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STUDY OF MIXED CONVECTION IN A SQUARE ENCLOSURE WITH A ROTATING CIRCULAR CYLINDER AT DIFFERENT VERTICAL LOCATIONS

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Abstract

Mixed convection in a square enclosure with a rotating circular cylinder located at different vertical locations is numerically studied. The horizontal and vertical walls are kept at constant temperature T_c, while the surface of the rotating circular cylinder is maintained at a constant temperature T_h. A two dimensional solution for steady laminar mixed convection is obtained by solving governing equations in stream function-vorticity form with finite difference technique for different Richardson number ($Ri=Gr/Re^2$) varying over the range of 0.0 to 10. Depending on the Richardson number the convection phenomena inside the enclosure becomes natural, mixed, and forced convection. For the first time this study goes to investigate the effect of changing the rotating circular cylinder positions along the vertical centerline on the fluid flow and heat transfer inside the enclosure. The phenomena inside the enclosure is analyzed through a streamlines, isothermal patterns and average Nusselt numbers. The results show that the flow field and temperature distribution inside the enclosure are strongly dependent on the Richardson numbers and the position of the rotating circular cylinder, and the enclosure has maximum heat transfer rate when the cylinder near the bottom wall (δ=0.25).

Keywords: Mixed Convection, Square Cavity, Rotating Circular Cylinder, Richardson Number, Vorticity Stream Method.

دراسة للحمل المختلط داخل حيز مربع بوجود اسطوانة دائرية دوارة في مواقع عمودية مختلفة

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الخلاصة:

أجريت دراسة عددية للحمل المختلط داخل حيز مربع يحتوي على اسطوانة دائرية دوارة وضعت في مواضع مختلفة بالاتجاه العمودي . حفظت الجدران العمودية و الأفقية للحيز المربع تحت درجة حرارة باردة (T_c) بينما حفظ سطح الاسطوانة تحت درجة حرارة ساخنة (T_h). الحل الثنائي البعد للحالة المستقرة ولحالة الحمل المختلط وجدت بواسطة حل المعادلات الحاكمة بصيغة دالة الجريان-الدوامية باستخدام تقنية ألفروقات المحددة لأرقام ريكاردسون مختلفة تتغير ضمن مدى من 0 إلى 10. بالاعتماد على رقم ريكاردسون فان ظاهرة الحمل داخل الحيز تصبح حمل حر أو حمل مختلط أو حمل قسري.لأول مرة تذهب هذه الدراسة لتوضيح تأثير تغيير موقع الاسطوانة الدائرية الدوارة على طول الخط العمودي للحيز و تأثير ذلك على جريان المائع وانتقال الحرارة داخل الحيز. مُثلت نتائج هذه الظاهرة بخطوط الجريان وخطوط الحرارة و أرقام نسلت. أظهرت النتائج أن خطوط الجريان وخطوط الحرارة تعتمد كثيرا على رقم ريكاردسون وموقع الاسطوانة الدائرية الدوارة وان الحيز يمتلك أعلى معدل لانتقال الحرارة عندما تكون الاسطوانة بالقرب من السطح السفلي للحيز (δ=0.25).

1. Introduction

Mixed convection in enclosure is relevant to many industrial and in environmental applications such as in heat exchanger, ventilation of building, design of solar collectors, nuclear and chemical reactors and cooling of electronic equipments etc. In engineering application the geometries that arise however are more complicated than simple cavity configuration filled with a convection fluid. The geometric configuration of interest can be enriched by adding a rotating cylinder, placed inside the enclosure, thus giving rise to a mixed convection problem. The applications are in numerous areas such as ventilation of enclosures, cooling of electronic systems, solar power collector. The resulting flow is due both to the natural convection component, associated to buoyancy and to the forced convection associated to the cylinder rotation. This can rotate in the direction corresponding to combined or opposite natural and forced convection, thus resulting on the intensification or on the attenuation of the convection flow, and of the corresponding overall thermal performance of the enclosure, respectively. This can be the model for real situations where a rotating shaft can be used to control the natural convection taking place on an enclosure, and thus to control the overall thermal performance of the enclosure, in order to increase or to reduce its effect.

Some investigators have deal the convection inside an enclosure that includes a cylinder, rotating in the center of the enclosure or stationary but for non centered locations, and in the presence of pure forced convection, mixed convection or pure natural convection. Buoyancy driven flow and heat transfer between a cylinder and its surrounding medium has been a problem of considerable importance. This problem has a wide range of applications. Energy storage devices, crop dryers, crude oil storage tanks and spent fuel storage of nuclear power plants are a few to mention. **Karim et al.[1]** have experimentally demonstrated the influence of horizontal confinement on heat transfer around a cylinder for Rayleigh numbers ranging from 10^3 to 10^5. These authors found that the heat flux around the cylinder increases with decreasing distance between the cylinder and the enclosure wall. **Abu-Hijleh and Heilen[2]** has studied numerically the Nusselt number due to laminar mixed convection from a rotating isothermal cylinder. The study covered a wide range of parameters:$5 \leq Re \leq 450$ and $0.1 \leq k \leq 10.0$. A correlation was proposed which can be used to accurately predict the Nusselt number for the range of parameters studied. **Cesini et al.[3]** carried out experiments similar to those of **Karim et al.[1]** but found that the heat flux from the cylinder reached a maximum for an optimal wall-to-wall distance of 2.9 times the cylinder diameter. **Mohamed et al.[4]** have investigated the effect of vertical confinement on the natural convection flow and heat transfer around a horizontal heated cylinder. It is shown that the primary effect of the vertical confinement is an increase heat flux on the upper of the cylinder for given separation distance between the cylinder and the fluid boundary. **Aydin Misirlioglu[5]** investigated numerically the heat transfer in square cavity with a rotating circular cylinder centered within it and filled with clear fluid or porous medium. The results indicate that rotation direction of the cylinder has significant effect on the natural and forced convection regimes, especially for the clear fluid case. **Sumon et al.[6]** carried out numerically study of natural convection in two-dimensional square enclosure containing adiabatic cylinder at the center by using finite element method. **Kim et al.[7]** numerically studied two-dimensional steady natural convection for cold outer square enclosure and a hot inner circular cylinder for different Rayleigh number varying over the range of 10^3-10^6. The study goes further to investigate the effect of the inner cylinder vertical location on the heat transfer and fluid flow. **Rahman et al.[8]** studied laminar mixed convection flow inside a vented square cavity without conducting horizontal solid cylinder placed at the center of the cavity by using a finite element method. The Richardson number is varied from 0.0 to 5.0 and the cylinder diameter is varied from 0.0 to 0.6. It is found that the stream lines, isotherms, average Nusselt number at the heated surface, average temperature of the fluid in the cavity and dimensionless temperature at the cylinder center strongly depend on the Richardson number as well as the diameter of the cylinder. **Rahman et al.[9]** conducted numerical simulation for mixed convection flow in a vented cavity with a heated conducting horizontal square cylinder. The results indicate that the flow field and temperature distribution inside the cavity are strongly dependent on the Richardson number and the position of the inner cylinder. **Shih et al.[10]** studied the periodic of laminar flow and heat transfer due to an insulated or isothermal various rotating objects (circle, square, and equilateral triangle) placed in the center of square cavity. Transient variations of the average Nusselt numbers of the respective systems show that for high Re numbers, a quasiperiodic behavior while for low Re numbers, periodicity of the system is clearly observed. **Costa and Raimudo[11]** analyzed the mixed convection in a square enclosure with rotating cylinder centered within it is numerically. The rotating cylinder participates on both the conductive and convective heat transfer process and exchanges heat with the fluid naturally, without imposition of a thermal condition at its surface. Results clearly show how the cylinder effects the thermal performance of the enclosure.

2.Physical Model and Coordinate System:

A schematic of the system considered in the present study is shown in **Fig.(1)**. The system consists of a square enclosure with sides of length L, within which a rotating circular cylinder with radius (R=0.2L) is located and moves along the vertical centerline in the range from -0.25L to 0.25L. The walls of the square enclosure was kept at constant low temperature of T_c, whereas the cylinder was kept at constant high temperature T_h, under the influence of the vertical gravitational field, the enclosure walls temperature and the temperature at the surface of rotating circular cylinder at different levels of temperature lead to a natural convection problem. Due to the non-slip boundary condition for velocity on it is surface, the rotating cylinder, of radius R, induced a forced flow, the overall resulting situation being a mixed convection problem. The free space in the enclosure is filled by a Newton fluid. The fluid density is not effected by pressure change(incompressible fluid). But it changes when temperature changes. Density on the buoyancy term is temperature dependent, and the Boussinesg approximation is used. All the remaining thermo physical properties of the fluid are assumed to be constant, including the density appearing in the convection term. The flow is assumed to be laminar. Thermal radiation heat transfer between the walls and the surface of circular cylinder is negligible, and the fluid assumed to be radioactively non-participating.

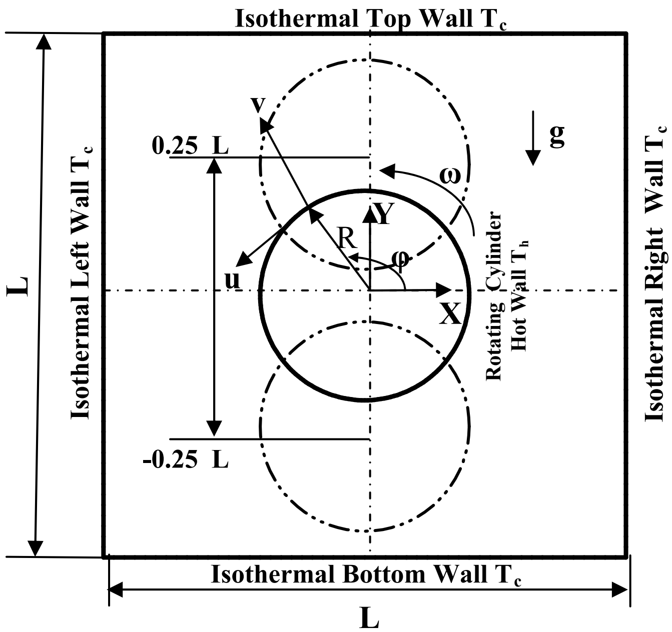

Fig. 1. Schematic diagram of the square enclosure with inner rotating circular cylinder with counterclockwise direction along with coordinate system and boundary conditions

The energy terms due to viscous dissipation and change of temperature due to reversible deformation(work of pressure forces) are not taken into account in the present study.

3. Numerical Modeling Equation

3.1. Governing Equations:

Mixed convection fluid flow inside an enclosure obeys the mass conservation equation that reads, in its dimensionless form:

Mass conservation [11]:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad \textbf{(1)}$$

The momentum equations:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad \textbf{(2)}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta \quad \textbf{(3)}$$

And the energy conservation equation:

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad \textbf{(4)}$$

The dimensionless parameter appearing in the forgoing equations are the space coordinates, the velocity components, the temperature and the driving pressure, defined, respectively as follows [10], [11]:

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{uL}{\alpha}\ ,\ V = \frac{vL}{\alpha},\ \theta = \frac{T - T_c}{T_h - T_c},\ P = \frac{p}{\rho[R\omega]^2} \quad \textbf{(5)}$$

And the Prandtl and Rayleigh numbers emerge as:

$$\Pr = \frac{\upsilon}{\alpha} \qquad Ra = \frac{g\beta(T_h - T_c)L^3 \Pr}{\upsilon^2} \quad \textbf{(6)}$$

For the problem under analysis it can be defined the following dimensionless parameters can be defined.[11]:

$$R^* = \frac{R}{L} \quad \textbf{(7)}$$

$$\omega^* = \frac{\omega L^2}{\alpha} \quad \textbf{(8)}$$

Over the rotating cylinder, velocity components are specified as:

$$U_c = \omega R \quad \textbf{(9)}$$

$$U = U_c \cos\varphi \quad \textbf{(9a)}$$

$$V = U_c \sin\varphi$$

Where φ varies from 0^o to 360^o. The typical dimensionless parameter used to evalute the relative importance of the natural and force convection is modified from of the Richardson number, defined elsewhere as[11]:

$Ri = \frac{(Ra/\Pr)}{\mathrm{Re}^2}$ **,** where Re is the Reynolds number ($\mathrm{Re} = \frac{(\omega R)D}{\nu}$ **),** where **D** is the characteristic length. For the present problem and dimensionless strategy, this parameter results.

$$Ri = \frac{Ra/\Pr}{[(\omega R)D/\upsilon]^2} = \frac{Ra\Pr}{4\omega^{*2}R^{*4}} \quad \textbf{(10)}$$

As stated in Eq.(10), Richardson number is an alternative parameter to either Ra, Pr, ω^* and will be only used to identify the heat transfer regime.

3.2. Boundary Conditions

The corresponding boundary conditions for the above problem are given by:

All walls U,V = 0 **(11)**

Surface of the cylinder $U = U_c \cos\varphi \quad V = U_c \sin\varphi$ **(12)**

All walls of the enclosure $\theta = 0$ **(13)**

Surface of the cylinder $\theta = 1$ **(14)**

Vorticity-stream function approach to two-dimensional problem of solving Navir-Stocks equations is rather easy to used to solve current problem . Stream vorticity (ζ) and stream function (ψ) must be obtained during the computation.

First let us provide some definition which will simplify steady, two-dimensional Navir-Stocks equations. The main goal of that is to remove explicitly from Navir-Stocks equations. We can do it with the procedure as follows. The definition of vorticity for two-dimensional case is:

$\zeta = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}$ and stream function definition is**:** $\frac{\partial \psi}{\partial Y} = U\ , \frac{\partial \psi}{\partial X} = -V$ **(15)**

$$\Psi = \frac{\psi \Pr}{\nu}, \; \varsigma = \frac{\xi (L)^2 \Pr}{\nu} \quad \textbf{(16)}$$

The above definition combined with eqations (1), (2), (3) and (4).
It will eliminate pressure from the momentum equations. That combination will give us non-pressure vorticity transport equation which in steady, two-dimensional form can be written as follow [11]:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\varsigma \quad \textbf{(17)}$$

$$\frac{\partial \varsigma}{\partial X}\frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial X}\frac{\partial \varsigma}{\partial Y} = \frac{1}{\mathrm{Re}}\nabla^2 \varsigma + Ri\frac{\partial \theta}{\partial X} \quad \textbf{(18)}$$

$$\frac{\partial \theta}{\partial X}\frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial X}\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Re}\Pr}\nabla^2 \theta \quad \textbf{(19)}$$

The boundary condition for this equations becomes:

All walls and surface of the cylinder $\Psi = 0$ **(20)**

All walls $\varsigma_w = \dfrac{2(\Psi_{w+1} - \Psi_w)}{(\Delta n)^2}$ **(21)**

Surface of the cylinder $\varsigma_{Ni,Nj} = \dfrac{2(\Psi_{Ni,Nj} - \Delta x - \Delta y - \Psi_{Ni,Nj-1})}{(\Delta x)^2 + (\Delta y)^2}$ **(22)**

3.3 Heat Transfer Calculations

The total dimensionless heat transferred through the enclosure, from the hot surface to the cold walls one is here represented by the overall Nusselt number, *Nu*, defined as [11]:

$$Nu = \frac{\int_0^L -k\left(\frac{\partial \theta}{\partial x}\right)_{x=0} dy}{k\left[{(T_h - T_c)}/{L}\right]L} = \int_0^1 -\left(\frac{\partial \theta}{\partial X}\right)_{X=0} dY \quad \textbf{(23)}$$

4. Numerical modeling and Verification

Governing equations of mixed convection (Eqs. (1)-(4)) which are written in stream function-vorticity form (Eqs. (17)-(19)) for laminar regime in two-dimensional form for steady, incompressible, and Newtonian fluid with Boussinesq approximation is used to characterize the buoyancy effect. It is assumed that radiation heat exchange is negligible according to other modes of heat transfer and the gravity acts in vertical direction, cylinder rotating with anti-clockwise direction. Finite difference is used to solve equations ((17)-(19)). Central difference method is applied for discretization of equations. The solution of linear algebraic equations was performed using successive under relaxation (SUR) method. As convergence criteria, 10^{-4} is chosen for all dependent variable and value of 0.1 is taken for under relaxation parameter. The computational geometry in the x-y plane with non-uniform grid distribution. A grid resolution of 200*100 along horizontal (x) and vertical (y) directions was employed as shown in **Fig(2a)** which presented the mesh corresponding to cylinder with R=0.2. The grid were uniformly distributed near the walls in order to account for the high gradients using the algebraic function. The denser grid lines were uniformly distributed with resolution of For the purpose of code validation, the natural convection problem for a low temperature outer square enclosure and high temperature inner circular cylinder with non-rotating condition was tested and compared the result of streamlines and isothermal lines with **Kim et al[7]** as shown in **Fig.(3)**. The result show a good agreement with results of **Kim et al[7]**.

