



THE EFFECT OF MAIN NONLINEARITIES ON SERVO SYSTEMS STABILITY

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ABSTRACT:

Servo systems are indispensable in modern industry. For example, they are widely used in robotics, electrical vehicles and automated factories. Therefore, the study of the stability of the system and its response to linear and nonlinear elements is an important control system problem. This paper focuses on determination of servo system transfer function and the effect of the main nonlinearities on its stability by developing an algorithm for the system and simulating using MATLAB. The algorithm used is the describing function algorithm and its analysis, since it is one of the methods used to analyze nonlinear systems stability.

KEYWORD:

Servo, Nonlinearity, Describing function, Dead-zone, Saturation.

تأثير اللاخطيات الرئيسية على استقرارية منظومة المؤازر

الخلاصة

منظومة المؤازر لا غنى عنها في الصناعة الحديثة. فعلى سبيل المثال، أنها تستخدم بشكل واسع في الإنسان الآلي، العربات الكهربائية والمصانع الآلية. لذا أصبح من المهم دراسة استقرارية هذه المنظومة وتأثرها بالعوامل الخطية واللاخطية. يركز هذا البحث على حساب الدالة الانتقالية لمنظومة المؤازرة وفحص استقرارية المنظومة بتأثير اللاخطيات الرئيسية وذلك ببناء نموذج رياضي للمنظومة وتطبيقه برمجياً باستخدام (MATLAB). الخوارزمية المستخدمة تعتمد على الدالة الوصفية وتحليلاتها، لأنها من أحد الطرق المستخدمة في تحليل الاستقرارية للأنظمة اللاخطية.

1 – INTRODUCTION:

A servo control systems is one of the most important and widely used forms of control system. Any machine or piece of equipment that has rotating parts will contain one or more servo control systems [Ali, Sami, and Karim, (2005), Elk Laubwald].

We have dealt only with linear systems, in order to establish firmly the basic fundamentals and characteristics of feedback control systems. While nonlinearities did exist in the systems studied, they were considered negligible in order to use the simpler linear methods of analysis. However, one occasionally meets a problem on which the nonlinearities are so important that they cannot be ignored [Graham, Stefen and Mario 2001, Ogata 1997]. So the mathematical approaches to nonlinear systems is different from the linear systems.

This paper is analyzed the main nonlinearities effect on servo systems using the describing- function method, which provides stability information for a system of any order. While other ways of analyzing such as phase-plane technique is limit to first and second order systems, and the second method of Liapunov's technique its application may be hampered because of difficulty in finding Liapunov's functions for complicated nonlinear systems.

2 - SERVO SYSTEM TRANSFER FUNCTION:

This paper is concentrate on the linear part of the servo system model, so referring to fig.(1a,1b) the transfer function form [Naresh 1992, Elke Laubwald]:

$$G(s) = K_p K_m / s(ts+1) \quad \dots\dots\dots(1)$$

Where K_m : is the servo system gain, t : is the time constant, K_p : amplifier gain.

The constants are taken from internet links[7 and Paul 2001]. Fig.(1b) shows these values of K_p , K_m , t



Fig. (a) General Block Diagram For Servo Unit

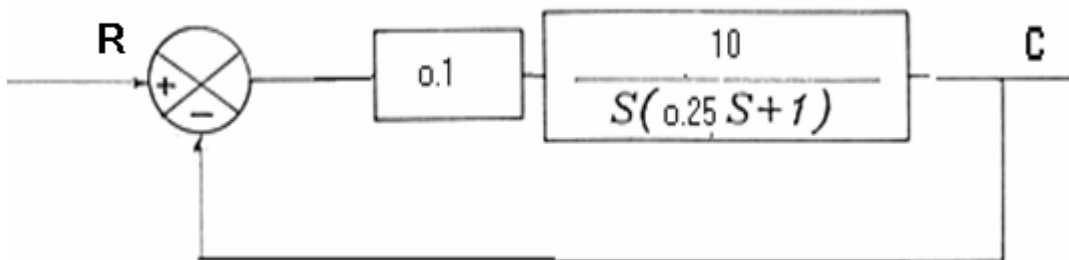


Fig. (b) Block Diagram For Servo Unit with optimum parameters

Fig.(2) shows the modified feedback system with gain H .

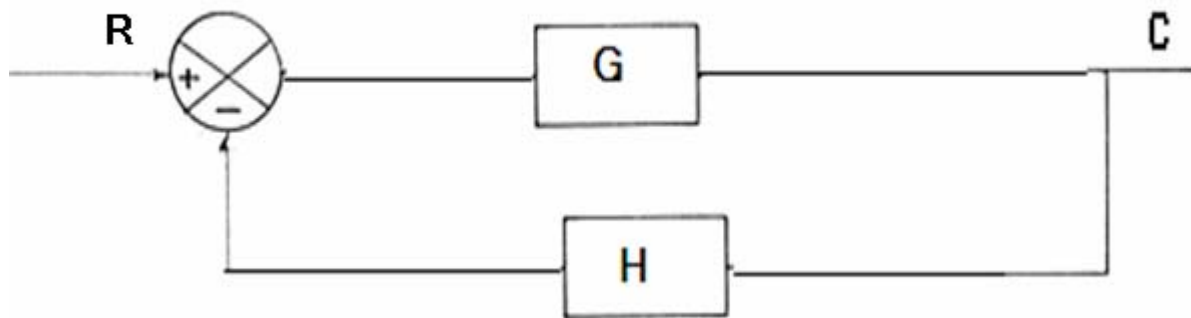


Fig. (2) Block Diagram For Feedback System

The closed loop transfer function for the servo system fig.(1b):

