Designing A Computer Program to Determine the Points and Planes in 3-Dimensional Projective Space

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Abstract

The purpose of this work is to determine the points and planes of 3-dimensional projective space PG(3,2) over Galois field GF(q), q=2,3 and 5 by designing a computer program.

Introduction

The study of finite projective spaces was at one time no more than an adjunct to algebraic geometry over the real and complex numbers. But , more recently, finite spaces were studied both for their application to practical topics such as coding theory and design experiments, and for their illumination of more abstract mathematical topics such as finite group theory and graph theory.

Perhaps the fastest growing area of modern mathematics is combinatories that is concerned with the study of arrangement of elements into sets. These elements are usually finite in number, and the arrangement is restricted by certain boundary conditions imposed by the particular problem under investigation.

Much of the growth of combinatory has gone hand in hand with the development of the computer. A major reason for this rapid growth of combinatories is its wealth of application, to computer science, communications, transportations, genetics, experimental design, and so on.

Many of the researchers worked to determine the pointes and lines in 2-dimensional projective planes by designing computer programs.

In this work, a computer program is designed to determine the points and planes in 3-dimensional projective spaces over Galois field GF (q), q=2, 3, 5.

Galois field

Definition (1)

Let κ be a finite set, κ has P elements {0, 1… p-1} where P is a prime number.
Define addition in κ by a +b =c if c is the remainder of a +b on division by p, i.e.
a+b=c if c is a +b reduced modulo p, or, a +b = c mod p.

Similarly, multiplication in κ is defined by ab=c if c is the remainder of ab on dividing by p, or , ab = c (mod p).

Then κ with the two operations, addition and multiplication, is defined above as a field called Galois field with characteristic p and is denoted by GF (p).

Thus GF (p) = {0, 1… p-1| p=0},
For GF (2) = {0, 1 | 2=0},
GF (3) = {0, 1, 2 | 3=0},
GF (5) = {0, 1,2,3,4 |5=0}

Projective 3-spaces

Definition (2,3)

A projective 3-space PG (3, q) over Galois field is a 3-dimensional projective space which consists of points, lines and planes with the incidence relation between them.

Any point in PG(3,q) has the form of a quadruple (x1,x2,x3,x4), where x1,x2,x3,x4 are elements in GF(q) with the exception of the quadruple consisting of four zero elements.

Two quadruples (x1,x2,x3,x4) and (y1,y2,y3,y4) represent the same point if there exists λ in GF(q), $\lambda \neq 0$ such that (x1,x2,x3,x4) = λ(y1,y2,y3,y4)

Similarly , any plane in PG(3,q) has the form of quadruple [x1,x2,x3,x4], where x1,x2,x3,x4 are elements in GF(q) with the exception of the quadruple consisting of four zero elements.

Two quadruples [x1,x2,x3,x4] and [y1,y2,y3,y4] represent the same plane if there exists λ in GF(q), $\lambda \neq 0$,such that :

[x1,x2,x3,x4] = λ [y1,y2,y3,y4] .

Also a point p(x1,x2,x3,x4) is incident with the plane π [a1, a2, a3, a4] if

a1x1 + a2x2 + a3x3 + a4x4 =0.

Program parts

procedure makepoints:

This procedure treats generating of points for modes 2,3 and 5. The reading of points or planes is difficult for the user, because the large number of inputs (four in puts in each point and there are 40 or 156 points in modes 3 and 5 respectively), but we can generate these points in programming by using counters increasing in some way.

The first step to generate the points by compute the number of points by the equation

Pono= $1+mo+mo^2+mo^3$

Where

pono is the maximum number of points

mo is the number of mode 3 or 5 .

then the result of equation if mode=3 is

pono=1+3+9+27=40

and if mode =5 the result of equation will be

pono=1+5+25+125=156.

There are some special points ,they are

Point 1 (1 0 0 0)

Points 2→mo+1, are generated in one for astatement

Points mo+2→ pono , are generated by some equations.

procedure mainwork

This procedure contains the execution of the main equation to find the Planes on each point

X1Y1+X2Y2+X3Y3+X4Y4 =0

If the result is 0 then the number of this Plane will be added to the array of Planes.

The number of Planes on each point in both modes should be equal to the result of equation $1+mo+mo^2$, if mo=3, then number of planes will be 1+3+9=13.and if mo=5 , then number of planes will be 1+5+25=31.

Main program

The main program consists of calling the two procedures by inputting the number of mode by the user, the first call to procedure Makepoint and the second call to Mainwork .

The last part of program is output results. The result consists of two tables:-

1- The first table contains the points and Planes of PG(3, 3).
2- The second table contains points and Planes of PG(3, 5).

The program language (4)

The language in which the program is executed is Pascal; it had become most widely used for scientific purposes. It's designed for teaching programming and other applications and this as based primarily on its remarkable combination of simplicity and expressivity.

Suggestion about the program

We can improve the way of generating points and planes and finding the planes on each line by many ways in programming such as:

1- Using files for saving points and Planes instead of arrays.

2-Using Matlab programming instead of Pascal language .

