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DYNAMIC RESPONSE ANALYSIS OF VISCOELASTIC MOVING BELTS

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Abstract

The dynamic response and stability of parametrically excited viscoelastic belts are investigated in the current study. In this work, the generalized equation of motion is obtained for a viscoelastic moving belt with geometric nonlinearity. Non-dimensional analysis of the model was built on some assumptions to simplify the problem. The viscoelasticity of the model was modelled using Kelvin-Voigt model, the dynamic equation of motion derived using perturbation technique. The displacement of vibration found using the zeroth order solution that was subdivided into two parts, real and imaginary parts, due to the nature of nonlinear system. In this research effects of many elastic and viscoelastic parameters are studied, it was shown that there exists an upper boundary for the existence condition of the summation parametric resonance due to the existence of viscoelasticity. Effects of viscoelastic parameters, excitation frequencies, excitation amplitudes, and the axial moving speed on dynamic responses and existence boundaries were investigated.

Keywords: dynamic response, viscoelastic, belt drive.

تحليل الاستجابة الديناميكية للأحزمة الناقلة للحركة المصنوعة من مواد لزجة مرنة

عصام زهير فاضل

قسم الهندسة الميكانيكية / جامعة بابل

الخلاصة

الاستجابة الديناميكية واستقرارية الأحزمة المهتزة والمصنوعة من مواد لزجة مرنة تم دراستها في هذا البحث. إن المعادلةَ ألعامه للحركة استحصلت للأحزمة الناقلة للحركة والمصنوعة من مواد لزجة مرنة وفي الحالة اللاخطيه للشكل الهندسي. تم استخدام التحليل اللابعدي للتمثيل الرياضي للنظام كما تم وضع مجموعة من الفرضيات لغرض التمكن من تمثيل الاهتزاز للحزام الناقل رياضيا باستخدام النموذج الميكانيكي (كيلفن– فوجت). تم اشتقاق المعادلة الديناميكية للحركة باستخدام طريقة التحليل القلق، استخدمت الدرجة الصفرية لنفس الطريقة لحساب سعة الاهتزاز وتم تقسيمها إلى جزأين، الأول حقيقي والأخر تخيلي بسبب طبيعة النظام وكذلك بسبب اختيار شكل الاهتزاز بجزأين. بحثت عدد من العوامل لكل من المواد المرنة واللزجة المرنة وتم توضيح تأثيرها على الاهتزاز. لقد تبين من النتائج إن وجود الطبيعة المرنة اللدنة للاحزمه ألناقله يؤدي إلى ظهور الحد الأعلى لشرطِ الوجودَ بحاله الرنين. تم دراسة تأثير المواد اللزجة المرنة، الترددات، السعه، والسرعة المحورية على الاستجابة الديناميكية والحدود الظاهرة.

1. Introduction

Belt drives are extensively used in mechanical engineering practice for the transmission of moments and power between axles located far away from each other. Its widespread application – in the automobile industry, a number of branches of the light industry, general engineering and machine tool industry, etc. can be explained by its inexpensive realisation, quiet operation, easy mounting, favourable vibration damping, and last but not least by its good efficiency.

In applications requiring higher accuracy, for example the main and feed drives of machine tools, it is not sufficient to dimension the particular machine elements. In such cases it is also essential to apply knowledge of vibrations that will facilitate the solution or elimination of dynamic problems in the design phase. However, one major problem in belt drive systems is that crank shaft-driven belt tension actually fluctuates, which leads to the occurrence of large transverse belt vibrations. Such a system with fluctuation tension as a source of excitation is called a parametrically excited moving belt system. With reliability, Wear, and noise of utmost concern, it is of great interest to recognize and understand this important source of dynamic response.

Moving belt is a typical axially moving system. The nonlinear vibration of axially moving system has been studied extensively by many investigators. Huang et. al. (1995) studied the dynamic response and stability of a moving string undergoing three dimensional vibration. Perkins (1996) obtained the expressions for amplitudes and stability boundaries nontrivial limit cycles. But in all of these works, the material is assumed to be linear elastic and damping is either ignored or introduce to any damping mechanism.

Viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. Viscous materials resist shear flow and strain linearly with time when a stress is applied. Elastic materials strain instantaneously when stretched and just as quickly return to their original state once the stress is removed. Viscoelastic materials have elements of both of these properties and exhibit time dependent strain [Meyers (1999)].

Viscoelasticity is an effective approach to model the dissipative mechanism because some string-like engineering devices are composed of some viscoelastic metallic or ceramic reinforcement materials like glass-cord and viscoelastic polymeric materials such as rubber. The damping due to the viscoelasticity of string material exists only in nonlinear terms. Therefore nonlinear vibration of an axially moving viscoelastic string should be studied.

Lixin (1999), assumed the belt has constant velocity to simplify the analysis procedure with little acceptable error ,that he studied the linear differential constitutive relation. Several commonly used models are discussed; it is concerned with the linear integral constitutive law and the relation between differential and integral constitutive laws. There are many engineering designs that require vescoelastic behavior of structures, for examples, creep analysis of magnetic tapes and vibration problem of conduits.

In this paper, the nonlinear dynamic model of viscoelastic axially moving belt with geometrical nonlinearity is established. The effects of material parameters, the steady-state velocity, and the perturbed axial velocity of the belt on the dynamic response of the belts are investigated by the research of digital simulation.

2. Equation of Motion

Kelvin-Voigt model ,that presented the material as elastic spring and damper connected in parallel [Lixin (1999)], was used to represents a solid undergoing reversible viscoelastic strain. Upon application of a constant stress, the material deforms at a decreasing rate, asymptotically approaching the steady-state strain. The constitutive relation, [LI et. al. (2003)], is expressed as a linear first-order differential equation:

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt} \tag{1}$$

Consider that the viscoelastic belt is in a state of uniform initial stress, and only the transverse vibration in the y direction is taken into consideration.

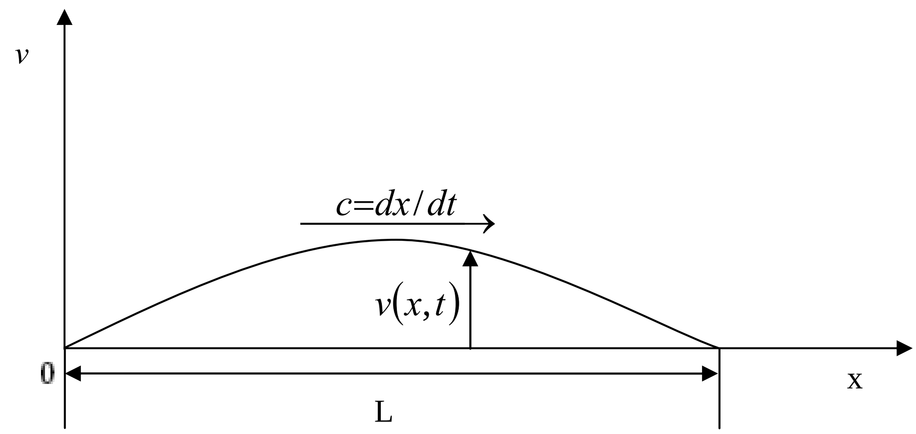


Figure 1. A mode of moving belt.

Figure (1) shows a prototypical model system of a viscoelastic moving belt used in this analysis. The Lagrangian strain component in the x-direction related to the transverse displacement, [LI et. al. (2003)], is $\varepsilon(x,t) = V_x^2(x,t)/2$. Thus, the equation of motion in the y-direction, [Lixin (1999)], can be obtained by Newton's second law of motion:

$$\rho \frac{\partial^2 V}{\partial t^2} + 2\rho c \frac{\partial^2 V}{\partial t \partial x} + \left(\rho c^2 - \frac{T}{A} \right) \frac{\partial^2 V}{\partial x^2} = E^* \left(\frac{1}{2} V_x^2 \right) V_{xx} + V_x \left\{ E^* \left(\frac{1}{2} V_x^2 \right) \right\}_x \tag{2}$$

with boundary conditions:

$$V(0,t) = 0 \qquad\qquad V(L,t) = 0 \tag{3}$$

The Kelvin viscoelastic model is chosen to describe the viscoelastic property of the belt material. The linear differential operator E^* for the Kelvin viscoelastic model, [Lixin (1999)], is given below:

$$E^* = E_0 + \eta \frac{\partial}{\partial t} \tag{4}$$

Introducing the following non-dimensional parameters:

$$v = \frac{V}{L} \qquad \zeta = \frac{x}{L} \qquad \tau = t\sqrt{\frac{T}{\rho A L^2}} \qquad \gamma = c\sqrt{\frac{\rho A}{T_0}}$$

$$a = \frac{T_1}{T_0} \qquad \omega = \Omega\sqrt{\frac{\rho A L^2}{T_0}} \qquad E_e = \frac{E_0 A}{T_0} \qquad E_v = \eta\sqrt{\frac{A}{\rho T_0 L^2}} \tag{5}$$

The corresponding non-dimensional equation of the transverse motion, [Lixin (1999)], is given by:

$$\frac{\partial^2 v}{\partial \tau^2} + 2\gamma \frac{\partial^2 v}{\partial \tau \partial \varsigma} + \left(\gamma^2 - 1 - a\cos\omega t\right)\frac{\partial^2 v}{\partial \varsigma^2} = N(v) \tag{6}$$

where the nonlinear operator *N(v)*, [Lixin (1999)], is defined as:

$$N(v) = E\left(\frac{1}{2}\right)v_\varsigma^2 v_{\varsigma\varsigma} + v_\varsigma\left[E\left(\frac{1}{2}v_\varsigma^2\right)\right]_\varsigma \tag{7}$$

Introduce the mass, gyroscopic, and linear stiffness operators as follows:

$$M = I\,, \qquad G = 2\gamma\frac{\partial}{\partial \varsigma}\;, \quad K = \left(\gamma^2 - 1\right)\frac{\partial^2}{\partial \varsigma^2} \tag{8}$$

Where operators M and K are symmetric and positive definite and G is skew-symmetric for sub-critical transport speeds. Equation (6) can be rewritten in a standard symbolic form:

$$Mv_{\tau\tau} + Gv_\tau + Kv = \varepsilon N(v) + \varepsilon a\cos\omega t\frac{\partial^2 v}{\partial \zeta^2} \tag{9}$$

Equation (9) is in the form of a continuous gyroscopic system with weakly nonlinearity and parameter excitation term.

The method of multiple scales is applied directly to solve the governing partial differential equation (9), which is in the form of a continuous gyroscopic system.

Here, to give more accurate results than the results studied by Lixin (1999),its preferred to use the harmonically fluctuating velocity that suggested by [LI et. al. (2003)],as followed :

$$v = C\{\cos\left[n\pi(1-c_0^2) + n\pi c_0\zeta + \theta\right]\sin n\pi\zeta + \frac{n\pi c_1}{\omega}\cos\omega\{\cos n\pi\zeta\cos[n\pi(1-c_0^2) + n\pi c_0\zeta + \theta] - c_0\sin n\pi\zeta\sin\left[n\pi(1-c_0^2) + n\pi c_0\zeta + \theta\right]\}\} \tag{10}$$

The values of $C = -0.5$, $\theta = 0$, $c_0 = 0.9$, and $c_1 = 0.06$, arbitrary constants, could be easily determined by initial conditions [LI et. al. (2003)].

The first term is usual constant velocity solution and the second term is the correction due to variation in velocity.

3. Limit Cycles and Existence Conditions

For nonlinear systems, limit cycles may exist in the vicinity of a parametric instability region. In this section, the interest is focused on the behavior of limit cycles around the parametric instability regions for elastic and viscoelastic nonlinear systems. Express A_n and A_l in the polar form as [Lixin (1999)]:

$$A_n = \frac{1}{2}\alpha_n e^{i\beta_n} \tag{11}$$

$$A_l = \frac{1}{2}\alpha_n e^{i\beta_l} \tag{12}$$

Note that α_k and β_k (*k*=*n, l*) represent the amplitude and the phase of the response, respectively.

3.1. Limit Cycles of Elastic Moving Belts

The response amplitude of steady state response of summation parametric resonance for elastic systems, [Lixin (1999)], are obtained:

$$\alpha_l^2 = \frac{n}{l}\alpha_n^2 \tag{13}$$

Where

$$\alpha_n^2 = \frac{\mu \pm \frac{a}{2\sqrt{nl\pi}}\sqrt{\mathrm{Re}(m_{\mathrm{nl}})^2 + \mathrm{Im}(m_{\mathrm{nl}})^2}}{-\left(\frac{3E_e m_{2\mathrm{n}}}{8n\pi} + \frac{3E_e m_{2\mathrm{l}} n}{8l^2\pi}\right)} \tag{14}$$

where m_{ln}, m_{nl} are solved by using direct multiple scales method by [Lixin (1999)] as:

$$m_{\mathrm{nl}} = m_{\mathrm{ln}} = \frac{4\pi n^2 l^2 v\left[-\sin(n+1)\pi v + i(1-\cos(n+1)\pi v)\right]}{(n+1)[(n+l)^2 v^2 - (n-l)^2]} \tag{15}$$

From the amplitude expression above of elastic problems, it can be seen that the first limit cycle exists if:

$$\mu \ge -\frac{\sqrt{\mathrm{Re}(m_{\mathrm{nl}})^2 + \mathrm{Im}(m_{\mathrm{nl}})^2}}{2\sqrt{nl\pi}}a \tag{16}$$

And the second limit cycle exists if

$$\mu \ge \frac{\sqrt{\mathrm{Re}(m_{\mathrm{nl}})^2 + \mathrm{Im}(m_{\mathrm{nl}})^2}}{2\sqrt{nl\pi}}a \tag{17}$$

As a special case. the response amplitude of principal parametric resonance (*n=l*) for elastic belts is given in the following:

$$\alpha_n^2 = \frac{4n\pi\mu \pm \frac{2n\pi\left|\sin n\pi\gamma\right|a}{\gamma}}{3E_e(-m_{2\mathrm{n}})} \tag{18}$$

The first limit cycle (select plus sign in equation(18))exists if the translation speed is sub-critical ($\gamma < 1$) and $\mu + \frac{\left|\sin n\pi\gamma\right|a}{2\gamma} > 0$. The second limit cycle (select negative sign in equation(18)) exists if the translation speed is sub-critical and $\mu - \frac{\left|\sin n\pi\gamma\right|a}{2\gamma} > 0$.