$$T.F. = G(s) / 1 + G(s) H(s) \quad \dots\dots\dots(2)$$

Since , $G(s) = 0.1 \cdot 10 / s(0.25s + 1)$

$H(s) = 1$ {unity feedback}

Then, $T.F. = 1 / (0.25s^2 + s + 1)$

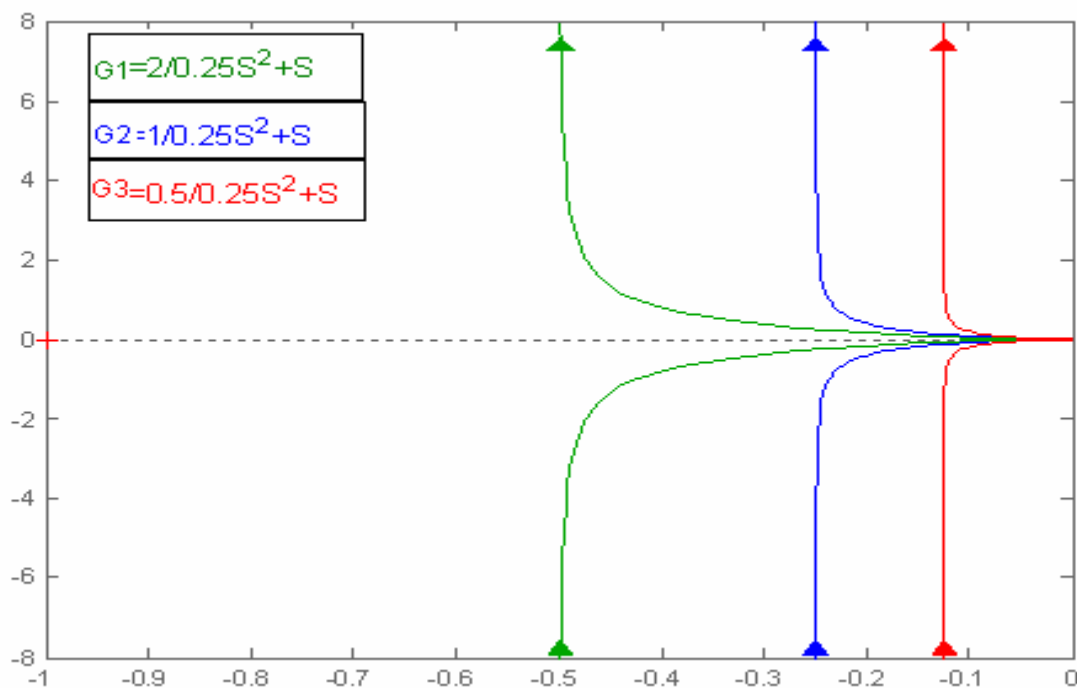


Fig. (3) the Nyquist's plot for different cases using different gains of servo system

3 – DESCRIBING FUNCTION:

The describing function is an useful frequency domain method for analyzing the stability of a nonlinear control system has hard nonlinear elements, such as relay, dead-zone, saturation, backlash, hysteresis and so on [Jau, Bing, Tien and Tsu 2007].

The describing function is an extension of frequency response analysis to systems containing a nonlinear element followed by linear elements in series fig.(4). The response of nonlinear element to an input sine wave is a distorted sinusoidal output with a fundamental frequency equal to that of the input, but with additional higher frequency harmonic components [A. Pollard 1981]. Using Fourier analysis, we can extract the fundamental component from the output waveform [Joel 2007,Ogata 1997]:

$$Y(t) = A_0 + \sum_{n=1}^{\infty} (A_n \cos(n\omega t) + B_n \sin(n\omega t)) \dots \dots \dots (3)$$

$$Y(t) = A_0 + \sum_{n=1}^{\infty} [Y_n \sin(n\omega t + \phi_n)] \dots \dots \dots (4)$$

$$A_n = 1/\pi \int_0^{2\pi} Y(t) \cos(n\omega t) d(\omega t)$$

$$B_n = 1/\pi \int_0^{2\pi} Y(t) \sin(n\omega t) d(\omega t)$$

$$Y_n = \sqrt{A_n^2 + B_n^2}$$

$$\Phi_n = \tan^{-1} (A_n / B_n)$$

If the nonlinear is symmetric, then $A_0 = 0$. The fundamental harmonic component of the output is

$$Y_1(t) = A_1 \cos(\omega t) + B_1 \sin(\omega t) \dots \dots \dots (5)$$

$$Y_1(t) = Y_1 \sin(\omega t + \phi_1) \dots \dots \dots (6)$$

The describing function is then given by

$$N = (Y_1 / X) \angle_{\phi_1} = \sqrt{A_1^2 + B_1^2} / X \angle_{\tan^{-1} (A_1 / B_1)} \dots \dots \dots (7)$$

Where

N: describing function.

X: amplitude of input sinusoid.

Y_1 : amplitude of the fundamental harmonic component of output.

ϕ_1 : phase shift of the fundamental harmonic component of output.

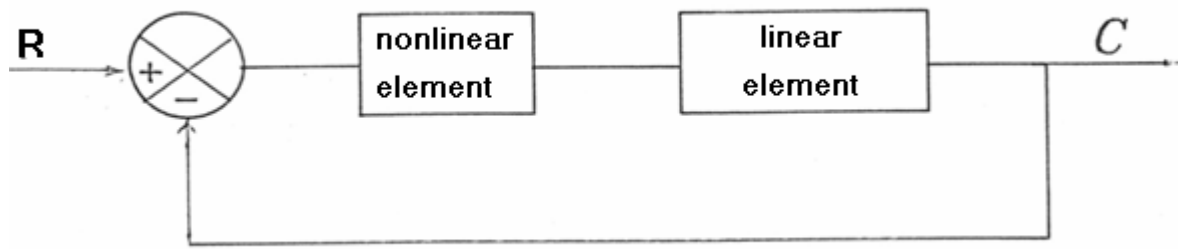


Fig. () Block Diagram For Nonlinear Control System

4 – DESCRIBING FUNCTION ANALYSIS:

The describing function of nonlinearity is N and can be used to determine the system's stability, provided that the harmonics are sufficiently attenuated. The control ratio of the closed-loop system of fig.(4) is given by:

$$C(j\omega) / R(j\omega) = NG(j\omega) / 1 + NG(j\omega) \quad \dots\dots\dots(8)$$

The stability can be determined from the zeros of the characteristic equation, which is

$$1 + NG(j\omega) = 0$$

Rewriting the above equation in form [Ogata 1997]:

$$G(j\omega) = -1/N$$

The first step in design and analysis of nonlinear control system is the stability assessment and in particular accurate prediction of any possible limit cycle [Maryam 2003]. In many mechanical systems, limit cycle occurs in the form of vibration or oscillation. The term limit cycle refers to an isolated closed orbit in the phase portrait of nonlinear systems. As the need of precision positioning systems becomes inevitable, the limit cycle has been the central problem in precision control designs. It successfully used describing function method to analyze the limit cycle [Nijmeijer and Vanessen 2000]. One of the good methods for prediction limit cycle based on describing function is Nyquist's stability criterion [Maryam 2003], in this paper we applied this method. The locus of $-1/N$ can be considered the locus of the critical points, which for linear systems is the $(-1, j0)$ point in complex frequency plane. When the critical point lies to the left of $G(j\omega)$ plot or is not enclosed by it, the system is then stable. Conversely, when the critical point lies to the right of $G(j\omega)$ locus and is therefore enclosed by it the system is unstable. If the $G(j\omega)$ plot passes through the critical point, the system may be either stable or unstable [Francis 1982].