The list of program

```
program modulo(input,output);
uses wincrt;
type arr1=array[1..156,1..4] of integer;
    arr2=array[1..156,1..32] of integer;
var points:arr1;
   lines:arr2;
   i,j,mo,pono,m:integer;
procedure makepoint(var points:arr1;mo:integer;var pono:integer);
var i,x,y,z,a:integer;
   xx:array[1..7,1..4]of integer;
begin
pono:=1+mo+SQR(mo)+SQR(mo)*mo;
  points[1,1]:=1;for i:=2 to 4 do points[1,i]:=0;
  for i:=2 to mo+1do
  begin
  points[i,1]:=i-2;points[i,2]:=1;points[i,3]:=0;points[i,4]:=0;end;
  a:=mo+2;
  y:=0;z:=1;
  for i:=a to pono do
  begin
   x:=(i mod mo)-2;
   if((x=-2) or (x=-1)) then x:=x+mo;
   points[i,1]:=x;
   if((x=0) and (i>(mo+2)))then y:=y+1;
   if((y mod mo)=0) then y:=0;
   points[i,2]:=y;
   if ((i>1+mo+sqr(mo))and (i<=1+2*sqr(mo))) then z:=0;
   if (i=(2+mo+(z+2)*sqr(mo))) then z:=z+1;
   points[i,3]:=z;
   if (i<=1+mo+sqr(mo))then points[i,4]:=0
                    else points[i,4]:=1;
  end;(* for i*)
end;(*procedure 1*)
procedure mainwork(var lines:arr2;points:arr1;pono:integer;var m:integer);
var y:array[1..4] of integer;
   i,j,k,sum,res:integer;
begin
```

```
 for i:=1 to pono do
```


```
begin
   m:=0;
   for j:=1 to pono do
   begin
   for k:=1 to 4 do
   y[k]:=points[j,k];
   sum:=0;
   for k:=1 to 4 do
   sum:=sum+points[i,k]*y[k];
   res:=sum mod mo;
   if res=0 then begin
               m:=m+1;
               lines[i,m]:=j;
              end;
   end;(*for j*)
   end;(*for i*)
end;(* procedure *)
begin(*main program*)
 write('enter the no. of mode please..?');
 readln(mo);
 makepoint(points,mo,pono);
 mainwork(lines,points,pono,m);
 case mo of
 3:writeln('                    Table (1)');
 5:writeln('                    Table (2)');
 end;
 writeln('              Points and planes of PG(3,',mo,')');
 writeln('-------------------------------------------------------------------');
 writeln(' i ',' Pi ','          PLi  ');
 writeln('-------------------------------------------------------------------');
 for i:=1 to pono do
 begin
  if (i<10) then write(' ',i,'   ')
  else if(i<100) then write(' ',i,'  ')
        else write(' ',i,' ');
 write('(');
 for j:=1 to 4 do
 if j<4 then write(points[i,j],',')
      else write(points[i,j],')');
 write('   ');
 if (m<=13) then for j:=1 to m do

            if(lines[i,j]<10)then   write(lines[i,j],'   ')
                        else write(lines[i,j],'  ')

         else begin
            for j:=1 to 16 do
            begin
```

```
          if(j<>16)  then
```



```
  if(lines[i,j]<10)  then write(lines[i,j],'    ')
          else if(lines[i,j]<100)  then write(lines[i,j],'   ')
                         else write(lines[i,j],'  ')
          else write(lines[i,j]);
         end;
         writeln;
        write('                ');
        for j:=17  to m do
        begin
        if(lines[i,j]<10)then  write(lines[i,j],'    ')
        else if(lines[i,j]<100)then  write(lines[i,j],'   ')
                  else write(lines[i,j],'  ');
        end;
        end;
writeln;
end;readln;
end.(*main program*)
```

References

1.Ismael,N.A.,(2005),"Complete (k,n)-arcs in the Projective Plane PG(2,13)",M.Sc.Thesis,College of Education Ibn Al-Haitham, University of Baghdad

2. Hirschefeld,J.W.P. (1998),"Projective Geometries Over Finite Fields",Second Edition ,Oxford University Press.

3. Al Mukhtar ,A.S.(2008)"Complete Arcs And Surfaces In Three Dimensional of Technology.

4. Robert, W. Sebesta (1993) "Concepts of Programming Languages" ,University of Colorado, Colarado Springs

Table(1) :Points and planes of PG(3,3)