It should be mentioned that existence conditions of non-trivial limit cycles are the same as the stability conditions of the trivial solution for elastic systems (Zhang, 1998). Thus, it is concluded that the non-trivial limit cycles bifurcate from the trivial limit cycle at the stability boundary of the trivial limit cycle for elastic summation parametric resonance.

3.2. Limit Cycles of Viscoelastic Moving Belts

The response amplitude of steady state response for vescoelastic systems, [Lixin (1999)], are obtained:

$$\alpha_l^2 = \frac{n^2}{l^2}\sqrt{\frac{n}{l}}\alpha_n^2 \tag{19}$$

It is seen that the relation between α_n and α_l of viscoelastic systems is different from that of elastic systems. The following amplitude modulation equation for steady state response, [Lixin (1999)], is obtained

$$c_1\alpha_n^6 + c_2\alpha_n^4 + c_3\alpha_n^2 = 0 \tag{20}$$

Where

$$c_1 = \left(\frac{3E_e m_{2n}}{2n} + \frac{3E_e m_{2l} n^2}{2l^3}\sqrt{\frac{n}{l}}\right)^2 \frac{\sqrt{\frac{n}{l}}}{\left(\frac{1}{l}\sqrt{\frac{n}{l}} + \frac{1}{n}\right)^2} + \left(E_v \omega_n m_{2n}\right)^2 \frac{l^2}{n^2}\sqrt{\frac{l}{n}} \tag{21}$$

$$c_2 = 12\pi\mu E_e \left(\frac{m_{2n}}{n} + \frac{m_{2l} n^2}{2l^3}\sqrt{\frac{n}{l}}\right) \frac{\sqrt{\frac{n}{l}}}{\left(\frac{1}{l}\sqrt{\frac{n}{l}} + \frac{1}{n}\right)^2} \tag{22}$$

$$c_3 = 16\pi^2\mu^2 \frac{\sqrt{\frac{n}{l}}}{\left(\frac{1}{l}\sqrt{\frac{n}{l}} + \frac{1}{n}\right)^2} - a^2\left(\mathrm{Im}(m_{nl})^2 + \mathrm{Re}(m_{nl})^2\right) \tag{23}$$

Where m_{nl}, m_{ln} are the same values in equation (15).

It is obvious that equation (20) possesses a singular point at the origin (trivial periodic solution). In addition, two non-trivial singular points may exist describing limit cycles with amplitudes

$$\alpha_n^2 = \frac{-c_2 \pm \sqrt{c_2^2 - 4c_1c_3}}{2c_1} \tag{24}$$

Equations (19) and (24) represent the amplitudes of the steady state response of the summation parametric resonance for viscoelastic systems. From the amplitude equation (24) of viscoelastic systems, it can be seen that the two non-trivial steady state solutions exist only when the following conditions are satisfied, the first limit cycle of viscoelastic systems exists if:

$$\frac{\left(\frac{1}{l}\sqrt[4]{\frac{n}{l}} + \frac{1}{n}\sqrt[4]{\frac{l}{n}}\right)\sqrt{\mathrm{Im}(m_{nl})^2 + \mathrm{Re}(m_{nl})^2}}{-4\pi} \le \frac{\mu}{a} \le \frac{\left(\frac{n}{l^2}\sqrt{\frac{n}{l}} + \frac{1}{l}\right)\sqrt{\mathrm{Im}(m_{nl})^2 + \mathrm{Re}(m_{nl})^2 c_1}}{-4\pi E_v \omega_n m_{2n}} \tag{25}$$

and the second limit cycle exists if:

$$\frac{\left(\frac{1}{l}\sqrt[4]{\frac{n}{l}} + \frac{1}{n}\sqrt[4]{\frac{l}{n}}\right)\sqrt{\mathrm{Im}(m_{nl})^2 + \mathrm{Re}(m_{nl})^2}}{4\pi} \le \frac{\mu}{a} \le \frac{\left(\frac{n}{l^2}\sqrt{\frac{n}{l}} + \frac{1}{l}\right)\sqrt{\mathrm{Im}(m_{nl})^2 + \mathrm{Re}(m_{nl})^2 c_1}}{-4\pi E_v \omega_n m_{2n}} \tag{26}$$

As a special case, the response amplitude of principal parametric resonance $(n = l)$ for viscoelastic belts is given in the following:

$$\alpha_n^2 = \frac{\frac{3E_e n\pi\mu}{8} \pm \sqrt{\frac{n^2\pi^2 a^2 \sin^2 n\pi\gamma}{4\gamma^2}\left(\frac{3E_e}{8}\right)^2 - \left(n^2\pi^2\mu^2 - \frac{n^2\pi^2 a^2 \sin^2 n\pi\gamma}{4\gamma^2}\right)\left(\frac{E_v\omega_n}{4}\right)^2}}{2\left[\left(\frac{3E_e}{8}\right)^2 + \left(\frac{E_v\omega_n}{4}\right)^2\right](-m_{2\mathrm{n}})} \tag{27}$$

The first limit cycle (select plus sign in equation (27)) exists if the translation speed is sub-critical and $\frac{3E_e\mu}{8} + \sqrt{\frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\left(\frac{3E_e}{8}\right)^2 - \left(\mu^2 - \frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\right)\left(\frac{E_v\omega_n}{4}\right)^2} > 0$. The second limit cycle (select negative sign in equation (27)) exists if the translation speed is sub-critical and $\frac{3E_e\mu}{8} - \sqrt{\frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\left(\frac{3E_e}{8}\right)^2 - \left(\mu^2 - \frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\right)\left(\frac{E_v\omega_n}{4}\right)^2} > 0$.

It can be seen from equations (25) and (26) that the existence conditions of non-trivial limit cycles have an upper boundary for viscoelastic models, which is different from the conclusion of the corresponding elastic systems. The upper boundaries of existence conditions for the first limit cycle and the second limit cycle are identical and are determined by the viscoelastic parameter E_v. The lower boundaries of existence conditions have no relation with the nonlinear parameter E_e and the viscoelastic parameter E_v, and are different from those of the corresponding elastic systems.

4. Results and Discussion

In this section, numerical results of steady state responses and existence boundaries for the summation parametric resonance of moving belts are presented. Effects of the viscoelastic parameter, the amplitude of excitation, the frequency of excitation, and the transport speed on the response of non-trivial limit cycles are investigated for the belt moving in the harmonic velocity. Belts are composed of cord reinforcement materials and the outer layer of rubber materials, which can be considered as a spring (reinforcement materials) and a dashpot (rubber materials) connected in parallel. Thus, Kelvin viscoelastic is a natural representation of the mechanical properties of belt materials.

Figure (1) compares the current results with the results in Lixin (1999). The system parameters are E_e=400, E_v=0, a=0.5 and γ=0.25. where give a good agreement between these results for both the first limit cycle and the second limit cycle. The amplitudes of non-trivial limit cycles of the first principal parametric resonance (*n=l=1*) are plotted in Figure (2) as a function of excitation frequency (detuning), μ, and excitation amplitude, a, for an elastic system. The non-dimensional transport speed (γ) is 0.2 and the nonlinear parameter of Young's modulus (E_e) is 400.

Figures (3) and (4) shows the analogous results for the second principal parametric resonance (*n=2, l=2*) and the first summation parametric resonance (*n=1, l=2*), respectively. From Figures (2) to (4), it can be seen that the amplitude increases without bound as detuning parameter increases. When the excitation amplitude grows, the response amplitude of the first limit cycle increases while the second limit cycle decreases. Only the trivial solution exists if the existence conditions of non-trivial solutions are not satisfied. The results obtained here are identical to those given by Mockensturm et. al. (1996).

The non-trivial limit cycles of the first summation parametric resonance (*n=1, l=2*) for a viscoelastic moving belt are shown Figure (5). The non-dimensional transport speed is 0.2. The nonlinear parameter (E_e) is 400, and the viscoelastic parameter (E_v) is 5. It is evident that though the amplitude increases with the growth of detuning parameter, there exists an upper bound. The non-trivial limit cycle will vanish when non-dimensional amplitude of perturbation tension and detuning parameter approach this bound, which indicates that damping introduced by the viscoelasticity enlarges the region of the trivial limit cycles. This phenomenon for viscoelastic moving belt is quite different from the corresponding elastic systems.

Translation speed not only influence the amplitude of the non-trivial limit cycles, but also influence the existence region of non-trivial limit cycles significantly. Figures (6) and (7) illustrate the effect of the translating speed on non-trivial limit cycles of the first principal (*n=l=1*) and the first summation (*n =1, l=2*) parametric resonance, respectively. The non-dimensional amplitude of perturbation tension (a) is chosen as 0.5 and the nonlinear parameter (E_e) is 400. From Figure (6), for the principal parametric resonance, it is seen that the amplitude of limit cycles decreases with the increase of transport speeds. The non-trivial amplitude grows more slowly with detuning parameter when translation speeds is larger. Moreover, for the translation speed unsatisfying equation (26) and (27), the non-trivial limit cycles no longer exist. These results indicate that by increasing the transport speed while keeping other parameters constant, an unstable belt can be stabilized. For the summation parametric resonance, the relation between the response and the transport speed is much more complicated. There exists a maximum value of response for the first limit cycle and a minimum value of response for the second limit cycle when non-dimensional transport speed is around 0.2.

6. Conclusions

From the above study, the following conclusions can be drawn:

1) The amplitude of the limit cycles decreases with increasing transport speeds for principal parametric resonance. There is no such a simple relation for the summation parametric resonance.

2) There exists an upper existence boundary for the viscoelastic model and this upper boundary of existence for limit cycles is determined by the viscoelastic property E_v.

3) The lower boundary of existence for limit cycles of elastic systems is identical to the stability boundary of the trivial solution. This suggests that non-trivial limit cycles of the summation parametric resonance bifurcate from the trivial limit cycle at the boundary of the trivial limit cycle.

4) The boundaries of existence have no relation with the nonlinear parameter E_e.

5) The most effects of the transverse amplitudes come from the frequency of the perturbed velocity when the belts moves with harmonic velocity.

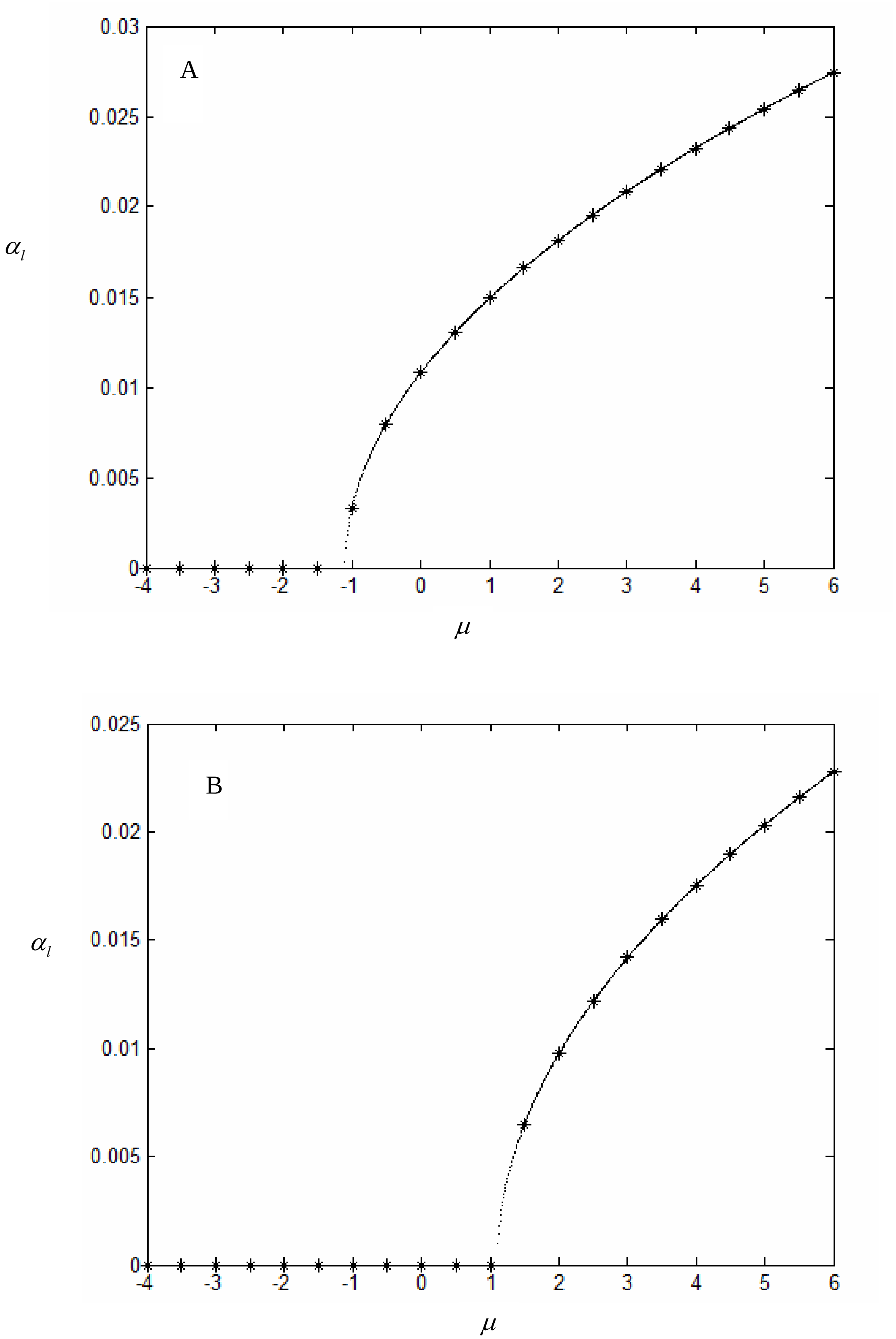


Figure 1. A Comparison of the current result with the Lixin result, ——— Present result, * Lixin result. (*n=1, l=2, E_e=400, E_v=0, γ=0.25, a=0.5*)
A: the first limit cycle. B: the second limit cycle.