5 – NONLINEARITIES IN CONTROL SYSTEM

Nonlinearities in control systems may appear due to one or more combination of the following [Choudhury 2008]:

- The process may be nonlinear in nature.
- The control system may have a nonlinear characteristic.
- The control system may develop nonlinear faults (the work in this paper focuses on this type of nonlinearities by studying each fault and its effect on the servo system).
- A nonlinear disturbance may enter the system.

The main nonlinearities which discussed in this paper are dead zone, saturation, on-off with hysteresis and backlash.

6 – SIMULATION RESULTS

The simulations are carried out under MATLAB7. To obtain the MATLAB code for this work, please contact the author.

A- (Dead Zone Element)

The dead zone nonlinearity is sometimes referred to as the threshold nonlinearity, the input-output waveforms is shown in fig.(5),while fig.(6) shown the input and output characteristic curve .In a dead zone element, there is no output for inputs within the dead zone amplitude. The output $y(t)$ for $0 \leq \omega t \leq \pi$ is given by : [Francis 1982,and Ogata 1970]

$$\begin{aligned}
 y(t) &= 0 && \text{For } 0 < t < t_1 \\
 y(t) &= K (X \sin \omega t - \Delta) && \text{For } t_1 < t < (\pi / \omega) - t_1 \\
 y(t) &= 0 && \text{For } (\pi / \omega) - t_1 < t < (\pi / \omega)
 \end{aligned}
 \tag{9}$$

Since the output $y(t)$ is once again an odd function, its Fourier series expansion has only sine terms. The fundamental harmonic component of the output is given by

$$y_1(t) = Y_1 \sin \omega t \tag{10}$$

where

$$Y_1 = 1 / \pi \int_0^{2\pi} y(t) \sin \omega t d(\omega t)$$

$$Y_1 = 4 / \pi \int_0^{\pi/2} y(t) \sin \omega t d(\omega t)$$

$$Y_1 = 4K / \pi \int_{\omega t_1}^{\pi/2} (X \sin \omega t - \Delta) \sin \omega t d(\omega t)$$

$$\Delta = X \sin \omega t_1$$

$$\omega t_1 = \sin^{-1}(\Delta / X)$$

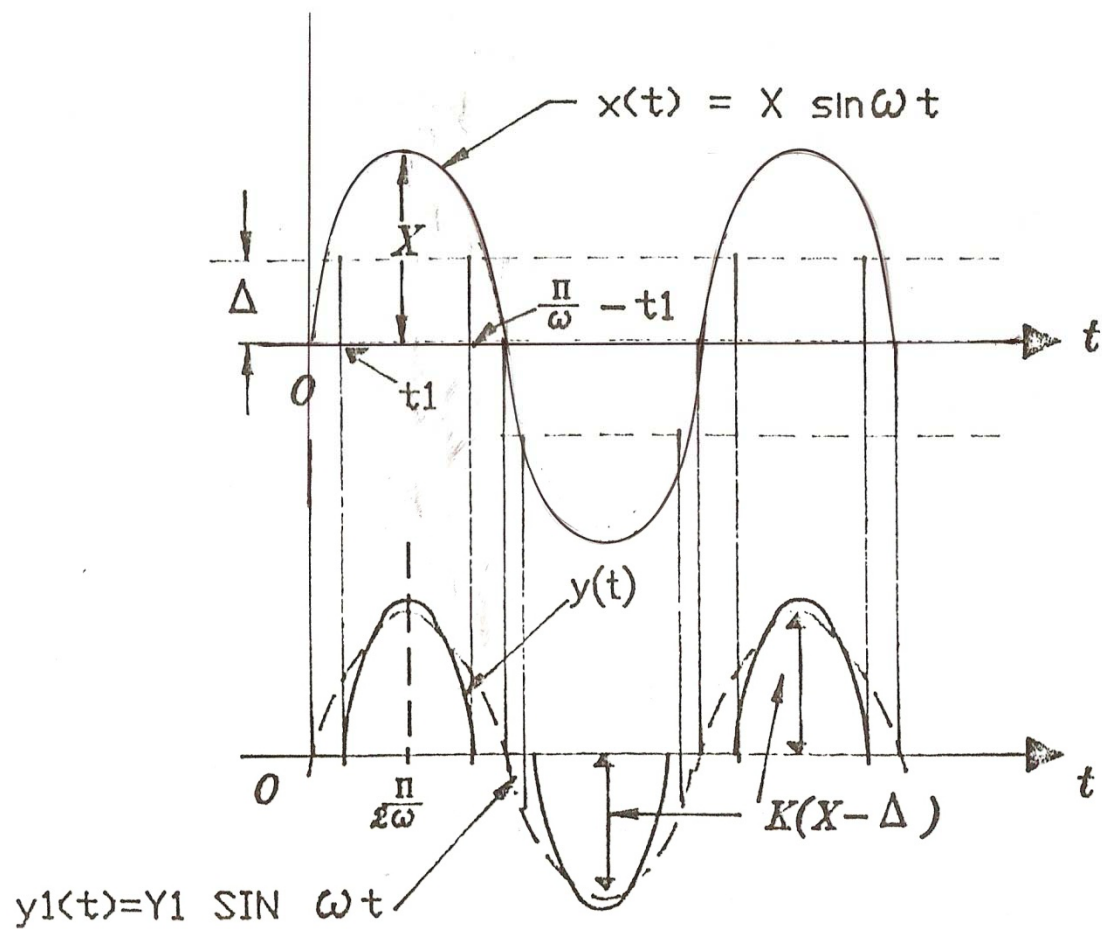


Fig. (5) Input and Output Waveforms for the Dead – Zone Nonlinearity[14,16]