I	Pi	πi
1	(1,0,0,0)	2 5 8 11 14 17 20 23 26 29 32 35 38
2	(0,1,0,0)	1 5 6 7 14 15 16 23 24 25 32 33 34
3	(1,1,0,0)	4 5 10 12 14 19 21 23 28 30 32 37 39
4	(2,1,0,0)	3 5 9 13 14 18 22 23 27 31 32 36 40
5	(0,0,1,0)	1 2 3 4 14 15 16 17 18 19 20 21 22
6	(1,0,1,0)	2 7 10 13 14 17 20 25 28 31 33 36 39
7	(2,0,1,0)	2 6 9 12 14 17 20 24 27 30 34 37 40
8	(0,1,1,0)	1 11 12 13 14 15 16 29 30 31 35 36 37
9	(1,1,1,0)	4 7 9 11 14 19 21 25 27 29 34 35 40
10	(2,1,1,0)	3 6 10 11 14 18 22 24 28 29 34 35 39
11	(0,2,1,0)	1 8 9 10 14 15 16 26 27 28 38 39 40
12	(1,2,1,0)	3 7 8 12 14 18 22 25 26 30 33 37 38
13	(2,2,1,0)	4 6 8 13 14 19 21 24 26 31 34 36 38
14	(0,0,0,1)	1 2 3 4 5 6 7 8 9 10 11 12 13
15	(1,0,0,1)	2 5 8 11 16 19 22 25 28 31 34 37 40
16	(2,0,0,1)	2 5 8 11 15 18 21 24 27 30 33 36 39
17	(0,1,0,1)	1 5 6 7 20 21 22 29 30 31 38 39 40
18	(1,1,0,1)	4 5 10 12 16 18 20 25 27 29 34 36 38
19	(2,1,0,1)	3 5 9 13 15 19 20 24 28 29 33 37 38
20	(0,2,0,1)	1 5 6 7 17 18 19 26 27 28 35 36 37
21	(1,2,0,1)	3 5 9 13 16 17 21 25 26 30 34 35 39
22	(2,2,0,1)	4 5 10 12 15 17 22 24 26 31 33 35 40

23	(0,0,1,1)	1 2 3 4 32 33 34 35 36 37 38 39 40
24	(1,0,1,1)	2 7 10 13 16 19 22 24 27 30 32 35 38
25	(2,0,1,1)	2 6 9 12 15 18 21 25 28 31 32 35 38
26	(0,1,1,1)	1 11 12 13 20 21 22 26 27 28 32 33 34
27	(1,1,1,1)	4 7 9 11 16 18 20 24 26 31 32 37 39
28	(2,1,1,1)	3 6 10 11 15 19 20 25 26 30 32 36 40
29	(0,2,1,1)	1 8 9 10 17 18 19 29 30 31 32 33 34
30	(1,2,1,1)	3 7 8 12 16 17 21 24 28 29 32 36 40
31	(2,2,1,1)	4 6 8 13 15 17 22 25 27 29 32 37 39
32	(0,0,2,1)	1 2 3 4 23 24 25 26 27 28 29 30 31
33	(1,0,2,1)	2 6 9 12 16 19 22 23 26 29 33 36 39
34	(2,0,2,1)	2 7 10 13 15 18 21 23 26 29 34 37 40
35	(0,1,2,1)	1 8 9 10 20 21 22 23 24 25 35 36 37
36	(1,1,2,1)	4 6 8 13 16 18 20 23 28 30 33 35 40
37	(2,1,2,1)	3 7 8 12 15 19 20 23 27 31 34 35 39
38	(0,2,2,1)	1 11 12 13 17 18 19 23 24 25 38 39 40
39	(1,2,2,1)	3 6 10 11 16 17 21 23 27 31 33 37 38
40	(2,2,2,1)	4 7 9 11 15 17 22 23 28 30 34 36 38

Table(2) : Points and planes of PG(3,5)

i	Pi	PLi
1	(1,0,0,0)	2 7 12 17 22 27 32 37 42 47 52 57 62 67 72 77 82 87 92 97 102 107 112 117 122 127 132 137 142 147 152
2	(0,1,0,0)	1 7 8 9 10 11 32 33 34 35 36 57 58 59 60 61 82 83 84 85 86 107 108 109 110 111 132 133 134 135 136
3	(1,1,0,0)	6 7 16 20 24 28 32 41 45 49 53 57 66 70 74 78 82 91 95 99 103 107 116 120 124 128 132 141 145 149 153
4	(2,1,0,0)	4 7 14 21 23 30 32 39 46 48 55 57 64 71 73 80 82 89 96 98 105 107 114 121 123 130 132 139 146 148 155
5	(3,1,0,0)	5 7 15 18 26 29 32 40 43 51 54 57 65 68 76 79 82 90 93 101 104 107 115 118 126 129 132 140 143 151 154
6	(4,1,0,0)	3 7 13 19 25 31 32 38 44 50 56 57 63 69 75 81 82 88 94 100 106 107 113 119 125 131 132 138 144 150 156
7	(0,0,1,0)	1 2 3 4 5 6 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56
8	(1,0,1,0)	2 11 16 21 26 31 32 37 42 47 52 61 66 71 76 81 85 90 95 100 105 109 114 119 124 129 133 138 143 148 153
9	(2,0,1,0)	2 9 14 19 24 29 32 37 42 47 52 59 64 69 74 79 86 91 96 101 106 108 113 118 123 128 135 140 145 150 155
10	(3,0,1,0)	2 10 15 20 25 30 32 37 42 47 52 60 65 70 75 80 83 88 93 98 103 111 116 121 126 131 134 139 144 149 154
11	(4,0,1,0)	2 8 13 18 23 28 32 37 42 47 52 58 63 68 73 78 84 89 94 99 104 110 115 120 125 130 136 141 146 151 156
12	(0,1,1,0)	1 27 28 29 30 31 32 33 34 35 36 77 78 79 80 81 97 98 99 100 101 117 118 119 120 121 137 138 139 140 141
13	(1,1,1,0)	6 11 15 19 23 27 32 41 45 49 53 61 65 69 73 77 85 89 93 97 106 109 113 117 126 130 133 137 146 150 154
14	(2,1,1,0)	4 9 16 18 25 27 32 39 46 48 55 59 66 68 75 77 86 88 95 97 104 108 115 117 124 131 135 137 144 151 153
15	(3,1,1,0)	5 10 13 21 24 27 32 40 43 51 54 60 63 71 74 77 83 91 94 97 105 111 114 117 125 128 134 137 145 148 156
16	(4,1,1,0)	3 8 14 20 26 27 32 38 44 50 56 58 64 70 76 77 84 90 96 97 103 110 116 117 123 129 136 137 143 149 155

