Effect of Spatial Variation of Soil Permeability on Pressure Heads in Zoned Dam

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Abstract

Two dimensional unsteady seepage through a zoned earth dam was investigated by the finite element based on Galerkin method. The coefficients of permeability are assumed to vary in terms of geometry, external load influences and the effect of head distribution in the flow domain and the resulted nonlinear seepage problem is solved. The effects of drawdown rates and various material parameters on pressure heads variation were discussed. The results show that the zoned dam permeability has important influence on pressure heads variation as the height of the dam increases. It is believed that a variable permeability analysis such as the one presented in this work should be taken into account.

الخلاصة :

تشمل الدراسة الحالية تحليل التسرب غير المستقر للمياه خلال جسم سد متعدد الطبقات إذ تم تطوير نموذج عددي يحكم ظاهرة التسرب خلال الطبقة المشبعة، واستخدمت طريقة العناصر المحددة لحل المعادلة الحاكمة. تستند الدراسة على محاولة معرفة تأثير التغاير المكاني اللاخطي لمعاملات التوصيل الهيدروليكي للتربة على توزيع شحنة الضغط داخل جسم السد، إذ تمت الدراسة بأخذ معدل هبوط سريع لماء بحيرة السد بالإضافة إلى تغيير في خصائص التربة المكونة لجسم السد. أظهرت النتائج أن لمعاملات التوصيل الهيدروليكي تأثيرا مهما في تغاير قيم شحنة الضغط وان التغير اللاخطي للموصلية الهيدروليكية يؤثر بشكل واضح مع زيادة ارتفاع السد.

1- INTRODUCTION

The flow of water through soil is one of the fundamental concepts in geotechnical and geo-environmental engineering. Flow quantity is often considered to be the key parameter in quantifying seepage losses from a reservoir. In engineering, the more important issue is the pore-water pressure. The importance should not be on how much water is flowing through the ground, but on the state of the pore-water pressure in the ground. The pore-water pressure, whether positive or negative, has a direct effect on the shear strength and volume change characteristics of the soil. Many researches have shown that even the flow of moisture in the unsaturated soil near the ground surface is directly related to the soil suction (negative water pressure). So, even when flow quantities are the main interest, it is important to accurately establish the pore-water pressures (Freeze, and Cherry, 1979).

The most common type of a rolled earth dam section is that in which a central impervious core is located by zones of materials considerably more pervious, called shells. These pervious zones or shells enclose, support, and protect the impervious core; the upstream pervious zone products stability against rapid drawdown; and the downstream pervious zone acts as a drain to control seepage and lower the free surface. In many cases, a filter between the impervious zone and downstream shell and a drainage layer under the downstream shell are necessary. These filter-drainage layers must meet filter criteria with adjacent fill and foundation materials. They are sometimes multilayered for capacity requirements(USBR,2007).

In flow problems, both the magnitude and direction of governing fluid flows are highly sensitive to the coefficient of soil permeability. For simplicity, this parameter is usually assumed to be a constant in space and time. In the present work, the coefficient of permeability is assumed to be spatially variable. This variation was defined for drawdown condition, and then the resulted governing differential equation was solved.

2- Governing equations

The governing differential equation for two dimensional flow, both saturated and unsaturated is (Bear , and Verruijt 1990):

$$\frac{\partial}{\partial x}(K_x \frac{\partial h}{\partial x}) + \frac{\partial}{\partial y}(K_y \frac{\partial h}{\partial y}) = S_s \frac{\partial h}{\partial t} - q \qquad (1)$$

In which K_x is the coefficient of permeability in x-direction, K_y is the coefficient of permeability in y-direction, h is the pressure head at any point (x,y) in flow region, S_s is the elastic specific storage, q is the replenishment rate per unit area (assumed zero in present work), and t is the time.

Initially the water in the reservoir is maintained at a constant level H_0 and the free surface has developed a steady position, then water level is drops instantaneously or at some prescribed rate to new elevation H_1, the free surface will start falling until new state of equilibrium is established.

This initial boundary value problem is subjected to the following set of boundary conditions:

$h(x,y,0)= h_{\circ}(x,y)$ (2)

$Z(x,y,0)= Z_{\circ}(x,y)$ (3)

$h(x,y,t)=\Delta H=H_{\circ}-H_1$ (4) on prescribed head boundaries

$k\ \dfrac{\partial h}{\partial y} n_i = -V(x,t)$ (5) on prescribed flux boundaries

$Z(x,y,t)=h(x,y,t)$ (6) on free surface

$k\dfrac{\partial h}{\partial y} n_i = (I - S_s \dfrac{\partial Z}{\partial t}) n_y$ (7) on free surface

$h(x,t)=y$ (8) on seepage face

where *Z* in equations (3), (6), and (7) represent the elevation of free surface above the horizontal datum from which head is measured, n_i is the component of unit normal to the boundary, and *I* is the net vertical specific rate of infiltration at free surface.