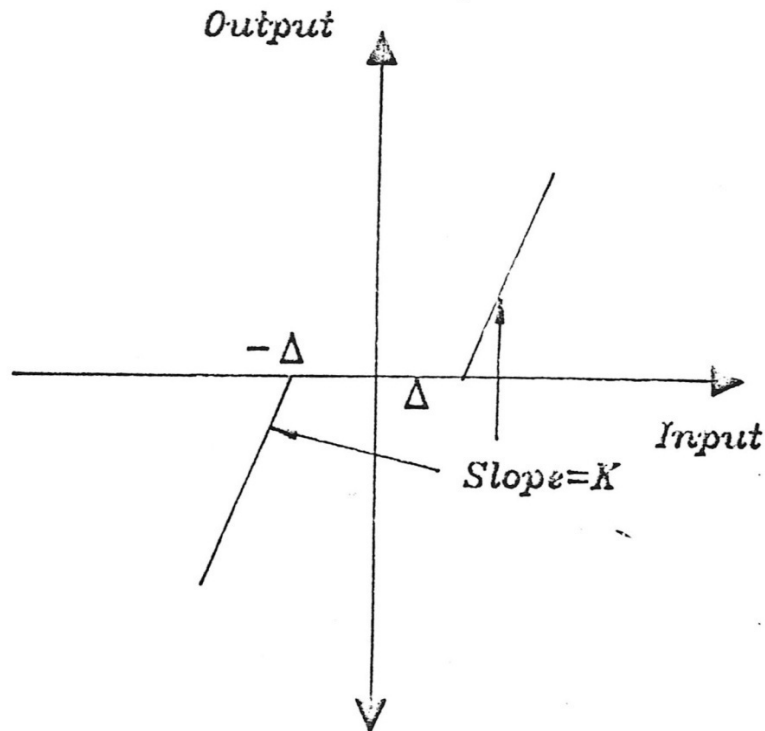


Fig. (6) Input - Output Characteristics Curve for the Dead - Zone Nonlinearity[14,16]

$$Y_1 = 4XK / \pi \left[\int_{wt_1}^{\pi/2} \sin^2 wt \, d(wt) - \sin wt_1 \int_{wt_1}^{\pi/2} \sin wt \, d(wt) \right] \dots\dots\dots(11)$$

$$Y_1 = 2XK / \pi \left[\pi / 2 - \sin^{-1} (\Delta / X) - \Delta / X \sqrt{1 - \left(\frac{\Delta}{X} \right)^2} \right]$$

The describing function for an element with dead zone can be obtained as

$$N = (Y_1 / X) / \underline{O}^\circ$$

$$N = K - 2K / \pi \left[\sin^{-1} \Delta / X + \Delta / X \sqrt{1 - \left(\frac{\Delta}{X} \right)^2} \right] \dots\dots\dots(12)$$

While

X : input signal amplitude (variant)

N : describing function value

1/N : inverse of describing function value

NOTE : All constants are needed for simulation taken from[Victor 2009,Ali,Sami and Karim 2005].

The stability test for the dead-zone is shown in fig.(7) for both $\Delta = 1.5$ and $\Delta = 2.5$

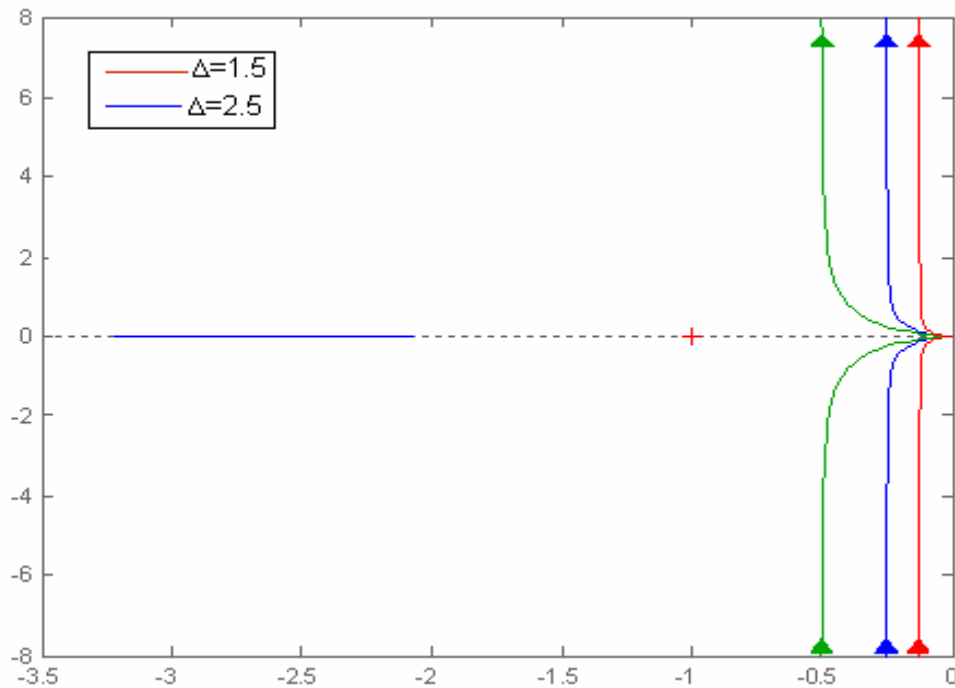


Fig. () Stability test for Dead – Zone

B - Saturation Element :

The input-output waveforms for the saturation nonlinearity is shown in fig.(8). For small input signals, the output of a saturation element is proportional to the input. For large input signals, the output will not increase proportionally, and finally for very large input signals the output is constant. Fig.(9) is shown the input and output characteristic curve for the saturation nonlinearity.