17	(0,2,1,0)	1 17 18 19 20 21 32 33 34 35 36 67 68 69 70 71 102 103 104 105 106 112 113 114 115 116 147 148 149 150 151
18	(1,2,1,0)	5 11 14 17 25 28 32 40 43 51 54 61 64 67 75 78 85 88 96 99 102 109 112 120 123 131 133 141 144 147 155
19	(2,2,1,0)	6 9 13 17 26 30 32 41 45 49 53 59 63 67 76 80 86 90 94 98 102 108 112 121 125 129 135 139 143 147 156
20	(3,2,1,0)	3 10 16 17 23 29 32 38 44 50 56 60 66 67 73 79 83 89 95 101 102 111 112 118 124 130 134 140 146 147 153
21	(4,2,1,0)	4 8 15 17 24 31 32 39 46 48 55 58 65 67 74 81 84 91 93 100 102 110 112 119 126 128 136 138 145 147 154
22	(0,3,1,0)	1 22 23 24 25 26 32 33 34 35 36 72 73 74 75 76 87 88 89 90 91 127 128 129 130 131 142 143 144 145 146
23	(1,3,1,0)	4 11 13 20 22 29 32 39 46 48 55 61 63 70 72 79 85 87 94 101 103 109 116 118 125 127 133 140 142 149 156
24	(2,3,1,0)	3 9 15 21 22 28 32 38 44 50 56 59 65 71 72 78 86 87 93 99 105 108 114 120 126 127 135 141 142 148 154
25	(3,3,1,0)	6 10 14 18 22 31 32 41 45 49 53 60 64 68 72 81 83 87 96 100 104 111 115 119 123 127 134 138 142 151 155
26	(4,3,1,0)	5 8 16 19 22 30 32 40 43 51 54 58 66 69 72 80 84 87 95 98 106 110 113 121 124 127 136 139 142 150 153
27	(0,4,1,0)	1 12 13 14 15 16 32 33 34 35 36 62 63 64 65 66 92 93 94 95 96 122 123 124 125 126 152 153 154 155 156
28	(1,4,1,0)	3 11 12 18 24 30 32 38 44 50 56 61 62 68 74 80 85 91 92 98 104 109 115 121 122 128 133 139 145 151 152
29	(2,4,1,0)	5 9 12 20 23 31 32 40 43 51 54 59 62 70 73 81 86 89 92 100 103 108 116 119 122 130 135 138 146 149 152
30	(3,4,1,0)	4 10 12 19 26 28 32 39 46 48 55 60 62 69 76 78 83 90 92 99 106 111 113 120 122 129 134 141 143 150 152
31	(4,4,1,0)	6 8 12 21 25 29 32 41 45 49 53 58 62 71 75 79 84 88 92 101 105 110 114 118 122 131 136 140 144 148 152
32	(0,0,0,1)	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31
33	(1,0,0,1)	2 7 12 17 22 27 36 41 46 51 56 61 66 71 76 81 86 91 96 101 106 111 116 121 126 131 136 141 146 151 156
34	(2,0,0,1)	2 7 12 17 22 27 34 39 44 49 54 59 64 69 74 79 84 89 94 99 104 109 114 119 124 129 134 139 144 149 154