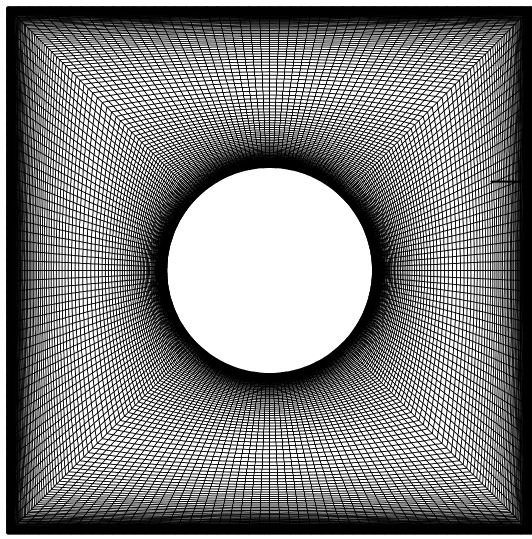

Fig. 2a. A typical grid distribution (200 x 100) with non-uniform and non- orthogonal distributions for δ = 0.

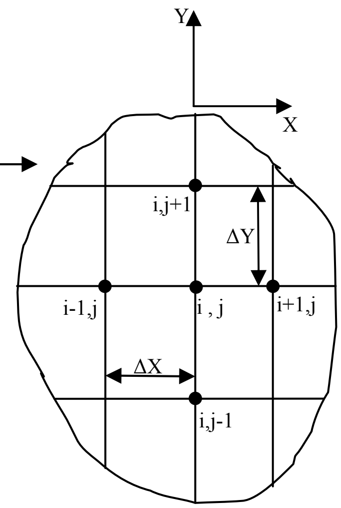

Fig.2b. Grid arrangement

4.1 Numerical procedure

Governing equations in stream-vorticity function form (Eqs.(17)-(19)) were solved using central difference method. Algebraic equations were obtained via Taylor series and they solved with Successive Under Relaxation (SUR) technique, iteratively, which is defined in Eq.24.

$$\phi = \phi^0 + r(\phi^{calculated} - \phi^0) \qquad \textbf{(24)}$$

In this equation, ϕ is any variable and ϕ^0 and $\phi^{\text{calclated}}$ are the old value and calculated value of variable. However, ϕ^1 is the value for next step and r shows the relaxation factor whose value is 0.1. Schematic of grid arrangement is given in **Fig.(2b)**. Discretization equations is performed according to this grid in **Appendix (A)**

4-2 Flow Chart

The flow chart has been shown in Fig.3. It starts with input data and initial value. Stream function, vorticity and temperatures are calculated by solving algebraic equations. After than the necessary values such as Nusselt number, pressure, velocity vector, heat vector is calculated. If a convergence criterion is obtained, the output file is printed. Otherwise, it goes to starting point. The program which used is Fortran90, and the time of the run ranges from 20 min to 40 min for each case.

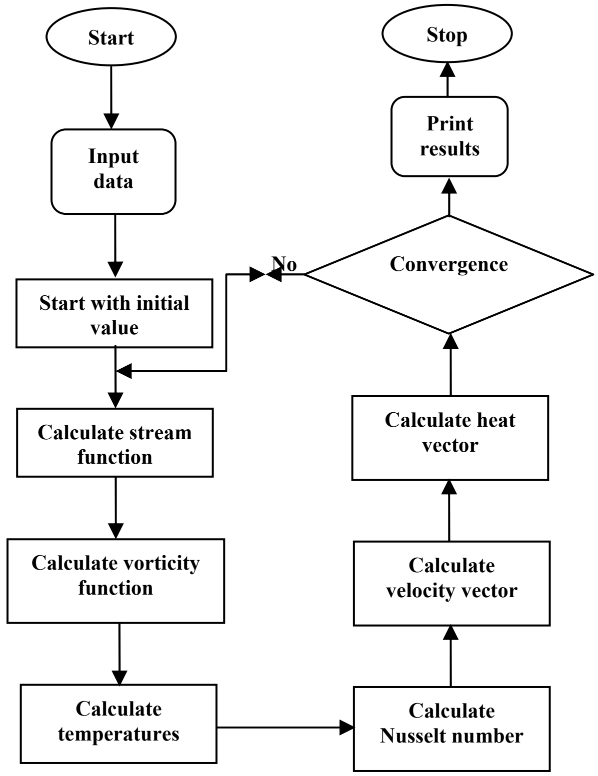

Fig.3. Flow chart for the problem

5. Results and Discussion

In order to understand the flow, temperature fields and heat transfer characteristics of the typical enclosure a total of 48 cases are considered. To study the effect of rotating cylinder speed ω=0.625, 1.25, 1.875 and 2.5 rad/sec are consider, the position of the rotating cylinder δ=0.25, 0.5 and 0.75 are considered. The fluid inside the enclosure is dry air with Pr=0.71 and Richardson number varied from 0 to 10, this selection of Richardson number provides to observe the different between forced convection ($Gr/Re^2<<1$), mixed convection ($Gr/Re^2=1$) and natural convection ($Gr/Re^2>>1$). Reynolds number Re=50, 100, 150 and 200 therefore Grashof number changing for each case, the value of cylinder diameter is 0.2 and rotating with counter-clock wise direction. Flow, temperature fields and Nusselt number are examined by stream lines and isotherm plots.

5-1 Flow structure and Temperature fields

The analysis of flow structure and temperature field are developed getting use of contour of the dimensionless stream function (stream lines) and temperature (isotherms).

Fig.4 is presented the stream lines (left) and isotherms (right) for R=0.2 Richardson number Ri=0, for different Reynolds number (vertical) and for different position δ (horizontal), in this case forced convection is dominant. The first column are the flow fields and isothermal lines for the case when δ=0.25 and different Reynolds number, the fluid pulled upward toward the motion of rotating cylinder (counter-clockwise) and small vortex appear above the rotating cylinder due to that the moving of rotating cylinder surface and stationary of the other walls of the enclosure, with increasing of Reynolds number the vortex moving to the left because the pulled force increase with increasing of rotating speed. Isothermal boundary conditions, which are defined as a neutral boundary condition (hot temperature on the surface of rotating cylinder and cold temperature of the other walls of the enclosure). the isotherms are clustered near the circle and fluid attached the cylinder surface is heated, therefore the temperature varied along the surface of the rotating cylinder where the temperature decreases with direction of rotating cylinder because the hot air attached to surface of the cylinder cooled by the fresh cold air above it, this variation becomes more uniform with increasing of the Reynolds number because the hot air have not enough time to cooled itself from fresh cold air. Because of the small space under the rotating cylinder therefore this region was hottest. The second column show the flow field and isotherms line for δ=0.5, because of the symmetrical boundary conditions on the horizontal and vertical walls the flow and temperature fields are symmetrical about the horizontal and vertical center line of the enclosure. In this case, considerably more space exists for the fluid flow between the cylinder and the enclosure walls therefore, the amount of mass influenced by the rotation of the cylinder increases. The circular flow patterns are approaching to the cavity walls creating clockwise rotating small vortices at each corner of the enclosure. As the Reynolds number increase the vortices size increase because the effect of cylinder rotation are weak. The isotherm plots also symmetrical about the horizontal and vertical mid plane. The isothermal lines is uniform distributed inside the enclosure and the temperature decreases from the surface of the rotating cylinder to the cold walls along the horizontal and vertical centerline of the enclosure and concentrates towards the hot surface indicating the presence of large temperature gradient, the isothermal contour are similitude for all value of Reynolds numbers. The third double column are the same absolute value of the rotating velocities to that presented in the first double column of Fig.4, but in the location (δ=0.75). In what concerns the flow fields and temperature distribution are similar to that when δ=0.25, the main difference being that the vortex appear on the opposite location because the rotating circular cylinder moving upward.

Results for the same conditions as before in Fig.4, but now for Ri=1, are presented in Fig.5. This means that $Re = \sqrt{Gr}$ **,** the natural convection in this case presented but with poorly effect in flow field and no effected appears in thermal field. The main difference being the convection front associated with the cylinder rotation, which appears on the opposite direction with the natural convection, this effect result as a vortex in the top left side when Re=200. For the case of δ=0.5 the symmetrical behavior vanish because the effect of natural convection. In the case of δ=0.75, small vortex result at bottom left side of the enclosure when Re=150 and 200.

Fig.6, show the result for the same values of the independent dimensionless parameters of the previous figures, namely the counter-clockwise rotation of the circular cylinder (ω) except now the Richardson number Ri=5. Due to the fact that, in this case, the natural convection and the forced convective have opposite effect, in what concerns fluid motion, some vortices appear in the flow. The main fluid motion take place in the clockwise direction, induced by natural convection, close to the rotating cylinder fluid motion occurs in counter-clockwise direction, an in between there are some vortices, where fluid motion is in the clockwise direction. In case of δ=0.25 and Re=50 two vortices result, one above the rotating cylinder and another at top left side of the enclosure, with the increasing Reynolds number a vortices result at bottom left side of the enclosure and the isothermal line show that the heat transfer across the enclosure occur mainly associated with the flow therefore the isothermal lines moves to the right as the effect of combined convection. When δ=0.5 two vortices result on the opposite side (right side) when compared with previous figure. The fluid moves faster as Reynolds number increase, the two vortices penetrate the stagnant fluid then merge to become a primary vortex. Small vortex result in left side when Re=200 because the fluid becomes faster. Isothermal lines uniform distributed at low Reynolds number (Re=50&100) but with increasing Reynolds number the hot region transfer from around rotating cylinder to top left side walls because the effect of combined convection increase. In the case of δ=0.75 the main flow consist of two vortices, this vortices spread with increasing of Reynolds number and third vortices appear besides the rotating cylinder. The isothermal lines clustered near the hot rotating cylinder therefore the high half becomes hot because the light fluid don't move to downward.

Fig.7, give the flow structure for the fluid flow when Ri=10. Even for high value of (Gr/Re^2), in this case of counter-clockwise rotation (right column of figure) there are big vortices formed around the rotating cylinder in the enclosure. These vortices prevent the mass to transfer easily, with increasing of Reynolds number this vortices squeezed thinner and ultimately is divided in two minor vortices. The isothermal lines moves to the right side of the enclosure and still the rotating cylinder the hottest region and heat transfer by natural convection to the top and left side walls. In case of δ=0.5 the flow contains two vortices one of it in top right side and its large in size and another in bottom left side and its small in size, with increasing of Reynolds number the small vortex vanish and the large vortex squeezed thinner and divided into two minor cells located near the top and bottom of the right side wall. With increasing of Reynolds number the effect of combined convection more increase in fluid flow behavior therefore, some vortices around the rotating cylinder appears. The isothermal lines was uniform distributed at low Reynolds number (Re=50&100) and this isothermal lines moves to right when Reynolds number increase and the light fluid moves to up cold wall and concentrates in this region. In case of δ=0.75, it is observed a considerable change on the flow field because the position of rotating cylinder and effect of mixed convection, there are some clockwise recirculation heat cells, in the form of clockwise rotating cells. The isotherms spread over the top half of the fluid domain. Due to the fact that, hot fluid was lightest from the cold fluid therefore the isotherms concentrates at top half side of the enclosure.

5-2 Heat Transfer

The variation of the average Nusselt number with Richardson number, Ri are investigated for the different locations, δ and Reynolds number, Re which are shown in Fig.8. The effect of Reynolds number on heat transfer is clearly demonstrated here. In general, it is observed that the average Nusselt number increases significantly with increasing Richardson number due to increasing the effect of convection. The average Nusselt number increases more rapidly as the Reynolds number increase. This phenomena observed for Ri>1. But for δ=0.25, the effect of Reynolds number is more significant between Richardson number 1 and 10 and for Ri<1, δ=0.5 the profile approach close to each other.

The results show that Nusselt number increases when the inner circular rotating cylinder moves to the top wall (δ=0.75), but when the circular rotating cylinder moves to the bottom wall (δ=0.25) the Nusselt number is higher because the combined convection becomes more effect in the flow field and heat transfer process.

6.Conclusions

A numerical investigation is performed for mixed convection in square enclosure with a rotating circular cylinder inside. A detailed analysis for the distribution of streamlines, isotherms and the average Nusselt number at the hot surface of the rotating circular cylinder is carried out to investigate the effect of the location of rotating circular cylinder on the fluid flow and heat transfer in the square enclosure for different Richardson number in the range of $0 \geq Ri \geq 10$.

It is observed that the location of the rotating circular cylinder has a strong influence on the resulting flow and heat transfer process, as it limits the space for fluid flow between the rotating circular cylinder and the enclosure wall, giving rise to higher or lower local fluid velocities. For centerline location of the rotating circular cylinder (δ=0.5) the average Nusselt number is small for all value of rotating velocity, and it considerable increase when the rotating circular cylinder moves to up or down locations. The value of average Nusselt number is the highest in the combined convection dominated area when the rotating circular cylinder is located near the bottom wall (δ=0.25).

