

PREDICTION OF CARRY- OVER COEFFICIENT FOR FLUID FLOW THROUGH TEETH ON ROTOR LABYRINTH SEALS USING COMPUTATIONAL FLUID DYNAMICS

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ABSTRACT :-

The design of the labyrinth seals depends on two main factors, namely, the reduction of leakage loss and the avoidance of rotor system instability. The most important parameter affecting the prediction of leakage mass flow rate of fluid through the labyrinth seal, namely, the carry over coefficients. Hence the computational fluid dynamics (CFD) and $k - \varepsilon$ model has been used to predict the carry-over coefficient of flow through teeth on rotor straight through labyrinth seal. The mathematical model proposed through this work has been solved using the finite volume technique using a FLUENT commercial computer program. The effect of turbulence, seal geometry such as (teeth width, clearance, and pitch cavity) and shaft rotational speed have been taken into consideration. A prediction equation for the carry-over coefficient of the labyrinth seal has been proposed through this work. The prediction equation of carry over coefficient has been found to be a function of (C/S), (W/S) and Reynolds number. It can be concluded from the results that obtain here that the carry over coefficient which describe the labyrinth seal efficiency is highly affected by clearance to pitch ratio (C/S) , while the tooth with to pitch ratio (W/S) has a secondary effect on carry over coefficient. The results obtained show that the carry over coefficient is increased about 42% with increasing of the (C/S) from (0.0075 to 0.0375). While this coefficient is decreased by 9% at Reynold's number more than 8000 with increasing of (W/S) from (0.0075) to (0.5). The effect of rotation has been studied on the carry over coefficient; therefore, the results showed that the carry over coefficient decreases with 10% after a certain value of rotation which is equal to (10000 rpm). Verification for the prediction models of the present work has been carried out by comparing some of the results obtained through this work with the experimental results published by Scharrer (1988) , Eser (2004) and the CFD analysis of Vamshi (2011). The results obtained through the present work found to be in a good agreement with the results of other workers.

KEY WORDS, Labyrinth seal, Carry-over coefficient, CFD.

تخمين معامل الترحيل لجريان المائع خلال مانعات التسرب المتهاهية ذات الأسنان على المحور الدوار باستخدام ديناميك الموائع الحسابية

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الخلاصة :

إن عملية تصميم مانعات التسرب ذات الشكل المتهاهي تعتمد على عاملين رئيسيين هما ، الحد من خسائر التسرب وتقادي عدم استقرار النظام الدوار. من أهم العوامل التي تؤثر على 1δ التنبؤ في معدل تسرب جريان الموائع من خلال مانع التسرب ما يسمى بعامل الطاقة المحمولة. وبالتالي تم استخدام حساب ديناميك الموائع (CFD) ونموذج الاضطراب $k-\epsilon$ لتنبؤ موديل لحساب عامل الطاقة المحمولة. تم حل النموذج الرياضي المقترح من خلال هذا العمل باستخدام تقنية متناهية الحجم (FV) باستخدام البرنامج الجاهز (FLUENT , 3 , 26) (6). فقد تم الاخذ بنظر الاعتبار تأثير الاضطراب و شكل مانع التسرب من حيث عرض الأسنان و عرض الفجوة و الخلوص مابين ال جزء الثابت (stator) و الجزء الدوار ال (rotor) و السرعة الدورانية للعمود الدوار على عامل الطاقة المحمولة. وقد اقترح معادلة التنبؤ لعامل الطاقة المحمولة لمانعات التسرب من خلال هذا العمل. حيث وجد من المعادله المستحصله ان عامل الطاقة المحمولة هو داله لكل من نسبة الخلوص الى الخطوة و نسبة عرض السن الى الخطوة وكذلك لعدد رينولدز. يمكن الاستنتاج من النتائج التي تم الحصول عليها ان معامل الطاقة المحمولة التي تصف كفاءة مانع التسرب يتأثر إلى حد كبير بنسبة (C / S) ، في حين أن نسبة عرض السن الى الخطوة (W / S) لديها تأثير ثانوي على عامل الطاقة المحمولة. و قد بينت النتائج أن عامل الطاقة المحمولة يزداد حوالي 42% مع زيادة (C / S) من (0.0075 - 0.00375). بينما انخفض هذا العامل بنسبة 9% في عدد رينولد أكثر من 8000 مع زيادة (W / S) من (0.0075 - 0.5). ومن دراسة تأثير الدوران هذا العامل ، أظهرت النتائج عامل الطاقة المحمولة يتناقص بمقدار 10% بعد قيمة معينة من دوران والتي تساوي (10000 دورة في الدقيقة). وقد تم التحقق من صحة نماذج التنبؤ ضد الأعمال التجريبية التي أجراها (Scharer (1988) و (Esser (2004) والتحليل العددي من قبل (Vamish (2011، وجدت النتائج التي تم الحصول عليها من خلال هذا العمل أن يكون في اتفاق جيد مع نتائج الباحثين.

Nomenclature

Latin Symbols

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
A	Clearance area	m ²
B	Height of tooth	m
C	Radial clearance	m
$C_{\varepsilon 1}$	k - ε turbulence model constant, 1.44	-
$C_{\varepsilon 2}$	k - ε turbulence model constant, 1.09	-
C_{μ}	k - ε turbulence model constant, 0.09	-
D	Shaft diameter	m
Re	Reynolds number based on clearance	-
S	Tooth pitch	m
U	Axial velocity	m/sec
V	Radial velocity	m/sec
W	Tooth width m	m

Greek symbols

<i>Symbol</i>	<i>Definition</i>	<i>Units</i>
α	Flow coefficient	-
β	Divergence angle of jet	radian
x	Percentage of kinetic energy carried over	-
ε	Dissipation of Turbulent Kinetic energy	m ² /sec ³
γ	Kinetic energy carry over coefficient	-
k	Turbulent Kinetic energy	m ² /sec ³
ρ	Fluid density at tooth inlet	kg/m ³

1- INTRODUCTION :-

Labyrinth seals are widely used in turbo machines such as turbines, compressors and pumps. The main function of the labyrinth seals is to reduce the leakage flow between components of different pressure and prevent the rotor from contacting the stator at a very high speed, because any contact between the rotor and stator causes seal damage, deterioration of seal performance and engine failure. Labyrinth seals have proved to be quite effective and long-lived as evidenced by their widespread use, chiefly on rotating machinery such as gas compressors and turbines. The straight type labyrinth seal is the most used in turbomachinery because of its greater kinetic energy carry over in comparison with other types of seals. The straight labyrinth seal can be classified according to arrangement of teeth into labyrinth seal with teeth on rotor or teeth on stator. Most previous works show that the modeling of seals was mainly based on the bulk flow model (**Child and Scharrer, (1986), Esser, and Kazakia, (1995), Yilmaz, and Esser,, (2004), Gamal, (2007)**). The main drawback of this model, due to the simplifying assumptions, is that sometimes it fails to predict the flow of labyrinth seal. Due to the bulk flow model limitation, it is claimed by **Pugachev, (2012)** that the computational fluid dynamics (CFD) method for solving Navier- Stokes equations were applied to obtain more satisfactory predictions for various seal types at different boundary and operating conditions. The previous studies on the application of (CFD) methods focus on modeling the fluid flow through labyrinth seals of small clearances with teeth on the stator. **Saikishan and Morrison (2009)** have been predicted of carry over for labyrinth seal with teeth on stator. **Sunil, (2010)**, simulated the leakage prediction of labyrinth seal having advanced cavity shape using commercial CFD software (FLUENT). It has been focused on seals with isosceles triangle shaped teeth, right triangle shaped teeth, and a NASA seal. **Jeng, (2011)** analyzed the compressible and incompressible flows through labyrinth seals using FLUENT software. It has been shown that the carry-over coefficient, based on the divergence angle of the jet, changed with flow parameters with fixed seal geometry only. **Vamish (2011)** studied the effect of shaft rotation on flow parameters without modeling carry-over coefficient for labyrinth seal. It can be concluded from above that there is a lack of information related to leakage prediction in labyrinth seal. Hence a model for labyrinth seal with large clearance has been developed. The above model is numerically solved in order to calculate the effect of flow and geometric parameters on the carry over coefficient. The solution is based on the finite volume method by using the commercial (CFD) code **FLUENT 6.3.26**. The continuity and momentum equations have been discretized using standard $k - \epsilon$ turbulence model which has been proved to be accurate for modeling the flow through seals with and without rotational effect (**Morrison and Al-Ghasem, 2007**).