35	(3,0,0,1)	2 7 12 17 22 27 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155
36	(4,0,0,1)	2 7 12 17 22 27 33 38 43 48 53 58 63 68 73 78 83 88 93 98 103 108 113 118 123 128 133 138 143 148 153
37	(0,1,0,1)	1 7 8 9 10 11 52 53 54 55 56 77 78 79 80 81 102 103 104 105 106 127 128 129 130 131 152 153 154 155 156
38	(1,1,0,1)	6 7 16 20 24 28 36 40 44 48 52 61 65 69 73 77 86 90 94 98 102 111 115 119 123 127 136 140 144 148 152
39	(2,1,0,1)	4 7 14 21 23 30 34 41 43 50 52 59 66 68 75 77 84 91 93 100 102 109 116 118 125 127 134 141 143 150 152
40	(3,1,0,1)	5 7 15 18 26 29 35 38 46 49 52 60 63 71 74 77 85 88 96 99 102 110 113 121 124 127 135 138 146 149 152
41	(4,1,0,1)	3 7 13 19 25 31 33 39 45 51 52 58 64 70 76 77 83 89 95 101 102 108 114 120 126 127 133 139 145 151 152
42	(0,2,0,1)	1 7 8 9 10 11 42 43 44 45 46 67 68 69 70 71 92 93 94 95 96 117 118 119 120 121 142 143 144 145 146
43	(1,2,0,1)	5 7 15 18 26 29 36 39 42 50 53 61 64 67 75 78 86 89 92 100 103 111 114 117 125 128 136 139 142 150 153
44	(2,2,0,1)	6 7 16 20 24 28 34 38 42 51 55 59 63 67 76 80 84 88 92 101 105 109 113 117 126 130 134 138 142 151 155
45	(3,2,0,1)	3 7 13 19 25 31 35 41 42 48 54 60 66 67 73 79 85 91 92 98 104 110 116 117 123 129 135 141 142 148 154
46	(4,2,0,1)	4 7 14 21 23 30 33 40 42 49 56 58 65 67 74 81 83 90 92 99 106 108 115 117 124 131 133 140 142 149 156
47	(0,3,0,1)	1 7 8 9 10 11 47 48 49 50 51 72 73 74 75 76 97 98 99 100 101 122 123 124 125 126 147 148 149 150 151
48	(1,3,0,1)	4 7 14 21 23 30 36 38 45 47 54 61 63 70 72 79 86 88 95 97 104 111 113 120 122 129 136 138 145 147 154
49	(2,3,0,1)	3 7 13 19 25 31 34 40 46 47 53 59 65 71 72 78 84 90 96 97 103 109 115 121 122 128 134 140 146 147 153
50	(3,3,0,1)	6 7 16 20 24 28 35 39 43 47 56 60 64 68 72 81 85 89 93 97 106 110 114 118 122 131 135 139 143 147 156
51	(4,3,0,1)	5 7 15 18 26 29 33 41 44 47 55 58 66 69 72 80 83 91 94 97 105 108 116 119 122 130 133 141 144 147 155
52	(0,4,0,1)	1 7 8 9 10 11 37 38 39 40 41 62 63 64 65 66 87 88 89 90 91 112 113 114 115 116 137 138 139 140 141
53	(1,4,0,1)	3 7 13 19 25 31 36 37 43 49 55 61 62 68

		74 80 86 87 93 99 105 111 112 118 124 130 136 137 143 149 155
54	(2,4,0,1)	5 7 15 18 26 29 34 37 45 48 56 59 62 70 73 81 84 87 95 98 106 109 112 120 123 131 134 137 145 148 156
55	(3,4,0,1)	4 7 14 21 23 30 35 37 44 51 53 60 62 69 76 78 85 87 94 101 103 110 112 119 126 128 135 137 144 151 153
56	(4,4,0,1)	6 7 16 20 24 28 33 37 46 50 54 58 62 71 75 79 83 87 96 100 104 108 112 121 125 129 133 137 146 150 154
57	(0,0,1,1)	1 2 3 4 5 6 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156
58	(1,0,1,1)	2 11 16 21 26 31 36 41 46 51 56 60 65 70 75 80 84 89 94 99 104 108 113 118 123 128 132 137 142 147 152
59	(2,0,1,1)	2 9 14 19 24 29 34 39 44 49 54 61 66 71 76 81 83 88 93 98 103 110 115 120 125 130 132 137 142 147 152
60	(3,0,1,1)	2 10 15 20 25 30 35 40 45 50 55 58 63 68 73 78 86 91 96 101 106 109 114 119 124 129 132 137 142 147 152
61	(4,0,1,1)	2 8 13 18 23 28 33 38 43 48 53 59 64 69 74 79 85 90 95 100 105 111 116 121 126 131 132 137 142 147 152
62	(0,1,1,1)	1 27 28 29 30 31 52 53 54 55 56 72 73 74 75 76 92 93 94 95 96 112 113 114 115 116 132 133 134 135 136
63	(1,1,1,1)	6 11 15 19 23 27 36 40 44 48 52 60 64 68 72 81 84 88 92 101 105 108 112 121 125 129 132 141 145 149 153
64	(2,1,1,1)	4 9 16 18 25 27 34 41 43 50 52 61 63 70 72 79 83 90 92 99 106 110 112 119 126 128 132 139 146 148 155
65	(3,1,1,1)	5 10 13 21 24 27 35 38 46 49 52 58 66 69 72 80 86 89 92 100 103 109 112 120 123 131 132 140 143 151 154
66	(4,1,1,1)	3 8 14 20 26 27 33 39 45 51 52 59 65 71 72 78 85 91 92 98 104 111 112 118 124 130 132 138 144 150 156
67	(0,2,1,1)	1 17 18 19 20 21 42 43 44 45 46 77 78 79 80 81 87 88 89 90 91 122 123 124 125 126 132 133 134 135 136
68	(1,2,1,1)	5 11 14 17 25 28 36 39 42 50 53 60 63 71 74 77 84 87 95 98 106 108 116 119 122 130 132 140 143 151 154
69	(2,2,1,1)	6 9 13 17 26 30 34 38 42 51 55 61 65 69 73 77 83 87 96 100 104 110 114 118 122 131 132 141 145 149 153
70	(3,2,1,1)	3 10 16 17 23 29 35 41 42 48 54 58 64 70 76 77 86 87 93 99 105 109 115 121 122 128 132 138 144 150 156
71	(4,2,1,1)	4 8 15 17 24 31 33 40 42 49 56 59 66 68