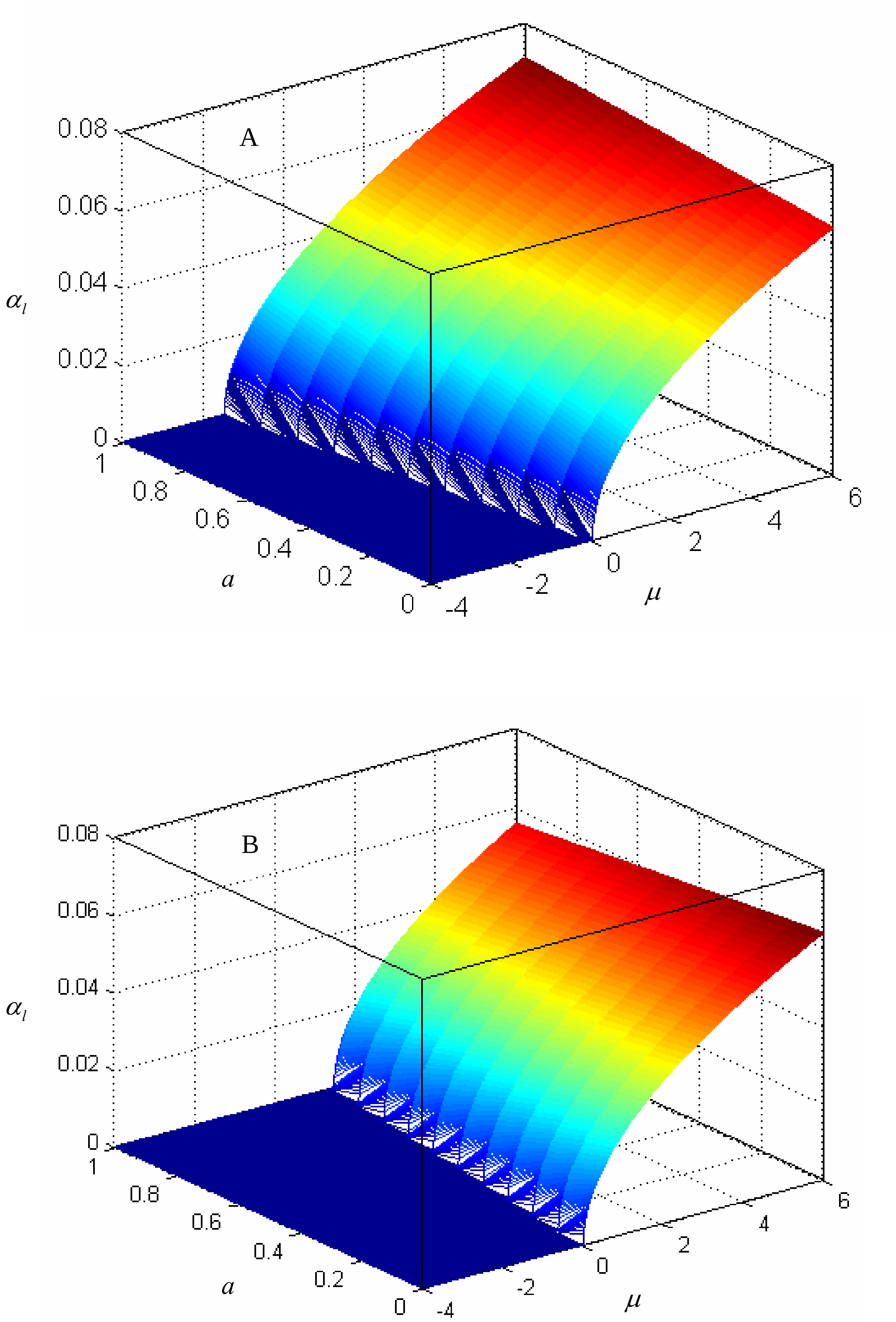


Figure 2. The nontrivial limit cycles that bifurcate from the boundary of the first principal parameter instability region (*γ=0.2, n=l=1, E_e=400, E_v=0*)
A: the first limit cycle. B: the second limit cycle

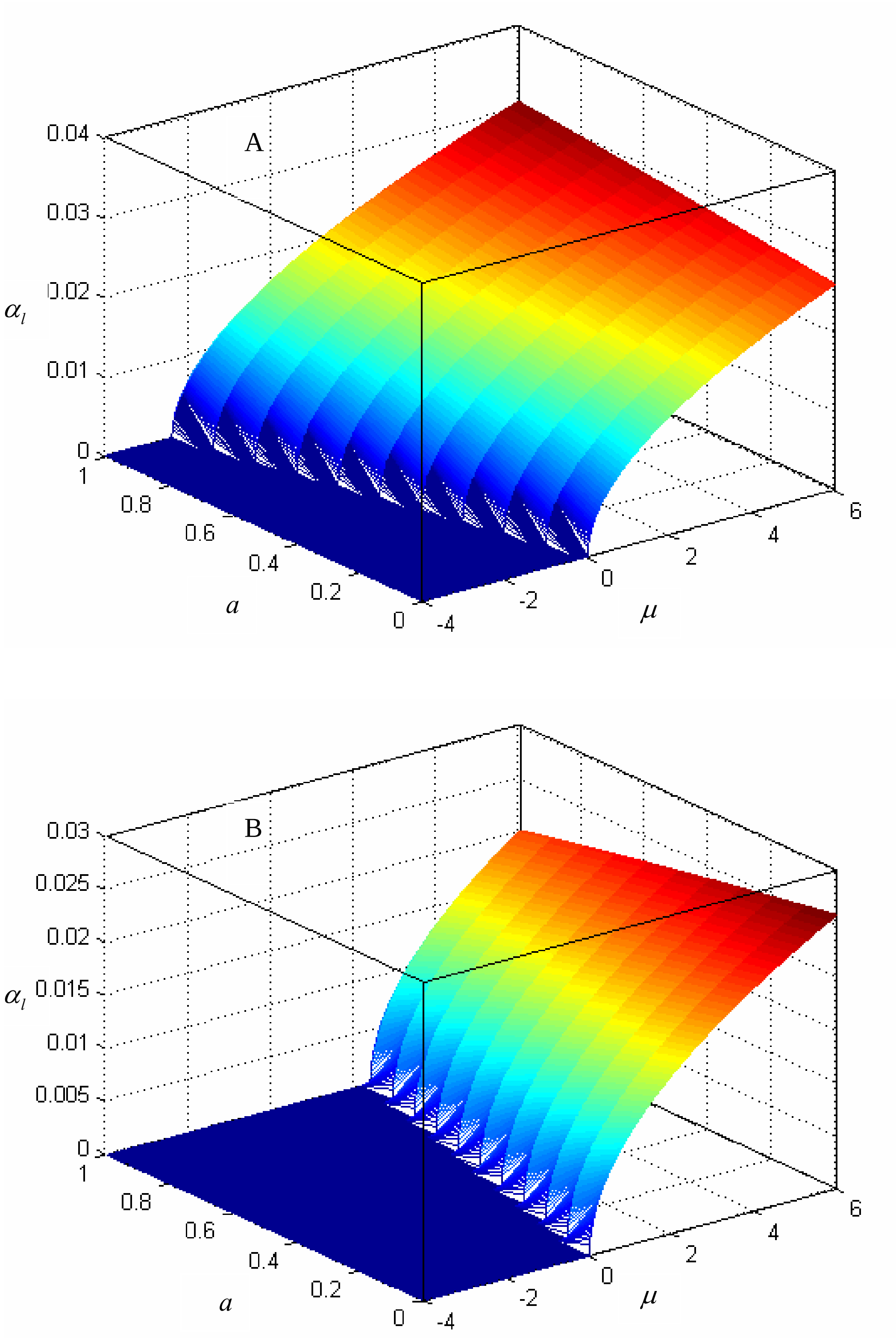
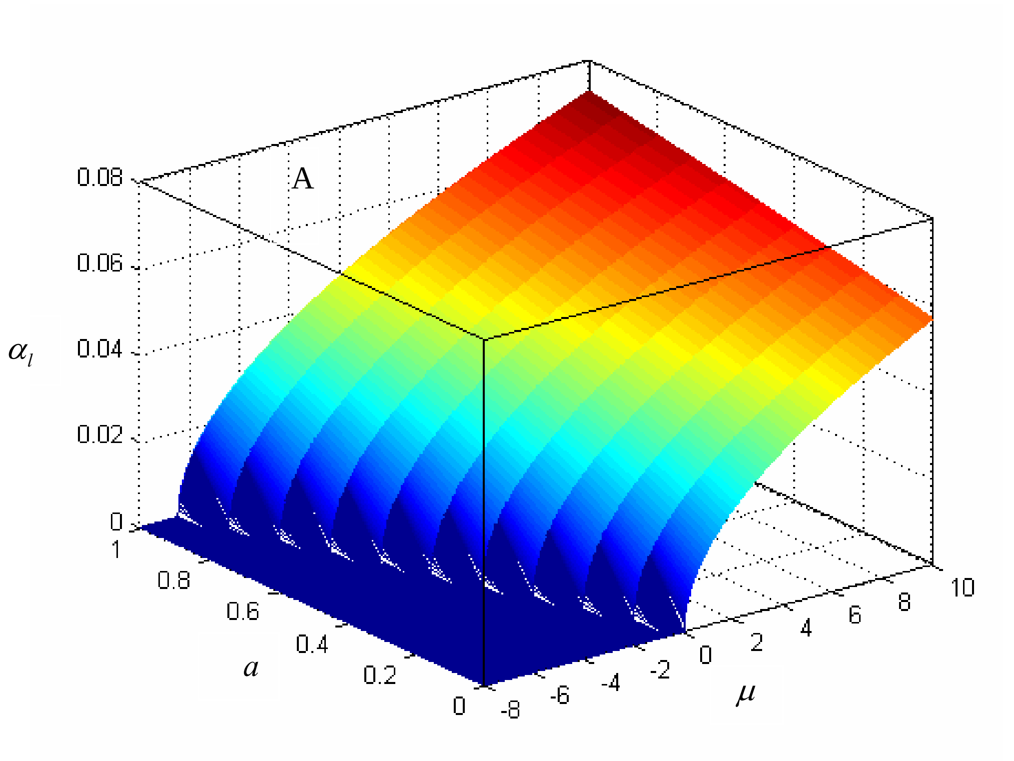


Figure 3. The nontrivial limit cycles that bifurcate from the boundary of the second principle parameter instability region (*γ=0.2, n=l=2, E_e=400, E_v=0*)
A: the first limit cycle. B: the second limit cycle