2. MATHEMATICAL MODELING :-

2-1 Labyrinth Seal Geometry and Coordinates.

The analysis presented here is applicable to straight labyrinth seal with teeth on the rotor. Figure (1) shows the geometry and coordinates system for a single cavity with two teeth on rotor labyrinth seal and axisymmetric flow (i.e., two dimensional flow simulation in radial and axial direction).

W is the tooth width (mm). C is the clearance (mm). H is the tooth height (mm), S is the tooth pitch (mm)

2-2 Governing Equations.

The leakage fluid flow driven by the pressure difference along the labyrinth seal is governed by the continuity and momentum equations,

Continuity Equation:

The continuity equation for incompressible fluid flow can be written as (Wie (2007)):

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial r} + \frac{V}{r} = 0 \quad (1)$$

Momentum equations

The momentum equation for 2-D axisymmetric geometry in axial direction can be written as:(Wie(2007))

$$\frac{\partial}{\partial x} (\rho U U) = -\frac{\partial P}{\partial x} + 2 \frac{\partial}{\partial x} \left(\mu_s \frac{\partial U}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left[\mu_s \left(r \frac{\partial V}{\partial x} + \frac{1}{r} \frac{\partial U}{\partial r} \right) - r \rho U V \right] \quad (2)$$

While the momentum equation in radial direction can be written as:-

$$\frac{\partial}{\partial x} (\rho U V) = -\frac{\partial P}{\partial r} + \frac{\partial}{\partial x} \mu_s \left[\frac{\partial U}{\partial r} + \frac{\partial V}{\partial x} \right] + \frac{\partial}{\partial r} \left[\mu_s \frac{\partial V}{\partial r} \frac{r^2+1}{r} - \rho V V \right] - \frac{2\mu_s V}{r^2} + \frac{\rho}{r} W^2 \quad (3)$$

$$\frac{\partial}{\partial x} (\rho U W) = \frac{\partial}{\partial x} \mu_s \left[\frac{\partial W}{\partial x} \right] + \frac{\partial}{\partial r} \left[\mu_s \frac{\partial W}{\partial r} - \rho V W \right] - \frac{W}{r^2} \frac{\partial}{\partial r} (r \mu_s) - \frac{\rho}{r} V W \quad (4)$$

Where U, V and W are the velocity components in the axial (x), radial (r) and circumferential (θ) directions respectively, μ_s represents summation of the molecular viscosity (μ) and the turbulent viscosity (μ_t). To solve the problem of turbulence, the standard k- ϵ model with wall functions was specified. The equations for the turbulent kinetic energy k and its dissipation rate ϵ can be written as [Vamish(2011)]:

$$\frac{\partial}{\partial x} (\rho U k) = \frac{\partial}{\partial x} \left[\left(\frac{\mu_s}{\sigma_k} \right) \frac{\partial k}{\partial x} \right] + \frac{\partial}{\partial r} \left[\left(\frac{\mu_s}{\sigma_k} \right) \frac{1}{r} \frac{\partial k}{\partial r} - r \rho V k \right] + G - \rho \epsilon \quad (5)$$

$$\frac{\partial}{\partial x} (\rho \epsilon U) = \frac{\partial}{\partial x} \left[\left(\frac{\mu_s}{\sigma_\epsilon} \right) \frac{\partial \epsilon}{\partial x} \right] + \frac{\partial}{\partial r} \left[\left(\frac{\mu_s}{\sigma_\epsilon} \right) \frac{1}{r} \frac{\partial \epsilon}{\partial r} - r \rho V \epsilon \right] + C_1 \frac{\epsilon G}{k} - C_2 \frac{\rho \epsilon^2}{k} \quad (6)$$

The turbulence eddy viscosity μ_t is determined by

$$\mu_t = \rho C_\mu \frac{k^2}{\epsilon} \quad (7)$$

The constants in Eqs. (5) - (7) are determined as

$$C_\mu = 0.09, C_1 = 1.44, C_2 = 1.92, \sigma_k = 1.0, \sigma_\epsilon = 1.22.$$

2.3 Solution procedure

Initially the seal geometry is meshed by using **GAMBIT 2.3.16**. In order to insure an equilibrium condition before the flow enters the cavity a 0.3 m long entrance and exit regions before and after the seal are considered. The mesh has been adapted with maximum pressure gradient set to **0.1**, near the rotor surface and the stator wall to insure that the Y^+ values are near or less than **5** to resolve the laminar sub-layer as shown in Figure(2).

To assure grid independent results for the CFD calculations, a preliminary CFD study was conducted. For this study, the seal geometry displayed in Figure (1) with $W/S = 0.0075$ and $H/S = 1$ was discretized with several grid resolutions. A grid resolutions was obtained by observing the mass flow rate predicted with various number of grids. At the inflow boundary condition is set at inlet pressure 122.5kPa with water as working fluid. At the outlet a pressure of 101 kPa was specified. The non slip condition is applied to all solid wall. The pressure field P was obtained by employing the SIMPLE algorithm. The maximum cell-based residual 10^{-4} was used as the convergence criteria. In computations, typically more than 10^3 iterations were sufficient to arrive convergence. It can be shown from fig.(4) that the change in mass flow rate was less than **0.5%** as the number of grids increase more than **20000**.

Hence there are five major components to optimize the procedure:

1. A grid generation software: **GAMBIT 2.3.16** has been used for this purpose.
2. A pre-processing software (**Fluent**) for the setup of the boundary and initial conditions.
3. The flow-solver (**Fluent**) to calculate the leakage mass flow rate then it transformed to a dimensionless parameter (Reynolds number) using equation().
4. A post-processing tool (**Tecplot 360**) for the evaluation of the divergence angle to calculate the kinetic energy carry over coefficient.
- 5- At specified conditions and seal geometry draw the relation $Re - \gamma$.

Carry over Coefficient

The carry over coefficient can be defined as the portion of the undissipated kinetic energy of the fluid exiting at the seal clearance that is converted to the next cavity (**Woo, 2011**). The carry over coefficient, γ , and the percentage of the kinetic energy carried over into the next cavity, χ , are defined by (**Hodkinson, 1939**) as follows:

$$\gamma^2 = \frac{1}{1-\chi} \quad (8)$$

where:

$$\chi = \frac{c}{c+s \tan \beta} \quad (9)$$

where β is the divergence angle defined by noting the position on the downstream tooth where the radial velocity is zero **Saikishan and Morrison (2009)**, **Jeng, (2011)**, and **Sunil,(2011)**. The exact position needs to be accurately determined for accurate determination of (β). **Tecplot 360** software has been used for this purpose after suitable magnification and limiting the range of measurement. Referring to Figure (5) the angle (β) can be calculated as follows:

$$\beta = \tan^{-1}\left(\frac{l_2}{l_1}\right) \quad (10)$$

The above definition of γ assumes one major recirculation zone in the cavity. This assumption is valid until rotor shaft speeds impart significant tangential velocities which produce a body force and a secondary recirculation zone which lead to an invalid definition of γ . The carry over coefficient cannot be less than one according to this definition. Since it is difficult to define γ when there is no dividing streamline that separates the flow in the cavity from the flow above tooth, the carry over coefficient(γ) is assumed to be unity when stream line is carried out into the cavity due to centrifugal effects.