		75 77 85 87 94 101 103 111 113 120 122 129 132 139 146 148 155
72	(0,3,1,1)	1 22 23 24 25 26 47 48 49 50 51 62 63 64 65 66 102 103 104 105 106 117 118 119 120 121 132 133 134 135 136
73	(1,3,1,1)	4 11 13 20 22 29 36 38 45 47 54 60 62 69 76 78 84 91 93 100 102 108 115 117 124 131 132 139 146 148 155
74	(2,3,1,1)	3 9 15 21 22 28 34 40 46 47 53 61 62 68 74 80 83 89 95 101 102 110 116 117 123 129 132 138 144 150 156
75	(3,3,1,1)	6 10 14 18 22 31 35 39 43 47 56 58 62 71 75 79 86 90 94 98 102 109 113 117 126 130 132 141 145 149 153
76	(4,3,1,1)	5 8 16 19 22 30 33 41 44 47 55 59 62 70 73 81 85 88 96 99 102 111 114 117 125 128 132 140 143 151 154
77	(0,4,1,1)	1 12 13 14 15 16 37 38 39 40 41 67 68 69 70 71 97 98 99 100 101 127 128 129 130 131 132 133 134 135 136
78	(1,4,1,1)	3 11 12 18 24 30 36 37 43 49 55 60 66 67 73 79 84 90 96 97 103 108 114 120 126 127 132 138 144 150 156
79	(2,4,1,1)	5 9 12 20 23 31 34 37 45 48 56 61 64 67 75 78 83 91 94 97 105 110 113 121 124 127 132 140 143 151 154
80	(3,4,1,1)	4 10 12 19 26 28 35 37 44 51 53 58 65 67 74 81 86 88 95 97 104 109 116 118 125 127 132 139 146 148 155
81	(4,4,1,1)	6 8 12 21 25 29 33 37 46 50 54 59 63 67 76 80 85 89 93 97 106 111 115 119 123 127 132 141 145 149 153
82	(0,0,2,1)	1 2 3 4 5 6 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106
83	(1,0,2,1)	2 10 15 20 25 30 36 41 46 51 56 59 64 69 74 79 82 87 92 97 102 110 115 120 125 130 133 138 143 148 153
84	(2,0,2,1)	2 11 16 21 26 31 34 39 44 49 54 58 63 68 73 78 82 87 92 97 102 111 116 121 126 131 135 140 145 150 155
85	(3,0,2,1)	2 8 13 18 23 28 35 40 45 50 55 61 66 71 76 81 82 87 92 97 102 108 113 118 123 128 134 139 144 149 154
86	(4,0,2,1)	2 9 14 19 24 29 33 38 43 48 53 60 65 70 75 80 82 87 92 97 102 109 114 119 124 129 136 141 146 151 156
87	(0,1,2,1)	1 22 23 24 25 26 52 53 54 55 56 67 68 69 70 71 82 83 84 85 86 122 123 124 125 126 137 138 139 140 141
88	(1,1,2,1)	6 10 14 18 22 31 36 40 44 48 52 59 63 67 76 80 82 91 95 99 103 110 114 118 122 131 133 137 146 150 154
89	(2,1,2,1)	4 11 13 20 22 29 34 41 43 50 52 58 65 67 74 81 82 89 96 98 105 111 113 120 122 129 135 137

		144 151 153
90	(3,1,2,1)	5 8 16 19 22 30 35 38 46 49 52 61 64 67 75 78 82 90 93 101 104 108 116 119 122 130 134 137 145 148 156
91	(4,1,2,1)	3 9 15 21 22 28 33 39 45 51 52 60 66 67 73 79 82 88 94 100 106 109 115 121 122 128 136 137 143 149 155
92	(0,2,2,1)	1 27 28 29 30 31 42 43 44 45 46 62 63 64 65 66 82 83 84 85 86 127 128 129 130 131 147 148 149 150 151
93	(1,2,2,1)	5 10 13 21 24 27 36 39 42 50 53 59 62 70 73 81 82 90 93 101 104 110 113 121 124 127 133 141 144 147 155
94	(2,2,2,1)	6 11 15 19 23 27 34 38 42 51 55 58 62 71 75 79 82 91 95 99 103 111 115 119 123 127 135 139 143 147 156

95	(3,2,2,1)	3 8 14 20 26 27 35 41 42 48 54 61 62 68 74 80 82 88 94 100 106 108 114 120 126 127 134 140 146 147 153
96	(4,2,2,1)	4 9 16 18 25 27 33 40 42 49 56 60 62 69 76 78 82 89 96 98 105 109 116 118 125 127 136 138 145 147 154
97	(0,3,2,1)	1 12 13 14 15 16 47 48 49 50 51 77 78 79 80 81 82 83 84 85 86 112 113 114 115 116 142 143 144 145 146
98	(1,3,2,1)	4 10 12 19 26 28 36 38 45 47 54 59 66 68 75 77 82 89 96 98 105 110 112 119 126 128 133 140 142 149 156
99	(2,3,2,1)	3 11 12 18 24 30 34 40 46 47 53 58 64 70 76 77 82 88 94 100 106 111 112 118 124 130 135 141 142 148 154
100	(3,3,2,1)	6 8 12 21 25 29 35 39 43 47 56 61 65 69 73 77 82 91 95 99 103 108 112 121 125 129 134 138 142 151 155
101	(4,3,2,1)	5 9 12 20 23 31 33 41 44 47 55 60 63 71 74 77 82 90 93 101 104 109 112 120 123 131 136 139 142 150 153
102	(0,4,2,1)	1 17 18 19 20 21 37 38 39 40 41 72 73 74 75 76 82 83 84 85 86 117 118 119 120 121 152 153 154 155 156
103	(1,4,2,1)	3 10 16 17 23 29 36 37 43 49 55 59 65 71 72 78 82 88 94 100 106 110 116 117 123 129 133 139 145 151 152
104	(2,4,2,1)	5 11 14 17 25 28 34 37 45 48 56 58 66 69 72 80 82 90 93 101 104 111 114 117 125 128 135 138 146 149 152
105	(3,4,2,1)	4 8 15 17 24 31 35 37 44 51 53 61 63 70 72 79 82 89 96 98 105 108 115 117 124 131 134 141 143 150 152
106	(4,4,2,1)	6 9 13 17 26 30 33 37 46 50 54 60 64 68 72 81 82 91 95 99 103 109 113 117 126 130 136 140 144 148 152