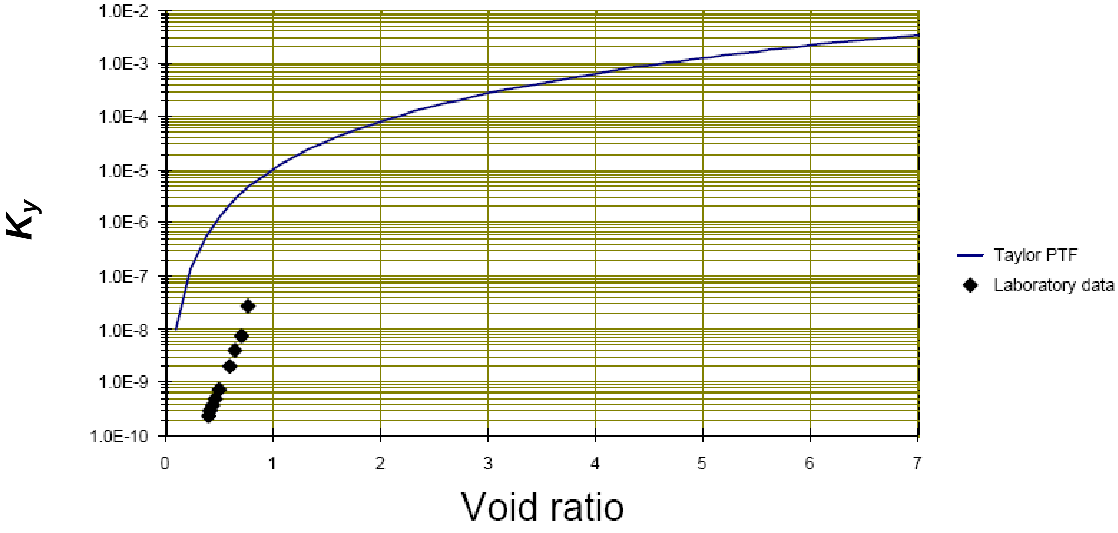
In order to formulate the stabilized finite element, we assumed that we have some appropriate defined finite-dimensional function spaces for the trial and weighting functions corresponding to pressure head, and the permeability function.

Generally K_y in vertical direction can vary by either the effect of overburden pressure of the natural soils or the influence of excess stresses due to an embankment load. For the first case, as the overburden pressure increases with depth, there would be a trend for the soil to become more compacted, therefore reducing K_y with depth. For the second case, as the depth increases the effect of embankment load decreases i.e. less consolidation, and thus K_y increases. The effect of the second condition is opposite to the first case, and these physical effects with depth should be superimposed in order to define variation in permeability for every starting point at interface of embankment and natural soil in the vertical direction(Smith, 2006).

Taylor (1948) indicates that the permeability for a homogeneous granular porous medium fit the following relationship (Shi and Chen, 2001):