3. RESULTS AND DISCUSSIONS :

In the present work a number of simulations, as presented in Table (1), are performed with fixed seal geometry and varying flow conditions. The carry over coefficient is calculated for each case. The effects of different parameters (Reynolds number, pressure ratio, and the shaft rotation...etc) on kinetic energy carry over are studied. The initial cases considered are for a single cavity (two teeth) seal. While a seal with multiple cavities will be considered later. The seal geometry has been fixed so as to consider only the effect of flow parameters. The initial analysis utilizes water to eliminate compressibility effects and considers a stationary shaft for a given seal geometry which has been shown in Table (1), the carry over coefficient (γ) was calculated for different mass flow rates. The Reynolds number based on clearance, was found to be the dominant parameter which can be defined as [**Dereli, Y. and Eser, D., 2004**] .

$$Re = \frac{2 \dot{m}}{\pi D_m \mu} \quad (11)$$

Where:

$$D_m = (D + 2B + C) .$$

D = Shaft diameter (m).

B = Tooth height (mm).

C = Clearance between stator and rotor (mm).

μ = Dynamic viscosity of water = 0.001003 N.s/m²

The effect of different flow parameters affecting the carry-over coefficient of straight through labyrinth seals working with water as incompressible fluid have been discussed as follows:

3.1. Effect of Reynolds number

A prediction model for carry over coefficient of incompressible fluid, needs a study to the relation between the carry over coefficient and Reynolds number at different exit pressures. Fig(6) shows such study for seal geometry **case (1)** in table (1) at values of exit pressures (**1, 2, 5 atm**) and inlet pressure calculate at define value of pressure ratio (ratio of outlet pressure to inlet pressure) which vary at five values(**0.1 , 0.25, 0.5, 0.75, 0.9**).

The effect of the Reynolds number on the carry over coefficient is effectively modeled using a power law curve fit of the following form

$$\gamma = C_1 \left[Re + C_2^{-1} \right]^{C_2} \quad (12)$$

where C_1 and C_2 are possibly functions of seal geometry and other flow parameters yet to be determined. The presence of C_2^{-1} in the model ensures that γ becomes 1 when the Reynolds number is 0. This is essential as the minimum value of γ is 1. For this baseline geometry (case 1 in table (1)), $C_1=0.9322$ and $C_2=0.019$.

3.2.Effect of seal geometry

Effect of the following main geometrical parameters on carry over coefficients has been studied as follows:

3.2.1. Effect of clearance

Since the clearance represent one of the major geometric parameter that affect on the carry over coefficient with the change of Reynolds number. So the relation of carry over coefficient with Reynolds number at different c/s has been shown in figure(7). It can be seen also from this figure at constant Re as the clearance increases the carry over coefficient increases too. This is according to Hodkinson theory. It can be stated for the same clearance for the same Re leads to higher inertial force of the jet and as the lower viscous force from above the tooth cause smaller divergence angle of the jet to generate larger portion of the jet's travelling the cavity (undissipated) and passing above the downstream tooth without being dissipated by

turbulence viscosity interactions. Therefore; a relatively smaller portion of the kinetic energy is dissipated in the cavity producing the larger carry over coefficient. In order to access the effect of c/s ratio, seals with four different clearances are simulated for a range of Reynolds numbers with a fixed pitch, which corresponded to cases (1 to 4) presented Table (1). The power law given by equation (12) is applied to each data series in fig (6) and constants C_1 and C_2 are found for the best fit as shown in equations (13) to (16).

$$\gamma = 0.9322(Re + 40.24)^{0.019} \text{ for } c/s=0.0075, R^2 = 0.93 \quad (13)$$

$$\gamma = 0.8596(Re + 56.51)^{0.0375} \text{ for } c/s=0.015, R^2=0.9677 \quad (14)$$

$$\gamma = 0.7811(Re + 72.36)^{0.0577} \text{ for } c/s=0.0225, R^2=0.93 \quad (15)$$

$$\gamma = 0.6404(Re + 85.0545)^{0.1003} \text{ for } c/s=0.0375, R^2=0.9877 \quad (16)$$

Figure (7) also shows that reducing the clearance to pitch ratio from 0.015 to 0.0075 reduces the values of γ by about 9 % for the same Reynolds number. It can be also observed that the simulation results in a larger deviation from the curve fit at higher Reynolds numbers and clearances. From the curve fits it is observed that C_1 decreases with increasing c/s ratio while C_2 increases with increasing c/s. The relationships are found to be linear, as shown in figure(8).

It is well known that the carry over coefficient approaches 1 as Re approaches 0 according to equation (12). In order to satisfy this, C_1 must be equal to (1) while C_2 should be equal to zero at $c/s = 0$ (as zero clearance implies no flow and zero Re). Considering the above observation, C_1 and C_2 in equation (12) are expressed in the form:

$$C_1 = 1 - R_1 \left[\frac{c}{s} \right] \quad (17)$$

and

$$C_2 = R_2 \left[\frac{c}{s} \right] \quad (18)$$

The constants R_1 and R_2 are determined as 9.67 and 2.6277 respectively. Substituting equations (17) and (18) in equation (12), leads to the following model for the carry over coefficient:

$$\gamma = \left(1 - 9.436565 \left(\frac{c}{s} \right) \right) (Re + (1 - 9.436565(c/s)))^{-1/2.6277(C/S)} 2.6277(C/S) \quad (19)$$

3.2.2.Effect of Tooth Width

The effect of (w/s) on the carry over coefficient for the cases (2, 5, 6 and 7) of Table (1) can be shown in Figure (9). It can be deduced from this figure that the tooth width has a little effect on

the carry over coefficient, spatially for Reynolds number higher than 2000. It has been found that the relationship between the carry over coefficient and the Reynolds number can be represented by power law relation. The following power law curve fit equations for the data series in fig(8) at a ratio of w/s from (0.0075 to 0.5).

$$\gamma = 0.875(\text{Re}+41.6)^{0.0358} \text{ for } w/s=0.0075 \quad (20)$$

$$\gamma = 0.8682(\text{Re}+49.2)^{0.036} \text{ for } w/s=0.1 \quad (21)$$

$$\gamma = 0.8444(\text{Re}+63.32)^{0.0408} \text{ for } w/s=0.25 \quad (22)$$

$$\gamma = 0.7923(\text{Re}+116.5)^{0.0489} \text{ for } w/s=0.5 \quad (23)$$

The relation between constants C_1 and C_2 and tooth width (w/s) for a certain (c/s=0.015) has been studied as shown in Figure (10).

The effect of tooth width can be added to the previous model of carry over coefficient presented in equation (19) as follows:

$$\gamma = (1 - 9.436565 \left[\frac{c}{s} \right] - R_3 f_1) (\text{Re} + R_0)^{(2.677 (C/S) + R_4 f_2)}$$

Where:

$$R_0 = \left(1 - 9.436565 \left[\frac{c}{s} \right] - R_3 f_1 \right)^{-(2.677 (C/S) + R_4 f_2)} \quad (24)$$

Both f_1 and f_2 represent the functions of W/S. R_3 and R_4 are constants for a given C/S, which are functions of c/s. The variation of C_1 and C_2 with tooth width for different values of c/s can be shown in Figure (11) for the cases (1, 4, 8 and 9) explained in Table (1).