107	(0,0,3,1)	1 2 3 4 5 6 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131
108	(1,0,3,1)	2 9 14 19 24 29 36 41 46 51 56 58 63 68 73 78 85 90 95 100 105 107 112 117 122 127 134 139 144 149 154
109	(2,0,3,1)	2 8 13 18 23 28 34 39 44 49 54 60 65 70 75 80 86 91 96 101 106 107 112 117 122 127 133 138 143 148 153
110	(3,0,3,1)	2 11 16 21 26 31 35 40 45 50 55 59 64 69 74 79 83 88 93 98 103 107 112 117 122 127 136 141 146 151 156
111	(4,0,3,1)	2 10 15 20 25 30 33 38 43 48 53 61 66 71 76 81 84 89 94 99 104 107 112 117 122 127 135 140 145 150 155
112	(0,1,3,1)	1 17 18 19 20 21 52 53 54 55 56 62 63 64 65 66 97 98 99 100 101 107 108 109 110 111 142 143 144 145 146
113	(1,1,3,1)	6 9 13 17 26 30 36 40 44 48 52 58 62 71 75 79 85 89 93 97 106 107 116 120 124 128 134 138 142 151 155
114	(2,1,3,1)	4 8 15 17 24 31 34 41 43 50 52 60 62 69 76 78 86 88 95 97 104 107 114 121 123 130 133 140 142 149 156
115	(3,1,3,1)	5 11 14 17 25 28 35 38 46 49 52 59 62 70 73 81 83 91 94 97 105 107 115 118 126 129 136 139 142 150 153
116	(4,1,3,1)	3 10 16 17 23 29 33 39 45 51 52 61 62 68 74 80 84 90 96 97 103 107 113 119 125 131 135 141 142 148 154
117	(0,2,3,1)	1 12 13 14 15 16 42 43 44 45 46 72 73 74 75 76 102 103 104 105 106 107 108 109 110 111 137 138 139 140 141
118	(1,2,3,1)	5 9 12 20 23 31 36 39 42 50 53 58 66 69 72 80 85 88 96 99 102 107 115 118 126 129 134 137 145 148 156
119	(2,2,3,1)	6 8 12 21 25 29 34 38 42 51 55 60 64 68 72 81 86 90 94 98 102 107 116 120 124 128 133 137 146 150 154
120	(3,2,3,1)	3 11 12 18 24 30 35 41 42 48 54 59 65 71 72 78 83 89 95 101 102 107 113 119 125 131 136 137 143 149 155
121	(4,2,3,1)	4 10 12 19 26 28 33 40 42 49 56 61 63 70 72 79 84 91 93 100 102 107 114 121 123 130 135 137 144 151 153
122	(0,3,3,1)	1 27 28 29 30 31 47 48 49 50 51 67 68 69 70 71 87 88 89 90 91 107 108 109 110 111 152 153 154 155 156
123	(1,3,3,1)	4 9 16 18 25 27 36 38 45 47 54 58 65 67 74 81 85 87 94 101 103 107 114 121 123 130 134 141 143 150 152
124	(2,3,3,1)	3 8 14 20 26 27 34 40 46 47 53 60 66 67 73 79 86 87 93 99 105 107 113 119 125 131 133 139 145 151 152

