1.Introduction

Sheet metal forming process is a widely used method in manufacturing complex shaped parts such as airplane fuselages, automative panels and parts of ship hulls Ming-Zhe Li et al. 2007. These products require that metal plates or sheets be formed into complex curvature. Acoordingly, the conventional sheet metal forming processes (deep drawing, stamping) became invaluable method in manufacturing these products. The non-conventional processes are the key to the solution of this problem. One such non-conventional sheet metal forming process is the digitized die forming (DDF) Ming-Zhe Li et al. 2007 and M. Z. Li 2002.

Digitized die forming concept is derived from multi point forming method (MPF). A DDF process of sheet metal is illustrated in **Fig.1**. Multi- point forming (MPF) is an advanced manufacturing technology for three dimensional sheet metal parts. In this process, the conventional stamping dies are replaced by a pair of matrices of punches; the relative positions of the punches can be varied and numerically controlled to suit the requirements of the parts shape. By controlling the position of each punch, the matrices of punches are transformed to multi- point dies (MPD) which approximate to a continuous forming surface of dies.

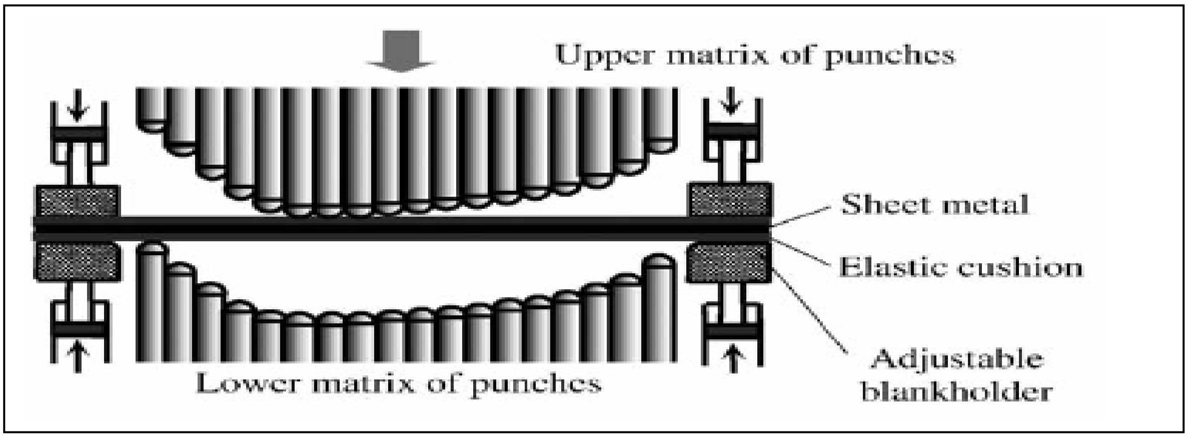

Figure1: A DDF of sheet metal[2].

Variety of methods are proposed for flattening or planar development of 3D surfaces. All these methods are depended on Zhong-Yi Cai et al. 2007:

- The kind of surface
- Propeties of material
- Forming method

Ming-Zhe Li et al. 2007 proposed an iterative scheme assuming that the straines in the final shape are evenly distributed. From the geometry of the initial blank and the desired part, intermediate shapes at a series of specific time are interpolated. Zong-Yi Cai et al. 2007 proposed an algorithm for generating planar development of blank by deviding the 3D surface into triangular finite elements and then adding in-plane strain of elemental edges.

An algorithm for surface development by approximating doubly curved surfaces by quadrilateral facets was introduced by B.K. Hinds 1991. That approximation was followed by flattening those facets. That surface approximation method produces some forms of gaps in the developed shapes.

R.M.C. Bodduluri and B. Ravani 1994 presented a method for flattening Bezier and B-spline surfaces assuming that these surfaces as developable surfaces. These developable surfaces can be flattening by rolling them onto the plane. C.H. Lee, H. Huh 1998 presented an inverse finite element method to develop the inintial blank shape by minimizing plastic work.

In this paper, the 3D curved surfaces are modeld using bi-cubic Bezier surfaces and flattening approach is based on the following assumptions:

1) The sheet is deformed based on pure shear deformation mechanism at a specific region of the surface and strech forming on the other regions.