$$K_y \propto D^2 \frac{e^3}{1+e} \qquad (9)$$

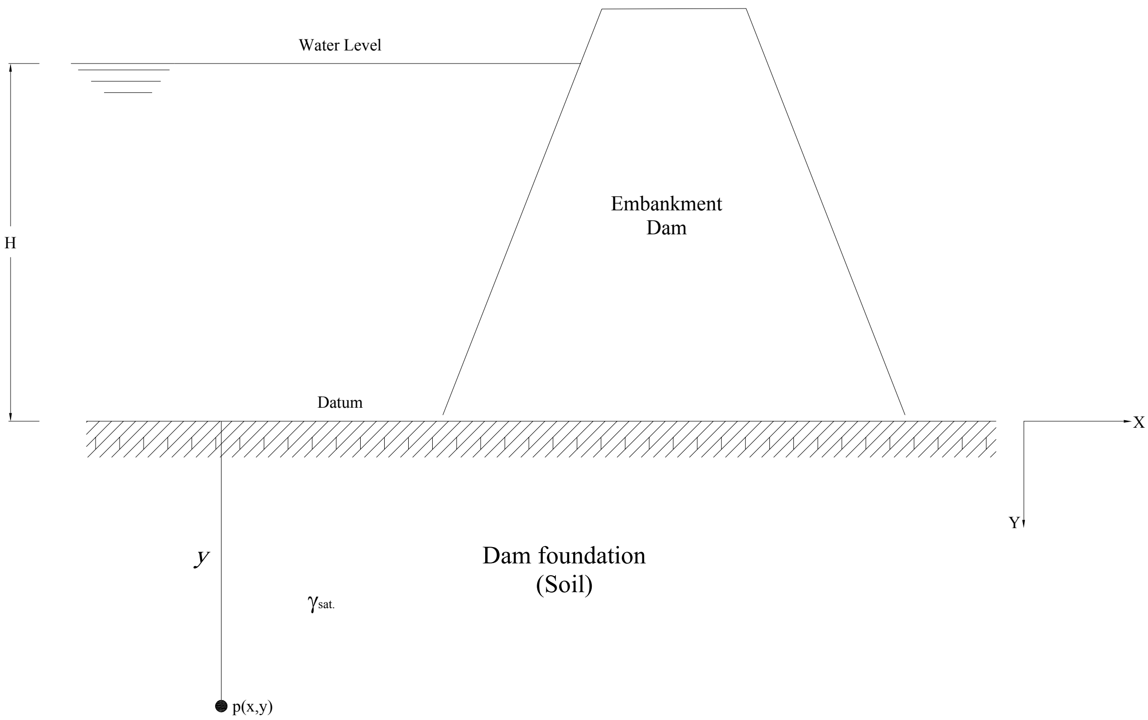
In which *D* is the soil particle diameter, *e* is the void ratio. Figure(1) shows the void ratio versus permeability for sandy clayey soil according to *Taylor* relationship.



Figure(1) : Void ratio versus coefficient of permeability for sandy clayey soil(Leroueil *et al*.,1992)

3- *Pressure heads versus permeability correlation*

From the effective stress *Terzaghi's* equation and from the information in Fig.(2) below, the effective stress at any point *p(x,y)* can be written as:



Figure(2): Effective stresses at a point below embankment dam

$$\sigma' = \sigma - u \tag{10}$$

In which σ' is the effective stress, σ is the total stress, and u is the pore water pressure. From Fig.(2) effective stress at point *p(x,y)* can be written as:

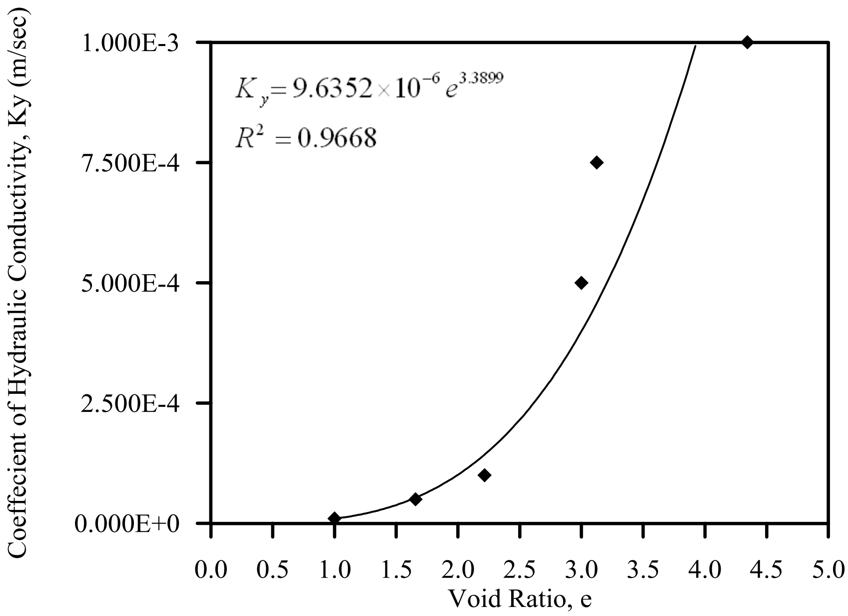
$$\sigma' = (y\gamma_{sat.} + \gamma_w H) - (h + y)\gamma_w \tag{11}$$

In which $\gamma_{sat.}$ is the saturated unit weight of the soil, h is the total head, $h + y$ is the pressure head, H is the upstream water height and γ_w is water unit weight. Table(1) presents ranges of permeability for various soils.

