said to be

í- Intiotionistic fuzzy special pre open set (IFSPOS for short) if there exist $G \in T$ such that $A \subseteq G \subseteq clA$

íí- Intuitionistic fuzzy special pre closed set (IFSPCS for short) if there exist B closed set in X such that $\text{Int } A \subseteq B \subseteq A$

Proposition 2.2 :

Let (X,T) be IFSTS, then $A \subseteq X$ is pre open in X iff $A \subseteq \text{Int cl } A$.

Now next proposition shows some relationships between IFSPOS and IFSPCS and it's prove is direct.

Proposition 2.3 :

Let (X,T) be IFSTS and A is IFSOS in X, then the following are equivalent .

í- A is IFSPOS

íí- $\bar{A}$ is IFSPCS

ííí- $A \subseteq \text{Int cl } A$

Remark 2.4 :

Every IFSOS (IFSCS) is IFSPOS (IFSPCS) and the converse is not true in general

Proof :

Let A be IFSOS then $A \subseteq \text{cl } A$

Since A is IFSOS in X then $A \subseteq A \subseteq \text{cl } A$

Example 2.5 :

Let X = {1,2,3) defined T by

T = { Ǿ, X, A, B,} when A = <x, Ø, {2,3} >

B = < x , {2}, Ø > then

$C = < x, \{1,2\}, Ø > \subseteq \text{Int cl } C = X$ is

IFSPOS but not IFSOS

Remark 2.6 :

Let (X,T) be IFSTS, if A is IFSPOS in X and $B \subseteq A \subseteq \text{cl } B$, then B is IFSPOS in X.

Proof :

Since A is IFSPOS in X, then there exist $G \in T$ such that $A \subseteq G \subseteq \text{cl } A$, but $B \subseteq A \subseteq \text{cl } B$

Then $B \subseteq A \subseteq G \subseteq \text{cl } A \subseteq \text{cl } B$, i.e. $B \subseteq G \subseteq \text{cl } B$

Then B is IFSPOS.

Remark 2.7 :

The set of all IFSPOS is not necessary to be IFSPOS, since the intersaction of two IFPOS is not necessary to be IFSPOS in general as shown by the example.

Example 2.8 :

Take X = {1,2,3} and A = < x, Ø , {2,3} >

B = <x , {2} , Ø > , C = <x , {1,2} , Ø> and D = <x , {3} , Ø >

And defined T on X by T = {Ǿ, X , A, B} then C, D are IFSPOS but $C \cap D$ is not.

3. Pre-Tichonov Separation axiom on Intuitionisitic Fuzzy special topological Spaces

In this section we introduce pre-Tichonov separation axiom on IFSTS.

Definition 3.1 :

Let (X,T) be IFSTS, then (X,T) is said to be satisfy pre-TIchonov separation axiom (P-Tich for short) if for each $x , y \in X$ such that $x \neq y$

Then there exist two pre open sets U and V such that ($x \in U$ and $y \notin U$) and ($x \notin V$ and $y \in V$)

Remark 3.2 :

Every Tich-space is P-Tich space but the converse is not true in general is shown by the example .

Example 3.3 :

Let X = {1,2,3} defined T = { Ø , X, A} where

A = < x , {1} , Ø > , B = <x , {1,2} , Ø > , C = < x , {1,3} , Ø >

Then p.o (X) = { Ó, X, A, B, C} , then (X,T) is

P-Tich space but not Tich space .

Theorem 3.4 :

Let (X,T) be IFSTS, then the following are equivalent ;

(i) (X,T) is P-Tich ;

(ii) (X,To,1) is P-Tich ;

(iii) (X,T1) is P-Tich .