2) The four edges of the plate are simply supported.This assumption is highly recommended in sheet metal forming processes T. J. Kim and D. Y. Yang 2000.

2.Third degree Bezier surface construction

Bezier surfaces are most famous tool in computer aided geometric design (CAGD) and widely used for free form surface construction E. Michael and Mortensor 2000. The Bezier bi-cubic surface can be defined using the following parametric equation E. Michael and Mortensor 2000:

$$\mathbf{S}(u,w) = \mathbf{M}_{beu}\, \mathbf{V}\, \mathbf{M}_{bew} \tag{1}$$

Where $\mathbf{M}_{beu}$, $\mathbf{M}_{bew}$ are the two blending function matrices;

$$\mathbf{M}_{beu} = [1\ u\ u^2\ u^3\ ]\ \mathbf{M}_{be} \tag{2}$$

$$\mathbf{M}_{bew} = [1\ w\ w^2\ w^3\ ]\ \mathbf{M}_{be} \tag{3}$$

$\mathbf{M}_{be}$ is the Bezier surface matrix;

$$\mathbf{M}_{be} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \end{pmatrix} \tag{4}$$

u,w are two independent variables ranging from 0 to 1,V is the vertix information matrix, containing 16 control points which are establishe the control polygon of the Bezier surface as shown in **Fig.2.a.** The control polygon determines the shape of the Bezier surface. Consequently, each Bezier surface has its own control polygon. Acordingly, changing the position of any control point will affect the shape of the produced surface (see **Fig.2.b**). This is one of the most important benefits of bezier surface.

$$\mathbf{V}= \begin{pmatrix} V_{oo}\ V_{o1}\ \ V_{o2}\ V_{o3} \\ V_{1o}\ V_{11}\ \ V_{12}\ V_{13} \\ {}_{32}V_{2o}\ V_{21}\ \ V_{22}\ V \end{pmatrix} \tag{5}$$

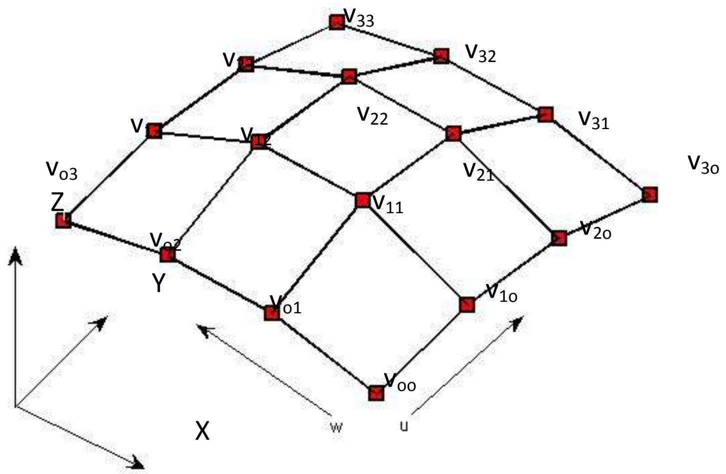

Figure2.a: The 16 control points of Bezier surface and their corresponding control polygon.

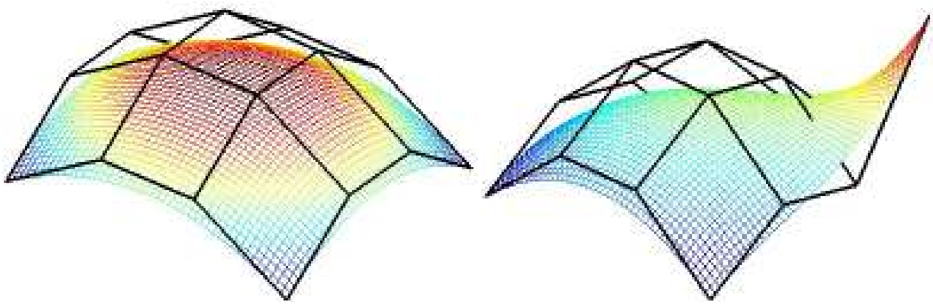

Figure2.b: Controlling the shape of the surface by changing the coordinates of the control points.