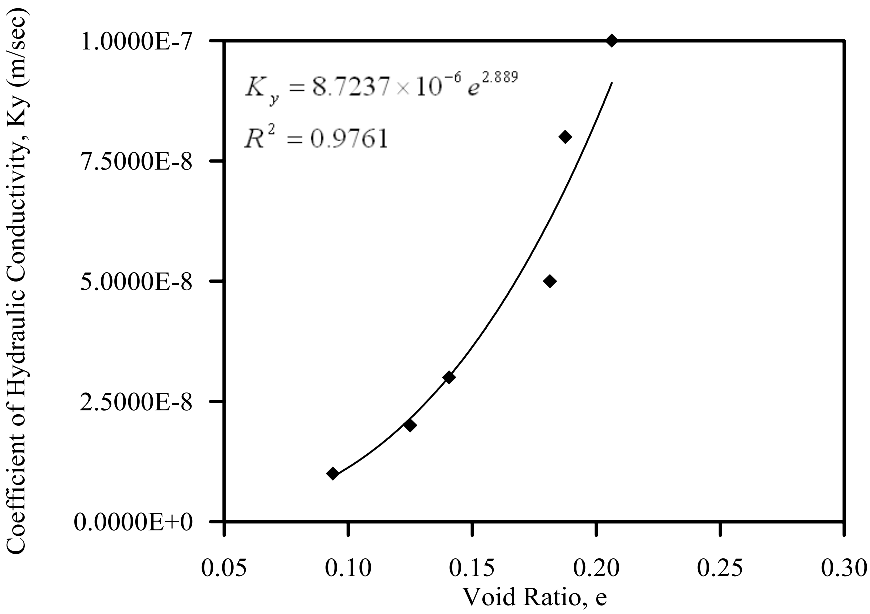
Table(1): Typical values of coefficients of hydraulic conductivity(Verruijt,2006)

Soil type	**Coefficient of permeability(cm/s)**
Gravel	$10^{-3} – 0^{-1}$
Sand	$10^{-6} – 10^{-3}$
Silt	$10^{-8} – 10^{-6}$
Clay	$10^{-10} – 10^{-8}$

Based on the *Taylor* relationship presented in Fig.(2) and Table(1), the present study related relations for both shells and central core materials of dam as shown in Figs.(3, and 4) below.



Figure(3): Vertical permeability vs. void ratio relationship for sandy soil



Figure(4): Vertical permeability vs. void ratio relationship for clayey soil

The equation which used for correlation may be expressed based on power procedure of regression as follows:

$$K_y = ae^b \tag{12}$$

In which *e* is the void ratio and ;

$$K_y = 9.6352 \times 10^{-6} e^{3.3899} \text{ for sandy soils} \tag{13}$$

$$K_y = 8.7237 \times 10^{-6} e^{2.889} \text{ for clayey soils} \tag{14}$$

The relationship between effective stress and void ratio can be verified in *e* vs. $log\,\sigma'$ (Lambe and Whitman, 1979; Das,2004) as:

$$e = C_c \log \frac{\sigma'}{\sigma'_0} + e_0 \tag{15}$$

in which σ' is the applied effective stresses (head) corresponding to *e* and σ'_0 is the known effective stress corresponding to e_0 , and C_c is the compression index.

Re-arrange Equation(15) yields:

$$e = \alpha \log \sigma' + \delta \tag{16}$$

where $\alpha = C_c$, and $\delta = e_0 - C_c \log \sigma'_0$

And with substitution of (16) into (12), it can be written as:

$$K_y = a(\alpha \log \sigma' + \delta)^b \tag{17}$$

By substituting (11) into (17), it can be calculated to the following form:

$$K_y = a[\alpha \log(y\gamma_{sat.} + \gamma_w (H - h - y) + \delta]^b \quad (18)$$

4- Finite element formulations

Galerkin method is considered to solve governing differential equation (1) by finite elements method, this method has the advantage of starting with the governing differential equation and eliminating the need for an alternate formulation of the physical problem (Smith, 2004). In order to simplify solution, saturated, and homogeneous porous material is considered i.e., S_s(x,y,t)=constant. The simplified form of governing equation is:

$$\frac{\partial K_x}{\partial x}\frac{\partial h}{\partial x} + K_x \frac{\partial^2 h}{\partial x^2} + \frac{\partial K_y}{\partial y}\frac{\partial h}{\partial y} + K_y \frac{\partial^2 h}{\partial y^2} - S_s \frac{\partial h}{\partial t} = 0 \quad (19)$$

The total head loss per unit length of flow path was referred by *hydraulic gradients* in the x and y directions as follow:

$$\left.\begin{array}{l} i_x = \dfrac{\partial h}{\partial x} \\ i_y = \dfrac{\partial h}{\partial y} \end{array}\right\} \quad (20)$$