The effect of tooth width on the carry over coefficient can be included in equation (24) as follows:

$$\gamma = (1 - 9.436565 \left[\frac{c}{s} \right] - A(c/s)^a (w/s)^b) (\text{Re} + R_0)^{\left(2.677 \left(\frac{c}{s} \right) + B \left(\frac{c}{s} \right)^c \left(\frac{w}{s} \right)^d \right)} \quad (25)$$

The a, b, c and d have been calculated using SPSS software. Hence equation (25) can be rewritten as:

$$\gamma = (1 - 9.436565 \left[\frac{c}{s} \right] - 0.401307 (c/s)^{0.2958546} (w/s)^{0.91784897}) * (\text{Re} + R_0)^{-1/cc}$$

$$R_0 = (1 - 9.436565 (c/s) - 0.401307 (c/s)^{0.2958546} (w/s)^{0.91784897})^{cc}$$

$$cc = \frac{-1}{2.6277 \frac{c}{s} + 0.2954893 \left(\frac{c}{s} \right)^{0.6863094} \left(\frac{w}{s} \right)^{0.964921808}}$$

The above modified equation has been proved to be a better equation for the model used since the relative error reduced from 1.07% to 0.9%.

3.2.3.Effect of Shaft Rotational Speed

A shaft rotational speed may changes the flow pattern within the seal which influences the carry over coefficient. The effect of rotational speed on the carry-over coefficient of the labyrinth seal has been studied by considering different shaft speeds at a given flow condition and seal geometry for the cases 1, 2 and 4 of Table (1). Reynolds number is assumed to be 1000 as shown in Figure (12). It can be seen from this figure that the carry over coefficient seems to be constant for the range of shaft speeds (0 -10000) rpm. For the range of shaft speeds (11000-20000) rpm the fluid is subjected to higher radial pressure gradient within the cavity leading to detachment of the streamlines that exit at the preceding tooth and being drawn into forming a secondary vortex. Hence for cases like that in Figure (13) the carry over coefficient cannot be defined by Hokinson's equation and the carry over coefficient is taken as the least possible value ($\gamma=1$ in this case).

3.3. Validation

The CFD models adopted in the present work are validated against previous experimental and theoretical works conducted by other workers. Figure (14) shows a comparison between the results obtained by the leakage mass flow rate of air as in through the straight - through labyrinth seal model with that obtained experimentally by **Scharrer (1988)** , **Derli and Eser (2004)** . The seal geometry and the operating conditions used in validation process are summarized in Table (2).The percentage of deviation between the results obtained with that for **Scharrer** and **Eser** was to be 5% and 11%, respectively.

Also the results obtained for the carry-over coefficient of labyrinth seal with air as a working fluid have been compared to that obtained by **Vamshi, Y. (2011)**, as shown in figure (15).

The carry over coefficient for the straight through labyrinth seal with a water as working fluid has been compared with that obtained by **Vamshi, (2011)**, as shown in Figure (16). The percentage of deviation for the results in Figure (15) was found to be 2.8 %. While for Figure (16) was found to be 0.4 % .

4. CONCLUSIONS :

It is clear from above results that the carry over coefficient affected by the seal geometry such as clearance and teeth width and flow parameters such as Reynolds number. The results obtained show that the carry over coefficient increased from 1 to 1.8 with increasing the clearance to pitch ratio from 0.0075 to 0.0375 at constant tooth width to pitch ratio and decreased

with increasing the tooth width to pitch ratio at constant clearance to pitch ratio. Effect of shaft rotation on carry over coefficient has been studied and show that the carry decreased after a certain value of rotational speed (10000 rpm) due to the secondary recirculation zone which appear at higher rotational speed prevent the fluid to leakage were dissipation energy increased . Hence the modified model of carry over coefficient for labyrinth seal with teeth on rotor has been show in equation (25). From this equation the carry over coefficient can be calculated at specified Reynolds number and seal geometry at a range of shaft rotation from 0 to 10000 rpm.

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Table (1). Seal geometries used for simulation

Case No.	No. of Teeth	Clearance (mm)	Pitch (mm)	Tooth Width (mm)	Tooth Height (mm)	Shaft Diameter (mm)
1	2	0.03	4	0.03	4	60
2	2	0.06	4	0.03	4	60
3	2	0.09	4	0.03	4	60
4	2	0.15	4	0.03	4	60
5	2	0.06	4	0.4	4	60
6	2	0.06	4	1	4	60
7	2	0.06	4	2	4	60
8	2	0.03	4	1	4	60
9	2	0.15	4	2	4	60

Seal Geometry	Operating Conditions
Number of teeth = 5,10,15,16	$P_{in} = 7.00E+5$ & $3.08 E+5$ N/m ²
Radius of Shaft = 0.0756 m	$Pe = 1.01 E+5$ N/m ²
Height of Teeth =0.03175 m	Temperature =300 K
Clearance = 0.000127 & 0.00033 m	Gas Constant 287.06 Nm/kg.k
Width of teeth = 2 E-4	Rotational velocity 16000 & 20000 rpm
Pitch = 0.002175 m & 0.003175 m	Fluid Air

TABLE (2) . SEAL GEOMETRIES AND OPERATING CONDITIONS

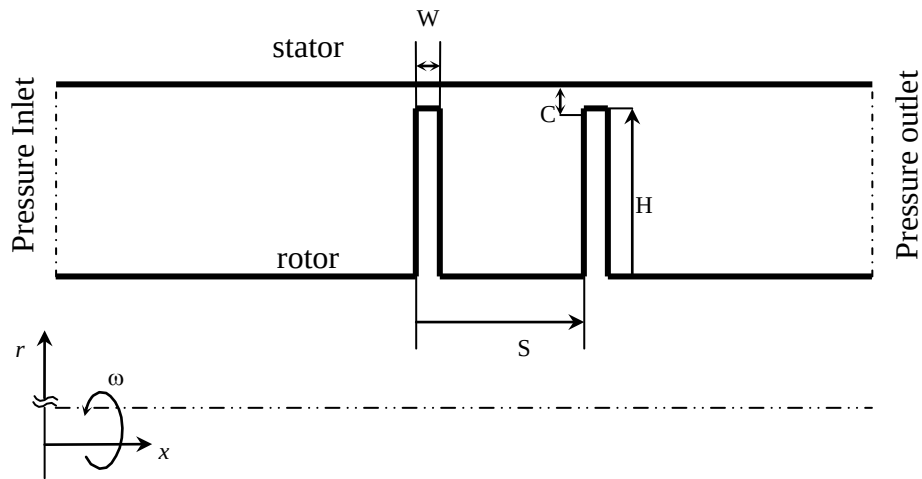
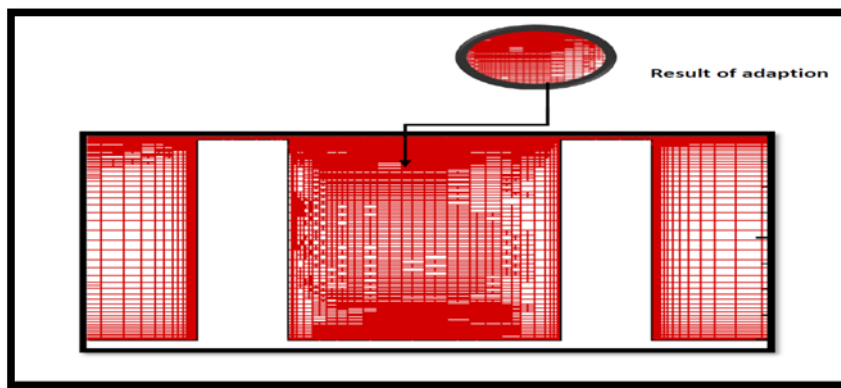


Fig.(1) Seal geometry and coordinates system.



Fig(2). Computational mesh.