3.Blank shape prediction and size determination

Any 3D Bezier control polygon can be idealized to a 2D polygon as shown in **Fig.3.** Consequently, the 16 control points of the Bezier surface can be grouped into (see **Fig.3**):

- 12 edge points, having square flags in **Fig.3**,
- 4 central points, having circular flags.

According to the two assumptions mentiond before in section 1, the central points have the ability to move vertically downward, so that the x,y coordinates of these points remain constant during the conversion from the part geometry to its flat blank (flattening scheme). Meanwhile, the edge points can be freely moved in space. Accordingly, the direction of motion of central points is known and the value of their motion is to be determined depending upon the number of forming steps. With this prespective, the value of motion of edge points are to be determined and the direction of motin is assumed to be tangent to the line segments joining the edge points with their central points (see **Fig.3**).

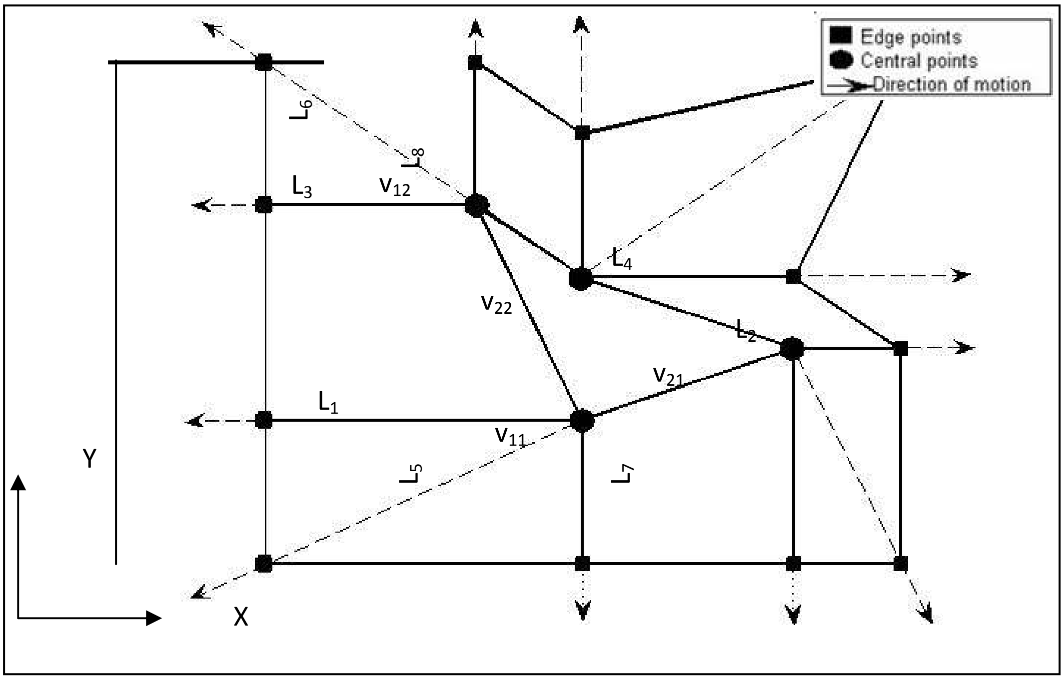

Figure3: Idealized Bezier polygon showing edge points ,central points and their direction of motion.

Mathematically, the motion of central points should satisfy the following condition (see **Fig.4**):

$$Z = 0 \ \forall \ V \ \in \ [v_{11} \ \ v_{21} \ \ v_{12} \ \ v_{22}] \tag{6}$$

The final x-coordinates of each edge point along the u-direction are:

$$\left.\begin{aligned} X_f^{o1} &= X^{11} - L_1 \\ X_f^{o2} &= X^{12} - L_3 \\ X_f^{31} &= X^{21} + L_2 \end{aligned}\right\} \tag{7}$$

$$X_f^{32} = X^{22} + L_4$$

The final Y-coordinates of each edge point along the w-direction are:

$$\left.\begin{aligned} Y_f^{1o} &= Y^{11} - L_5 \\ Y_f^{2o} &= Y^{21} - L_7 \\ Y_f^{13} &= Y^{12} + L_6 \end{aligned}\right\} \tag{8}$$

$$Y_f^{23} = Y^{22} + L_8$$

The X and Y-coordinates of each corner point :

$$\left.\begin{aligned} X_{oo} &= X_{o1} \\ Y_{oo} &= Y_{1o} \\ X_{o3} &= X_{o2} \\ Y_{o3} &= Y_{13} \end{aligned}\right\} \tag{9}$$