By assuming $\lambda_x = \dfrac{\partial K_x}{\partial x}, \lambda_y = \dfrac{\partial K_y}{\partial y}$ (21)

Substitution of Eqs.(20) and (21) into (19), yields:

$$\lambda_x i_x + K_x \frac{\partial^2 h}{\partial x^2} + \lambda_y i_y + K_y \frac{\partial^2 h}{\partial y^2} - S_s \frac{\partial h}{\partial t} = 0 \quad (22)$$

The field equation describing an approximate variation of pressure head within a finite element is:

$$h^e(x,y,t) = \sum_{i=1}^{n} h_i^{\,e}(t) N_i^{\,e}(x,y) \quad (23)$$

in which $h_i^{\,e}$ is the nodal value of pressure head of element (*e*), *n* is the number of nodes per element, and N_i is the shape function of the element, at node *i*. Equation (23) may also be written in matrix form as :

$$h^e = [N_i]\{h_i\} \quad (24)$$

The approximate solution for pressure head variation, *h*, over the whole flow domain is given as follows:

$$h = \sum_{e=1}^{n_e} h^e = \sum_{e=1}^{n_e} \sum_{1}^{n} N_i h_i \quad (25)$$

$$h = \sum_{e=1}^{n_e} [N_i]\{h_i\} \tag{26}$$

in which n_e is the total number of elements in the problem domain.

Applying the weighted residual method with Galerkin's method to equation (22) yields in matrix form:

$$[D^e]\{h^e\} + [M^e]\left\{\frac{\partial h^e}{\partial t}\right\} = \{F^e\} \tag{27}$$

In which,

$$D^e{}_{ij} = \int_{\Omega^e} [K_x \frac{\partial N_i}{\partial x}\frac{\partial N_j}{\partial x} + K_y \frac{\partial N_i}{\partial y}\frac{\partial N_j}{\partial y}]dxdy \tag{28}$$

$$M^e{}_{ij} = \int_{\Omega^e} SN_i N_j dxdy \tag{29}$$

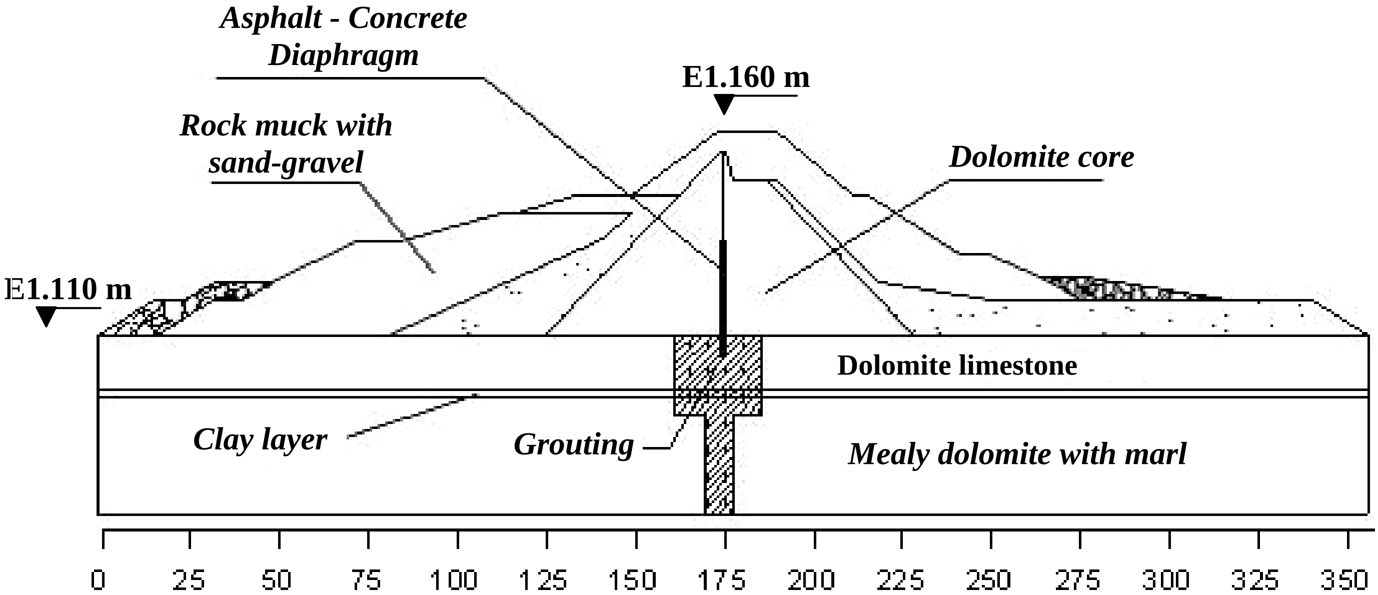
$$F^e{}_i = \oint_{\Gamma^e} N_i q_n ds \tag{30}$$

which q_n is the discharge occurred in the direction cosine between the normal and (x) direction, S is the boundary portion over which integration takes place.