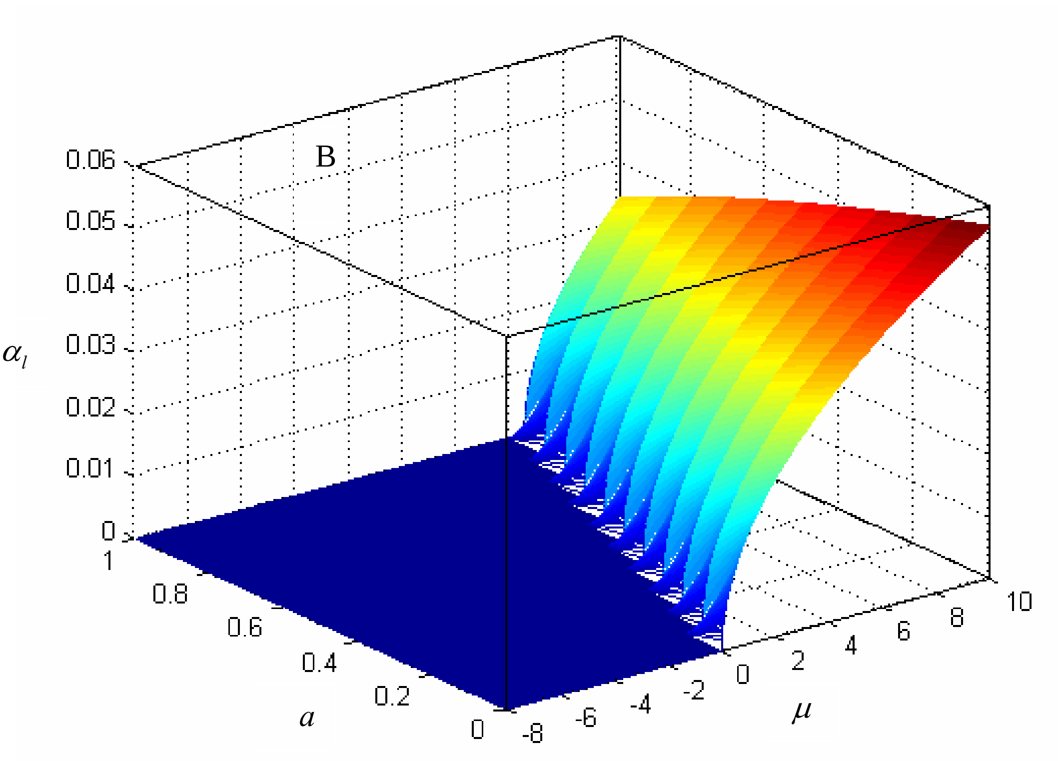
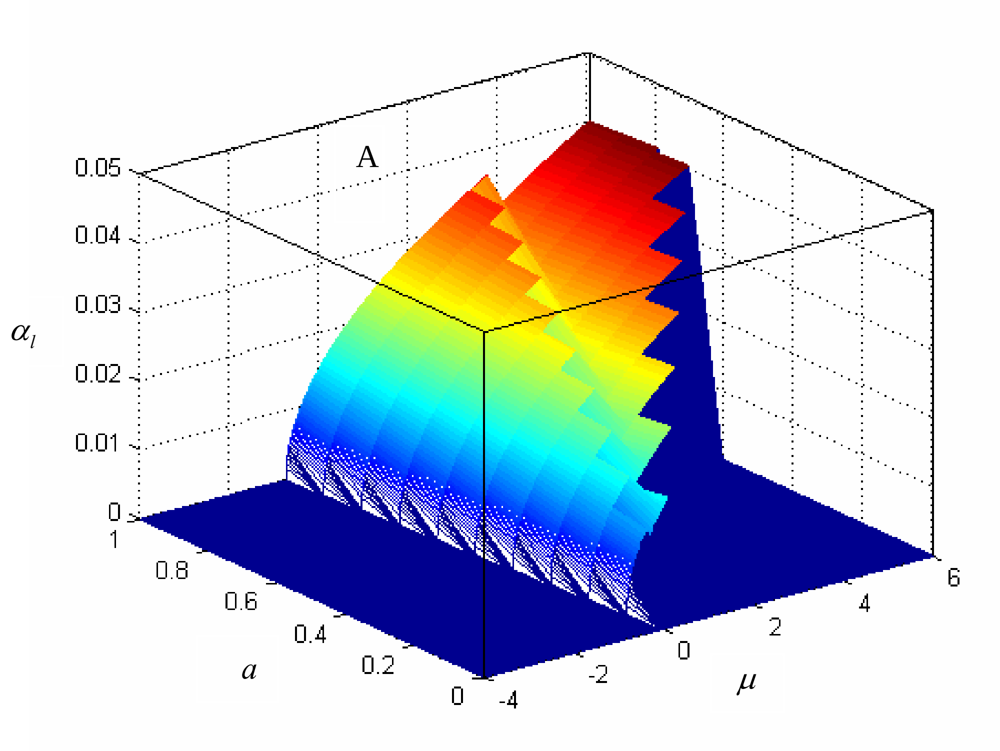


Figure 4. The nontrivial limit cycles that bifurcate from the boundary of the first summation parameter instability region ($\gamma=0.2$, $n=1$, $l=2$, $E_e=400$, $E_v=0$)
A: the first limit cycle. B: the second limit cycle



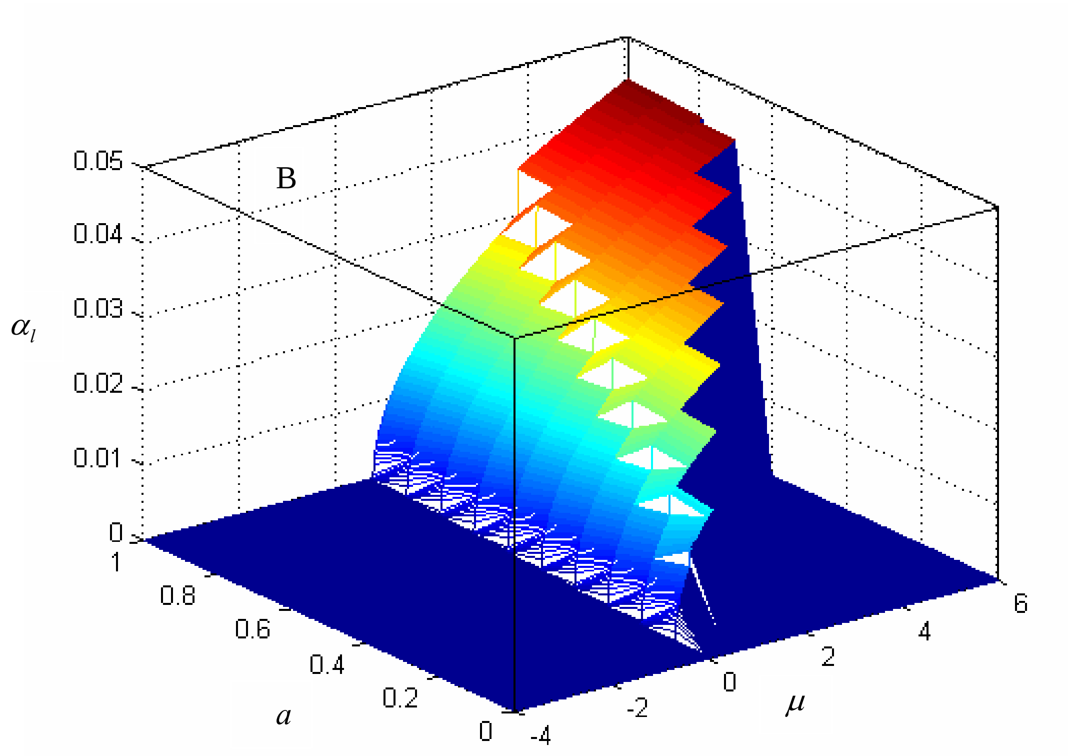


Figure 5. The response amplitude of nontrivial limit cycles for the summation parameter resonance of a viscoelastic moving belt (*$\gamma=0.2$, $n=1$, $l=2$, $E_e=400$, $E_v=5$*)
A: the first limit cycle. B: the second limit cycle

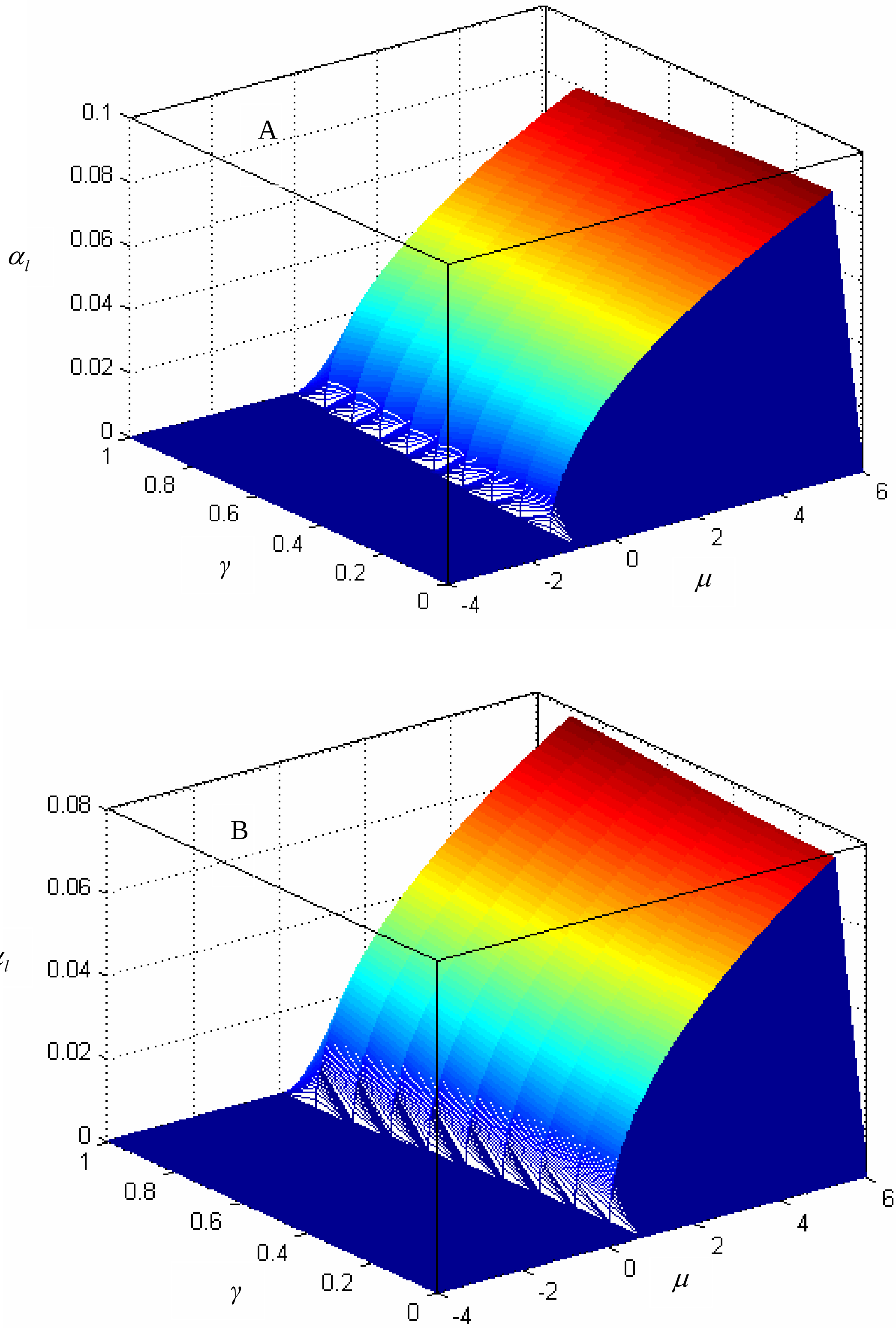


Figure 6. Effect of the transport speed on nontrivial limit cycles for the first principle parameter resonance (*a=0.5, n=l=1,* E_e*=400,* E_v*=5*)
A: the first limit cycle. B: the second limit cycle

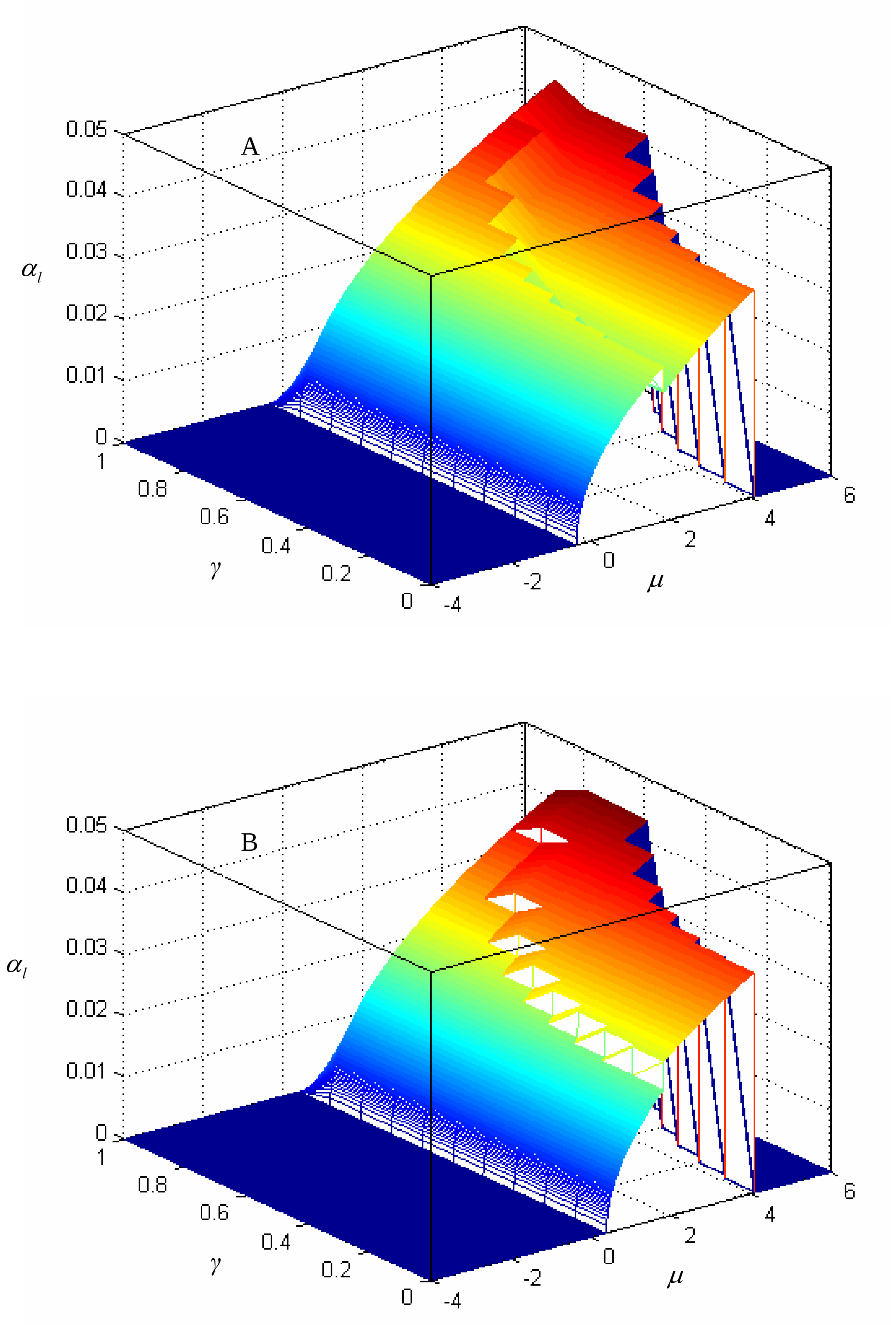


Figure 7. Effect of the transport speed on nontrivial limit cycles for the first summation parameter resonance (*a=0.5, n=1, l=2, E_e=400, E_v=5*)
A: the first limit cycle. B: the second limit cycle

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NOMENCLATURE

Symbol	Definition	Units
a	Non-dimensional amplitude of perturbation tension	-
A	Cross-sectional area of belt	m^2
c	Axial velocity of belt	m/s
c_0	constant	-
c_1	constant	-
C	constant	-
E	Non-dimensional equivalent Young's modulus	-
E^*	Equivalent Young's modulus	N/m^2
E_0	Initial Young's modulus	N/m^2
E_e	Non-dimensional Young's modulus	-
E_v	Non-dimensional viscoelastic parameter	-
G	Non-dimensional gyroscopic operator	-
K	Non-dimensional stiffness operator	-
L	Length of moving belts	m
$m_{\ln}$, m_{nl}	Non-dimensional parameter	-
M	Non-dimensional mass operator of moving belts	-
N	Nonlinear terms	-
t	time of moving belt	sec
T	Initial tension of moving belts	N
T0	Steady state tension	N
T1	Perturbation tension	N
v	Non-dimensional transverse displacement of moving belts	-
V	Transverse displacement of moving belt	m
x	Local coordinate in longitudinal direction	-
α_n , α_m	Response amplitude of the nth and mth mode	-
ε	Non-dimensional small parameter	-
ϕ_i	The ith eigenfunction of moving belts	-
θ	constant	-
γ	Non-dimensional translating speed	-
ρ	Belt mass per unit volume	Kg/m^3
η	Dynamic viscosity of the dashpot	-
λ_n	Eigenvalue of mode n	-
μ	Non-dimensional detuning parameter	-
τ	Non-dimensional time	-
ω	Excitation frequency	Rad/sec
ω_n	Natural frequency for mode n	Rad/sec
Ω	Excitation frequency	Rad/sec
ζ	Non-dimensional coordinate in longitudinal direction	-

1. Introduction

Belt drives are extensively used in mechanical engineering practice for the transmission of moments and power between axles located far away from each other. Its widespread application – in the automobile industry, a number of branches of the light industry, general engineering and machine tool industry, etc. can be explained by its inexpensive realisation, quiet operation, easy mounting, favourable vibration damping, and last but not least by its good efficiency.