Proof :

($i \Rightarrow ii$) let $x,y \in X$ s.t $x \neq y \Rightarrow \exists$ Upx = < x, A1, A2 >

Vpy = < y, B1, B2 > s.t $x \in$ Upx , $x \notin$ Vpy and

$Y \in$ Vpy , $y \notin$ Upx , thus $x \in A1$ and $x \notin A2$

And $y \in B1$, $y \notin B2$ also $x \notin B1$, $y \in A1$

Since FUpx = <x, A1, $A1^c$ > & FVpy = <y,B1, $B1^c$ >

Then $x \in A1$ and $x \notin A1^c$ so $x \in$ FUpx , $x \notin$ Vpy

$\Rightarrow x \notin B1$ when $x \in B2$ and since $B1 \cap B2 = Ø$ so $x \in B1^C$

Thus $x \notin$ FVpy . Similarly $y \in$ FVpy

And $x \notin$ FVpy . Therefore (X, To, 1) is P-Tich. ;

($ii \Rightarrow iii$) take $x,y \in X$ such that $x \neq y$

$\Rightarrow \exists$ FUpx = < x , A1, $A1^c$ > & FVpy = <y, B1, $B1^c$ >

In To,1 when two open sets Upx = < x , A1, A2 > ;

And Vpy = < y, B1, B2 in T s.t $x \in$ FUpx

$x \notin$ FVpy and $y \in$ FVpy and $y \notin$ FUpx . Thus $x \in A1$ and not in B1 and $y \in B1$ not in A1

therefore (X,T1) is P-Tich. ;

$iii \rightarrow i$ let $x \neq y$ and $x, y \in X \Rightarrow$

$\exists$ $x \in A1$, $x \notin B1$ and $y \in B1$ and $y \notin A1$

Where A1, B1 in T1 .

Put Upx and Vpx in T where

Upx = < x , A1, A2 > and Vpy = <y , B1, B2 >

So Upx and Vpy satisfy P-Tich axiom,

Therefore (X,T) is P-Tich.

Proposition 3.5 :

Let (X,T) be P-Tich, then

(i) (X, To,2) is P-Tich ;

(ii) (X, T2) is P-Tich.

Proof :

(i) let (X, T) be P-Tich and take two elements in X say x,y such that $x \neq y$

$\Rightarrow \exists$ Upx = <x, A1, A2 > , Vpy = <y, B1, B2 >

s.t $x \in$ Upx , $y \in$ Vpy and $x \notin$ Vpy , $y \notin$ Upx

Thus $x \in A1$ and $x \notin A2$ and $y \in B1$

And $y \notin B2$ also $x \notin B1$, $y \notin A1$

Since SUpx = < x, $A2^c$, A2 > and

SVpy = <y, $B2^c$, B2 > . So $x \in A2^c$ and $y \in B2^c$ thus $x \in$ SVpx Λ $y \in$ SVpy

similarly $y \notin$ SUpx Λ $x \notin$ SVpy

therefore (X, To,2) is P-Tich.

In similarly we can prove (ii)

Remark 3.6 :

The converse of the last proposition is not true in general. The following example shows the case.

Example 3.7 :

Let X = {1,2,3} and put A = < x, {2}, {1} > B = < x, Ø, {3} >

C = < x, {3} , {2} > defined T on X by

T = { Ó, X, A, B, C, AUB , BUC , A∩B , A∩C, B∩ C } we can see that (X, T) is not P-Tich. , but (X, To, 2) and

(X, T2) is P-Tich.

4. Pre-Hausdorff Space on Intuitionist Fuzzy Special Topological Spaces

Definition 4.1 :

Let (X, T) be IFSTS, then (X, T) is satisfy pre-Hausdroff separation axiom (P-Haus. For short) if for each $x , y \in x$ such that $x \neq y$ then there exist two pre open sets U and V s.t $x \in U$ and $y \in V$, U∩V = Ø*

Proposition 4.2 :

Let (X, T) be P_Haus. IFSTS, then ;

(i)- (X, To, 1) , (ii)- (X, T1) , (iii)- (X, To, 2)

(iv)- (X, T2) are P-Haus.

Proof ; direct

Remark 4.3 :

The converse of proposition 4.2 is not true in general , is shown by the example .

Example 4.4 :

(i) Take X = {1,2,3} and Z = < x, {1} , {3} > , W = < x, {3} , {2}> ,

K = < x, {2} , {1} > and

Defined T = { Ó, X, Z, W, K, ZUW , ZUK, WUK , Z∩W , Z∩K , W∩K}

We see that T is not P-Haus but To, 1 and T2 are P-Haus.