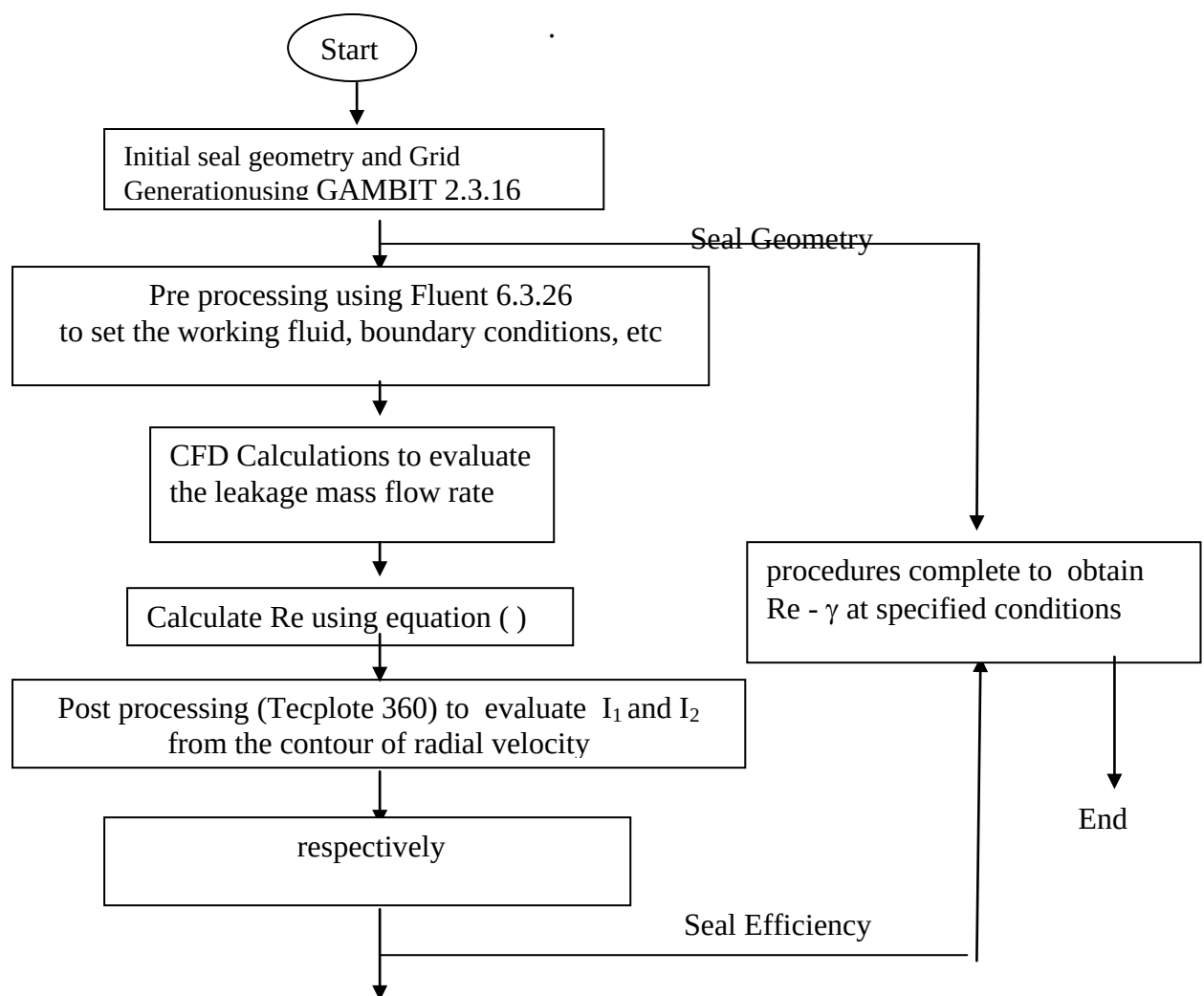


Figure (3) Outline of the optimization procedure.

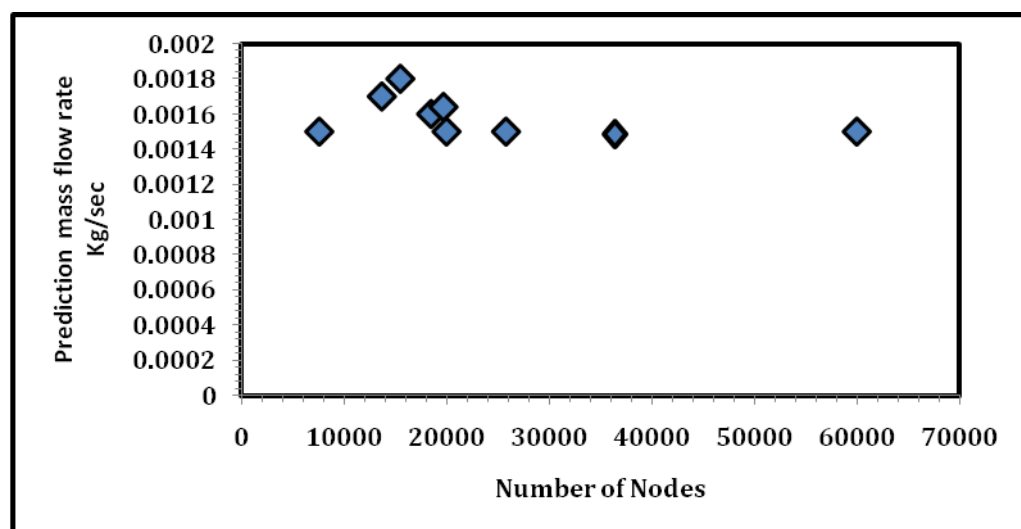


Figure (4) . Accuracy of mass flow rate prediction against number of nodes.

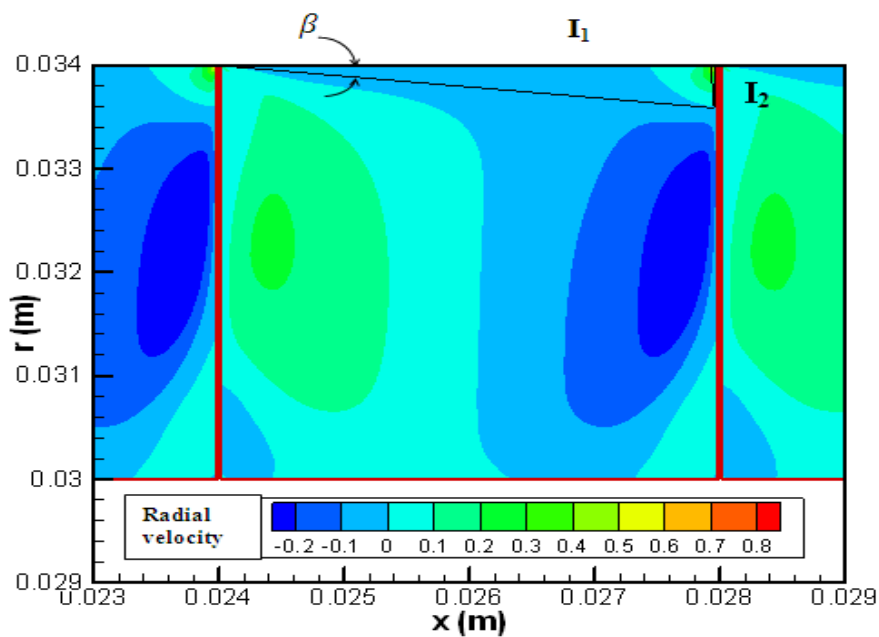


Fig.(5):Shows The Contour Plots of The Radial Velocity and Measurement of β .