In applications requiring higher accuracy, for example the main and feed drives of machine tools, it is not sufficient to dimension the particular machine elements. In such cases it is also essential to apply knowledge of vibrations that will facilitate the solution or elimination of dynamic problems in the design phase. However, one major problem in belt drive systems is that crank shaft-driven belt tension actually fluctuates, which leads to the occurrence of large transverse belt vibrations. Such a system with fluctuation tension as a source of excitation is called a parametrically excited moving belt system. With reliability, Wear, and noise of utmost concern, it is of great interest to recognize and understand this important source of dynamic response.

Moving belt is a typical axially moving system. The nonlinear vibration of axially moving system has been studied extensively by many investigators. Huang et. al. (1995) studied the dynamic response and stability of a moving string undergoing three dimensional vibration. Perkins (1996) obtained the expressions for amplitudes and stability boundaries nontrivial limit cycles. But in all of these works, the material is assumed to be linear elastic and damping is either ignored or introduce to any damping mechanism.

Viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. Viscous materials resist shear flow and strain linearly with time when a stress is applied. Elastic materials strain instantaneously when stretched and just as quickly return to their original state once the stress is removed. Viscoelastic materials have elements of both of these properties and exhibit time dependent strain [Meyers (1999)].

Viscoelasticity is an effective approach to model the dissipative mechanism because some string-like engineering devices are composed of some viscoelastic metallic or ceramic reinforcement materials like glass-cord and viscoelastic polymeric materials such as rubber. The damping due to the viscoelasticity of string material exists only in nonlinear terms. Therefore nonlinear vibration of an axially moving viscoelastic string should be studied.

Lixin (1999), assumed the belt has constant velocity to simplify the analysis procedure with little acceptable error ,that he studied the linear differential constitutive relation. Several commonly used models are discussed; it is concerned with the linear integral constitutive law and the relation between differential and integral constitutive laws. There are many engineering designs that require vescoelastic behavior of structures, for examples, creep analysis of magnetic tapes and vibration problem of conduits.

In this paper, the nonlinear dynamic model of viscoelastic axially moving belt with geometrical nonlinearity is established. The effects of material parameters, the steady-state velocity, and the perturbed axial velocity of the belt on the dynamic response of the belts are investigated by the research of digital simulation.

2. Equation of Motion

Kelvin-Voigt model ,that presented the material as elastic spring and damper connected in parallel [Lixin (1999)], was used to represents a solid undergoing reversible viscoelastic strain. Upon application of a constant stress, the material deforms at a decreasing rate, asymptotically approaching the steady-state strain. The constitutive relation, [LI et. al. (2003)], is expressed as a linear first-order differential equation:

$$\sigma(t) = E\varepsilon(t) + \eta \frac{d\varepsilon(t)}{dt} \tag{1}$$

Consider that the viscoelastic belt is in a state of uniform initial stress, and only the transverse vibration in the y direction is taken into consideration.

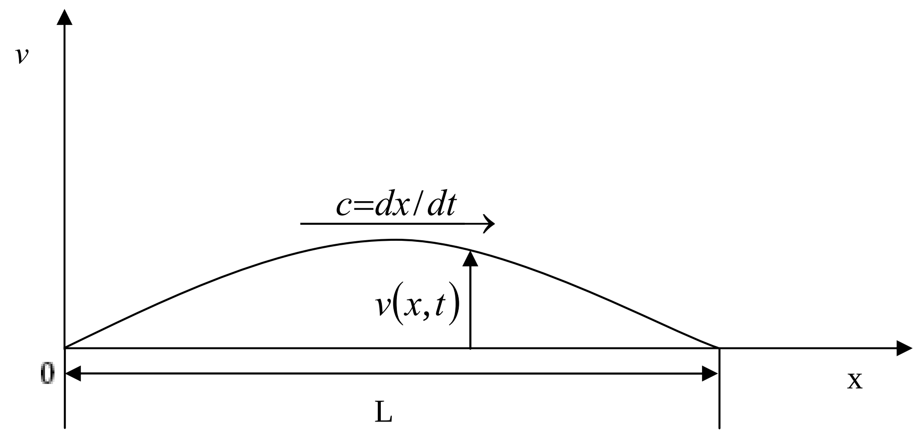


Figure 1. A mode of moving belt.

Figure (1) shows a prototypical model system of a viscoelastic moving belt used in this analysis. The Lagrangian strain component in the x-direction related to the transverse displacement, [LI et. al. (2003)], is $\varepsilon(x,t) = V_x^2(x,t)/2$. Thus, the equation of motion in the y-direction, [Lixin (1999)], can be obtained by Newton's second law of motion:

$$\rho \frac{\partial^2 V}{\partial t^2} + 2\rho c \frac{\partial^2 V}{\partial t \partial x} + \left(\rho c^2 - \frac{T}{A}\right)\frac{\partial^2 V}{\partial x^2} = E^*\left(\frac{1}{2}V_x^2\right)V_{xx} + V_x\left\{E^*\left(\frac{1}{2}V_x^2\right)\right\}_x \tag{2}$$

with boundary conditions:

$$V(0,t) = 0 \qquad\qquad V(L,t) = 0 \tag{3}$$

The Kelvin viscoelastic model is chosen to describe the viscoelastic property of the belt material. The linear differential operator E^* for the Kelvin viscoelastic model, [Lixin (1999)], is given below:

$$E^* = E_0 + \eta \frac{\partial}{\partial t} \tag{4}$$

Introducing the following non-dimensional parameters:

$$v = \frac{V}{L} \qquad \zeta = \frac{x}{L} \qquad \tau = t\sqrt{\frac{T}{\rho A L^2}} \qquad \gamma = c\sqrt{\frac{\rho A}{T_0}}$$

$$a = \frac{T_1}{T_0} \qquad \omega = \Omega\sqrt{\frac{\rho A L^2}{T_0}} \qquad E_e = \frac{E_0 A}{T_0} \qquad E_v = \eta\sqrt{\frac{A}{\rho T_0 L^2}} \tag{5}$$

The corresponding non-dimensional equation of the transverse motion, [Lixin (1999)], is given by:

$$\frac{\partial^2 v}{\partial \tau^2}+2\gamma\frac{\partial^2 v}{\partial \tau \partial \varsigma}+\left(\gamma^2-1-a\cos\omega t\right)\frac{\partial^2 v}{\partial \varsigma^2}=N(v) \tag{6}$$

where the nonlinear operator *N(v)*, [Lixin (1999)], is defined as:

$$N(v)=E\left(\frac{1}{2}\right)v_{\varsigma}^2 v_{\varsigma\varsigma}+v_{\varsigma}\left[E\left(\frac{1}{2}v_{\varsigma}^2\right)\right]_{\varsigma} \tag{7}$$

Introduce the mass, gyroscopic, and linear stiffness operators as follows:

$$M=I\,, \qquad G=2\gamma\frac{\partial}{\partial \varsigma}\;, \quad K=\left(\gamma^2-1\right)\frac{\partial^2}{\partial \varsigma^2} \tag{8}$$

Where operators M and K are symmetric and positive definite and G is skew-symmetric for sub-critical transport speeds. Equation (6) can be rewritten in a standard symbolic form:

$$Mv_{\tau\tau}+Gv_{\tau}+Kv=\varepsilon N(v)+\varepsilon a\cos\omega t\frac{\partial^2 v}{\partial \zeta^2} \tag{9}$$

Equation (9) is in the form of a continuous gyroscopic system with weakly nonlinearity and parameter excitation term.

The method of multiple scales is applied directly to solve the governing partial differential equation (9), which is in the form of a continuous gyroscopic system.

Here, to give more accurate results than the results studied by Lixin (1999),its preferred to use the harmonically fluctuating velocity that suggested by [LI et. al. (2003)],as followed :

$$v=C\{\cos\left[n\pi(1-c_0^2)+n\pi c_0\zeta+\theta\right]\sin n\pi\zeta+\frac{n\pi c_1}{\omega}\cos\omega\{\cos n\pi\zeta\cos[n\pi(1-c_0^2)+ \\ n\pi c_0\zeta+\theta]-c_0\sin n\pi\zeta\sin\left[n\pi(1-c_0^2)+n\pi c_0\zeta+\theta\right]\}\} \tag{10}$$

The values of $C=-0.5$, $\theta=0$, $c_0=0.9$, and $c_1=0.06$, arbitrary constants, could be easily determined by initial conditions [LI et. al. (2003)].

The first term is usual constant velocity solution and the second term is the correction due to variation in velocity.

3. Limit Cycles and Existence Conditions

For nonlinear systems, limit cycles may exist in the vicinity of a parametric instability region. In this section, the interest is focused on the behavior of limit cycles around the parametric instability regions for elastic and viscoelastic nonlinear systems. Express A_n and A_l in the polar form as [Lixin (1999)]:

$$A_n=\frac{1}{2}\alpha_n e^{i\beta_n} \tag{11}$$

$$A_l=\frac{1}{2}\alpha_n e^{i\beta_l} \tag{12}$$

Note that α_k and β_k (*k=n, l*) represent the amplitude and the phase of the response, respectively.

3.1. Limit Cycles of Elastic Moving Belts

The response amplitude of steady state response of summation parametric resonance for elastic systems, [Lixin (1999)], are obtained:

$$\alpha_l^2 = \frac{n}{l}\alpha_n^2 \tag{13}$$

Where

$$\alpha_n^2 = \frac{\mu \pm \dfrac{a}{2\sqrt{nl\pi}}\sqrt{\mathrm{Re}(m_{\mathrm{nl}})^2 + \mathrm{Im}(m_{\mathrm{nl}})^2}}{-\left(\dfrac{3E_e m_{2\mathrm{n}}}{8n\pi} + \dfrac{3E_e m_{2\mathrm{l}} n}{8l^2\pi}\right)} \tag{14}$$

where m_{ln}, m_{nl} are solved by using direct multiple scales method by [Lixin (1999)] as:

$$m_{\mathrm{nl}} = m_{\mathrm{ln}} = \frac{4\pi n^2 l^2 v\left[-\sin(n+1)\pi v + i(1-\cos(n+1)\pi v)\right]}{(n+1)[(n+l)^2 v^2 - (n-l)^2]} \tag{15}$$

From the amplitude expression above of elastic problems, it can be seen that the first limit cycle exists if:

$$\mu \geq -\frac{\sqrt{\mathrm{Re}(m_{\mathrm{nl}})^2 + \mathrm{Im}(m_{\mathrm{nl}})^2}}{2\sqrt{nl\pi}}a \tag{16}$$

And the second limit cycle exists if

$$\mu \geq \frac{\sqrt{\mathrm{Re}(m_{\mathrm{nl}})^2 + \mathrm{Im}(m_{\mathrm{nl}})^2}}{2\sqrt{nl\pi}}a \tag{17}$$

As a special case. the response amplitude of principal parametric resonance (*n=l*) for elastic belts is given in the following:

$$\alpha_n^2 = \frac{4n\pi\mu \pm \dfrac{2n\pi\left|\sin n\pi\gamma\right|a}{\gamma}}{3E_e(-m_{2\mathrm{n}})} \tag{18}$$

The first limit cycle (select plus sign in equation(18))exists if the translation speed is sub-critical $(\gamma < 1)$ and $\mu + \dfrac{\left|\sin n\pi\gamma\right|a}{2\gamma} > 0$. The second limit cycle (select negative sign in equation(18)) exists if the translation speed is sub-critical and $\mu - \dfrac{\left|\sin n\pi\gamma\right|a}{2\gamma} > 0$.

It should be mentioned that existence conditions of non-trivial limit cycles are the same as the stability conditions of the trivial solution for elastic systems (Zhang, 1998). Thus, it is concluded that the non-trivial limit cycles bifurcate from the trivial limit cycle at the stability boundary of the trivial limit cycle for elastic summation parametric resonance.