The describing function for such an element can be obtained as: [Francis 1982, and Ogata 1970]

$$N = 2K / \pi \left[\sin^{-1} (S / X) + S / X \sqrt{1 - \left(\frac{S}{X} \right)^2} \right] \quad \dots\dots\dots(13)$$

Fig.(10) shows the stability test for saturation element (s=1)

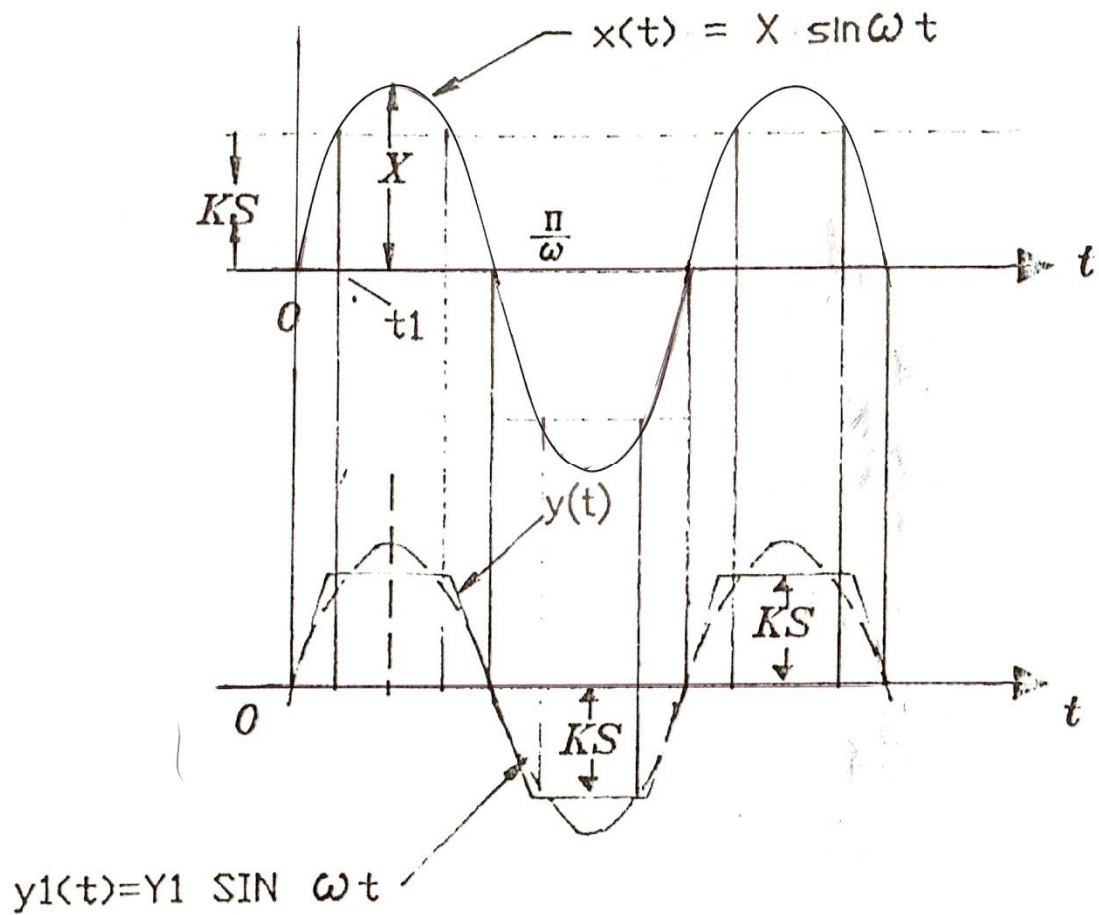


Fig. (8) Input and Output Waveforms for the Saturation Nonlinearity[14,16]

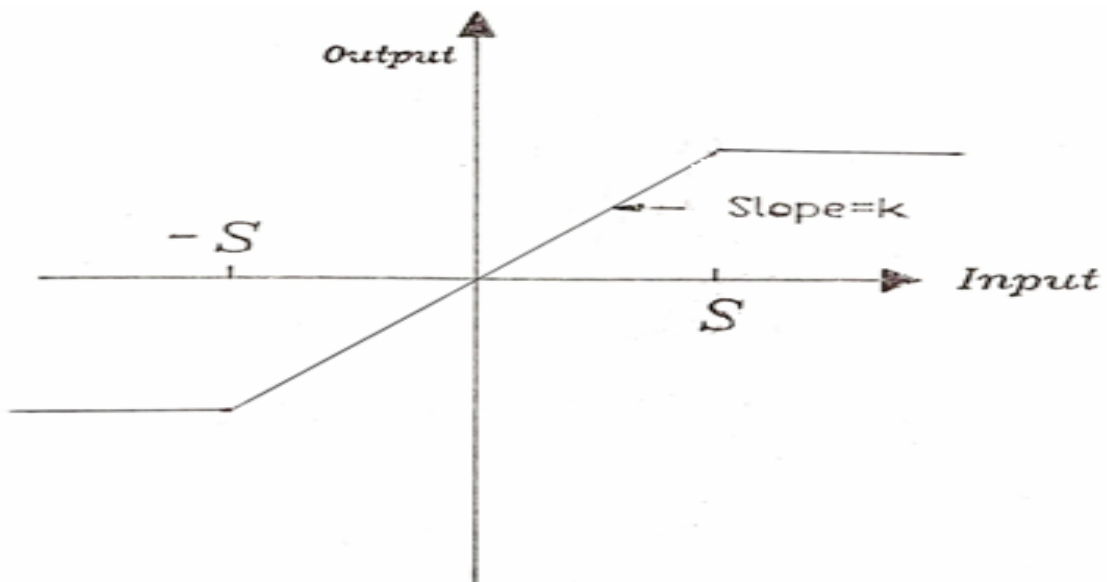


Fig. (9) Input - Output Characteristics Curve
for the Saturation Nonlinearity[14,16]