(ii) let X = {1,2,3,4} and Z = < x , {4}, {2,3} > ,

W = < x, {3} , {1,2} > , K = < x, {2} , {1,4} > ,

M = < x , {1}, {3, 4} > and T = { Ó, X, Z, W, K, M}

U { all pair wise intersection and union }

We see that T0,2 is P-Haus but T is not .

(iii) let X = {1,2,3} and Z = < x , {1} , {2} >

W = < x , {2} , {1} > , K= <x, {3} , {1,3} > and

T = { Ó, X, Z, W, K, ZUK, WUK, Z∩W, Z∩K3 , we see that T1 is P-Haus. But T is not.

Proposition 4.5 :

Let (X,T) be IFSTS such that (X,T1) , ((X, T2)) is P-Haus. Then (X,T) is P-Haus.

Proof :

Let (X, T1) be P-Haus. Then for each $x, y \in x$ and $x \neq y$,

$\exists$ U = <x, A1, A2 > and V = < y, B1, B2 > $\in T$ such that

$x \in A1$, $y \in B1$, $A1 \cap B1 = Ø$ so $x \in U$

and $y \in V$ and $U \cap V = Ø$ i.e. (X, T) is P_Haus.

To prove (X, T) is P_Haus, when (X, T2) is P_Haus.

Take $x, y \in X$ and $x \neq y$

$\Rightarrow\ \exists$ U = < x, A1, A2 > and V = < y, B1, B2 >
And U, V$\in$T s.t x $\in$A2^c , y$\in$B2^c
And A2$^c \bigcap$ B2^c = Ø , then A2UB2 = X and we have
A1$\bigcap$ A2 = Ø ,
B1$\bigcap$ B2 = Ø then
[(A1$\bigcap$ A2) $\bigcap$ B1] U [(A1$\bigcap$ A2) $\bigcap$ B2)] = Ø
We get U$\bigcap$ V = Ø* i.e. (X1, T) is P-Haus.

<u>Proposition 4.6 :</u>
Let (X, T) be IFSTS such that (X, To,1) ,
((X, To, 2)) is P- Haus. , Then (X,T) is P-Haus.

<u>Proof :</u>
To prove (X, T) is P-Haus. Let (X, To,1) be P-Haus $\Rightarrow\ \forall$ x, $\forall$ y$\in$X
and x $\neq$ y
$\exists$ Upx = < x, A1, A2 > and Vpy = < y, B1, B2 > $\in$T
s.t x $\in$ FUpx and y$\in$ FVpy and
FUpx $\bigcap$ FVpy = Ø* , So x $\in$ Upx , y$\in$Vpy and
Upx$\bigcap$ Vpy = Ø* So (X,T) is P-Haus.
Similarly we can prove (X,T) is P-Haus.
When (X,To,2) is P-Haus.

<u>Remark 4.7 :</u>
- í- (X,T1) and (X,T2) are independent notion, the examples 3.7 and 4.4 shows this case.
- íí- Every P-Haus space is P-Tich, but the converse is not true in general. The next example shows this case.

<u>Example 4.8 :</u>
Let X = {1,2} and U = <x, {1} , {2} >
V = <x , {2} , Ø > and T = { Ǿ , X, U, V, U Λ V }
Then (X,T) is P-Tich , but not P-Haus.

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بديهيات الفصل بري تيكنوف وبري هاوزدورف في الفضاءات التبولوجية الحدسية الخاصة

فاطمة محمود محمد

قسم الرياضيات، كلية التربية، جامعة تكريت، تكريت، جمهورية العراق

الملخص:

يتناول هذا البحث بديهية بري تكينوف وبري هاوزدورف في الفضاءات البتولوجية الحدسية الخاصة وبعض العلاقات في هذه الفضاءات مع فضاءات البتولوجية الخاصة (X, To, 1) و (X, To, 2) والفضاءات البتولوجية (X, T1) و (X, T2) .