Appendix A

$$-\varsigma_{i,j} = \frac{\Psi_{i+1,j} + \Psi_{i-1,j} - 2\Psi_{i,j}}{\Delta X^2} + \frac{\Psi_{i,j+1} + \Psi_{i,j-1} - 2\Psi_{i,j}}{\Delta Y^2}$$

$$\Psi_{i,j} = \frac{\left[\left(\dfrac{\Psi_{i+1,j} + \Psi_{i-1,j}}{\Delta X^2}\right) + \left(\dfrac{\Psi_{i,j+1} + \Psi_{i,j-1}}{\Delta Y^2}\right)\right] + \varsigma_{i,j}}{\left(\dfrac{2}{\Delta X^2} + \dfrac{2}{\Delta Y^2}\right)}$$

$$\varsigma_{i,j} = \left\{ \begin{array}{l} \left(\dfrac{-\mathrm{Re}}{4\Delta X \Delta Y}\right)\left[\left(\Psi_{i,j+1} - \Psi_{i,j-1}\right)\left(\varsigma_{i+1,j} - \varsigma_{i-1,j}\right) - \left(\Psi_{i+1,j} - \Psi_{i-1,j}\right)\left(\varsigma_{i,j+1} - \varsigma_{i,j-1}\right)\right] + \\ \left(\dfrac{\varsigma_{i+1,j} + \varsigma_{i-1,j}}{\Delta X^2}\right) + \left(\dfrac{\varsigma_{i,j+1} + \varsigma_{i,j-1}}{\Delta Y^2}\right) - Ri\,\mathrm{Re}\left(\dfrac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta X}\right) \end{array} \right\} \Bigg/ \left(\frac{2}{\Delta X^2} + \frac{2}{\Delta Y^2}\right)$$

$$\theta_{i,j} = \left\{ \begin{array}{l} \left(\dfrac{-\mathrm{Re}\,\mathrm{Pr}}{4\Delta X \Delta Y}\right)\left[\left(\Psi_{i,j+1} - \Psi_{i,j-1}\right)\left(\theta_{i+1,j} - \theta_{i-1,j}\right) - \left(\Psi_{i+1,j} - \Psi_{i-1,j}\right)\left(\theta_{i,j+1} - \theta_{i,j-1}\right)\right] + \\ \left(\dfrac{\theta_{i+1,j} + \theta_{i-1,j}}{\Delta X^2}\right) + \left(\dfrac{\theta_{i,j+1} + \theta_{i,j-1}}{\Delta Y^2}\right) \end{array} \right\} \Bigg/ \left(\frac{2}{\Delta X^2} + \frac{2}{\Delta Y^2}\right)$$

Ri	Re	ω	Nu			
0	50	0.625	5.046557			
	100	1.25	5.0527472			
	150	1.875	5.05889			
	200	2.5	5.064836			
01	50	0.625	5.0477			
	100	1.25	5.058705			
	0150	1.875	5.071098			
	0200	2.5	5.08316			
05	50	0.625	5.0761397			
	100	1.25	5.265095			
	150	1.875	5.983949			
	200	2.5	7.014728			
010	50	0.625	5.17749			
	100	1.25	6.436944			
	150	1.875	8.195753			
	200	2.5	9.474571			

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Nomenclature

d Dimensional cylinder length (m)
D Dimensionless cylinder length
Gr Grashof number
g Gravitational acceleration (m/s^2)
k Thermal conductivity of fluid (W/m.K)
L Length of the enclosure (m)
n Normal direction
Nu Nusselt number
p Pressure (N/m^2)
P Dimensionless pressure
Pr Prandtl number
r Relaxation function
R Radius of circular cylinder (m)
Re Reynolds number
Ri Richardson number
T Temperature (K)
T_c Cold temperature (K)
T_h Hot temperature (K)
u, v Cartesian velocity components (m/s)
X, Y Cartesian coordinates

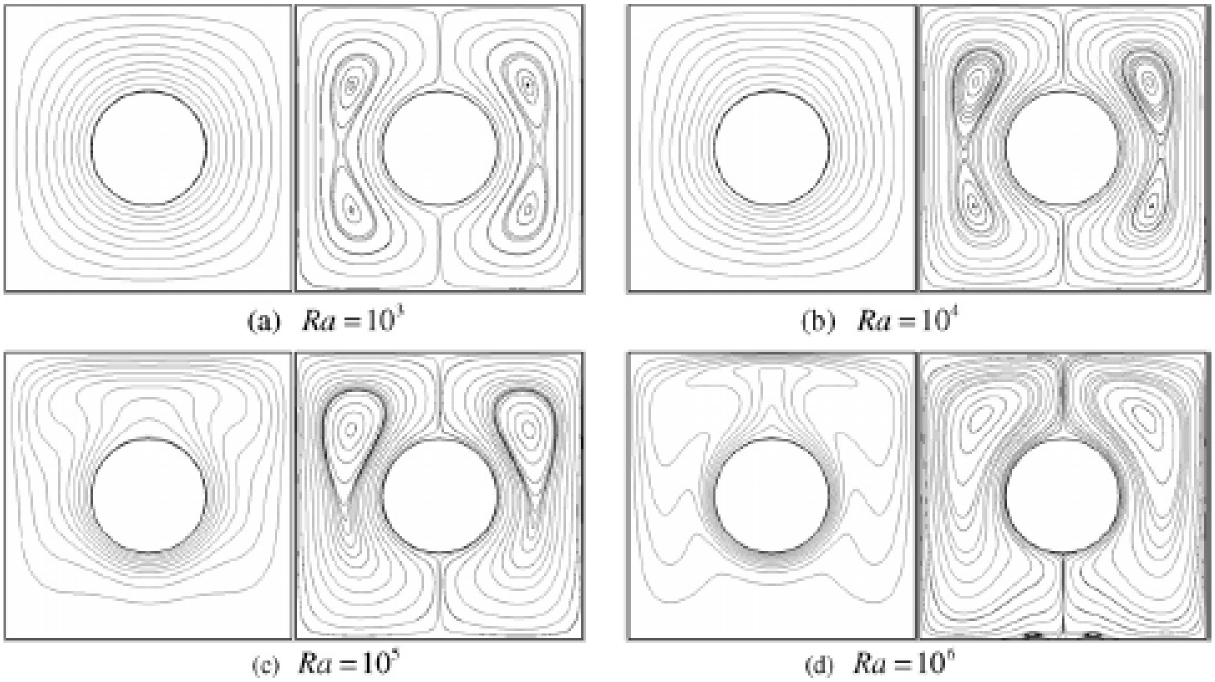
Greek Symbols

α Thermal diffusivity (m^2/s)
β Thermal expansion coefficient (1/K)
ξ Vorticity function
ζ Dimensionless vorticity function
υ Kinematic viscosity (m^2/s)
φ angle of circular cylinder (degree)
θ Dimensionless temperature
ρ Density of the fluid (kg/m^3)
ψ Stream function
Ψ Dimensionless stream function
δ Distance from bottom wall to the circular cylinder center
ω angular velocity (rad/sec)

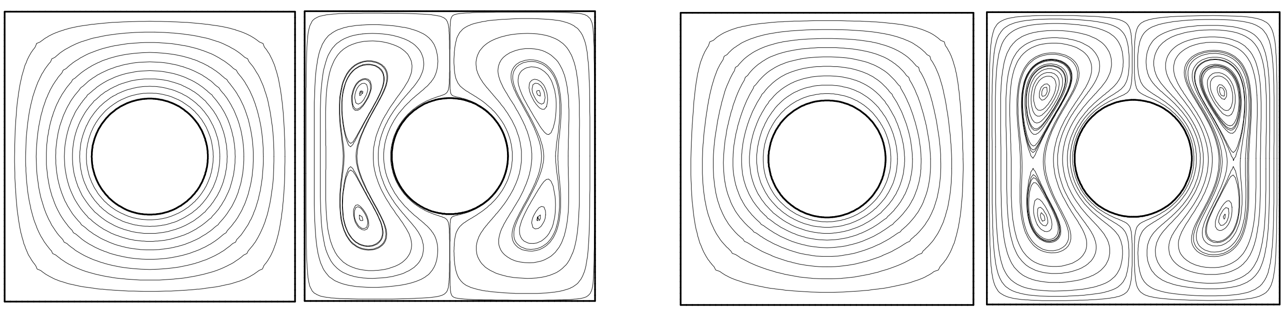
Superscripts

* dimensionless value

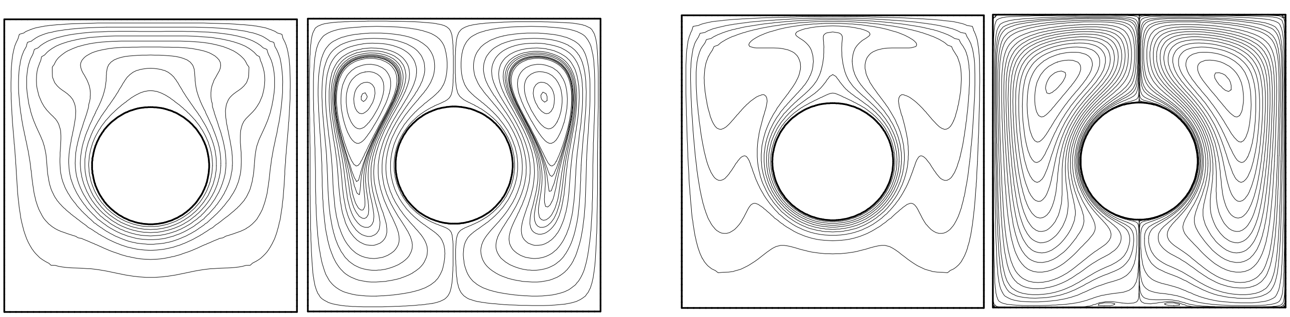
.



Kim *et al*. [7] Results



(*a*) **(*b*) *Ra*=10^4^**



(*c*) *Ra*=10^5 **(*d*) *Ra*=10^6**

Present Work Results

Fig. 3. Comparison of the temperature contours and streamlines between the present work and that of Kim *et al*. [7] at δ = 0.5 for four different Rayleigh numbers of (*a*) 10^3, (*b*) 10^4, (*c*) 10^5 and (*d*) 10^6