$X_{3o}=X_{31}$

$Y_{3o}=Y_{2o}$

$X_{33}=X_{32}$

$Y_{33}=Y_{23}$

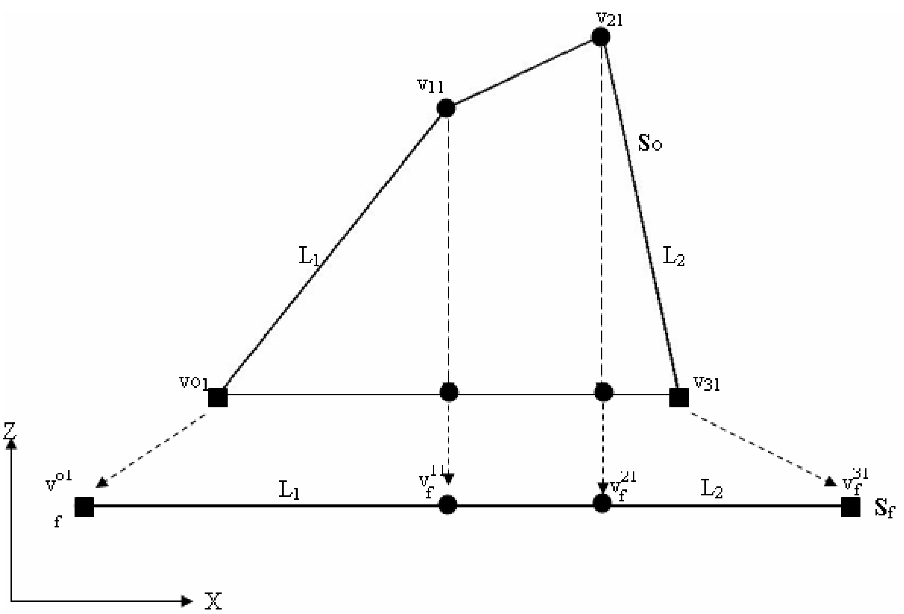

Figure4.a The flattening mechanism of a sample of control polygon projected in x-z plane

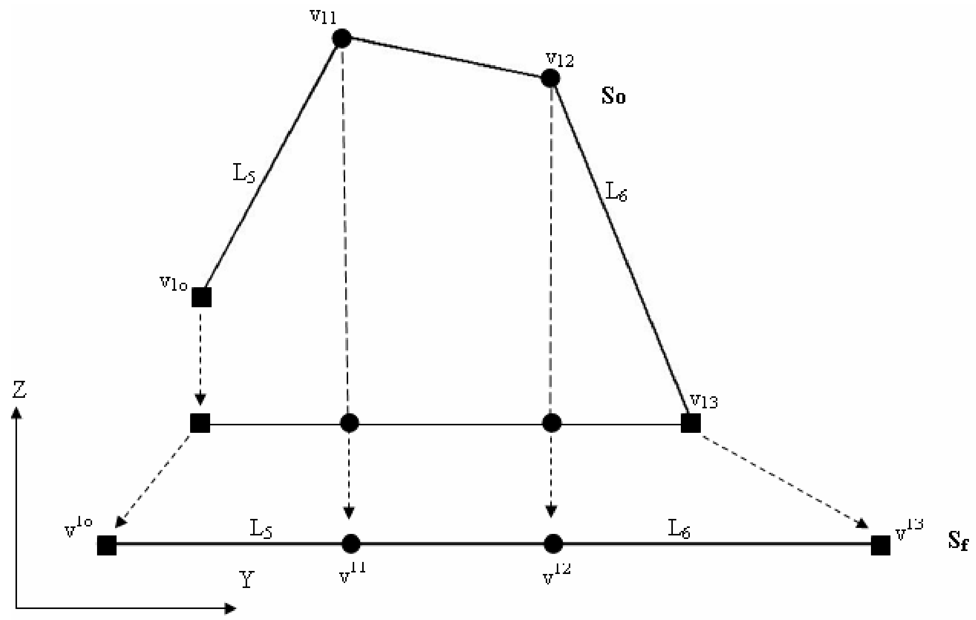

Figure4.b The flattening mechanism of a sample of control polygon projected in y-z plane.