125	(3,3,3,1)	6 11 15 19 23 27 35 39 43 47 56 59 63 67 76 80 83 87 96 100 104 107 116 120 124 128 136 140 144 148 152
126	(4,3,3,1)	5 10 13 21 24 27 33 41 44 47 55 61 64 67 75 78 84 87 95 98 106 107 115 118 126 129 135 138 146 149 152
127	(0,4,3,1)	1 22 23 24 25 26 37 38 39 40 41 77 78 79 80 81 92 93 94 95 96 107 108 109 110 111 147 148 149 150 151
128	(1,4,3,1)	3 9 15 21 22 28 36 37 43 49 55 58 64 70 76 77 85 91 92 98 104 107 113 119 125 131 134 140 146 147 153
129	(2,4,3,1)	5 8 16 19 22 30 34 37 45 48 56 60 63 71 74 77 86 89 92 100 103 107 115 118 126 129 133 141 144 147 155
130	(3,4,3,1)	4 11 13 20 22 29 35 37 44 51 53 59 66 68 75 77 83 90 92 99 106 107 114 121 123 130 136 138 145 147 154
131	(4,4,3,1)	6 10 14 18 22 31 33 37 46 50 54 61 65 69 73 77 84 88 92 101 105 107 116 120 124 128 135 139 143 147 156
132	(0,0,4,1)	1 2 3 4 5 6 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81
133	(1,0,4,1)	2 8 13 18 23 28 36 41 46 51 56 57 62 67 72 77 83 88 93 98 103 109 114 119 124 129 135 140 145 150 155
134	(2,0,4,1)	2 10 15 20 25 30 34 39 44 49 54 57 62 67 72 77 85 90 95 100 105 108 113 118 123 128 136 141 146 151 156
135	(3,0,4,1)	2 9 14 19 24 29 35 40 45 50 55 57 62 67 72 77 84 89 94 99 104 111 116 121 126 131 133 138 143 148 153
136	(4,0,4,1)	2 11 16 21 26 31 33 38 43 48 53 57 62 67 72 77 86 91 96 101 106 110 115 120 125 130 134 139 144 149 154
137	(0,1,4,1)	1 12 13 14 15 16 52 53 54 55 56 57 58 59 60 61 87 88 89 90 91 117 118 119 120 121 147 148 149 150 151
138	(1,1,4,1)	6 8 12 21 25 29 36 40 44 48 52 57 66 70 74 78 83 87 96 100 104 109 113 117 126 130 135 139 143 147 156
139	(2,1,4,1)	4 10 12 19 26 28 34 41 43 50 52 57 64 71 73 80 85 87 94 101 103 108 115 117 124 131 136 138 145 147 154
140	(3,1,4,1)	5 9 12 20 23 31 35 38 46 49 52 57 65 68 76 79 84 87 95 98 106 111 114 117 125 128 133 141 144 147 155
141	(4,1,4,1)	3 11 12 18 24 30 33 39 45 51 52 57 63 69 75 81 86 87 93 99 105 110 116 117 123 129 134 140 146 147 153
142	(0,2,4,1)	1 22 23 24 25 26 42 43 44 45 46 57 58 59 60 61 97 98 99 100 101 112 113 114 115 116 152 153

		154 155 156
143	(1,2,4,1)	5 8 16 19 22 30 36 39 42 50 53 57 65 68 76 79 83 91 94 97 105 109 112 120 123 131 135 138 146 149 152
144	(2,2,4,1)	6 10 14 18 22 31 34 38 42 51 55 57 66 70 74 78 85 89 93 97 106 108 112 121 125 129 136 140 144 148 152
145	(3,2,4,1)	3 9 15 21 22 28 35 41 42 48 54 57 63 69 75 81 84 90 96 97 103 111 112 118 124 130 133 139 145 151 152
146	(4,2,4,1)	4 11 13 20 22 29 33 40 42 49 56 57 64 71 73 80 86 88 95 97 104 110 112 119 126 128 134 141 143 150 152
147	(0,3,4,1)	1 17 18 19 20 21 47 48 49 50 51 57 58 59 60 61 92 93 94 95 96 127 128 129 130 131 137 138 139 140 141
148	(1,3,4,1)	4 8 15 17 24 31 36 38 45 47 54 57 64 71 73 80 83 90 92 99 106 109 116 118 125 127 135 137 144 151 153
149	(2,3,4,1)	3 10 16 17 23 29 34 40 46 47 53 57 63 69 75 81 85 91 92 98 104 108 114 120 126 127 136 137 143 149 155
150	(3,3,4,1)	6 9 13 17 26 30 35 39 43 47 56 57 66 70 74 78 84 88 92 101 105 111 115 119 123 127 133 137 146 150 154
151	(4,3,4,1)	5 11 14 17 25 28 33 41 44 47 55 57 65 68 76 79 86 89 92 100 103 110 113 121 124 127 134 137 145 148 156
152	(0,4,4,1)	1 27 28 29 30 31 37 38 39 40 41 57 58 59 60 61 102 103 104 105 106 122 123 124 125 126 142 143 144 145 146
153	(1,4,4,1)	3 8 14 20 26 27 36 37 43 49 55 57 63 69 75 81 83 89 95 101 102 109 115 121 122 128 135 141 142 148 154
154	(2,4,4,1)	5 10 13 21 24 27 34 37 45 48 56 57 65 68 76 79 85 88 96 99 102 108 116 119 122 130 136 139 142 150 153
155	(3,4,4,1)	4 9 16 18 25 27 35 37 44 51 53 57 64 71 73 80 84 91 93 100 102 111 113 120 122 129 133 140 142 149 156
156	(4,4,4,1)	6 11 15 19 23 27 33 37 46 50 54 57 66 70 74 78 86 90 94 98 102 110 114 118 122 131 134 138 142 151 155

تصميم برنامج حاسوبي لتعيين النقاط والمستويات في الفضاء الاسقاطي الثلاثي الابعاد

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الخلاصة

الغرض من هذا العمل هو تصميم برنامج حاسوبي لتعيين النقاط والمستويات لفضاء اسقاطي ذي ثلاثة ابعاد PG(3,q) في حقل كالو GF(q), q=2,3 and 5