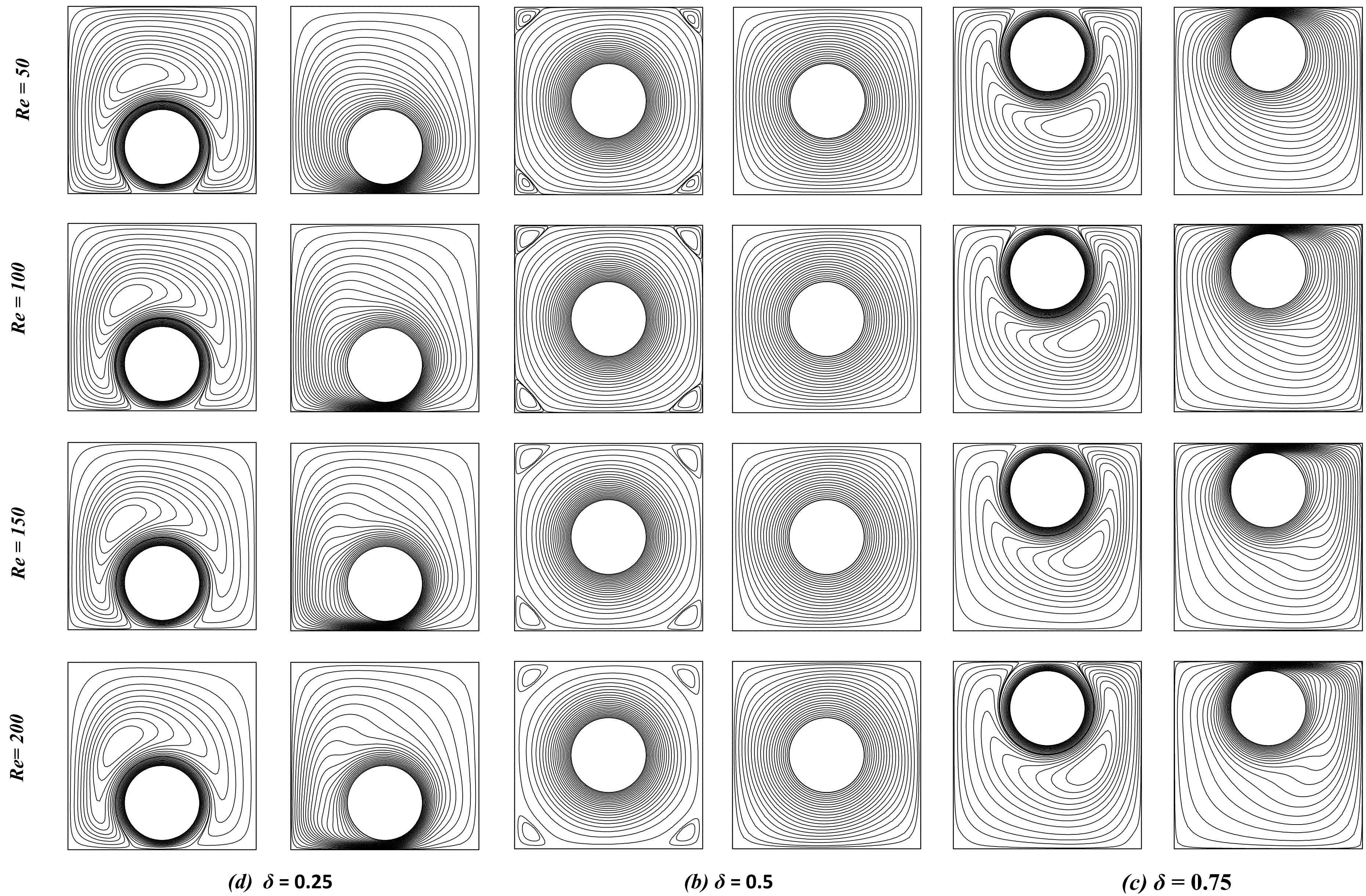


Fig. 4. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 0

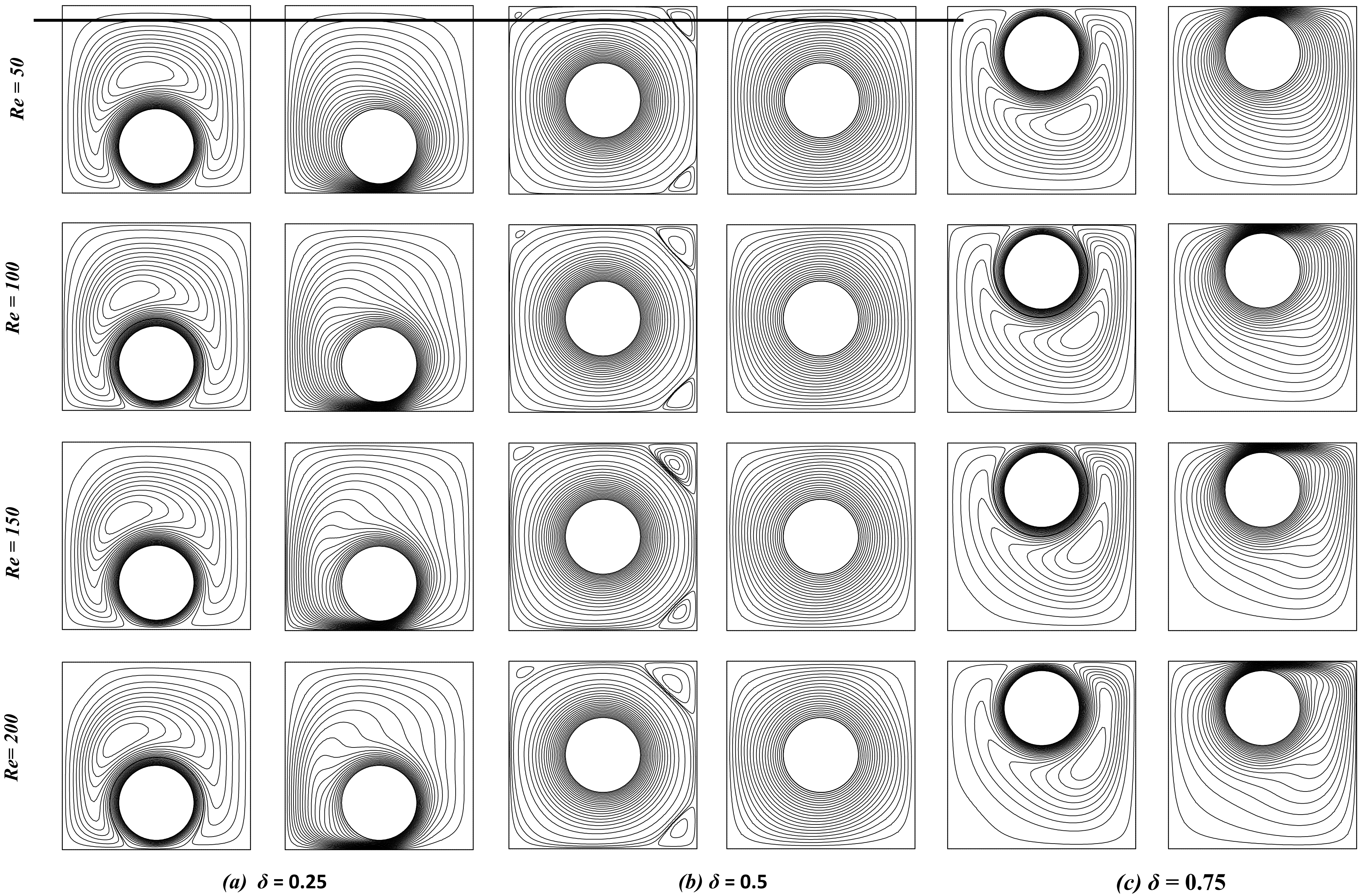


Fig. 5. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 1

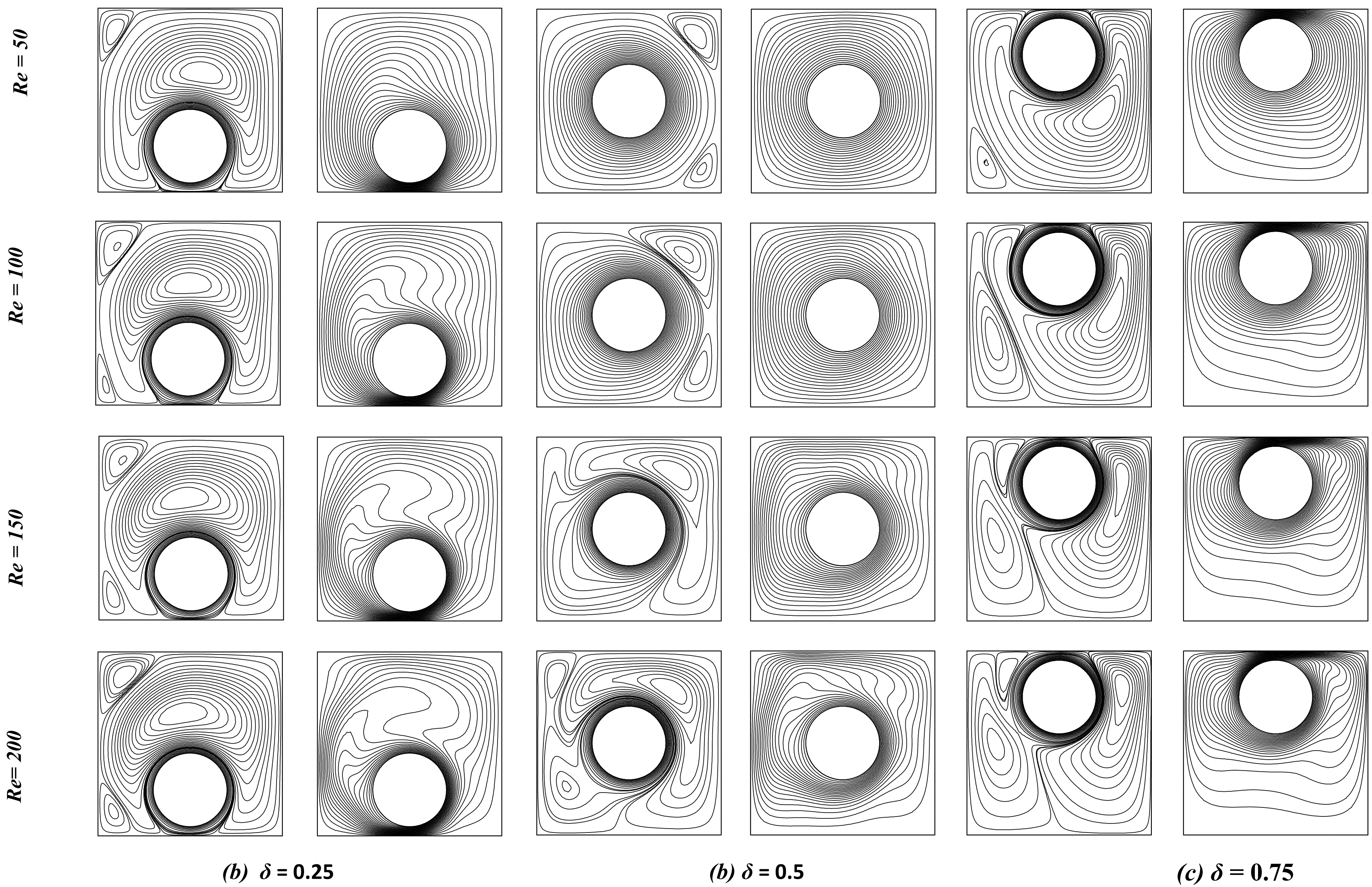


Fig. 6. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 5

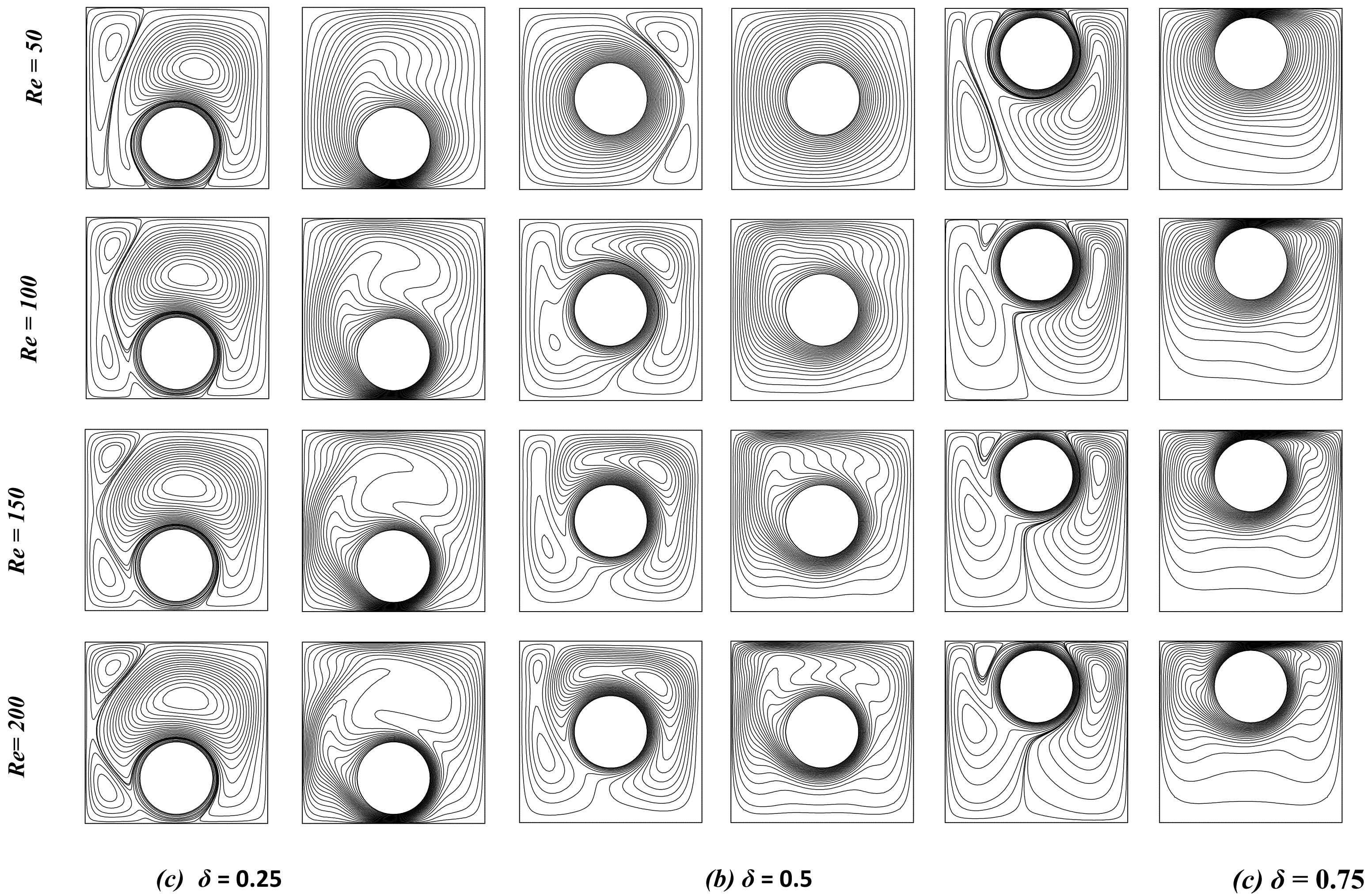
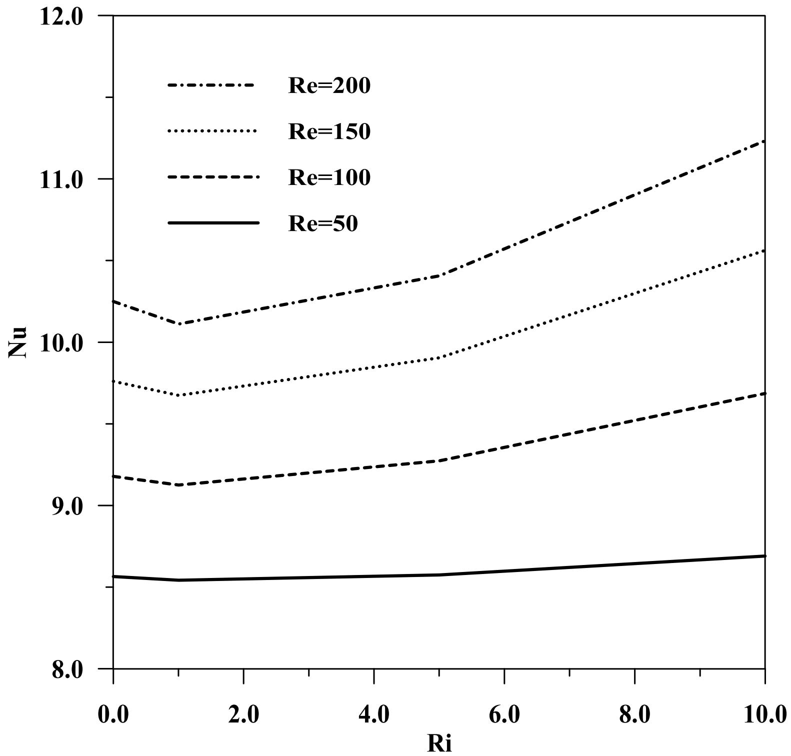
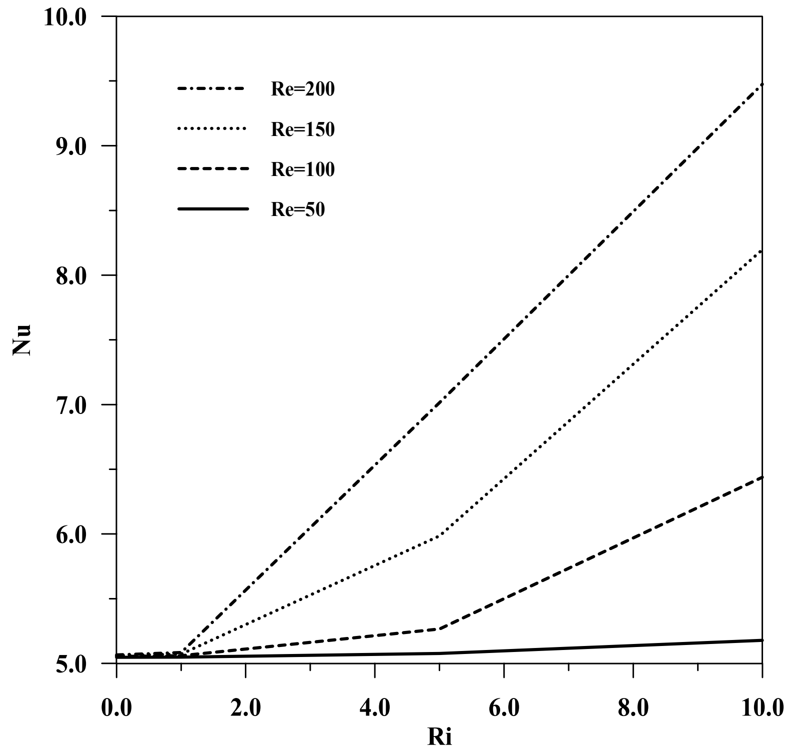


Fig. 7. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 10


12.0
11.0
10.0
9.0
8.0
Nu
Re=200
Re=150
Re=100
Re=50
0.0
2.0
4.0
6.0
8.0
10.0
Ri

δ=0.75


10.0
9.0
8.0
7.0
6.0
5.0
Nu
Re=200
Re=150
Re=100
Re=50
0.0
2.0
4.0
6.0
8.0
10.0
Ri

δ=0.5

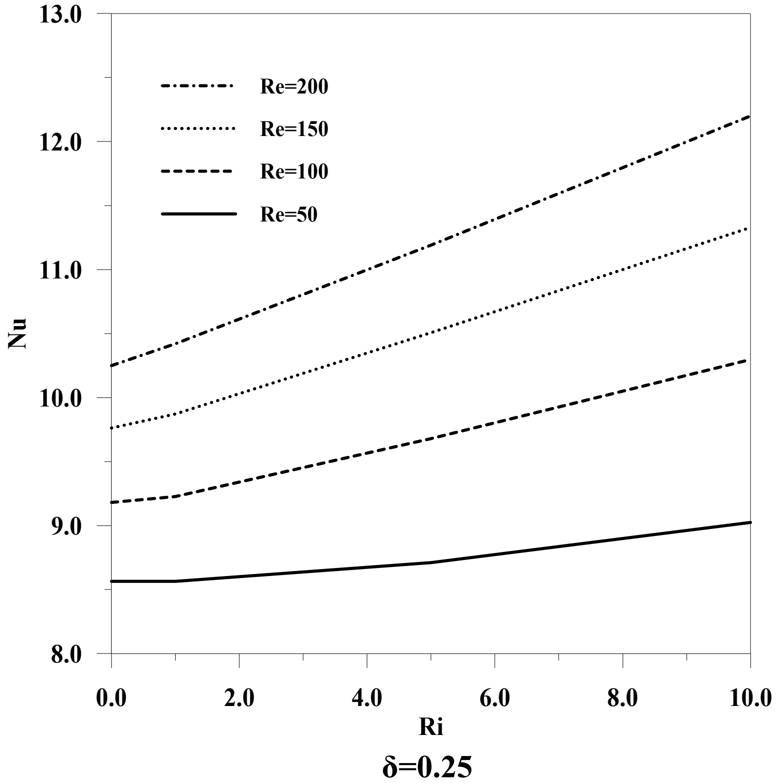


Fig. 8. Variation of average Nusselt number (Nu) with different locations (δ), Reynolds number (Re) and Richardson number (Ri).