Because it is assumed that the initial control points are transformed into the final control points by only shear deformation, x and y coordinates are coincident between the initial and final state. Accordingly, the v_{11} is transformed to v_f^{11} and v_{12} is transformed to v_f^{12} based on this assumption.

4.Results and discusion

For the purpose of completeness, an example of a 3D curved surface has been modeled using the adobted Bezier technique assuming the following control points:

$$V_x = \begin{pmatrix} 0 & 3 & 5 & 6 \\ 0 & 3 & 5 & 6 \\ 0 & 2 & 3 & 6 \end{pmatrix}; \quad V_y = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 2 & 2 & 3 & 3 \\ 5 & 5 & 4 & 4 \end{pmatrix}; \quad V_z = \begin{pmatrix} 5 & 8 & 5 & 8 \\ 10 & 14 & 15 & 10 \\ 10 & 13 & 15 & 10 \end{pmatrix}$$

The control polygon and its required surface are shown in **Fig.5**. Table 1 shows the length of each linear segment of the control polygon. Aplying equations 6-9 to the given control polygon, the control points of the developed planar blank are;

$$V_x = \begin{pmatrix} -2 & 3 & 5 & 10 \\ -2 & 3 & 5 & 10 \\ -1.6 & 2 & 3 & 8.8 \end{pmatrix}; \quad V_y = \begin{pmatrix} -4.3 & -4.3 & -4.6 & -4.6 \\ 2 & 2 & 3 & 3 \\ 5 & 5 & 4 & 4 \end{pmatrix}; \quad V_z = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Table 1: The length of each linear segment of surface polygon

Segment	L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8
Length (length unit)	5	$(26)^{1/2}$	$(13)^{1/2}$	$(34)^{1/2}$	$(40)^{1/2}$	$(68)^{1/2}$	$(58)^{1/2}$	$(109)^{1/2}$

The control polygon of the curved surface and the developed planar blank are projected and overlaped as **Fig.6** depects. Assuming that the number of steps required to develop planar shape (forming steps) is four, the steps of planar development can be simulated as shown in **Fig.7**.

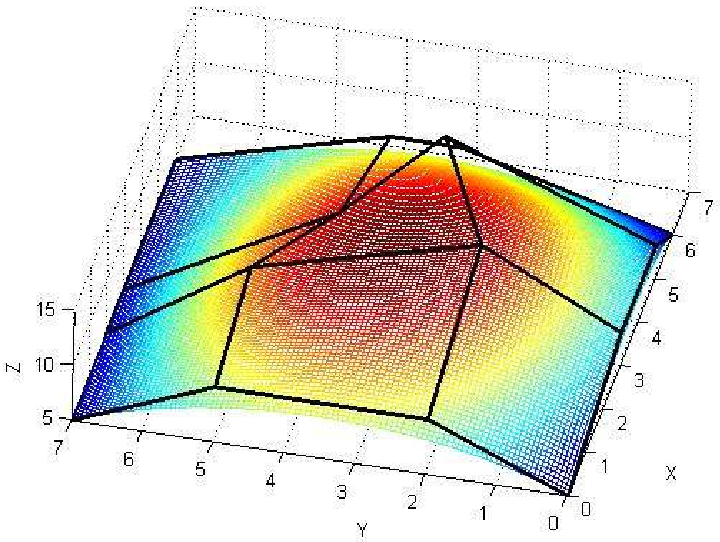

Figure 5: The control polygon and its required Bezier surface .