5- Haditha dam

Haditha dam was constructed in 1988 on the Euphrates River in the Middle West of Iraq 7km upstream from Haditha town. The project generates (660 Mw) of electrical power a side from performing its flood control function. Central and southern parts of Iraq get the benefit of irrigation water from its reservoir.

The project comprises mainly of an earth dam, 9km long. Because of its considerable length and diversity of its topography and geological conditions, the design of the dam embankment varies from section to section but in general it preserves the features of the basic type which cover most of the dam length .The body of the dam consists of a central dolomite core and shells made of sand/gravel material and/or a rock muck (random rock material). An asphaltic concrete diaphragm through the core was provided as an anti-seepage measure through the body of the dam. A grout curtain was constructed to provide treatment for the foundation against seepage, cross-section of Haditha dam is shown in Fig.5(Al-Shuker,2008).



Figure(5): Typical cross section of Haditha Dam with ground profile

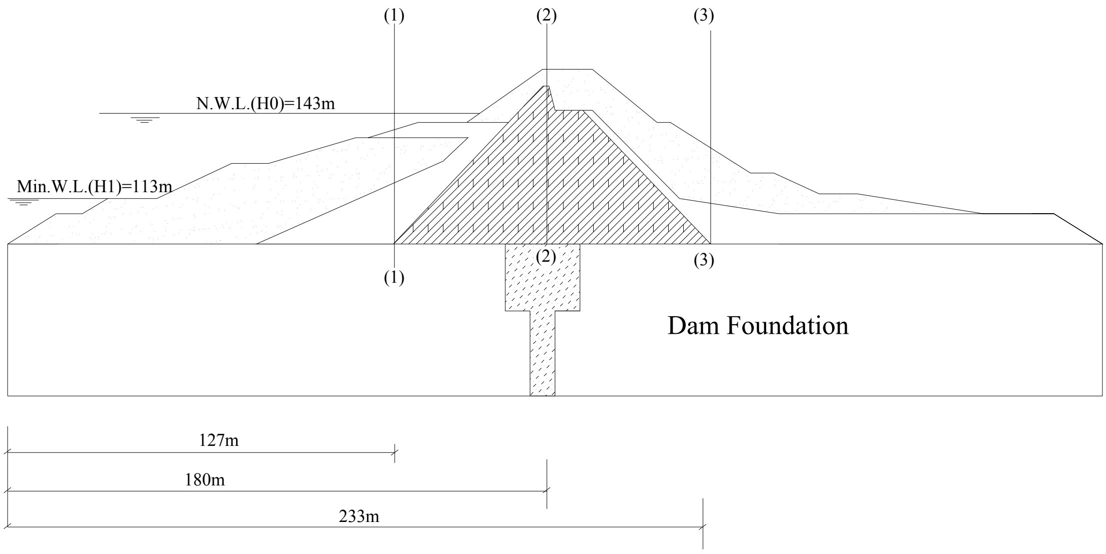
The zoned embankment and the ground physical mechanics parameters are presented in table 2.

Table(2): Material parameters (Irzooki,1998; Obead,2002)