1. Introduction

Mixed convection in enclosure is relevant to many industrial and in environmental applications such as in heat exchanger, ventilation of building, design of solar collectors, nuclear and chemical reactors and cooling of electronic equipments etc. In engineering application the geometries that arise however are more complicated than simple cavity configuration filled with a convection fluid. The geometric configuration of interest can be enriched by adding a rotating cylinder, placed inside the enclosure, thus giving rise to a mixed convection problem. The applications are in numerous areas such as ventilation of enclosures, cooling of electronic systems, solar power collector. The resulting flow is due both to the natural convection component, associated to buoyancy and to the forced convection associated to the cylinder rotation. This can rotate in the direction corresponding to combined or opposite natural and forced convection, thus resulting on the intensification or on the attenuation of the convection flow, and of the corresponding overall thermal performance of the enclosure, respectively. This can be the model for real situations where a rotating shaft can be used to control the natural convection taking place on an enclosure, and thus to control the overall thermal performance of the enclosure, in order to increase or to reduce its effect.

Some investigators have deal the convection inside an enclosure that includes a cylinder, rotating in the center of the enclosure or stationary but for non centered locations, and in the presence of pure forced convection, mixed convection or pure natural convection. Buoyancy driven flow and heat transfer between a cylinder and its surrounding medium has been a problem of considerable importance. This problem has a wide range of applications. Energy storage devices, crop dryers, crude oil storage tanks and spent fuel storage of nuclear power plants are a few to mention. **Karim et al.[1]** have experimentally demonstrated the influence of horizontal confinement on heat transfer around a cylinder for Rayleigh numbers ranging from 10^3 to 10^5. These authors found that the heat flux around the cylinder increases with decreasing distance between the cylinder and the enclosure wall. **Abu-Hijleh and Heilen[2]** has studied numerically the Nusselt number due to laminar mixed convection from a rotating isothermal cylinder. The study covered a wide range of parameters:$5 \leq Re \leq 450$ and $0.1 \leq k \leq 10.0$. A correlation was proposed which can be used to accurately predict the Nusselt number for the range of parameters studied. **Cesini et al.[3]** carried out experiments similar to those of **Karim et al.[1]** but found that the heat flux from the cylinder reached a maximum for an optimal wall-to-wall distance of 2.9 times the cylinder diameter. **Mohamed et al.[4]** have investigated the effect of vertical confinement on the natural convection flow and heat transfer around a horizontal heated cylinder. It is shown that the primary effect of the vertical confinement is an increase heat flux on the upper of the cylinder for given separation distance between the cylinder and the fluid boundary. **Aydin Misirlioglu[5]** investigated numerically the heat transfer in square cavity with a rotating circular cylinder centered within it and filled with clear fluid or porous medium. The results indicate that rotation direction of the cylinder has significant effect on the natural and forced convection regimes, especially for the clear fluid case. **Sumon et al.[6]** carried out numerically study of natural convection in two-dimensional square enclosure containing adiabatic cylinder at the center by using finite element method. **Kim et al.[7]** numerically studied two-dimensional steady natural convection for cold outer square enclosure and a hot inner circular cylinder for different Rayleigh number varying over the range of 10^3-10^6. The study goes further to investigate the effect of the inner cylinder vertical location on the heat transfer and fluid flow. **Rahman et al.[8]** studied laminar mixed convection flow inside a vented square cavity without conducting horizontal solid cylinder placed at the center of the cavity by using a finite element method. The Richardson number is varied from 0.0 to 5.0 and the cylinder diameter is varied from 0.0 to 0.6. It is found that the stream lines, isotherms, average Nusselt number at the heated surface, average temperature of the fluid in the cavity and dimensionless temperature at the cylinder center strongly depend on the Richardson number as well as the diameter of the cylinder. **Rahman et al.[9]** conducted numerical simulation for mixed convection flow in a vented cavity with a heated conducting horizontal square cylinder. The results indicate that the flow field and temperature distribution inside the cavity are strongly dependent on the Richardson number and the position of the inner cylinder. **Shih et al.[10]** studied the periodic of laminar flow and heat transfer due to an insulated or isothermal various rotating objects (circle, square, and equilateral triangle) placed in the center of square cavity. Transient variations of the average Nusselt numbers of the respective systems show that for high Re numbers, a quasiperiodic behavior while for low Re numbers, periodicity of the system is clearly observed. **Costa and Raimudo[11]** analyzed the mixed convection in a square enclosure with rotating cylinder centered within it is numerically. The rotating cylinder participates on both the conductive and convective heat transfer process and exchanges heat with the fluid naturally, without imposition of a thermal condition at its surface. Results clearly show how the cylinder effects the thermal performance of the enclosure.

2.Physical Model and Coordinate System:

A schematic of the system considered in the present study is shown in **Fig.(1)**. The system consists of a square enclosure with sides of length L, within which a rotating circular cylinder with radius (R=0.2L) is located and moves along the vertical centerline in the range from -0.25L to 0.25L. The walls of the square enclosure was kept at constant low temperature of T_c, whereas the cylinder was kept at constant high temperature T_h, under the influence of the vertical gravitational field, the enclosure walls temperature and the temperature at the surface of rotating circular cylinder at different levels of temperature lead to a natural convection problem. Due to the non-slip boundary condition for velocity on it is surface, the rotating cylinder, of radius R, induced a forced flow, the overall resulting situation being a mixed convection problem. The free space in the enclosure is filled by a Newton fluid. The fluid density is not effected by pressure change(incompressible fluid). But it changes when temperature changes. Density on the buoyancy term is temperature dependent, and the Boussinesg approximation is used. All the remaining thermo physical properties of the fluid are assumed to be constant, including the density appearing in the convection term. The flow is assumed to be laminar. Thermal radiation heat transfer between the walls and the surface of circular cylinder is negligible, and the fluid assumed to be radioactively non-participating.

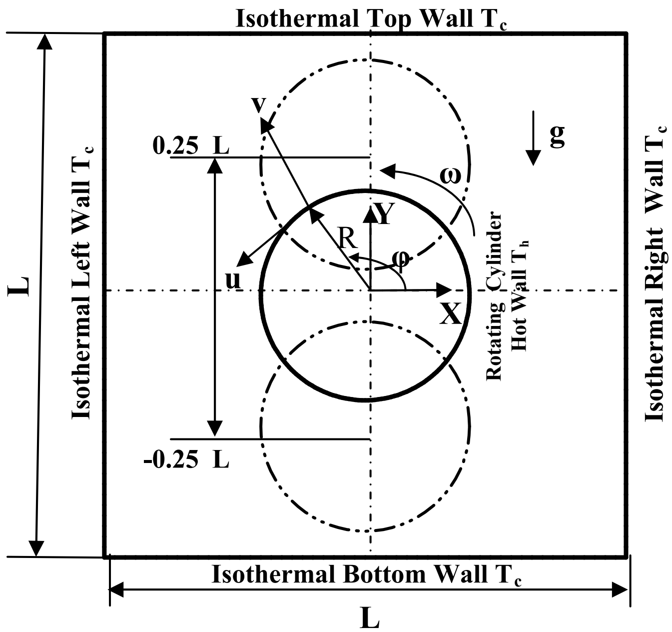


Fig. 1. Schematic diagram of the square enclosure with inner rotating circular cylinder with counterclockwise direction along with coordinate system and boundary conditions

The energy terms due to viscous dissipation and change of temperature due to reversible deformation(work of pressure forces) are not taken into account in the present study.

3. Numerical Modeling Equation

3.1. Governing Equations:

Mixed convection fluid flow inside an enclosure obeys the mass conservation equation that reads, in its dimensionless form:

Mass conservation [11]:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad \textbf{(1)}$$

The momentum equations:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\Pr\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \quad \textbf{(2)}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\Pr\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ra\Pr\theta \quad \textbf{(3)}$$

And the energy conservation equation:

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \quad \textbf{(4)}$$

The dimensionless parameter appearing in the forgoing equations are the space coordinates, the velocity components, the temperature and the driving pressure, defined, respectively as follows [10], [11]:

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{uL}{\alpha}\ ,\ V=\frac{vL}{\alpha},\ \theta=\frac{T-T_c}{T_h-T_c},\ P=\frac{p}{\rho[R\omega]^2} \quad \textbf{(5)}$$

And the Prandtl and Rayleigh numbers emerge as:

$$\Pr=\frac{\upsilon}{\alpha} \quad Ra=\frac{g\beta(T_h-T_c)L^3\Pr}{\upsilon^2} \quad \textbf{(6)}$$

For the problem under analysis it can be defined the following dimensionless parameters can be defined.[11]:

$$R^*=\frac{R}{L} \quad \textbf{(7)}$$

$$\omega^*=\frac{\omega L^2}{\alpha} \quad \textbf{(8)}$$

Over the rotating cylinder, velocity components are specified as:

$$U_c=\omega R \quad \textbf{(9)}$$

$$U=U_c\cos\varphi \quad \textbf{(9a)}$$

$$V=U_c\sin\varphi$$

Where φ varies from 0^o to 360^o. The typical dimensionless parameter used to evalute the relative importance of the natural and force convection is modified from of the Richardson number, defined elsewhere as[11]:

$Ri=\frac{(Ra/\Pr)}{\text{Re}^2}$ **,** where Re is the Reynolds number ($\text{Re}=\frac{(\omega R)D}{\nu}$ **),** where **D** is the characteristic length. For the present problem and dimensionless strategy, this parameter results.

$$Ri=\frac{Ra/\Pr}{[(\omega R)D/\upsilon]^2}=\frac{Ra\Pr}{4\omega^{*2}R^{*4}} \quad \textbf{(10)}$$

As stated in Eq.(10), Richardson number is an alternative parameter to either Ra, Pr, $\omega*$ and will be only used to identify the heat transfer regime.

3.2. Boundary Conditions

The corresponding boundary conditions for the above problem are given by:

All walls U,V = 0 **(11)**

Surface of the cylinder $U=U_c\cos\varphi \quad V=U_c\sin\varphi$ **(12)**

All walls of the enclosure $\theta=0$ **(13)**

Surface of the cylinder $\theta=1$ **(14)**

Vorticity-stream function approach to two-dimensional problem of solving Navir-Stocks equations is rather easy to used to solve current problem . Stream vorticity (ζ) and stream function (ψ) must be obtained during the computation.

First let us provide some definition which will simplify steady, two-dimensional Navir-Stocks equations. The main goal of that is to remove explicitly from Navir-Stocks equations. We can do it with the procedure as follows. The definition of vorticity for two-dimensional case is:

$\zeta=\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}$ and stream function definition is**:** $\frac{\partial\psi}{\partial Y}=U\ ,\frac{\partial\psi}{\partial X}=-V$ **(15)**

$$\Psi = \frac{\psi \Pr}{\nu}, \ \varsigma = \frac{\xi (L)^2 \Pr}{\nu} \tag{16}$$

The above definition combined with eqations (1), (2), (3) and (4).
It will eliminate pressure from the momentum equations. That combination will give us non-pressure vorticity transport equation which in steady, two-dimensional form can be written as follow [11]:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\varsigma \tag{17}$$

$$\frac{\partial \varsigma}{\partial X}\frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial X}\frac{\partial \varsigma}{\partial Y} = \frac{1}{\mathrm{Re}}\nabla^2 \varsigma + Ri\frac{\partial \theta}{\partial X} \tag{18}$$

$$\frac{\partial \theta}{\partial X}\frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial X}\frac{\partial \theta}{\partial Y} = \frac{1}{\mathrm{Re}\Pr}\nabla^2 \theta \tag{19}$$

The boundary condition for this equations becomes:

All walls and surface of the cylinder $\Psi = 0$ **(20)**

All walls $\varsigma_w = \dfrac{2(\Psi_{w+1} - \Psi_w)}{(\Delta n)^2}$ **(21)**

Surface of the cylinder $\varsigma_{Ni,Nj} = \dfrac{2(\Psi_{Ni,Nj} - \Delta x - \Delta y - \Psi_{Ni,Nj-1})}{(\Delta x)^2 + (\Delta y)^2}$ **(22)**

3.3 Heat Transfer Calculations

The total dimensionless heat transferred through the enclosure, from the hot surface to the cold walls one is here represented by the overall Nusselt number, *Nu*, defined as [11]:

$$Nu = \frac{\int_0^L -k\left(\frac{\partial \theta}{\partial x}\right)_{x=0} dy}{k\left[{(T_h - T_c)}/{L}\right]L} = \int_0^1 -\left(\frac{\partial \theta}{\partial X}\right)_{X=0} dY \tag{23}$$

4. Numerical modeling and Verification

Governing equations of mixed convection (Eqs. (1)-(4)) which are written in stream function-vorticity form (Eqs. (17)-(19)) for laminar regime in two-dimensional form for steady, incompressible, and Newtonian fluid with Boussinesq approximation is used to characterize the buoyancy effect. It is assumed that radiation heat exchange is negligible according to other modes of heat transfer and the gravity acts in vertical direction, cylinder rotating with anti-clockwise direction. Finite difference is used to solve equations ((17)-(19)). Central difference method is applied for discretization of equations. The solution of linear algebraic equations was performed using successive under relaxation (SUR) method. As convergence criteria, 10^{-4} is chosen for all dependent variable and value of 0.1 is taken for under relaxation parameter. The computational geometry in the x-y plane with non-uniform grid distribution. A grid resolution of 200*100 along horizontal (x) and vertical (y) directions was employed as shown in **Fig(2a)** which presented the mesh corresponding to cylinder with R=0.2. The grid were uniformly distributed near the walls in order to account for the high gradients using the algebraic function. The denser grid lines were uniformly distributed with resolution of For the purpose of code validation, the natural convection problem for a low temperature outer square enclosure and high temperature inner circular cylinder with non-rotating condition was tested and compared the result of streamlines and isothermal lines with **Kim et al[7]** as shown in **Fig.(3)**. The result show a good agreement with results of **Kim et al[7]**.