3.2. Limit Cycles of Viscoelastic Moving Belts

The response amplitude of steady state response for vescoelastic systems, [Lixin (1999)], are obtained:

$$\alpha_l^2 = \frac{n^2}{l^2}\sqrt{\frac{n}{l}}\alpha_n^2 \tag{19}$$

It is seen that the relation between α_n and α_l of viscoelastic systems is different from that of elastic systems. The following amplitude modulation equation for steady state response, [Lixin (1999)], is obtained

$$c_1\alpha_n^6 + c_2\alpha_n^4 + c_3\alpha_n^2 = 0 \tag{20}$$

Where

$$c_1 = \left(\frac{3E_e m_{2\mathrm{n}}}{2n} + \frac{3E_e m_{2\mathrm{l}} n^2}{2l^3}\sqrt{\frac{n}{l}}\right)^2 \frac{\sqrt{\frac{n}{l}}}{\left(\frac{1}{l}\sqrt{\frac{n}{l}} + \frac{1}{n}\right)^2} + \left(E_v\omega_n m_{2\mathrm{n}}\right)^2 \frac{l^2}{n^2}\sqrt{\frac{l}{n}} \tag{21}$$

$$c_2 = 12\pi\mu E_e\left(\frac{m_{2\mathrm{n}}}{n} + \frac{m_{2\mathrm{l}} n^2}{2l^3}\sqrt{\frac{n}{l}}\right)\frac{\sqrt{\frac{n}{l}}}{\left(\frac{1}{l}\sqrt{\frac{n}{l}} + \frac{1}{n}\right)^2} \tag{22}$$

$$c_3 = 16\pi^2\mu^2 \frac{\sqrt{\frac{n}{l}}}{\left(\frac{1}{l}\sqrt{\frac{n}{l}} + \frac{1}{n}\right)^2} - a^2\left(\mathrm{Im}(m_{\mathrm{nl}})^2 + \mathrm{Re}(m_{\mathrm{nl}})^2\right) \tag{23}$$

Where m_{nl} , m_{ln} are the same values in equation (15).

It is obvious that equation (20) possesses a singular point at the origin (trivial periodic solution). In addition, two non-trivial singular points may exist describing limit cycles with amplitudes

$$\alpha_n^2 = \frac{-c_2 \pm \sqrt{c_2^2 - 4c_1c_3}}{2c_1} \tag{24}$$

Equations (19) and (24) represent the amplitudes of the steady state response of the summation parametric resonance for viscoelastic systems. From the amplitude equation (24) of viscoelastic systems, it can be seen that the two non-trivial steady state solutions exist only when the following conditions are satisfied, the first limit cycle of viscoelastic systems exists if:

$$\frac{\left(\frac{1}{l}\sqrt[4]{\frac{n}{l}} + \frac{1}{n}\sqrt[4]{\frac{l}{n}}\right)\sqrt{\mathrm{Im}(m_{\mathrm{nl}})^2 + \mathrm{Re}(m_{\mathrm{nl}})^2}}{-4\pi} \le \frac{\mu}{a} \le \frac{\left(\frac{n}{l^2}\sqrt{\frac{n}{l}} + \frac{1}{l}\right)\sqrt{\mathrm{Im}(m_{\mathrm{nl}})^2 + \mathrm{Re}(m_{\mathrm{nl}})^2 c_1}}{-4\pi E_v\omega_n m_{2\mathrm{n}}} \tag{25}$$

and the second limit cycle exists if:

$$\frac{\left(\frac{1}{l}\sqrt[4]{\frac{n}{l}} + \frac{1}{n}\sqrt[4]{\frac{l}{n}}\right)\sqrt{\mathrm{Im}(m_{\mathrm{nl}})^2 + \mathrm{Re}(m_{\mathrm{nl}})^2}}{4\pi} \le \frac{\mu}{a} \le \frac{\left(\frac{n}{l^2}\sqrt{\frac{n}{l}} + \frac{1}{l}\right)\sqrt{\mathrm{Im}(m_{\mathrm{nl}})^2 + \mathrm{Re}(m_{\mathrm{nl}})^2 c_1}}{-4\pi E_v\omega_n m_{2\mathrm{n}}} \tag{26}$$

As a special case, the response amplitude of principal parametric resonance $(n = l)$ for viscoelastic belts is given in the following:

$$\alpha_n^2 = \frac{\frac{3E_e n\pi\mu}{8} \pm \sqrt{\frac{n^2\pi^2 a^2 \sin^2 n\pi\gamma}{4\gamma^2}\left(\frac{3E_e}{8}\right)^2 - \left(n^2\pi^2\mu^2 - \frac{n^2\pi^2 a^2 \sin^2 n\pi\gamma}{4\gamma^2}\right)\left(\frac{E_v\omega_n}{4}\right)^2}}{2\left[\left(\frac{3E_e}{8}\right)^2 + \left(\frac{E_v\omega_n}{4}\right)^2\right](-m_{2n})} \tag{27}$$

The first limit cycle (select plus sign in equation (27)) exists if the translation speed is sub-critical and $\frac{3E_e\mu}{8} + \sqrt{\frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\left(\frac{3E_e}{8}\right)^2 - \left(\mu^2 - \frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\right)\left(\frac{E_v\omega_n}{4}\right)^2} > 0$. The second limit cycle (select negative sign in equation (27)) exists if the translation speed is sub-critical and $\frac{3E_e\mu}{8} - \sqrt{\frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\left(\frac{3E_e}{8}\right)^2 - \left(\mu^2 - \frac{a^2 \sin^2 n\pi\gamma}{4\gamma^2}\right)\left(\frac{E_v\omega_n}{4}\right)^2} > 0$.

It can be seen from equations (25) and (26) that the existence conditions of non-trivial limit cycles have an upper boundary for viscoelastic models, which is different from the conclusion of the corresponding elastic systems. The upper boundaries of existence conditions for the first limit cycle and the second limit cycle are identical and are determined by the viscoelastic parameter E_v. The lower boundaries of existence conditions have no relation with the nonlinear parameter E_e and the viscoelastic parameter E_v, and are different from those of the corresponding elastic systems.

4. Results and Discussion

In this section, numerical results of steady state responses and existence boundaries for the summation parametric resonance of moving belts are presented. Effects of the viscoelastic parameter, the amplitude of excitation, the frequency of excitation, and the transport speed on the response of non-trivial limit cycles are investigated for the belt moving in the harmonic velocity. Belts are composed of cord reinforcement materials and the outer layer of rubber materials, which can be considered as a spring (reinforcement materials) and a dashpot (rubber materials) connected in parallel. Thus, Kelvin viscoelastic is a natural representation of the mechanical properties of belt materials.

Figure (1) compares the current results with the results in Lixin (1999). The system parameters are E_e=400, E_v=0, a=0.5 and γ=0.25. where give a good agreement between these results for both the first limit cycle and the second limit cycle. The amplitudes of non-trivial limit cycles of the first principal parametric resonance (*n=l=1*) are plotted in Figure (2) as a function of excitation frequency (detuning), μ, and excitation amplitude, a, for an elastic system. The non-dimensional transport speed (γ) is 0.2 and the nonlinear parameter of Young's modulus (E_e) is 400.

Figures (3) and (4) shows the analogous results for the second principal parametric resonance (*n=2, l=2*) and the first summation parametric resonance (*n=1, l=2*), respectively. From Figures (2) to (4), it can be seen that the amplitude increases without bound as detuning parameter increases. When the excitation amplitude grows, the response amplitude of the first limit cycle increases while the second limit cycle decreases. Only the trivial solution exists if the existence conditions of non-trivial solutions are not satisfied. The results obtained here are identical to those given by Mockensturm et. al. (1996).

The non-trivial limit cycles of the first summation parametric resonance (*n=1, l=2*) for a viscoelastic moving belt are shown Figure (5). The non-dimensional transport speed is 0.2. The nonlinear parameter (E_e) is 400, and the viscoelastic parameter (E_v) is 5. It is evident that though the amplitude increases with the growth of detuning parameter, there exists an upper bound. The non-trivial limit cycle will vanish when non-dimensional amplitude of perturbation tension and detuning parameter approach this bound, which indicates that damping introduced by the viscoelasticity enlarges the region of the trivial limit cycles. This phenomenon for viscoelastic moving belt is quite different from the corresponding elastic systems.

Translation speed not only influence the amplitude of the non-trivial limit cycles, but also influence the existence region of non-trivial limit cycles significantly. Figures (6) and (7) illustrate the effect of the translating speed on non-trivial limit cycles of the first principal (*n=l=1*) and the first summation (*n =1, l=2*) parametric resonance, respectively. The non-dimensional amplitude of perturbation tension (a) is chosen as 0.5 and the nonlinear parameter (E_e) is 400. From Figure (6), for the principal parametric resonance, it is seen that the amplitude of limit cycles decreases with the increase of transport speeds. The non-trivial amplitude grows more slowly with detuning parameter when translation speeds is larger. Moreover, for the translation speed unsatisfying equation (26) and (27), the non-trivial limit cycles no longer exist. These results indicate that by increasing the transport speed while keeping other parameters constant, an unstable belt can be stabilized. For the summation parametric resonance, the relation between the response and the transport speed is much more complicated. There exists a maximum value of response for the first limit cycle and a minimum value of response for the second limit cycle when non-dimensional transport speed is around 0.2.

6. Conclusions

From the above study, the following conclusions can be drawn:

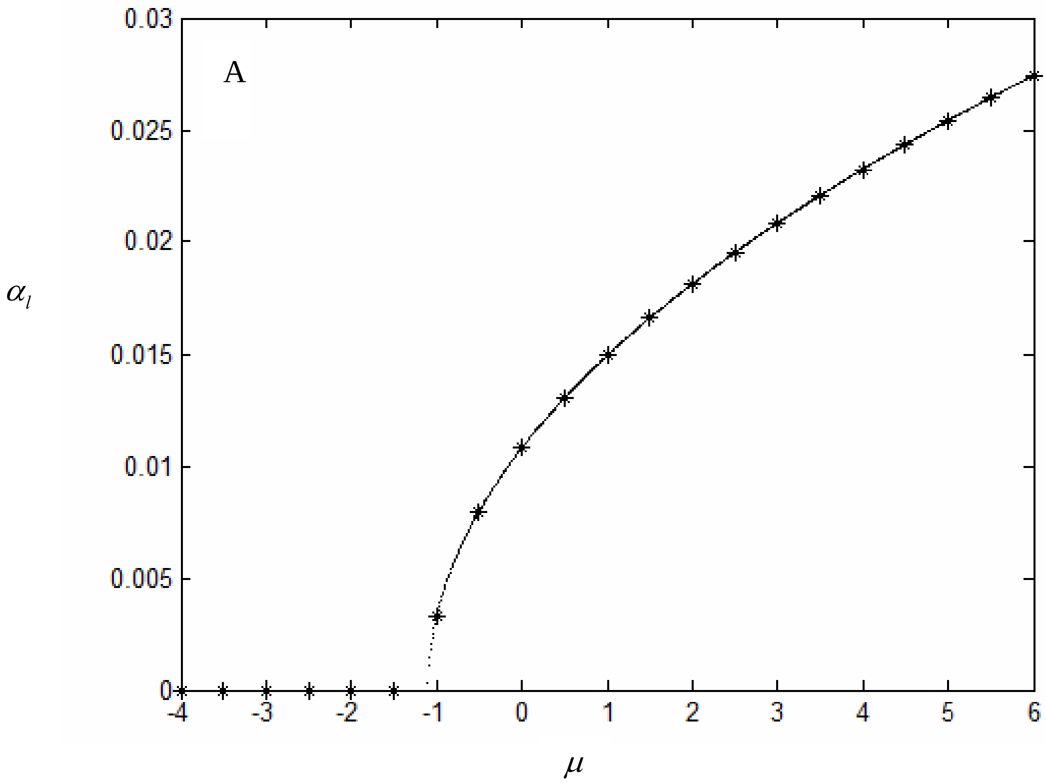
1) The amplitude of the limit cycles decreases with increasing transport speeds for principal parametric resonance. There is no such a simple relation for the summation parametric resonance.

2) There exists an upper existence boundary for the viscoelastic model and this upper boundary of existence for limit cycles is determined by the viscoelastic property E_v.

3) The lower boundary of existence for limit cycles of elastic systems is identical to the stability boundary of the trivial solution. This suggests that non-trivial limit cycles of the summation parametric resonance bifurcate from the trivial limit cycle at the boundary of the trivial limit cycle.

4) The boundaries of existence have no relation with the nonlinear parameter E_e.

5) The most effects of the transverse amplitudes come from the frequency of the perturbed velocity when the belts moves with harmonic velocity.