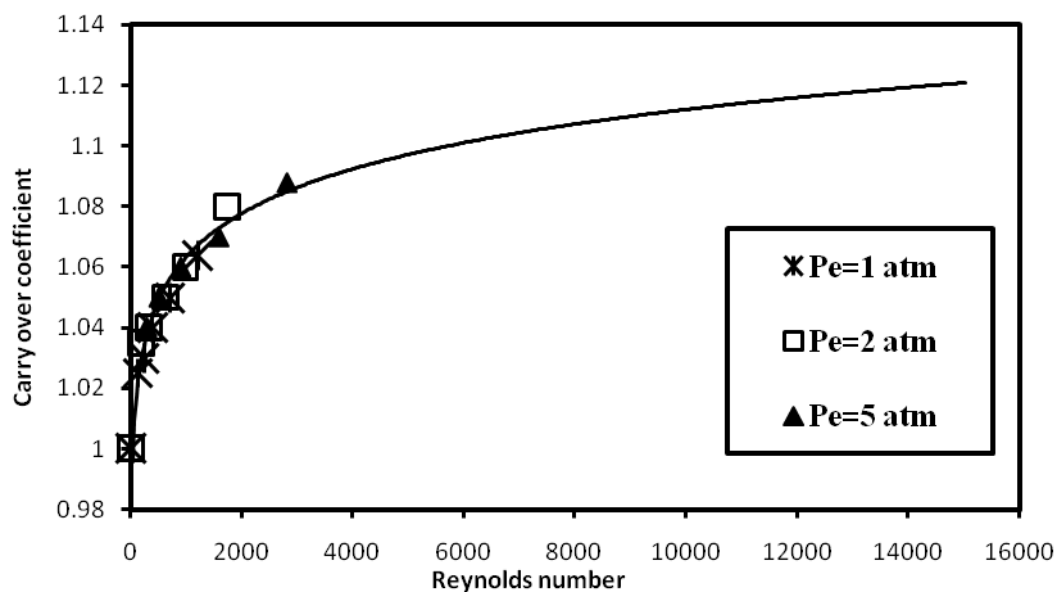
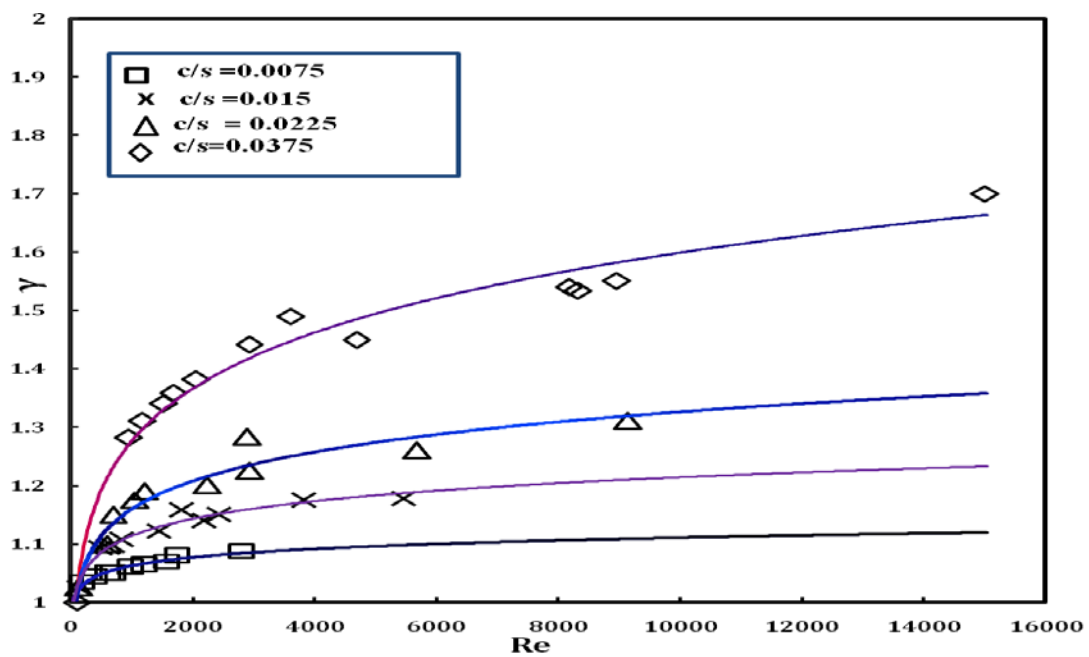


Figure (6). Carry over coefficient vs. Reynolds Number for water at different exit pressures for case (1) in Table (1).



Figure(7). Reynolds number VS. carry over coefficient for cases (1- 4) of Table (1)

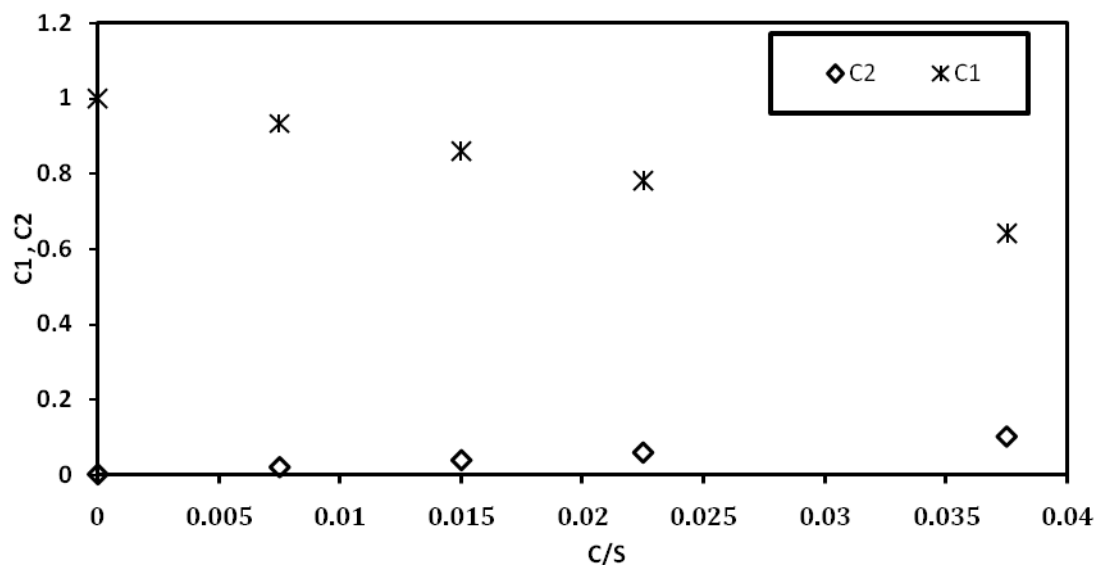
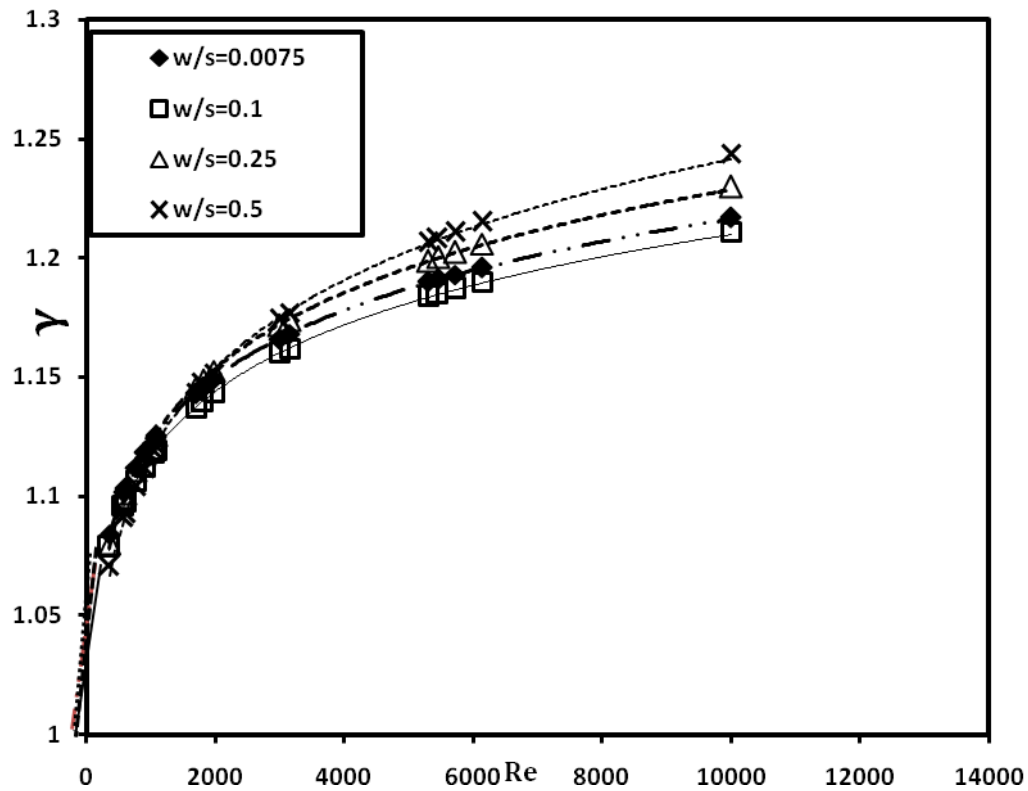


Figure (8) Variation of C1 and C2 with c/s ratios.