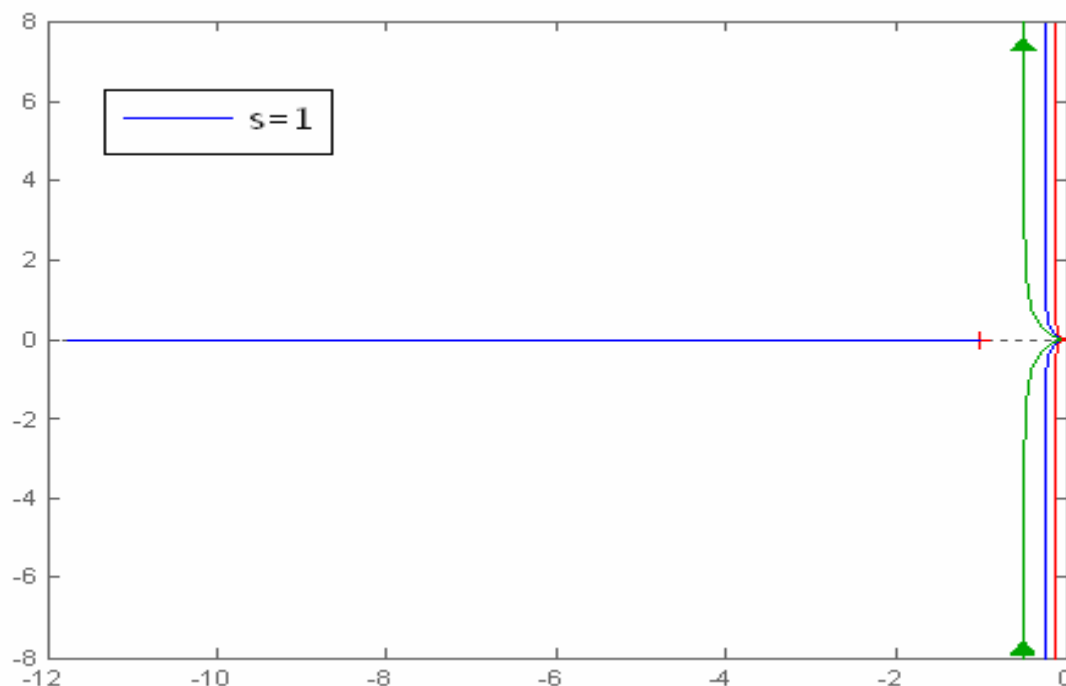


Fig. (10) Stability test for Saturation Element

C- On – Off with Hysteresis Element:

The input-output waveforms is shown in fig.(11),while the input and output characteristic curve are shown in fig.(12) . Clearly, the output is a square wave, but it lags behind the input by: [Francis 1982,and Ogata 1970]

$$[wt_1 = \sin^{-1} (h / x)] \dots\dots\dots(14)$$

Hence the describing function for this nonlinear element is

$$N = 4M / \pi X \quad \underline{ / - \sin^{-1} (h/x) } \dots\dots\dots(15)$$

The stability test for this case is shown in fig.(13) for both $h=1$ and $h=2$.

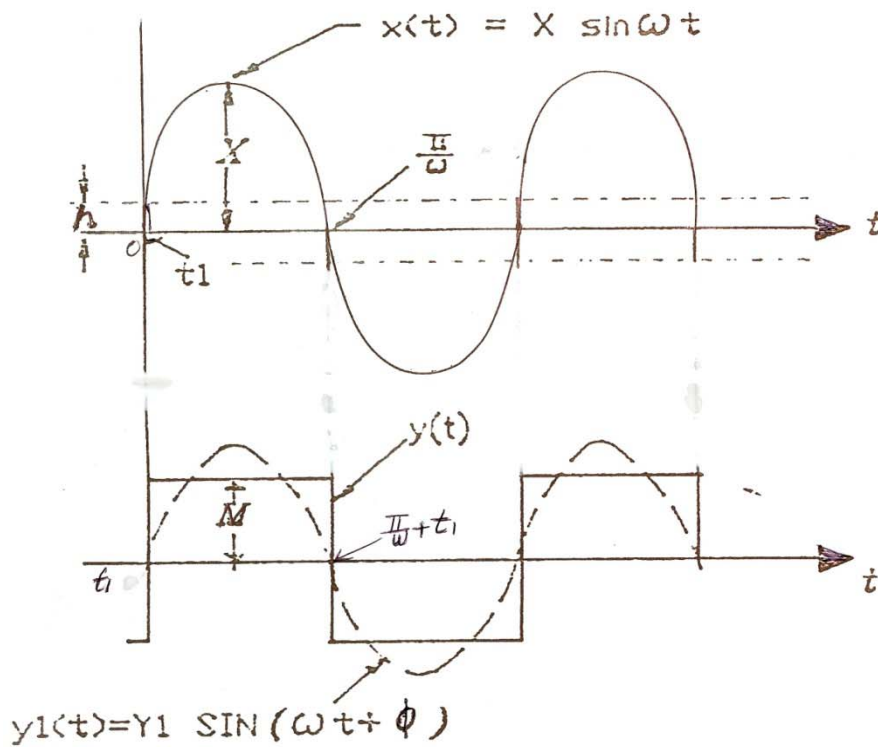


Fig. (11) Input - Output Waveforms for the ON-OFF Nonlinearity with Hysteresis[14,16]