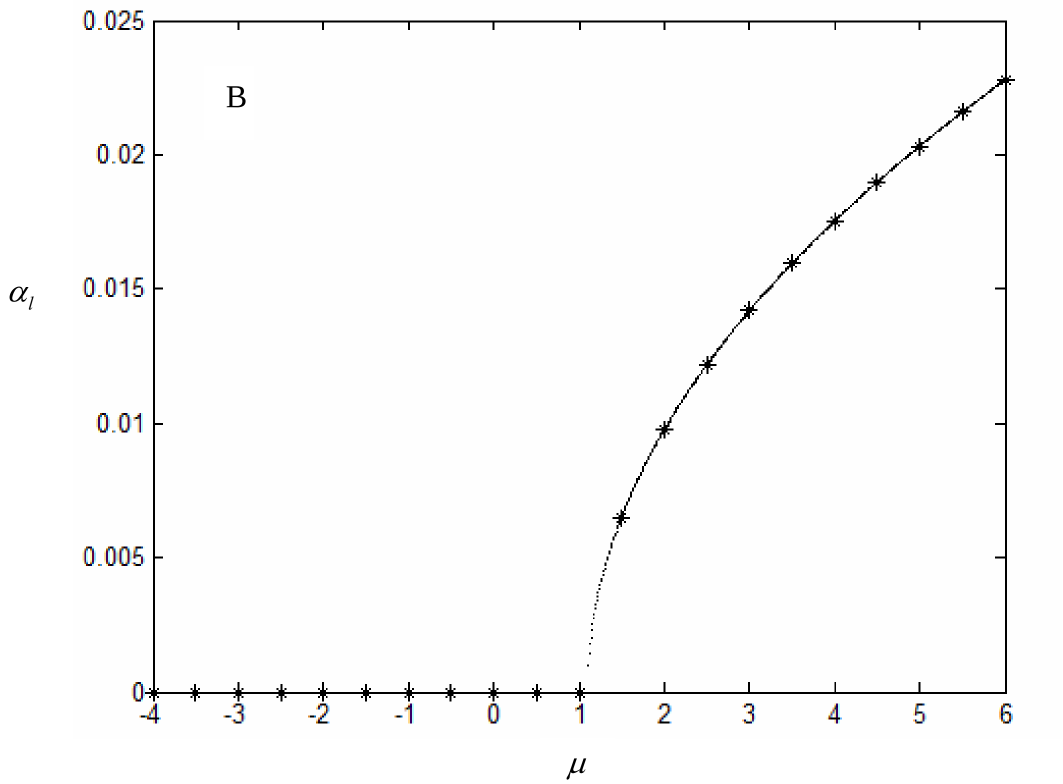


Figure 1. A Comparison of the current result with the Lixin result, ——Present result, * Lixin result. (*n=1, l=2, E_e=400, E_v=0, γ=0.25, a=0.5*)
A: the first limit cycle. B: the second limit cycle.

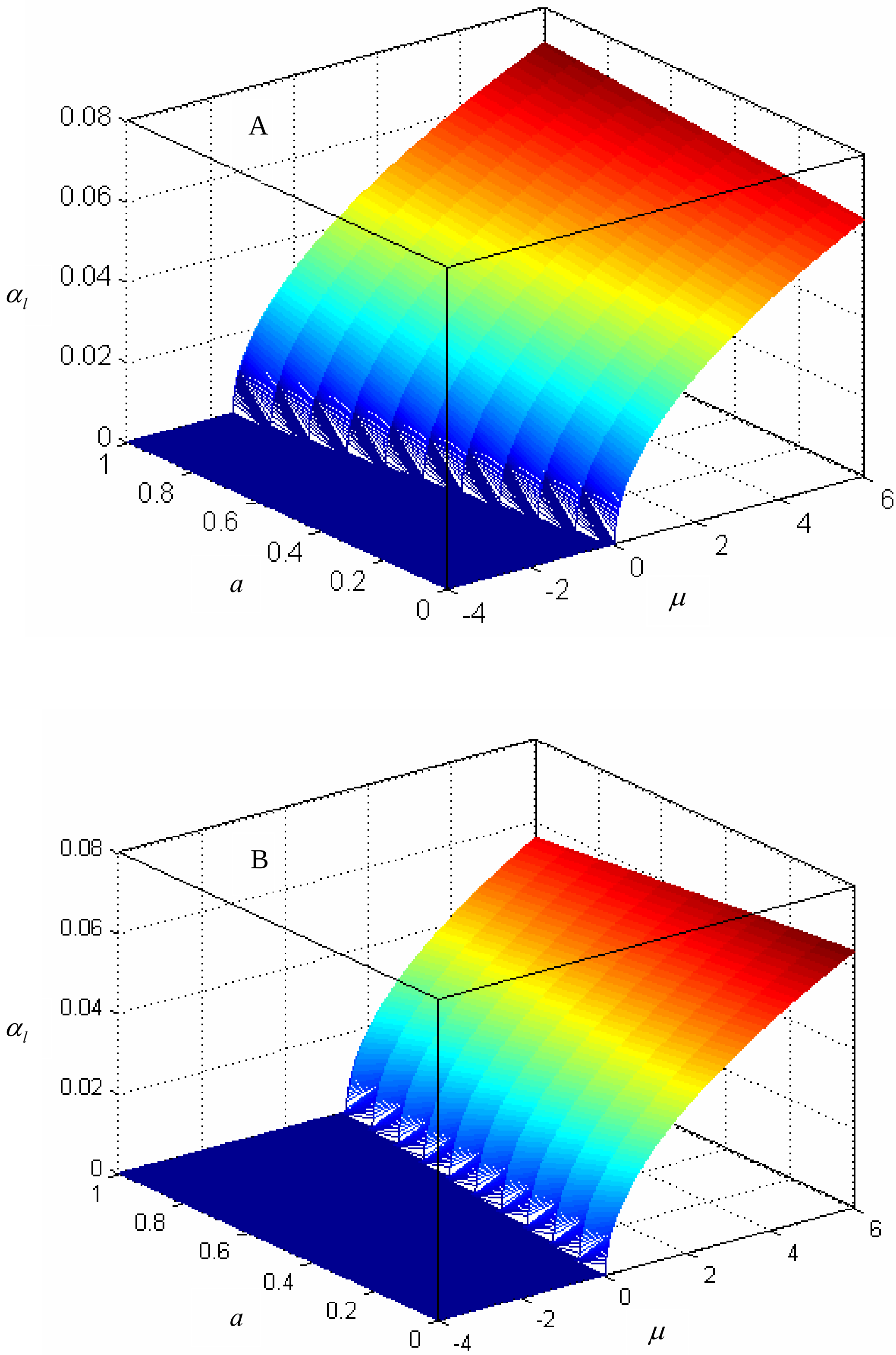


Figure 2. The nontrivial limit cycles that bifurcate from the boundary of the first principal parameter instability region (*γ=0.2, n=l=1, E_e=400, E_v=0*)
A: the first limit cycle. B: the second limit cycle

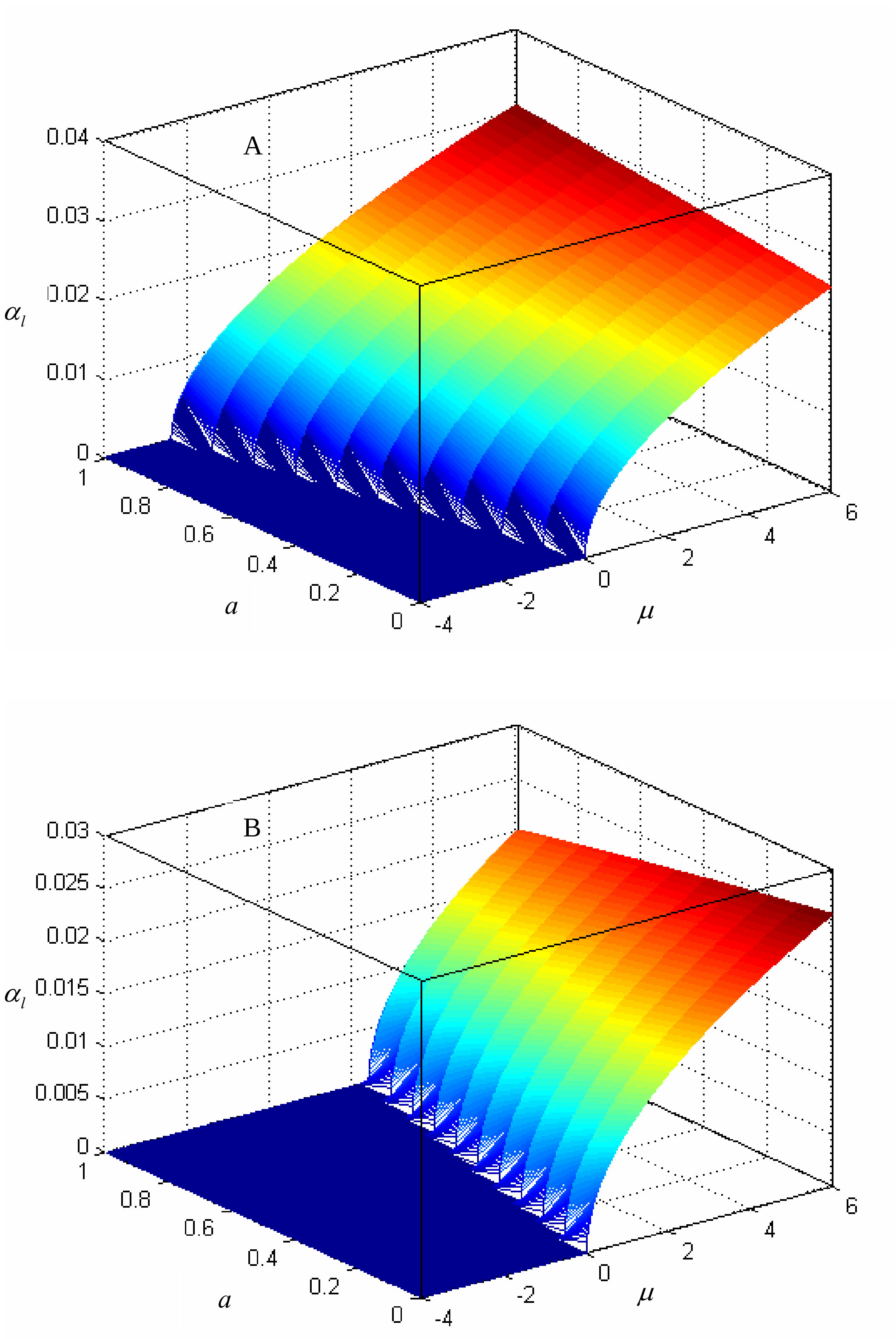


Figure 3. The nontrivial limit cycles that bifurcate from the boundary of the second principle parameter instability region (*γ=0.2, n=l=2, E_e=400, E_v=0*)
A: the first limit cycle. **B: the second limit cycle**

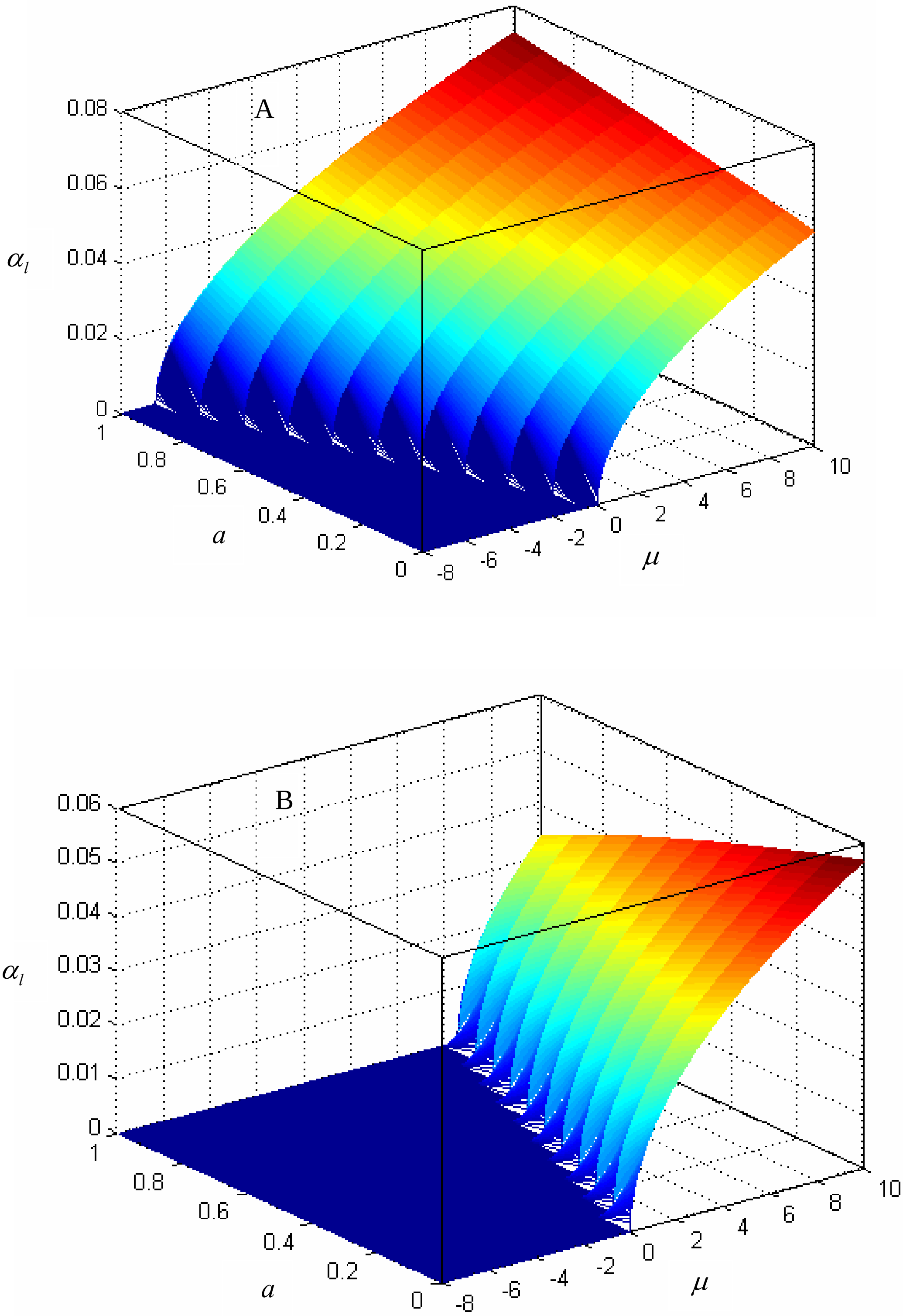


Figure 4. The nontrivial limit cycles that bifurcate from the boundary of the first summation parameter instability region (*γ=0.2, n=1, l=2, E_e=400, E_v=0*)
A: the first limit cycle. B: the second limit cycle