Figure(9). Relationship between carry over coefficient and Reynolds number for different tooth widths (for $c/s = 0.015$).

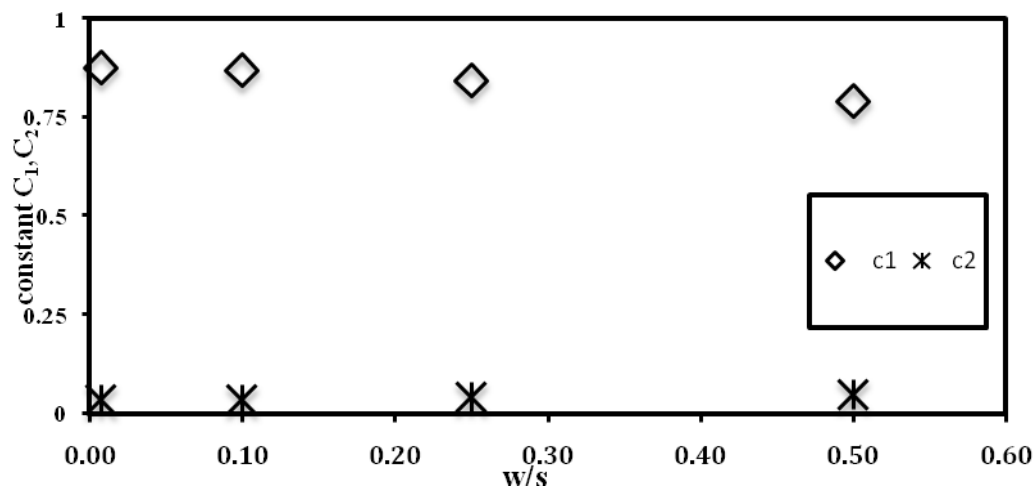
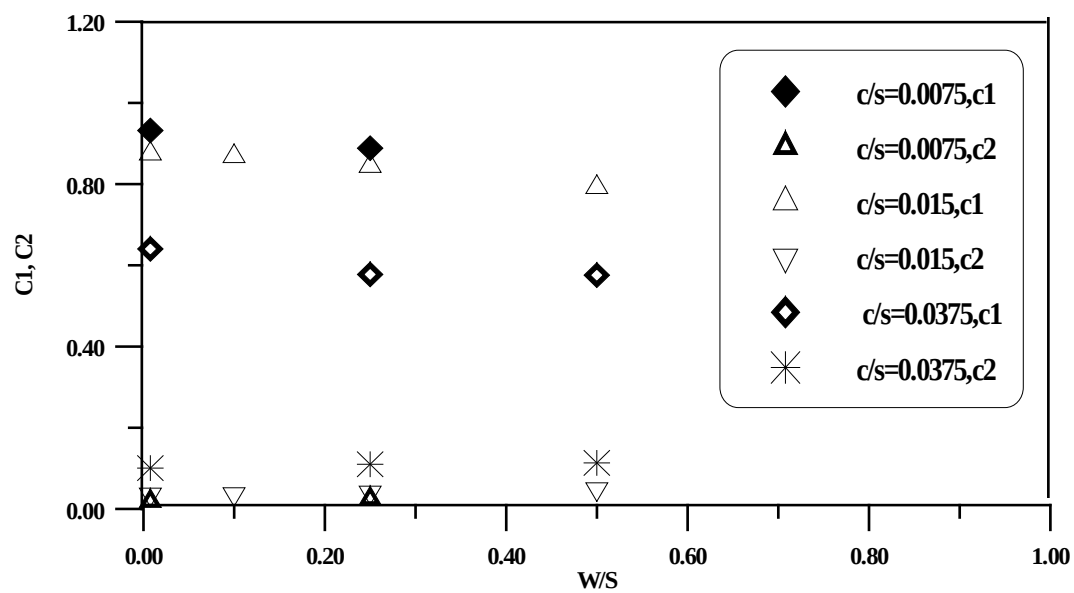
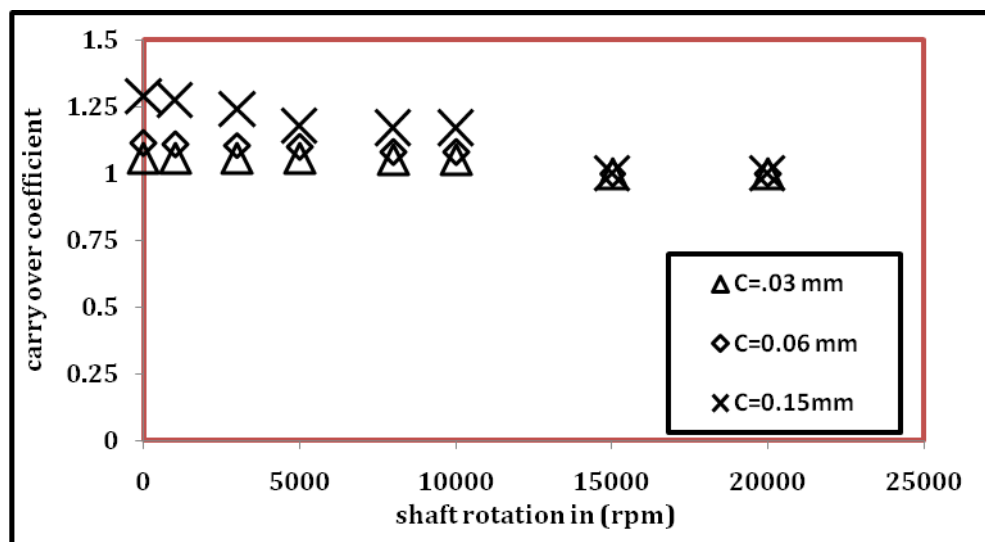


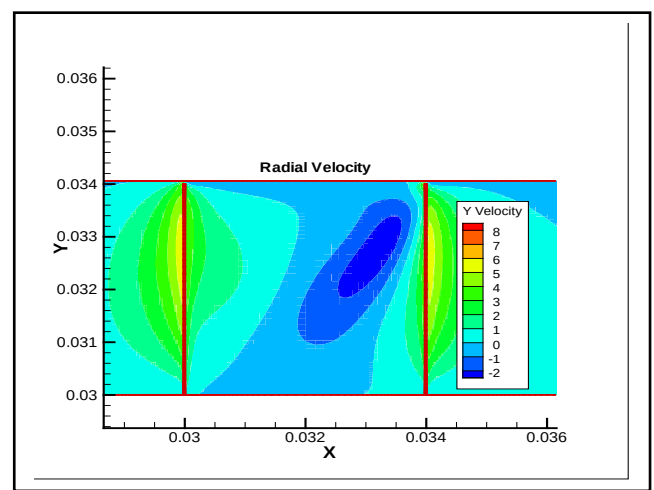
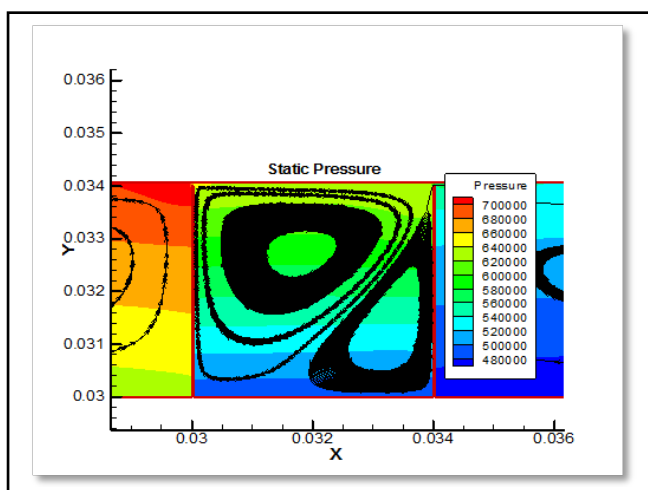
Figure (10) .Variation of c_1 , c_2 with tooth width (for $c/s = 0.015$).