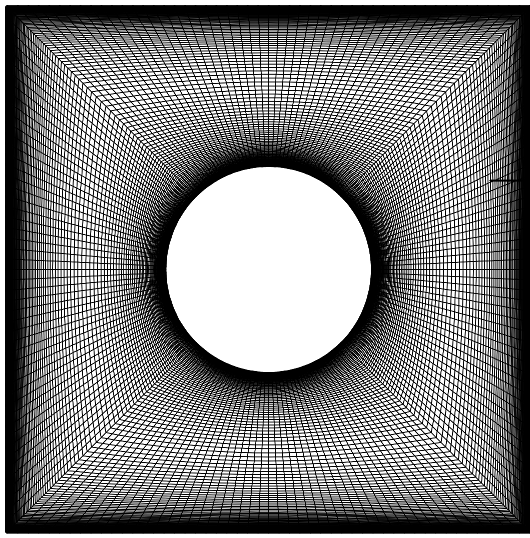


Fig. 2a. A typical grid distribution (200 x 100) with non-uniform and non- orthogonal distributions for δ = 0.

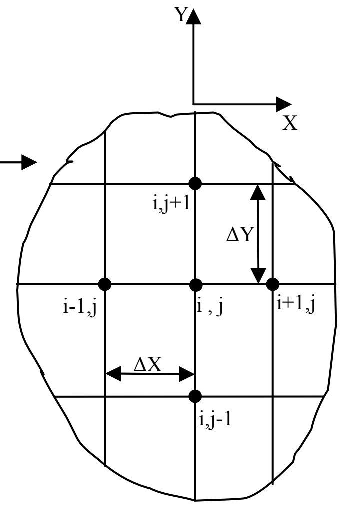


Fig.2b. Grid arrangement

4.1 Numerical procedure

Governing equations in stream-vorticity function form (Eqs.(17)-(19)) were solved using central difference method. Algebraic equations were obtained via Taylor series and they solved with Successive Under Relaxation (SUR) technique, iteratively, which is defined in Eq.24.

$$\phi = \phi^0 + r(\phi^{calculated} - \phi^0) \qquad \textbf{(24)}$$

In this equation, ϕ is any variable and ϕ^0 and $\phi^{\text{calclated}}$ are the old value and calculated value of variable. However, ϕ^1 is the value for next step and r shows the relaxation factor whose value is 0.1. Schematic of grid arrangement is given in **Fig.(2b)**. Discretization equations is performed according to this grid in **Appendix (A)**

4-2 Flow Chart

The flow chart has been shown in Fig.3. It starts with input data and initial value. Stream function, vorticity and temperatures are calculated by solving algebraic equations. After than the necessary values such as Nusselt number, pressure, velocity vector, heat vector is calculated. If a convergence criterion is obtained, the output file is printed. Otherwise, it goes to starting point. The program which used is Fortran90, and the time of the run ranges from 20 min to 40 min for each case.

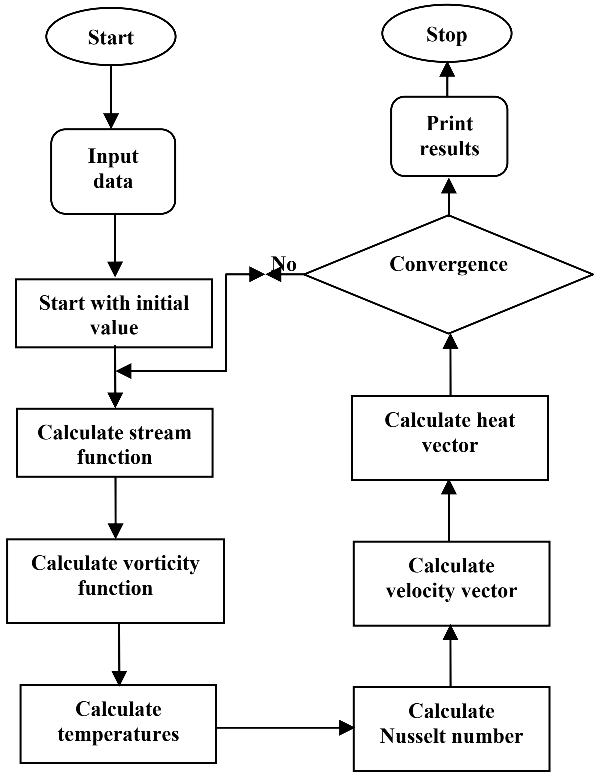


Fig.3. Flow chart for the problem

5. Results and Discussion

In order to understand the flow, temperature fields and heat transfer characteristics of the typical enclosure a total of 48 cases are considered. To study the effect of rotating cylinder speed ω=0.625, 1.25, 1.875 and 2.5 rad/sec are consider, the position of the rotating cylinder δ=0.25, 0.5 and 0.75 are considered. The fluid inside the enclosure is dry air with Pr=0.71 and Richardson number varied from 0 to 10, this selection of Richardson number provides to observe the different between forced convection ($Gr/Re^2<<1$), mixed convection ($Gr/Re^2=1$) and natural convection ($Gr/Re^2>>1$). Reynolds number Re=50, 100, 150 and 200 therefore Grashof number changing for each case, the value of cylinder diameter is 0.2 and rotating with counter-clock wise direction. Flow, temperature fields and Nusselt number are examined by stream lines and isotherm plots.

5-1 Flow structure and Temperature fields

The analysis of flow structure and temperature field are developed getting use of contour of the dimensionless stream function (stream lines) and temperature (isotherms).

Fig.4 is presented the stream lines (left) and isotherms (right) for R=0.2 Richardson number Ri=0, for different Reynolds number (vertical) and for different position δ (horizontal), in this case forced convection is dominant. The first column are the flow fields and isothermal lines for the case when δ=0.25 and different Reynolds number, the fluid pulled upward toward the motion of rotating cylinder (counter-clockwise) and small vortex appear above the rotating cylinder due to that the moving of rotating cylinder surface and stationary of the other walls of the enclosure, with increasing of Reynolds number the vortex moving to the left because the pulled force increase with increasing of rotating speed. Isothermal boundary conditions, which are defined as a neutral boundary condition (hot temperature on the surface of rotating cylinder and cold temperature of the other walls of the enclosure). the isotherms are clustered near the circle and fluid attached the cylinder surface is heated, therefore the temperature varied along the surface of the rotating cylinder where the temperature decreases with direction of rotating cylinder because the hot air attached to surface of the cylinder cooled by the fresh cold air above it, this variation becomes more uniform with increasing of the Reynolds number because the hot air have not enough time to cooled itself from fresh cold air. Because of the small space under the rotating cylinder therefore this region was hottest. The second column show the flow field and isotherms line for δ=0.5, because of the symmetrical boundary conditions on the horizontal and vertical walls the flow and temperature fields are symmetrical about the horizontal and vertical center line of the enclosure. In this case, considerably more space exists for the fluid flow between the cylinder and the enclosure walls therefore, the amount of mass influenced by the rotation of the cylinder increases. The circular flow patterns are approaching to the cavity walls creating clockwise rotating small vortices at each corner of the enclosure. As the Reynolds number increase the vortices size increase because the effect of cylinder rotation are weak. The isotherm plots also symmetrical about the horizontal and vertical mid plane. The isothermal lines is uniform distributed inside the enclosure and the temperature decreases from the surface of the rotating cylinder to the cold walls along the horizontal and vertical centerline of the enclosure and concentrates towards the hot surface indicating the presence of large temperature gradient, the isothermal contour are similitude for all value of Reynolds numbers. The third double column are the same absolute value of the rotating velocities to that presented in the first double column of Fig.4, but in the location (δ=0.75). In what concerns the flow fields and temperature distribution are similar to that when δ=0.25, the main difference being that the vortex appear on the opposite location because the rotating circular cylinder moving upward.

Results for the same conditions as before in Fig.4, but now for Ri=1, are presented in Fig.5. This means that $\mathrm{Re} = \sqrt{Gr}$ **,** the natural convection in this case presented but with poorly effect in flow field and no effected appears in thermal field. The main difference being the convection front associated with the cylinder rotation, which appears on the opposite direction with the natural convection, this effect result as a vortex in the top left side when Re=200. For the case of δ=0.5 the symmetrical behavior vanish because the effect of natural convection. In the case of δ=0.75, small vortex result at bottom left side of the enclosure when Re=150 and 200.

Fig.6, show the result for the same values of the independent dimensionless parameters of the previous figures, namely the counter-clockwise rotation of the circular cylinder (ω) except now the Richardson number Ri=5. Due to the fact that, in this case, the natural convection and the forced convective have opposite effect, in what concerns fluid motion, some vortices appear in the flow. The main fluid motion take place in the clockwise direction, induced by natural convection, close to the rotating cylinder fluid motion occurs in counter-clockwise direction, an in between there are some vortices, where fluid motion is in the clockwise direction. In case of δ=0.25 and Re=50 two vortices result, one above the rotating cylinder and another at top left side of the enclosure, with the increasing Reynolds number a vortices result at bottom left side of the enclosure and the isothermal line show that the heat transfer across the enclosure occur mainly associated with the flow therefore the isothermal lines moves to the right as the effect of combined convection. When δ=0.5 two vortices result on the opposite side (right side) when compared with previous figure. The fluid moves faster as Reynolds number increase, the two vortices penetrate the stagnant fluid then merge to become a primary vortex. Small vortex result in left side when Re=200 because the fluid becomes faster. Isothermal lines uniform distributed at low Reynolds number (Re=50&100) but with increasing Reynolds number the hot region transfer from around rotating cylinder to top left side walls because the effect of combined convection increase. In the case of δ=0.75 the main flow consist of two vortices, this vortices spread with increasing of Reynolds number and third vortices appear besides the rotating cylinder. The isothermal lines clustered near the hot rotating cylinder therefore the high half becomes hot because the light fluid don't move to downward.

Fig.7, give the flow structure for the fluid flow when Ri=10. Even for high value of (Gr/Re^2), in this case of counter-clockwise rotation (right column of figure) there are big vortices formed around the rotating cylinder in the enclosure. These vortices prevent the mass to transfer easily, with increasing of Reynolds number this vortices squeezed thinner and ultimately is divided in two minor vortices. The isothermal lines moves to the right side of the enclosure and still the rotating cylinder the hottest region and heat transfer by natural convection to the top and left side walls. In case of δ=0.5 the flow contains two vortices one of it in top right side and its large in size and another in bottom left side and its small in size, with increasing of Reynolds number the small vortex vanish and the large vortex squeezed thinner and divided into two minor cells located near the top and bottom of the right side wall. With increasing of Reynolds number the effect of combined convection more increase in fluid flow behavior therefore, some vortices around the rotating cylinder appears. The isothermal lines was uniform distributed at low Reynolds number (Re=50&100) and this isothermal lines moves to right when Reynolds number increase and the light fluid moves to up cold wall and concentrates in this region. In case of δ=0.75, it is observed a considerable change on the flow field because the position of rotating cylinder and effect of mixed convection, there are some clockwise recirculation heat cells, in the form of clockwise rotating cells. The isotherms spread over the top half of the fluid domain. Due to the fact that, hot fluid was lightest from the cold fluid therefore the isotherms concentrates at top half side of the enclosure.

5-2 Heat Transfer

The variation of the average Nusselt number with Richardson number, Ri are investigated for the different locations, δ and Reynolds number, Re which are shown in Fig.8. The effect of Reynolds number on heat transfer is clearly demonstrated here. In general, it is observed that the average Nusselt number increases significantly with increasing Richardson number due to increasing the effect of convection. The average Nusselt number increases more rapidly as the Reynolds number increase. This phenomena observed for Ri>1. But for δ=0.25, the effect of Reynolds number is more significant between Richardson number 1 and 10 and for Ri<1, δ=0.5 the profile approach close to each other.

The results show that Nusselt number increases when the inner circular rotating cylinder moves to the top wall (δ=0.75), but when the circular rotating cylinder moves to the bottom wall (δ=0.25) the Nusselt number is higher because the combined convection becomes more effect in the flow field and heat transfer process.

6.Conclusions

A numerical investigation is performed for mixed convection in square enclosure with a rotating circular cylinder inside. A detailed analysis for the distribution of streamlines, isotherms and the average Nusselt number at the hot surface of the rotating circular cylinder is carried out to investigate the effect of the location of rotating circular cylinder on the fluid flow and heat transfer in the square enclosure for different Richardson number in the range of $0 \geq Ri \geq 10$.

It is observed that the location of the rotating circular cylinder has a strong influence on the resulting flow and heat transfer process, as it limits the space for fluid flow between the rotating circular cylinder and the enclosure wall, giving rise to higher or lower local fluid velocities. For centerline location of the rotating circular cylinder (δ=0.5) the average Nusselt number is small for all value of rotating velocity, and it considerable increase when the rotating circular cylinder moves to up or down locations. The value of average Nusselt number is the highest in the combined convection dominated area when the rotating circular cylinder is located near the bottom wall (δ=0.25).