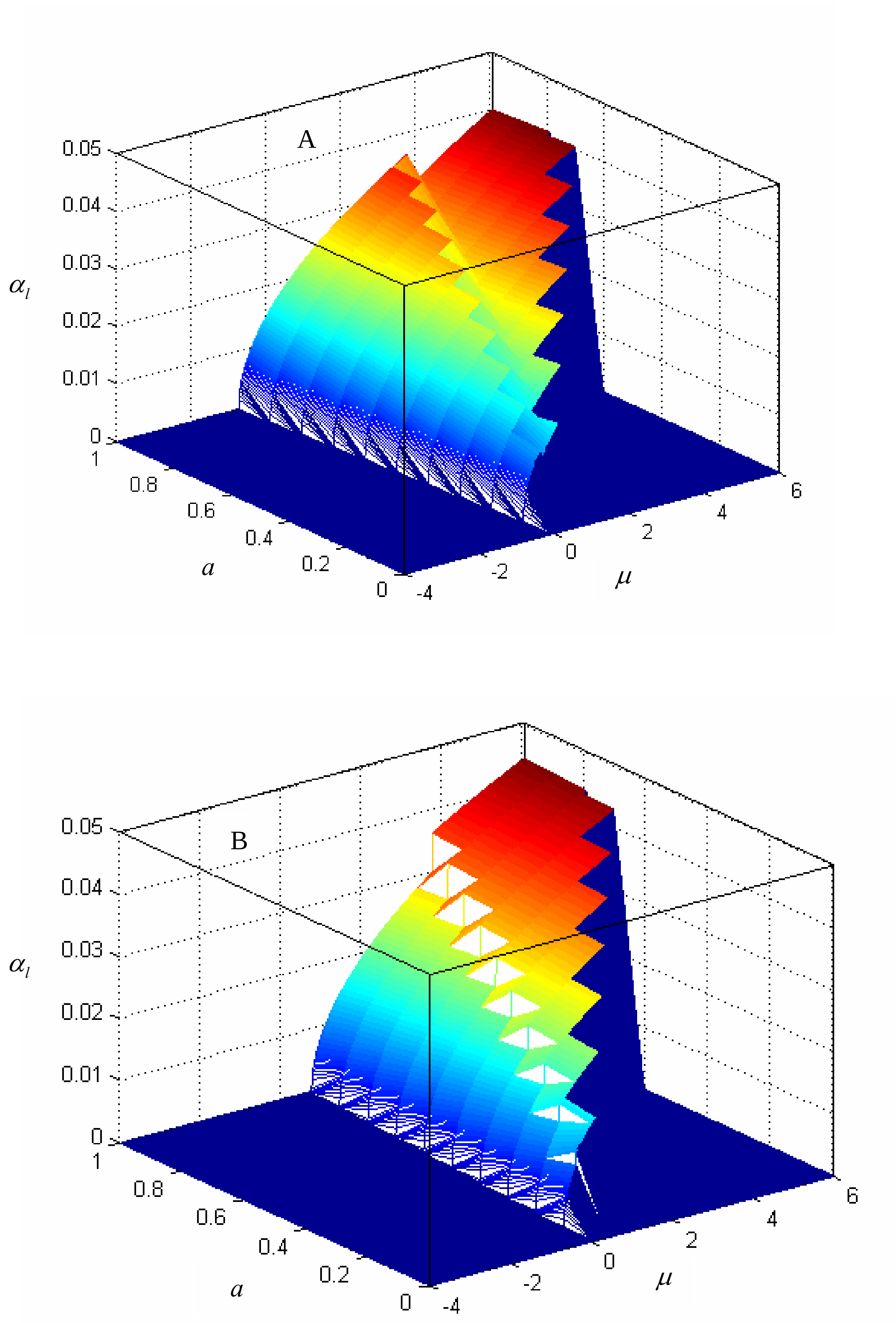


Figure 5. The response amplitude of nontrivial limit cycles for the summation parameter resonance of a viscoelastic moving belt (*γ=0.2, n=1, l=2, E_e=400, E_v=5*)
A: the first limit cycle. B: the second limit cycle

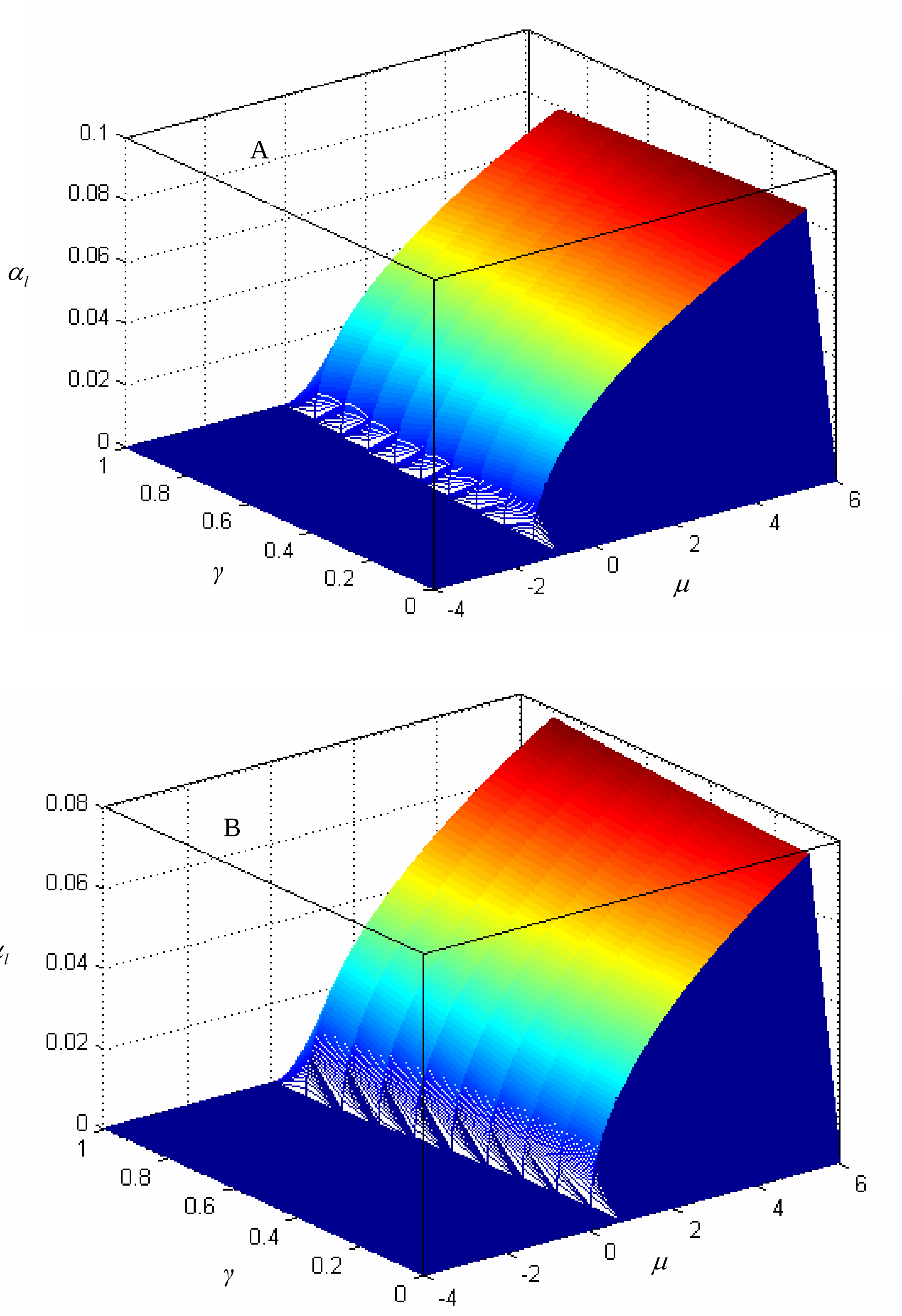


Figure 6. Effect of the transport speed on nontrivial limit cycles for the first principle parameter resonance (*a=0.5, n=l=1, E_e=400, E_v=5*)
A: the first limit cycle. B: the second limit cycle

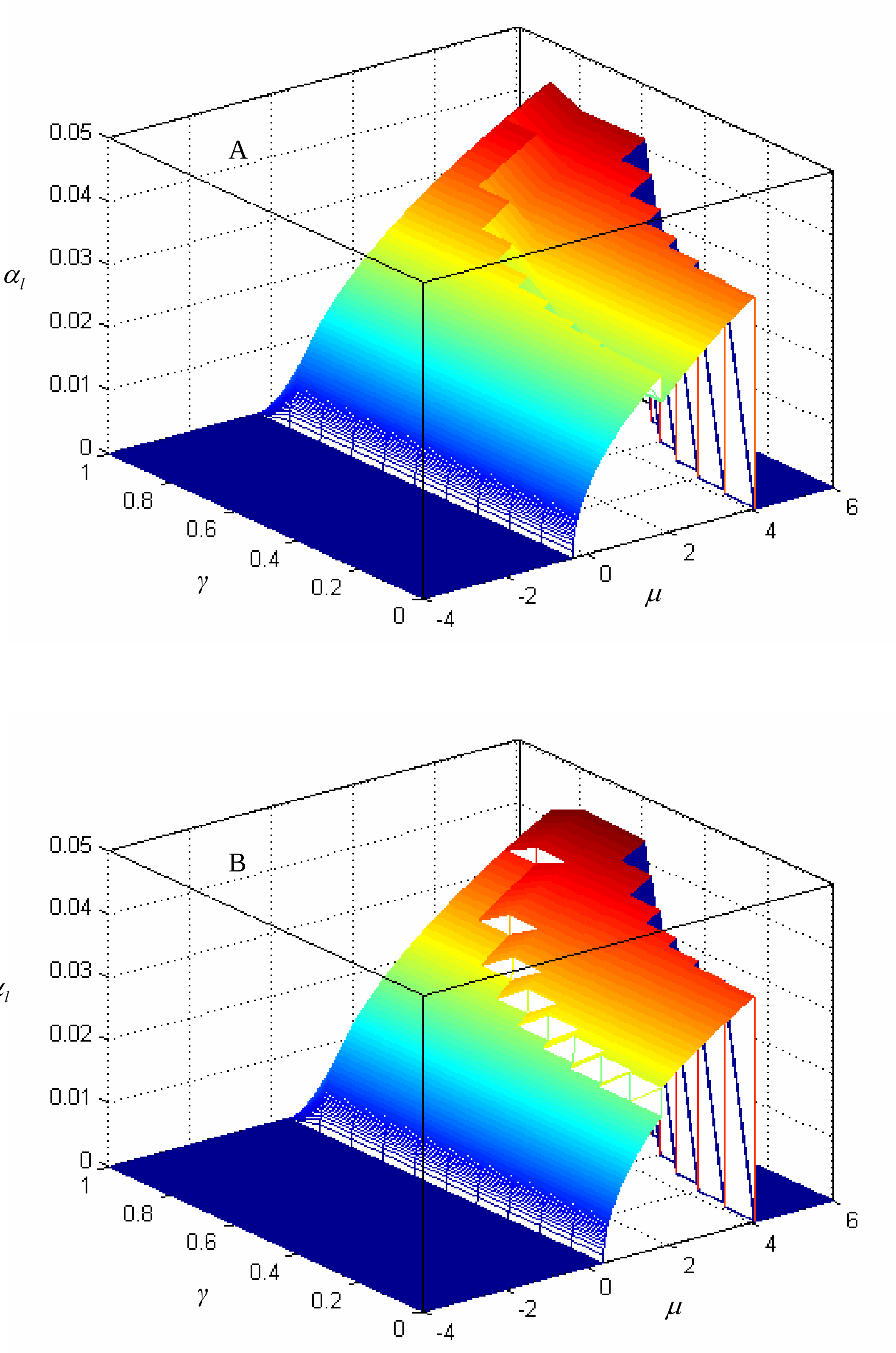


Figure 7. Effect of the transport speed on nontrivial limit cycles for the first summation parameter resonance (*a=0.5, n=1, l=2, E_e=400, E_v=5*)
A: the first limit cycle. B: the second limit cycle

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NOMENCLATURE

Symbol	Definition	Units
a	Non-dimensional amplitude of perturbation tension	-
A	Cross-sectional area of belt	m^2
c	Axial velocity of belt	m/s
c_0	constant	-
c_1	constant	-
C	constant	-
E	Non-dimensional equivalent Young's modulus	-
E^*	Equivalent Young's modulus	N/m^2
E_0	Initial Young's modulus	N/m^2
E_e	Non-dimensional Young's modulus	-
E_v	Non-dimensional viscoelastic parameter	-
G	Non-dimensional gyroscopic operator	-
K	Non-dimensional stiffness operator	-
L	Length of moving belts	m
$m_{\ln}$, m_{nl}	Non-dimensional parameter	-
M	Non-dimensional mass operator of moving belts	-
N	Nonlinear terms	-
t	time of moving belt	sec
T	Initial tension of moving belts	N
T0	Steady state tension	N
T1	Perturbation tension	N
v	Non-dimensional transverse displacement of moving belts	-
V	Transverse displacement of moving belt	m
x	Local coordinate in longitudinal direction	-
α_n , α_m	Response amplitude of the nth and mth mode	-
ε	Non-dimensional small parameter	-
ϕ_i	The ith eigenfunction of moving belts	-
θ	constant	-
γ	Non-dimensional translating speed	-
ρ	Belt mass per unit volume	Kg/m^3
η	Dynamic viscosity of the dashpot	-
λ_n	Eigenvalue of mode n	-
μ	Non-dimensional detuning parameter	-
τ	Non-dimensional time	-
ω	Excitation frequency	Rad/sec
ω_n	Natural frequency for mode n	Rad/sec
Ω	Excitation frequency	Rad/sec
ζ	Non-dimensional coordinate in longitudinal direction	-