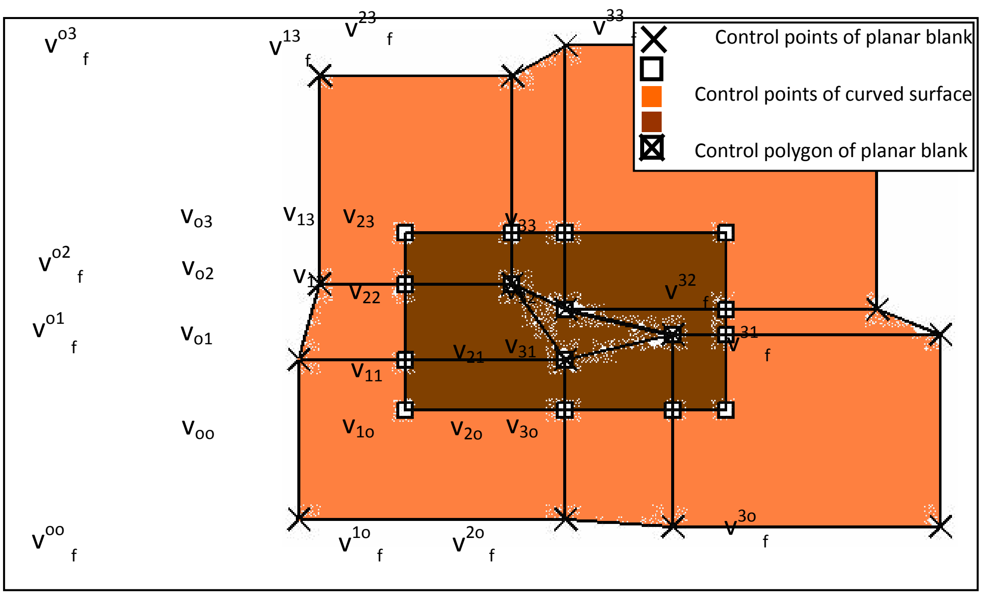

Figure 6: The duplicated control polygons of both curved surface and the developed planar blank .