Appendix A

$$-\varsigma_{i,j} = \frac{\Psi_{i+1,j} + \Psi_{i-1,j} - 2\Psi_{i,j}}{\Delta X^2} + \frac{\Psi_{i,j+1} + \Psi_{i,j-1} - 2\Psi_{i,j}}{\Delta Y^2}$$

$$\Psi_{i,j} = \frac{\left[\left(\dfrac{\Psi_{i+1,j} + \Psi_{i-1,j}}{\Delta X^2}\right) + \left(\dfrac{\Psi_{i,j+1} + \Psi_{i,j-1}}{\Delta Y^2}\right)\right] + \varsigma_{i,j}}{\left(\dfrac{2}{\Delta X^2} + \dfrac{2}{\Delta Y^2}\right)}$$

$$\varsigma_{i,j} = \left\{ \begin{gathered} \left(\frac{-\mathrm{Re}}{4\Delta X\Delta Y}\right)\left[\left(\Psi_{i,j+1} - \Psi_{i,j-1}\right)\left(\varsigma_{i+1,j} - \varsigma_{i-1,j}\right) - \left(\Psi_{i+1,j} - \Psi_{i-1,j}\right)\left(\varsigma_{i,j+1} - \varsigma_{i,j-1}\right)\right] + \\ \left(\frac{\varsigma_{i+1,j} + \varsigma_{i-1,j}}{\Delta X^2}\right) + \left(\frac{\varsigma_{i,j+1} + \varsigma_{i,j-1}}{\Delta Y^2}\right) - Ri\,\mathrm{Re}\left(\frac{\theta_{i+1,j} - \theta_{i-1,j}}{2\Delta X}\right) \end{gathered} \right\} \Bigg/ \left(\frac{2}{\Delta X^2} + \frac{2}{\Delta Y^2}\right)$$

$$\theta_{i,j} = \left\{ \begin{gathered} \left(\frac{-\mathrm{Re}\,\mathrm{Pr}}{4\Delta X\Delta Y}\right)\left[\left(\Psi_{i,j+1} - \Psi_{i,j-1}\right)\left(\theta_{i+1,j} - \theta_{i-1,j}\right) - \left(\Psi_{i+1,j} - \Psi_{i-1,j}\right)\left(\theta_{i,j+1} - \theta_{i,j-1}\right)\right] + \\ \left(\frac{\theta_{i+1,j} + \theta_{i-1,j}}{\Delta X^2}\right) + \left(\frac{\theta_{i,j+1} + \theta_{i,j-1}}{\Delta Y^2}\right) \end{gathered} \right\} \Bigg/ \left(\frac{2}{\Delta X^2} + \frac{2}{\Delta Y^2}\right)$$

Ri	Re	ω	Nu			
0	50	0.625	5.046557			
	100	1.25	5.0527472			
	150	1.875	5.05889			
	200	2.5	5.064836			
01	50	0.625	5.0477			
	100	1.25	5.058705			
	0150	1.875	5.071098			
	0200	2.5	5.08316			
05	50	0.625	5.0761397			
	100	1.25	5.265095			
	150	1.875	5.983949			
	200	2.5	7.014728			
010	50	0.625	5.17749			
	100	1.25	6.436944			
	150	1.875	8.195753			
	200	2.5	9.474571			

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Nomenclature

d Dimensional cylinder length (m)
D Dimensionless cylinder length
Gr Grashof number
g Gravitational acceleration (m/s^2)
k Thermal conductivity of fluid (W/m.K)
L Length of the enclosure (m)
n Normal direction
Nu Nusselt number
p Pressure (N/m^2)
P Dimensionless pressure
Pr Prandtl number
r Relaxation function
R Radius of circular cylinder (m)
Re Reynolds number
Ri Richardson number
T Temperature (K)
T_c Cold temperature (K)
T_h Hot temperature (K)
u, v Cartesian velocity components (m/s)
X, Y Cartesian coordinates

Greek Symbols

α Thermal diffusivity (m^2/s)
β Thermal expansion coefficient (1/K)
ξ Vorticity function
ζ Dimensionless vorticity function
υ Kinematic viscosity (m^2/s)
φ angle of circular cylinder (degree)
θ Dimensionless temperature
ρ Density of the fluid (kg/m^3)
ψ Stream function
Ψ Dimensionless stream function
δ Distance from bottom wall to the circular cylinder center
ω angular velocity (rad/sec)

Superscripts

$*$ dimensionless value

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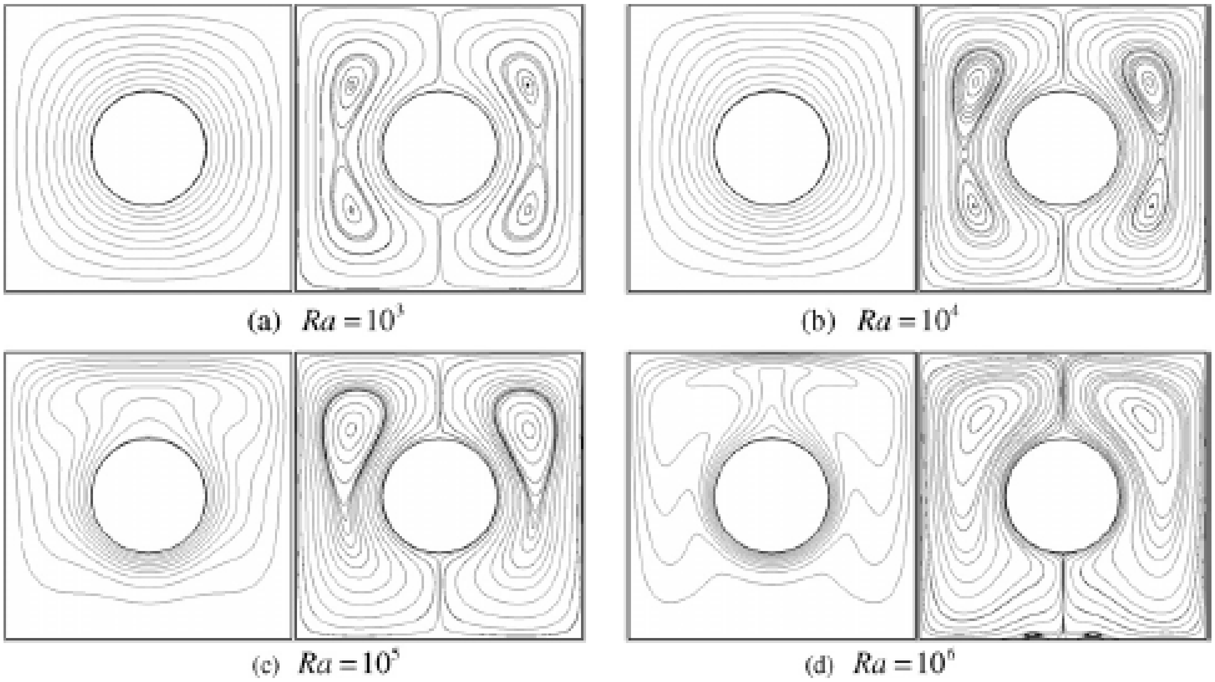
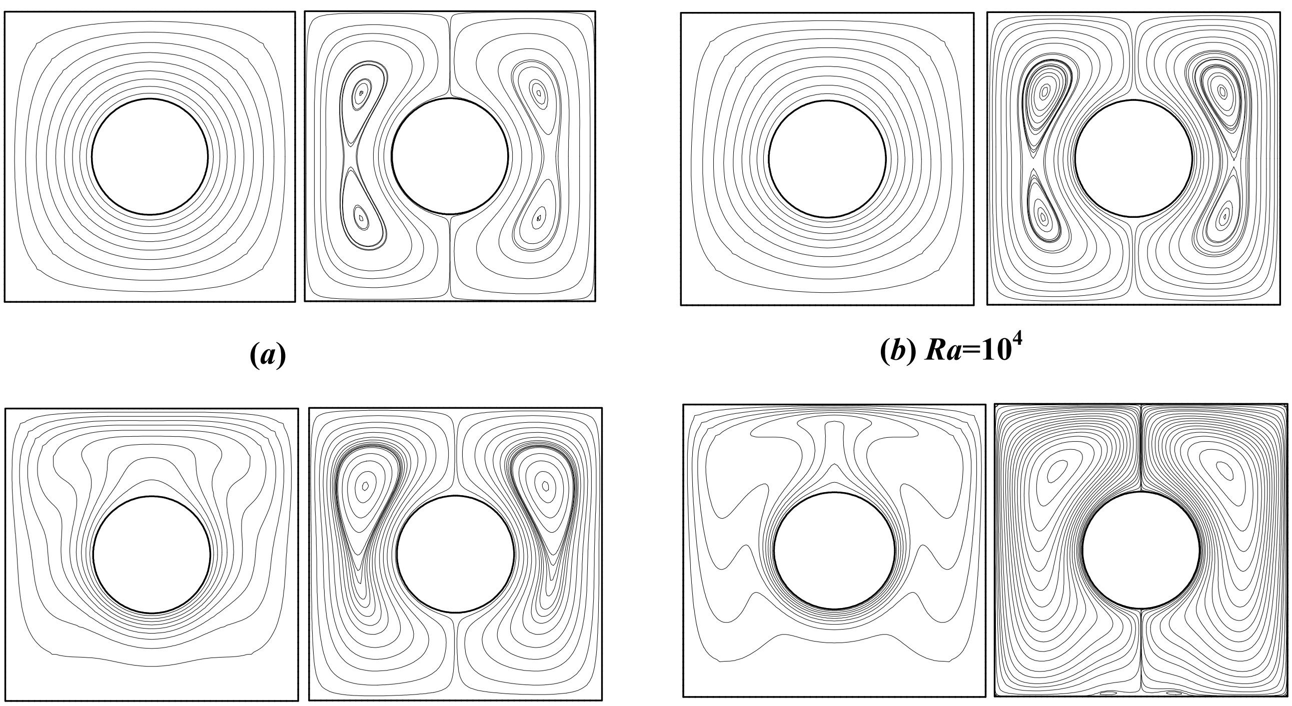


Kim *et al*. [7] Results

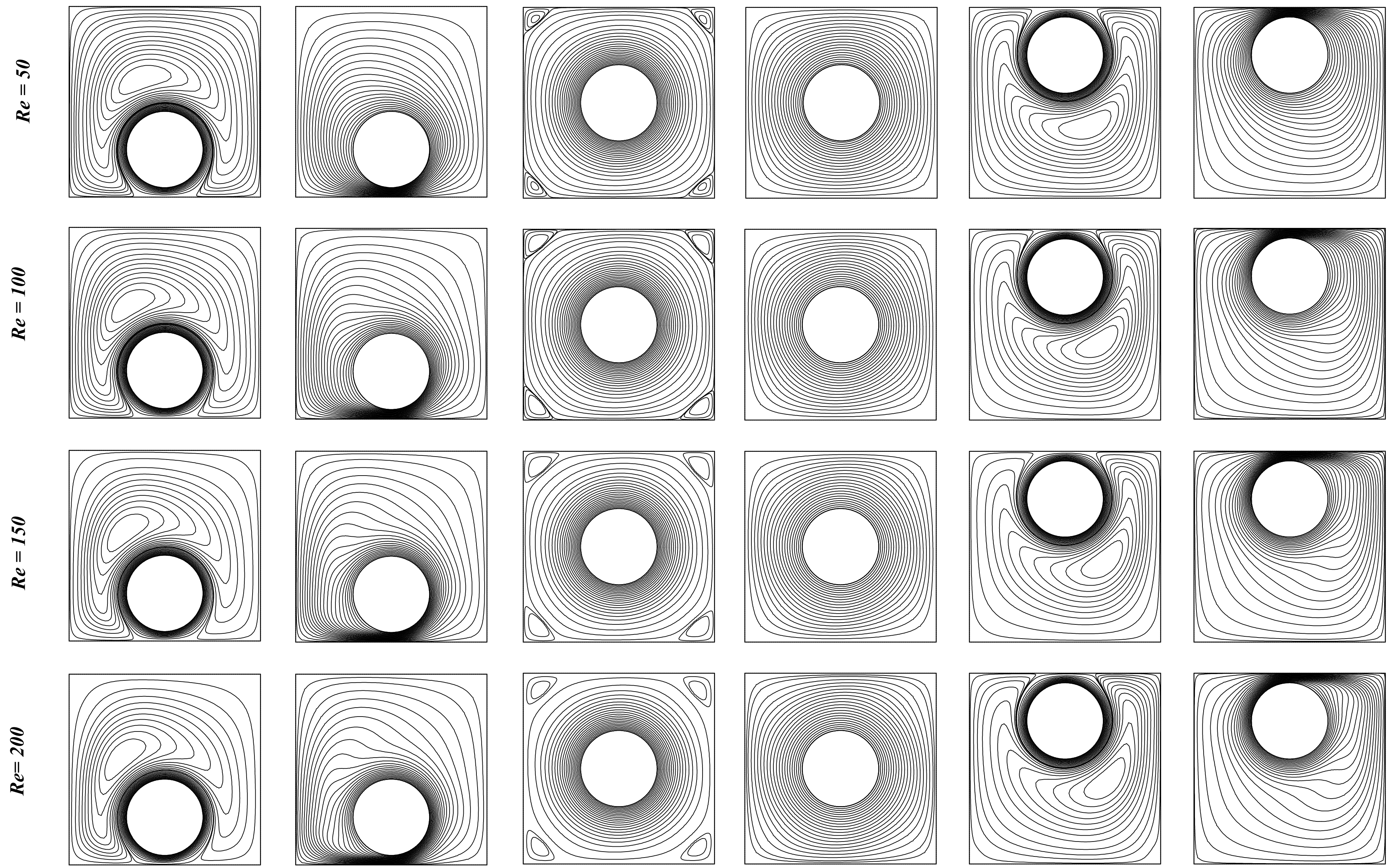


(*a*) **(*b*) *Ra*=10^4**

(*c*) *Ra*=10^5 **(*d*) *Ra*=10^6**

Present Work Results

Fig. 3. Comparison of the temperature contours and streamlines between the present work and that of Kim *et al*. [7] at δ = 0.5 for four different Rayleigh numbers of (*a*) 10^3, (*b*) 10^4, (*c*) 10^5 and (*d*) 10^6