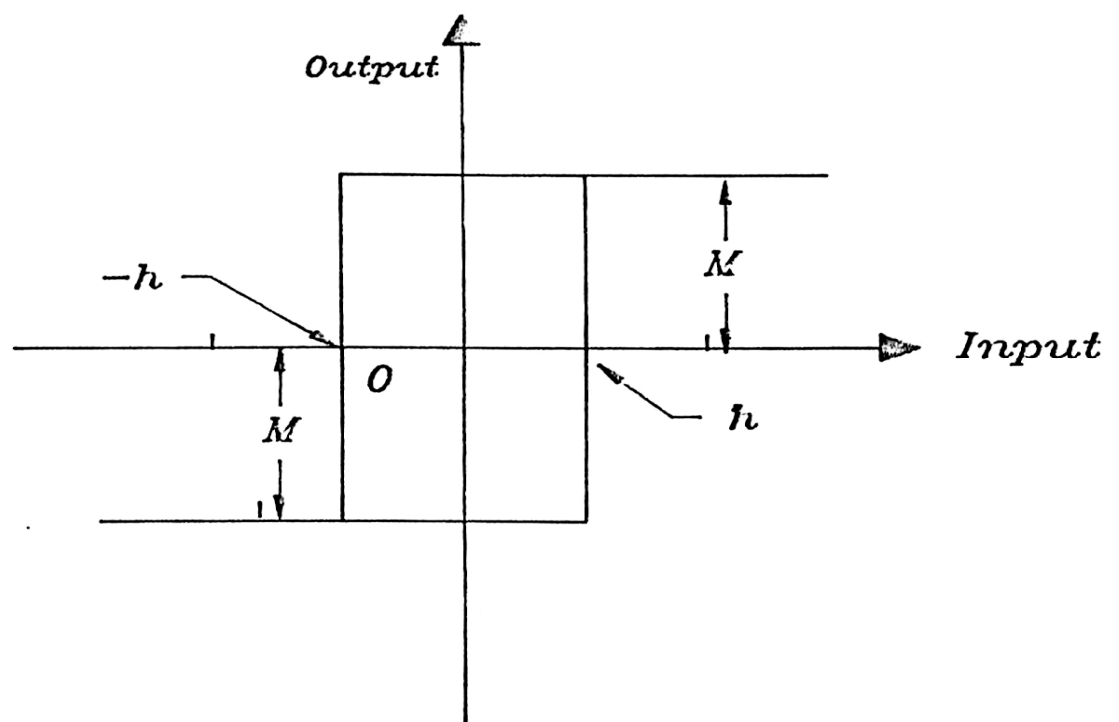


Fig. (2) Input - Output Characteristics Curve for the ON-OFF Nonlinearity with Hysteresis[14,16]

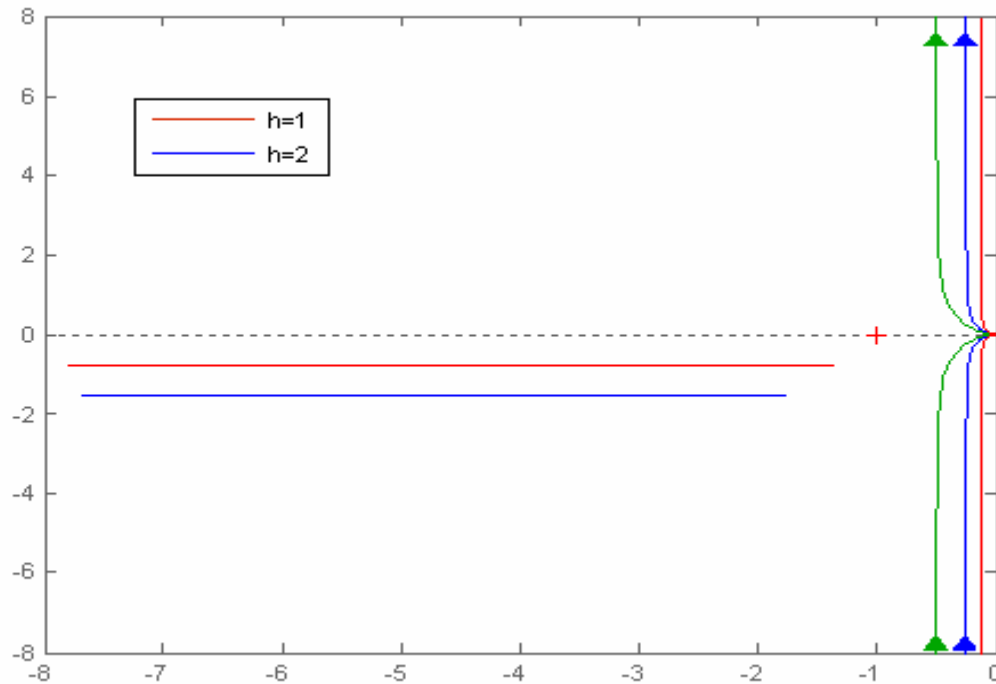


Fig. (3) Stability test for Hysteresis

D – Backlash Element

The input and output waveforms for a backlash nonlinearity is shown in fig.(14), while input-output characteristic curves for this element is shown in fig.(15). The describing function for this nonlinear element is:[Victor 2009]

$$N = A^2 + B^2 / \tan^{-1} A/B \quad \dots\dots\dots(16)$$

$$R = h / x , \quad A = (R / \pi) (R - 2)$$

$$B = 1 / \pi [\pi/2 + \sin^{-1} (1 - R) + (1 - R) \sqrt{2R - R^2}]$$

The stability test of backlash element is shown in fig.(16) for h=1 and h=2.

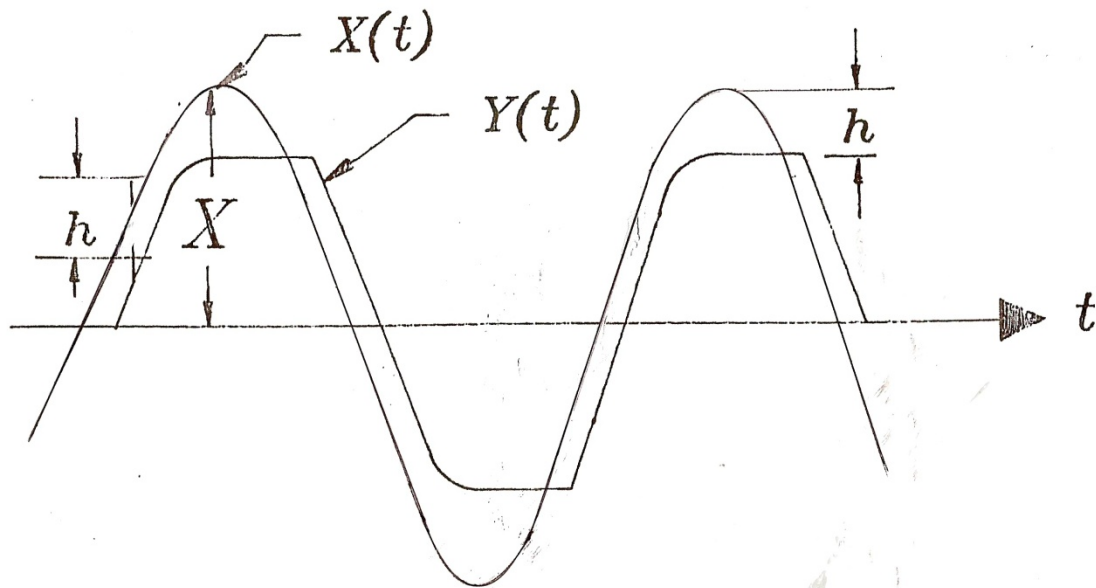


Fig. (14) Input – Output Waveforms for a Backlash Nonlinearity[14,16]