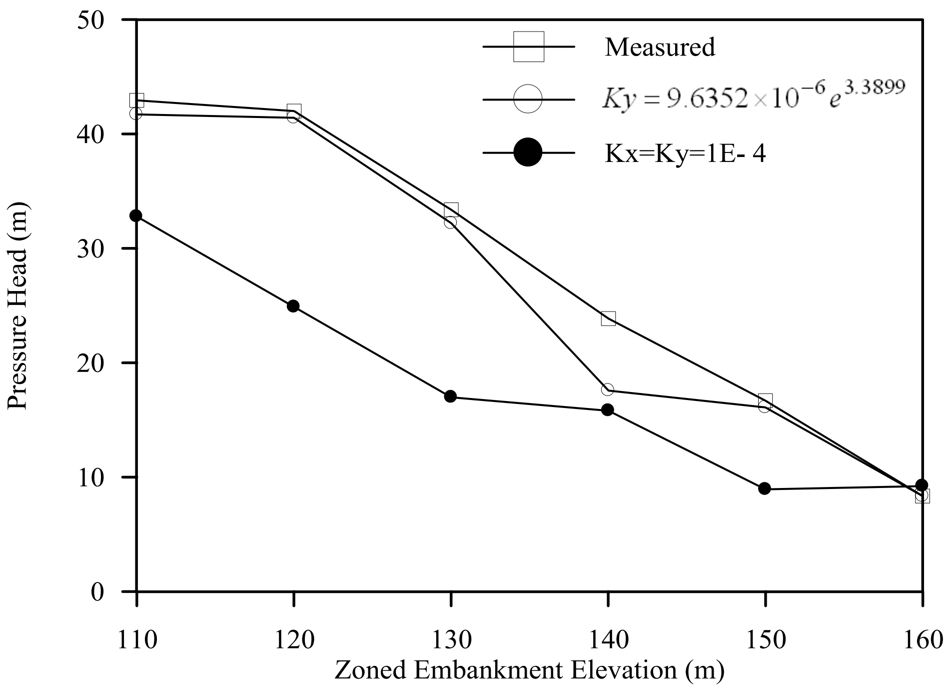
Material type	γ_t (kN/m^3)	$\gamma_{sat.}$ (kN/m^3)	C (kpa)	ϕ (Deg.)	E (Mpa)	Slope of critical state line	Slope of normal consolidation line, λ_c	Initial void ratio, $e_\circ$
Sand-Gravel Rock muck (shell)	17	20.3	0	27-30	30-80	1.15	0.1576	1.0
Dolomite (Core)	20.3	21.4	0	30-33	70-100	1.2	0.12	1.0
Limestone (Foundation)	22	22	100-200	30-35	100-300	1.15	0.01	0.965
Clay , Marl (Foundation)	18.7	19	30-50	14-16	20-50	1.35	0.158	0.86
Mealy Dolomite (Foundation)	22	22	20-40	24-28	50-100	1.25	0.096	1.0

6- Results and discussion

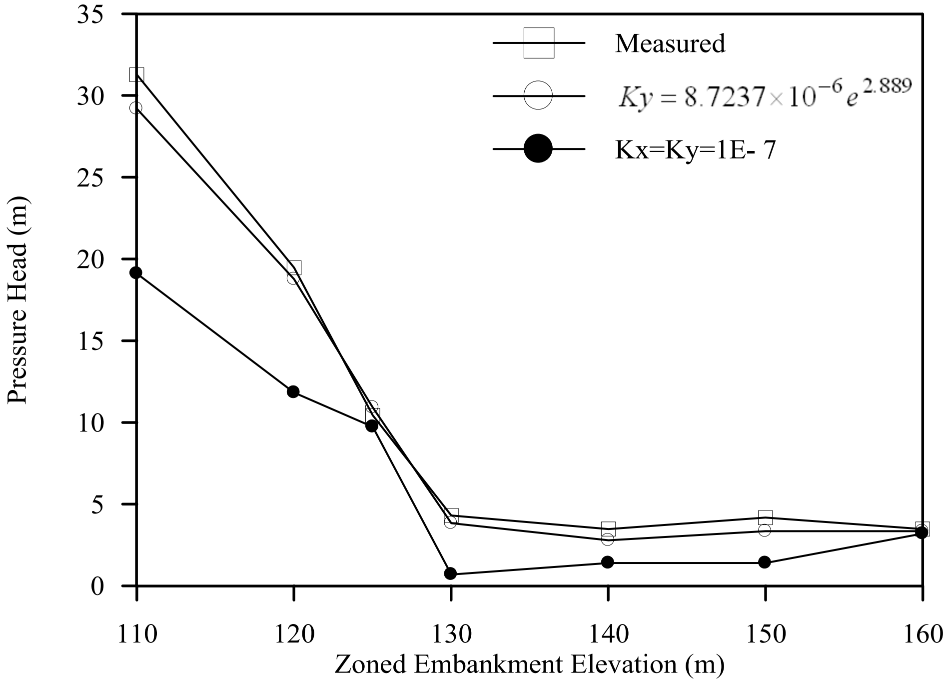
The influence of nonlinear permeability coupled with unsteady drawdown process were performed in the locations shown in figure (6).