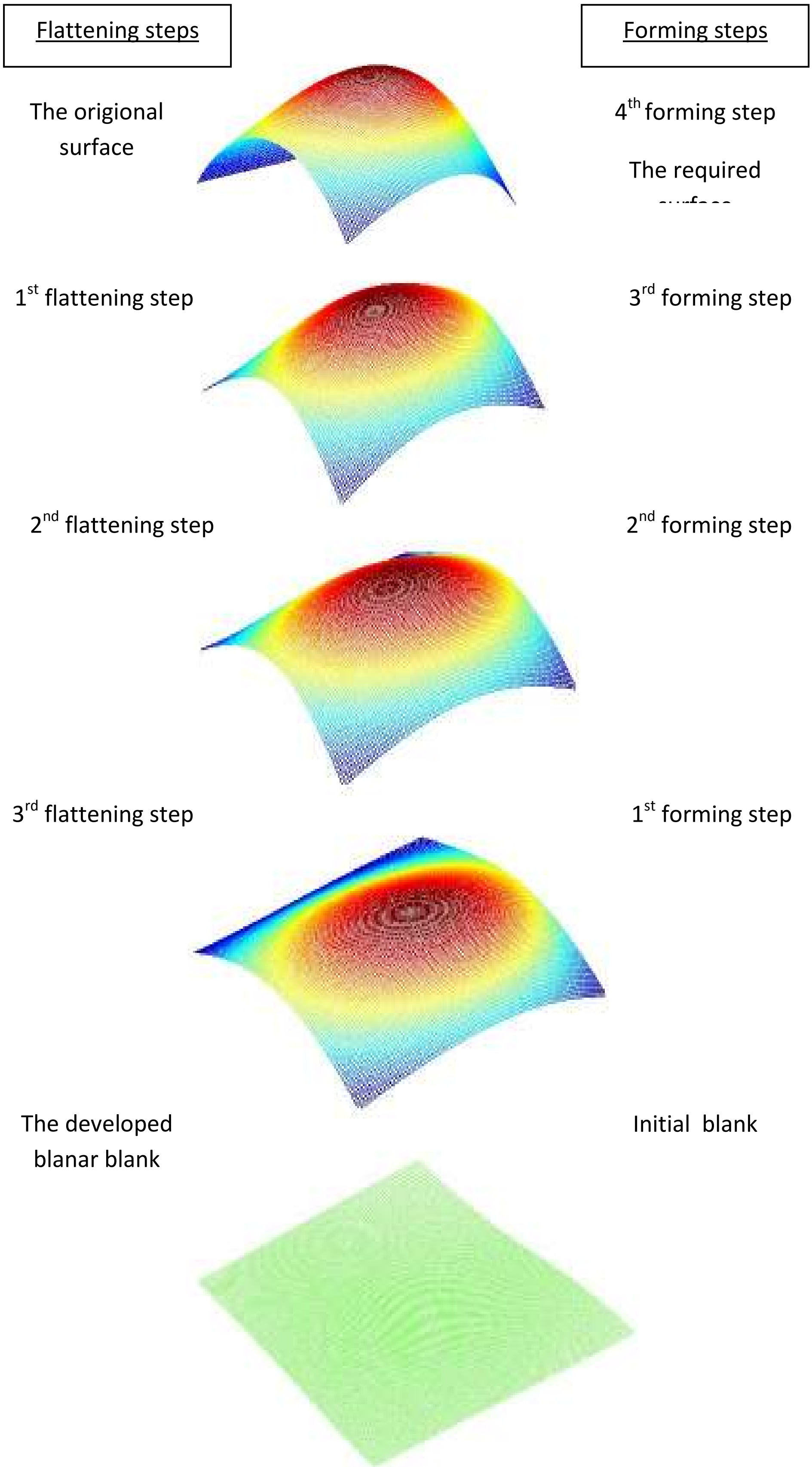

Figure7: Simulation of the four forming steps, showing the planar development steps.

The finite element technique is used in this paper to compare between the proposed method and the conventional one, taking the blank thickness distribution as a criterion to asses the proposed method. Based on volume constancy rule T. J. Kim and D. Y. Yang 2000;

$A_o t_o = A\ t$

or $t = A_o t_o / A$ (10)

where A_o , t_o are the initial area and thickness of the finite element respectively. A, t are the final area and thickness of specified element (see **Fig.8**). The finite elements mesh is generated for conventional planar blank, the proposed planar blank and the curved surface (see **Fig.8**) using MATLAB programing environment Rudar Pratap 2002.