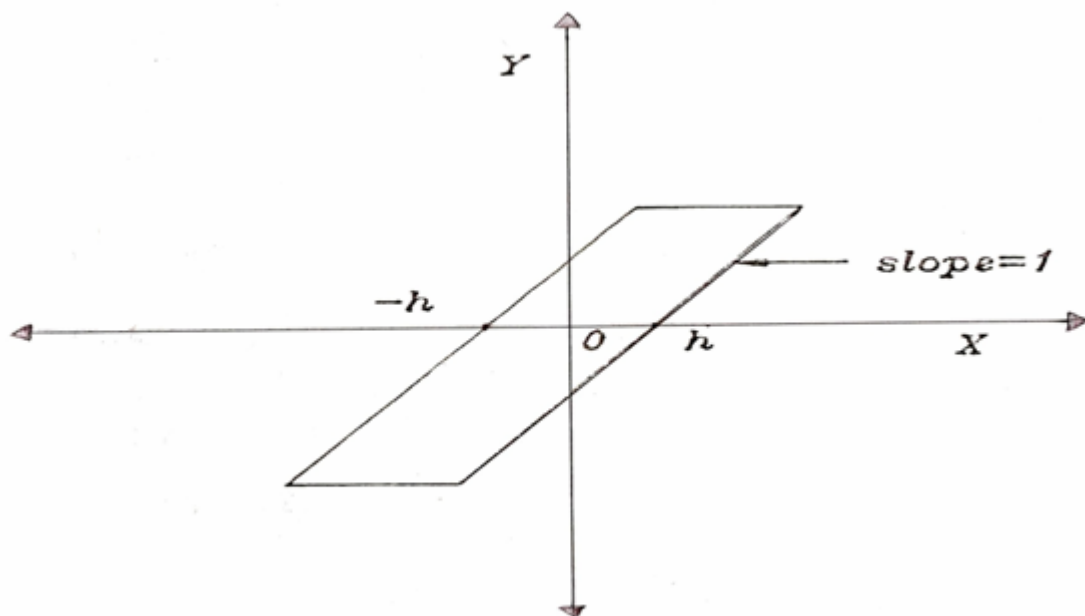


Fig.(15) Input – Output Characteristics Curve for a Backlash Nonlinearity[17].

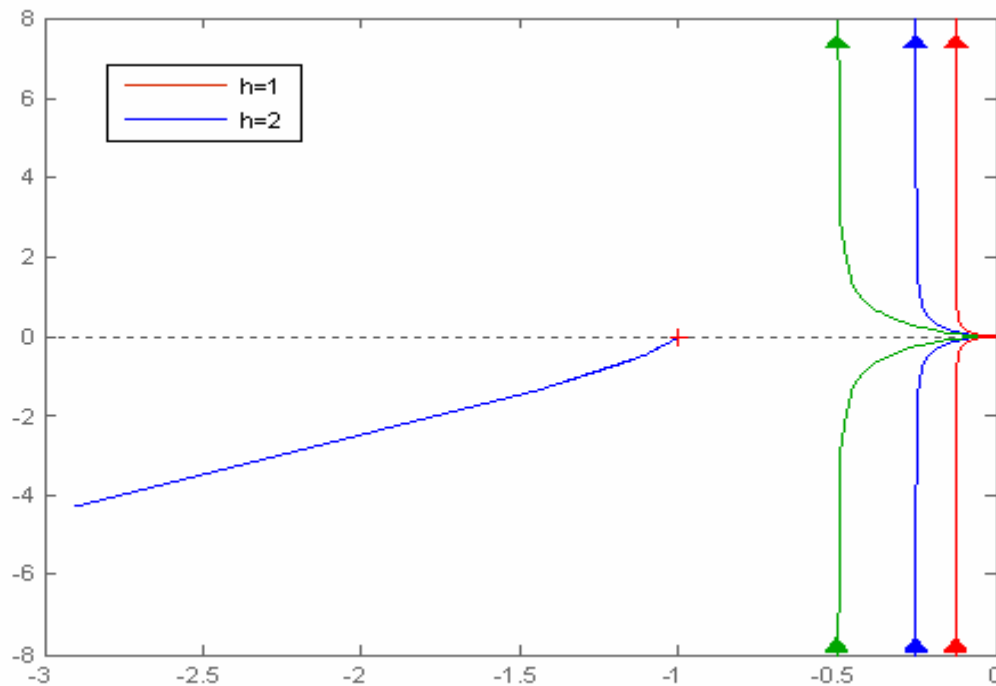


Fig. (16) Stability Test for Backlash Element

7- CONCLUSION:

The describing function of nonlinearity can be used to determine the systems stability, provided that the harmonics are sufficiently attenuated. In this work, by using describing function analysis show that for variable input amplitude:

- 1) The $(-1/N)$ locus for dead-zone element lies on the negative real axis and extends to $-\infty$ for both cases ($\Delta = 1.5$ and $\Delta = 2.5$) and not intersects with Nyquist's plot. So the system will be stable.
- 2) The $(-1/N)$ locus for saturation element lies also on the negative real axis and extends to $-\infty$ for ($s=1$) and not intersects with the Nyquist's plot. Therefore the system will be stable also.
- 3) For On-Off with hysteresis element the $(-1/N)$ locus for ($h=1$ and $h=2$) be a straight line lies to the left of Nyquist's plot and never intersects with it, hence the system is stable.
- 4) For backlash element the $(-1/N)$ locus for ($h=1$ and $h=2$) be a curve because of ϕ (the phase shift angle between input and output), but also does not intersect the Nyquist's plot; i.e. the system is stable.

The results are obtained above explain that the main nonlinearities are not effect on servo system stability even when changing the system gains or the nonlinear element constants the servo system remain stable. Therefore, the servo system can be assumed as a linear system and all linear analyses can be apply on it.

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