Figure(6): Locations of present study calculation

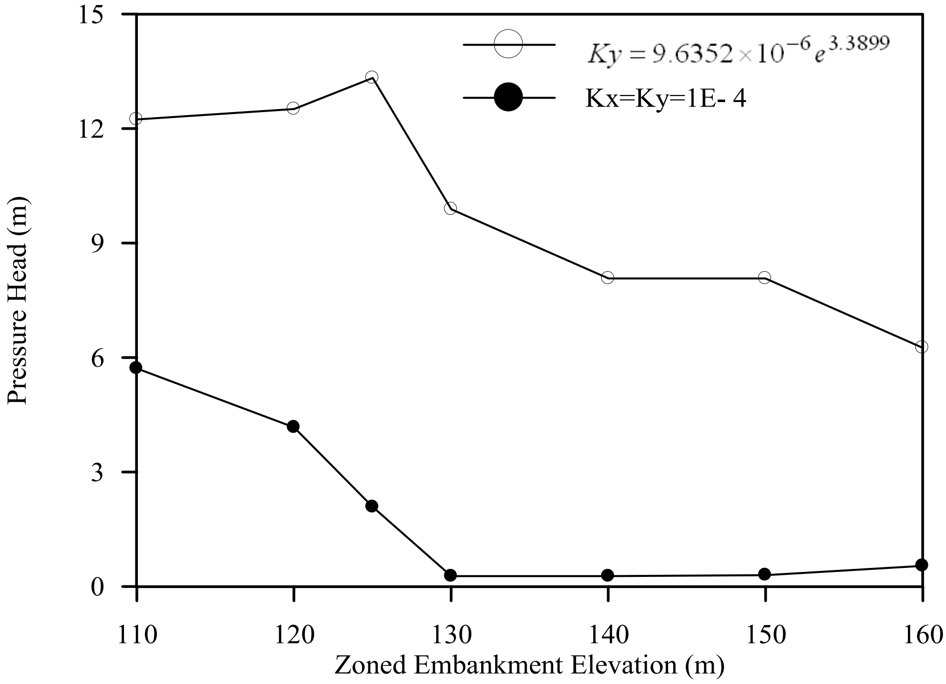


Figure(7):Influence of variation of permeability on pressure heads distribution in section (1-1) for various dam levels in the U/S face.



Figure(8):Influence of variation of permeability on pressure heads distribution in section (2 - 2) for various dam levels in the central core.

Results show that (Figs. 7,8 and 9), when the water level falls from normal operation of 143 m to 113 m (minimum water level) with a saturated permeability of construction soil (1×10^{-7} m/s for Dolomite clay core, and 1×10^{-4} m/s for Sand-Gravel shells) with a drawdown rate 0.25 m/h, large area of pressure head in the interior flow region still has not promptly drained out. It induces the seepage gradient to rapidly increase at upstream side, which reaches(10% - 16%). When the upstream head falls to minimum water level 113m, in the embankment there is still a small amount of pressure head that is dissipating slowly, and supplying to slope in upstream and downstream faces.



Figure(9):Influence of variation of permeability on pressure heads distribution in section (3 - 3) for various dam levels in the D/S face.

We can know that the reductions of pressure heads on upstream surface are corresponding with the water level falling, but in middle part of dam the reductions delay extremely for higher saturation (*Dolomite core*). While the pressure heads have supporting to the upstream face and formed uplift seepage pressure. Two types of curves are shown in these figures, one with constant permeability and the other with variable permeability(present method). In the one with constant hydraulic conductivity, similar to most classical seepage problems, permeability is assumed constant throughout the analysis and the system, and if it varies, it is not due to the effect of upstream head. But in the other one variable permeability refers to the influence of head on discharge rate. It can be seen from Figs.(7, 8 and 9) that for both cases when upstream head decrease, the pressure head also decreases. It should be explained, however, that in the actual case, head effects influenced hydraulic conductivity. The pressure heads are different from that of the constant permeability case. The difference would be higher for longer values of upstream head, i.e. for H_0 = 143m the effect of the total head on pressure head is about more than 2%. This is mainly due to the effect of upstream head on hydraulic conductivity.

7- Conclusions

It is clear from above discussion that:

1. When the head at any location within saturated flow domain varies, it would influenced by the permeability of that position.

2. As the head increases the values of permeability decrease, and when the head at any point increases, consolidation of the material occurs resulting in reduced the hydraulic conductivity, this variation of $K_{sat.}$ against head is non-linear.

3. Generally, results of pressure head changes either by constant or variable permeability conditions compares adequate to the case when the permeability is assumed to be nonlinear and a appreciate difference than constant conductivity is observed.

4. Results of pressure heads for a drawdown condition show that the values are less affected for rapid drawdown rates (more than 0.25 m/h), but the effect increases for higher head. The results, assuming variable permeability conditions, would give a lower safety factor regarding piping, etc.

8- References

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