Figure(11) . Variation of C_1 and C_2 with tooth width for different values of C/S .



Fig(12).Variation of γ with shaft speed (for case1, 2, 4 in Table (1), $Re = 1000$).



Fig(13).Contour of Static pressure and Radial Velocity at(15000rpm).

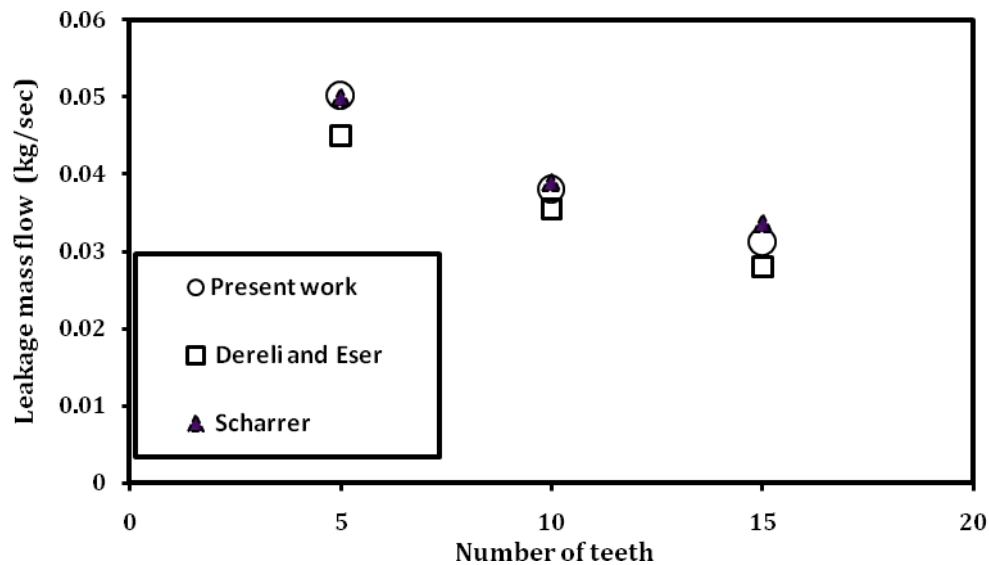


Figure (14). Comparison of leakage flow rate at $P_{in}=7.00E+5 \text{ N/m}^2$.

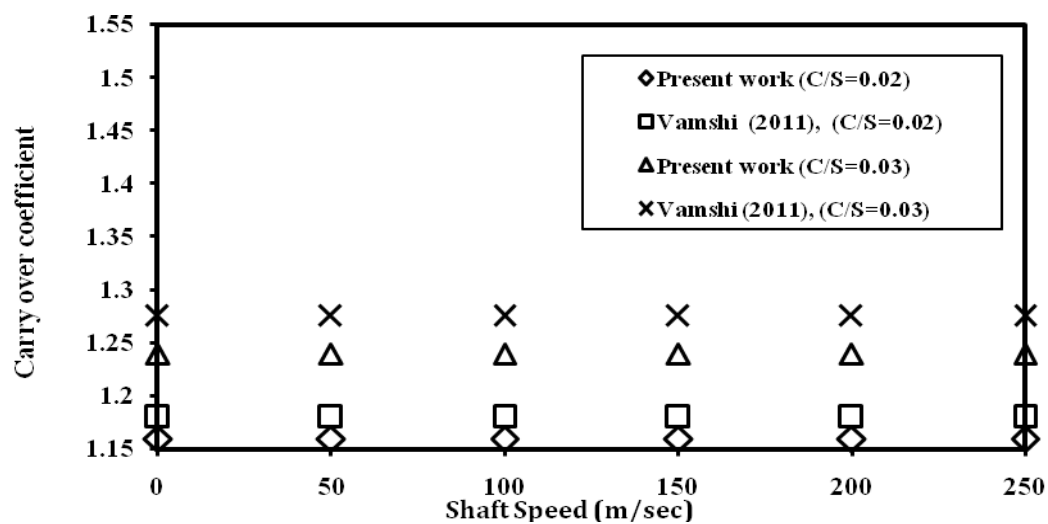


Figure (15). Effect of shaft rotation on carry over coefficient at
 $Re = 1000$ of Air .

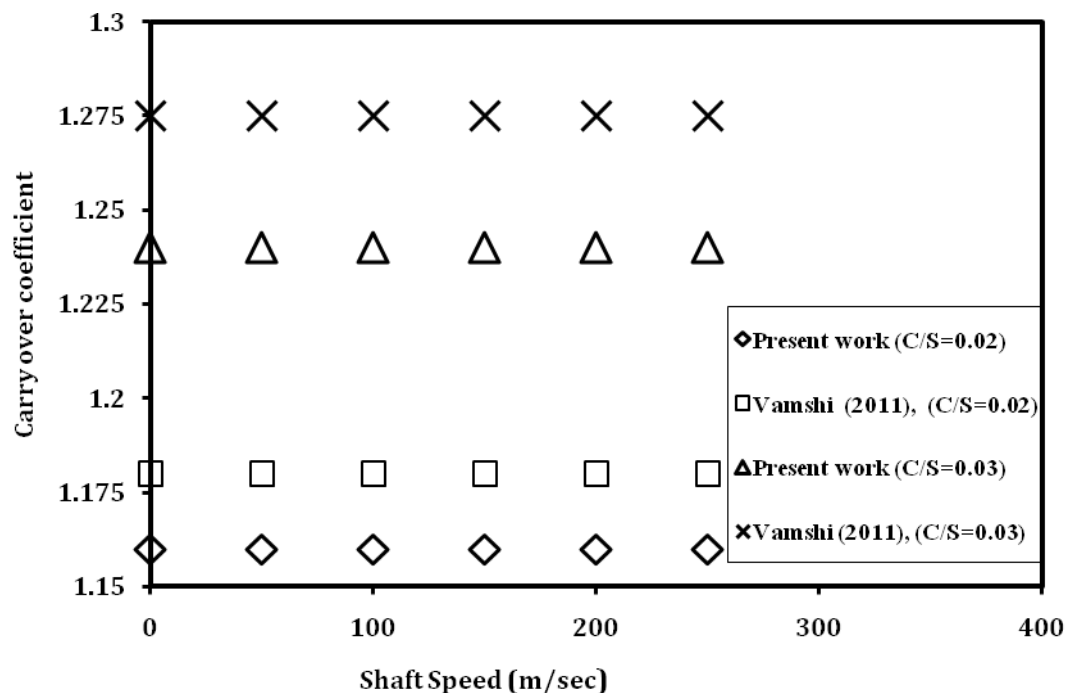


Figure (16).Effect of shaft rotation on carry over coefficient at
Re = 1000 of water .

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