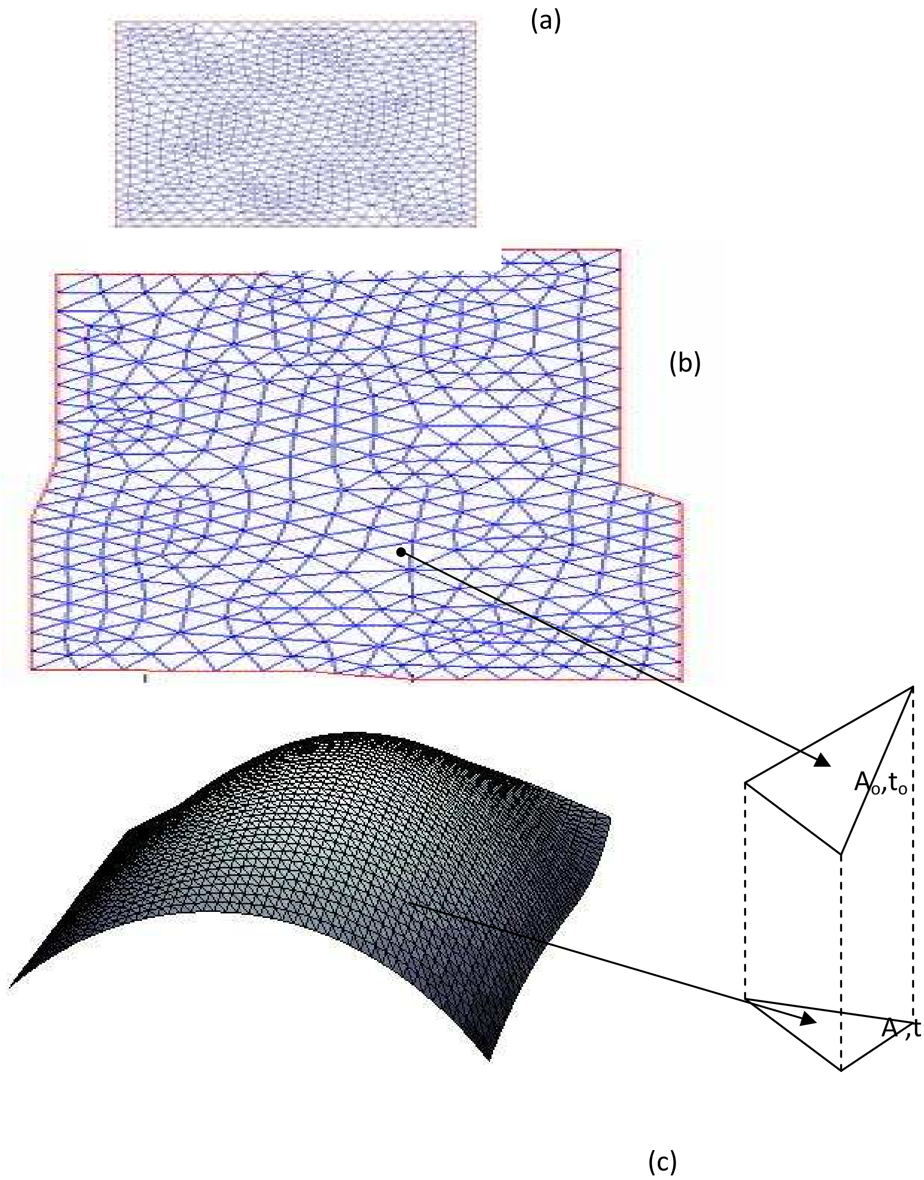

Figure8: The finite element mesh generation for (a) conventional blank, (b) proposed blank and (c) curved surface.

Fig.9 shows a comparison between thickness distribution of conventional blank and proposed blank assuming that (t_o = 1 mm). From this figure it can be seen that the thickness is evenly distributed using the proposed method and the risks of thining can be greatly reduced.

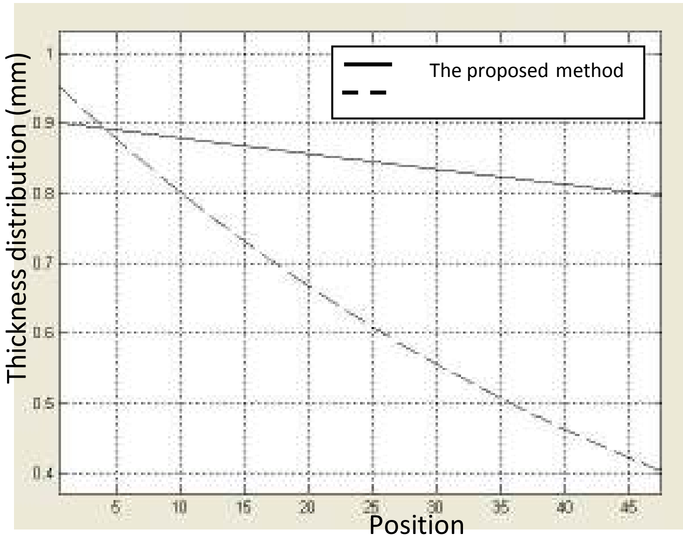

Figure9: A comparison between the thickness distribution of conventional blank and proposed method.

5.Conclusion

In this work, a devised method for developing a planar blank from a given 3D curved surface has been disclosed. The pure shear deformation mechanism used in this study is infallible. The compined pure shear deformation mechanism and stretch forming at the edges of the blank has incendiary results in predicting the shape of the initial blank and simulation of forming steps. The motion of edges control points may be a good and simple way to produce a strech forming at these edges. The proposed method to develop planar shape from its curved surface is greatly inhancing the thickness of the sheet and reduces the chances of sheet failure by thining.

The presented method is especially suited for thin plate obey small deflection theory. In view of this fact, if the thin plate obeied large deflection theory, it might produce detrimental results since that some of surface control points incarcerated from spatial movements. Impartially, the presented method is more suitable for 3D plane surfaces.

6. Future works

1. The large deflection theory greatly affects the mode of deformation and so the shape of the blank. This affect will be investigated.
2. The arbitrary 3D curved surfaces will be invistigated rather than 3D plane surfaces.

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Nomencluture

S(u,w)	Parametric Surface Equation	S_o	Initial surface
M_{beu}	Bezier Surface Fundamental along u-direction	S_f	Final surface
V	Control point matrix	A_o	Initial area
M_{bew}	Bezier Surface Fundamental along w-direction	A	Final area
u, w	Independent parameters	t_o	Initial thickness
L	Linear segment length	t	Final area