(d) δ = 0.25 *(b)* δ = 0.5 *(c)* δ = 0.75

Fig. 4. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 0

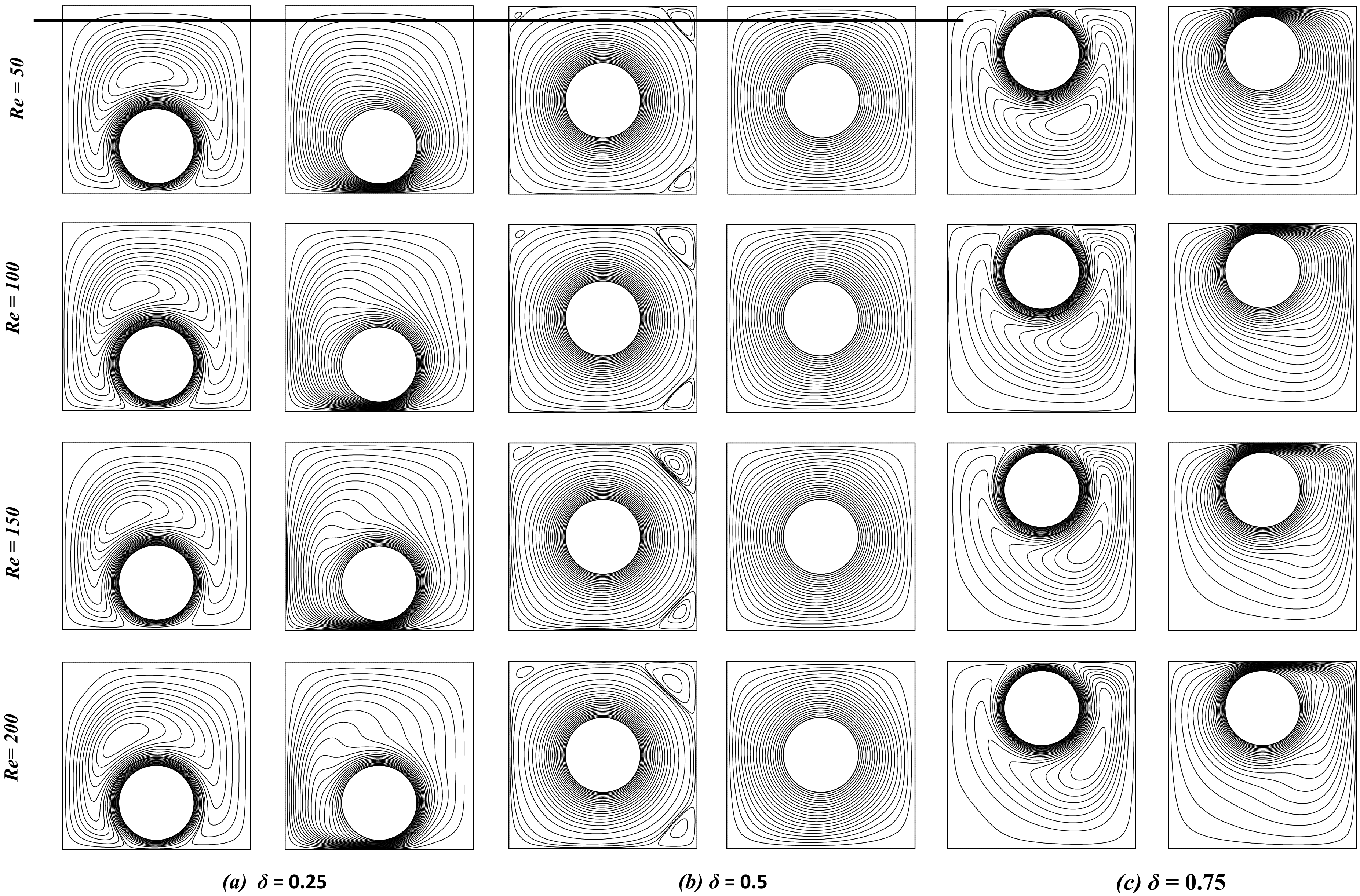
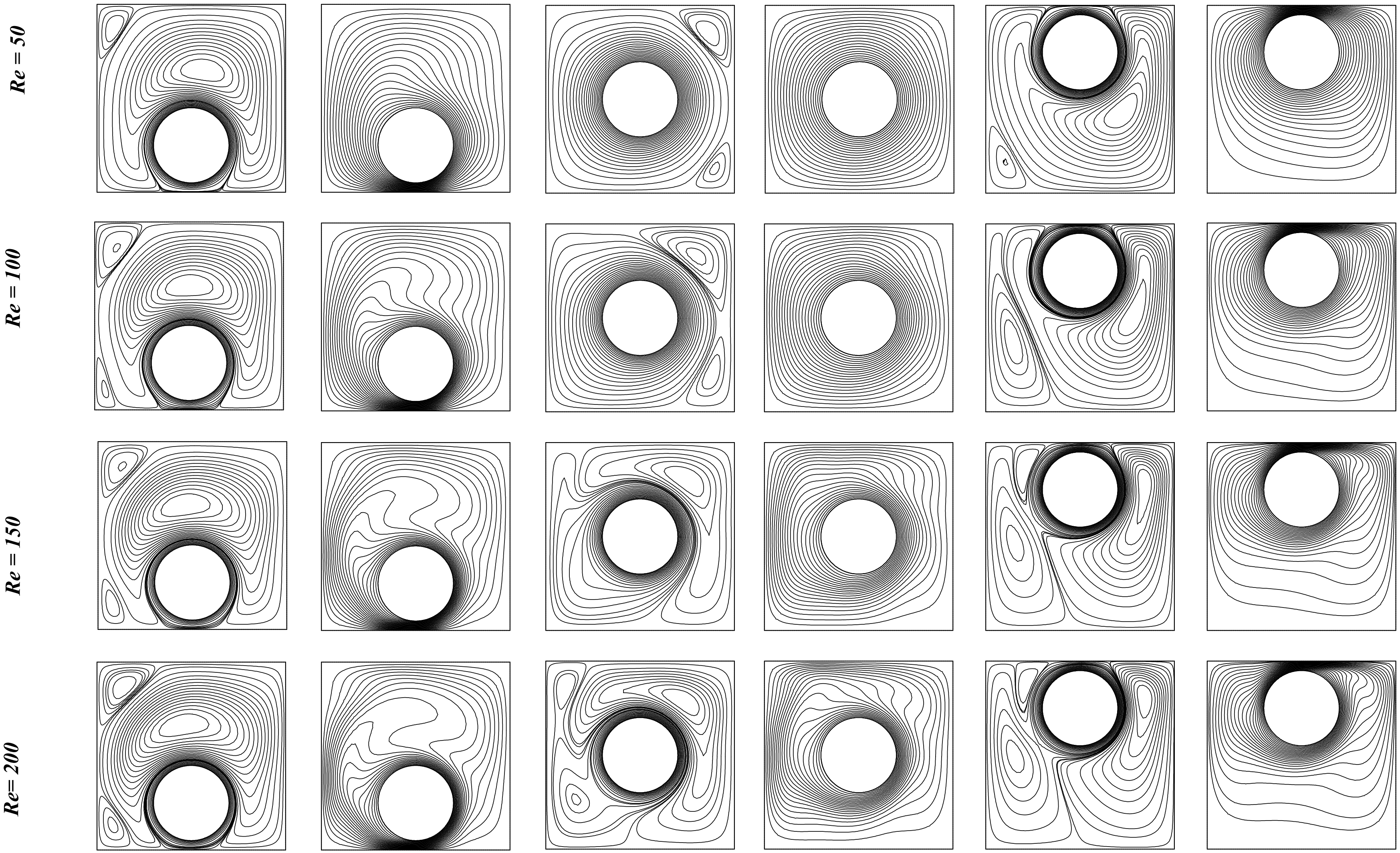
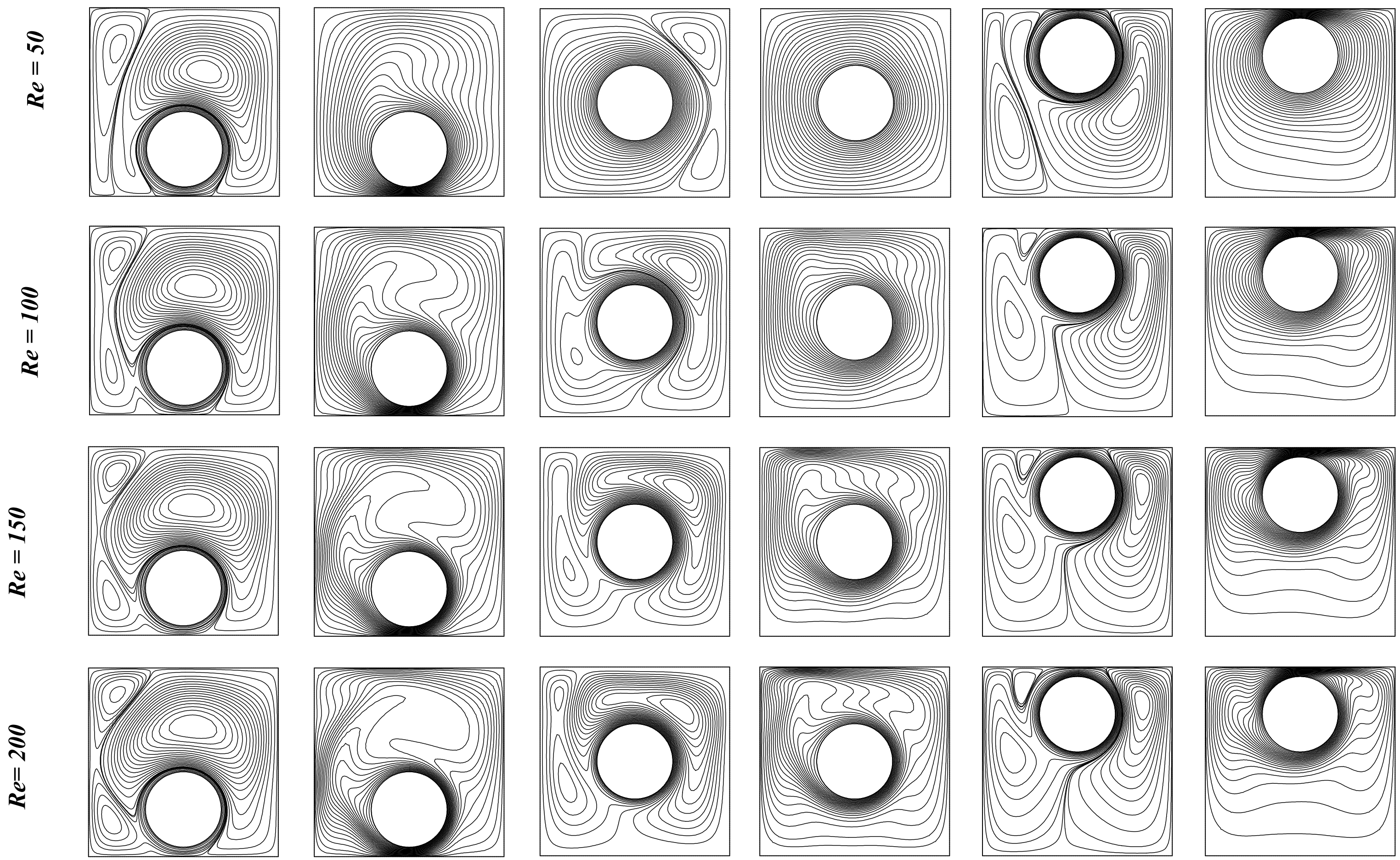


Fig. 5. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 1



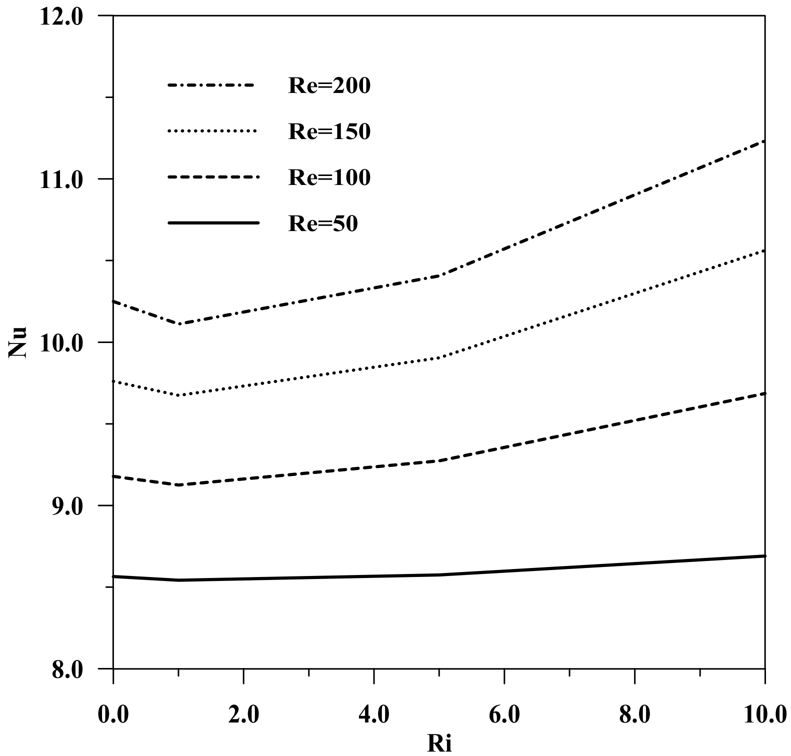
(b) δ = 0.25 *(b)* δ = 0.5 *(c)* δ = 0.75

Fig. 6. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 5

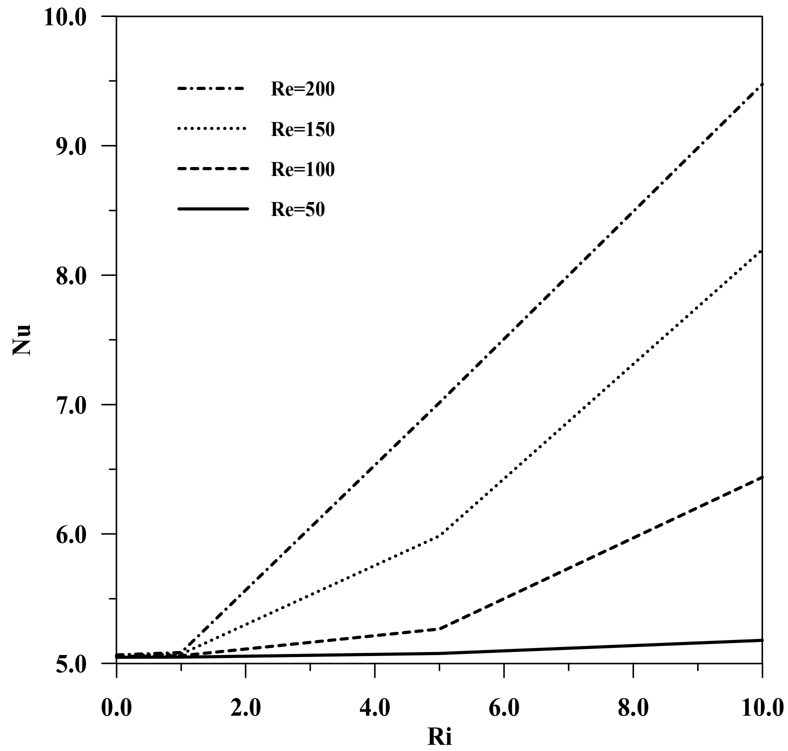


(c) δ = 0.25 *(b)* δ = 0.5 *(c)* δ = 0.75

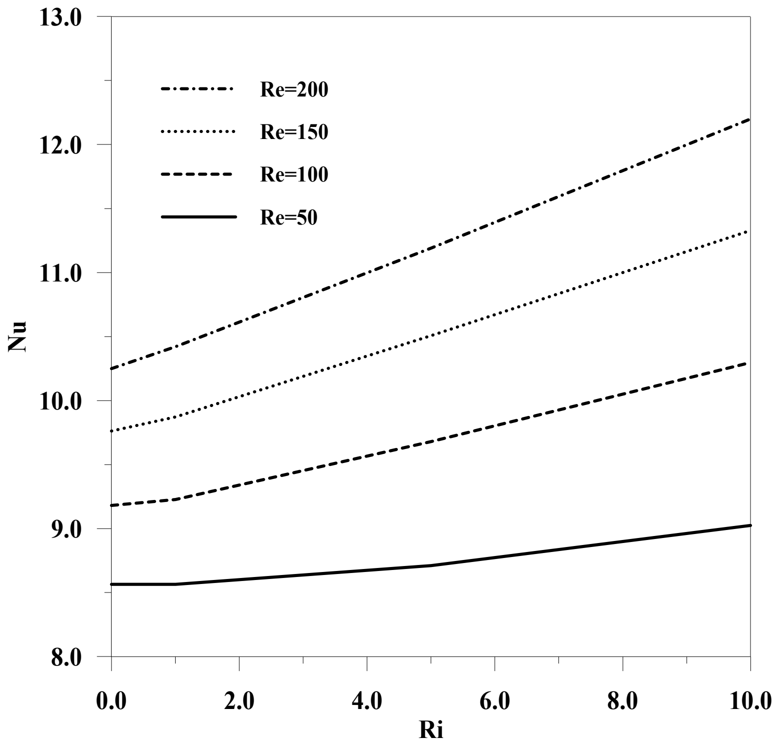
Fig. 7. streamlines (on the left) and Isotherms (on the right) for different vertical locations of the inner rotating circular cylinder and Reynolds numbers (*Re*) at *Ri* = 10


12.0
11.0
10.0
9.0
8.0
Nu
Re=200
Re=150
Re=100
Re=50
0.0
2.0
4.0
6.0
8.0
10.0
Ri

δ=0.75


10.0
9.0
8.0
7.0
6.0
5.0
Nu
Re=200
Re=150
Re=100
Re=50
0.0
2.0
4.0
6.0
8.0
10.0
Ri

δ=0.5



δ=0.25

Fig. 8. Variation of average Nusselt number (Nu) with different locations (δ), Reynolds number (Re) and Richardson number (Ri).