

## STUDY OF LAMINAR MIXED CONVECTION FOR TWO-SIDED NON-FACING LID-DRIVEN DIFFERENTIALLY HEATED SQUARE CAVITY

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### ABSTRACT:

Laminar mixed convection for incompressible flow in two sided non-facing lid driven square cavities are numerically investigated in this study using ANSYS 12.1 commercial software. Depending on the movement direction for the walls and various temperature locations on these walls, cases have been considered. Vortices are generated by moving the upper and the right walls in the first case, while in the second case, bottom and left walls are using with different locations for hot and cold temperatures on the moving walls. For each case, the non-facing walls that which in opposite side to the moving walls are thermally insulated and stationary. Natural and forced convections are distinguished by an important relative value of Richardson number ( $Ri=Gr/Re^2$ ) which relates the fluid flow and the heat transfer in the cavity. Governing parameters are varied as  $0.1 < Ri < 10$  and  $Pr=0.7$ . The obtained results show that the fluid flow and heat transfer in the cavity are effected by Richardson numbers, locations and directions of moving walls, and locations of hot and cold temperatures. Two sided non-facing lid driven for the cavity is give the same results for streamlines and isotherms with the another opposite two sided non-facing lid-driven, but inversely and upside down for different Richardson numbers. The results of this investigation illustrate that when install the hot temperature on the lower moving wall and the cold temperature on the left moving wall in the second case, the local Nusselt number in this situation is to be relatively higher and heat transfer is enhance, as compared with the other differential heated cavities orientations.

**Keywords:** incompressible flow, mixed convection, non-facing lid-driven, square cavity.

### دراسة الحمل الطبقي المختلط لسواقة جانبيين غير متقابلين في التجويف الساخن بشكل تفاضلي

الخلاصة :

تم حل الحمل الطبقي المختلط للجريان الا انضغاطي في سواقة تجاويف مربعة الشكل ذات جدارين غير متقابلين عددياً باستخدام البرنامج التجاري انسز ANSYS 12.1. اعتماداً على اتجاه الحركة للجدران واختلاف مواقع درجات الحرارة على هذه الجدران تتكون الدوامات بحركة الجدران العلوي والأيمن

في الحالة الأولى بينما الحالة الثانية تكون الحركة للجدارين الأيسر والسفلي حيث أُستعملت مواقع مختلفة لدرجات الحرارة الحارة والباردة على الجدارين المتحركين لكل حالة مدروسة، الجدران الغير متواجئة التي في الجانب المعاكس إلى الجدران المتحركة المعزولة والساكنة. الحمل الحراري الحر والقسري مُيَّز بقيمة نسبية مهمة كعدد ريكاردسون Richardson ( $Ri = Gr/Re^2$ ) الذي يربط بين جريان المائع وانتقال الحرارة في الفجوة. المتغيرات الحاكمة في هذه الدراسة كانت  $0.1 < Ri < 10$  و  $Pr = 0.7$ . النتائج التي تم الحصول عليها تبين ان تدفق الجريان وانتقال الحرارة يتأثران بأعداد Richardson، ومواقع واتجاهات الجدران المتحركة ومواقع درجات الحرارة الحارة والباردة. سواقة الجانبين غير المتقابلين للتجفيف تعطي نفس النتائج لخطوط الانسياب والحرارة مع سواقة الجانبين الغير متقابلين المعاكسين الآخرين لكن بشكل عكسي ومقلوب لمختلف إعداد ريكاردسون. نتائج هذا التحقيق تبين أنه عند تثبيت درجة الحرارة الساخنة في الجدار السفلي المتحرك ودرجة الحرارة الباردة على الجدار الأيسر المتحرك في الحالة الثانية، فإن عدد نسلت Nusselt الموقعي سيصبح أعلى نسبياً ونقل حرارة أكثر، مقارنة بالتجاويف الساخنة الأخرى ذات المواقع المختلفة للسطحين البارد والساخن.

#### NOMENCLATURE :-

Unit	Description	Symbols
J/kg. °C	specific heat at constant pressure	$C_p$
-	Grashof number	$Gr = \frac{g\beta\Delta TL^3}{\nu^2}$
m/s <sup>2</sup>	gravitational acceleration	$g$
W/m <sup>2</sup> . °C	heat transfer coefficient	$h$
W/m. °C	thermal conductivity of fluid	$k$
m	side length	$L$
m	characteristic length	$L_c$
-	local Nusselt number	$Nu$
N/m <sup>2</sup>	Pressure	$P$
-	Richardson number	$Ri$
°C	temperature	$T$
m/s	lid velocity	$U_p, V_p$
m/s	velocity component in x-direction	$V_x$
m/s	velocity component in y-direction	$V_y$
		<b>Greek symbols</b>
N.s/m <sup>2</sup>	dynamic viscosity of the fluid	$\mu$
m <sup>2</sup> /s	kinematic viscosity of the fluid	$\nu$
kg/m <sup>3</sup>	density of the fluid	$\rho$
m <sup>2</sup> /s	stream function	$\Psi$
-	depended variable	$\Phi$
K <sup>-1</sup>	Volumetric coefficient of thermal expansion	$\beta$
		<b>Subscripts</b>

-	cold wall	<b>c</b>
-	hot wall	<b>h</b>
-	plate	<b>p</b>
m	cartesian coordinate in horizontal direction	x
m	cartesian coordinate in vertical direction	y
		<b>Abbreviations</b>
-	Figure	Fig.

## 1-INTRODUCTION :-

The fluid flow and the heat transfer phenomenon in lid-driven cavity is taking by fluid mechanics researchers, that studied effects of dynamic vortices on natural, forced and mixed convection flows in channels, ducts and cavities in several applications of engineering. such as cooling of electronic devices, furnaces, lubrication technologies, chemical processing equipment, drying technologies, these applications are listed in the study of Oztop and Dagtekin [2004] and Mahapatra et al. [2006] , also heat exchangers and crystal growth, solar collectors, storage of nuclear waste etc, reviewed by Oztop and Varol [2009] and float glass production reviewed by Al-Amiri et al. [2007] and T.S. Cheng [ 2011].

Some studies have been fulfilled the behavior for fluid flow and heat transfer in lid-driven cavity with several categories:

The first category is interested by one sided moving wall lid driven as { Franco and Ganzarolli [1996], Prasad and Koseff [1996], Shi and Khodadadi [2005], Zoran et al. [2006], Al-Amiri et al. [2007], Basak et al.[2009] and Lioua et al. [2011] }.The second category is interested by two walls which are move in parallel or opposing lid driven differentially heated cavities as {Oztop and Dagtekin [ 2004 ] and Alinia et al. [2011]}. The third category is a two non-facing moving walls as {Beya and Lili [2008], Wahba [2008]} , also two and four sided non-facing moving walls as{Perumal and Dass [2011]}.

For each category, review the work of each researcher is summarized in the following paragraphs:

Franco and Ganzarolli [1996] studied a numerical simulation by using the finite volume – SOLA ( **SOL**ution **AL**gorithm ) method for combined forced and natural convection in a one sided lid driven square cavity. Numerical results was presented for Grashof number ranging from  $10^2$  to  $10^7$  , Reynolds number 100 , 400 and 1000. and Prandtl number varying from 0.01 to 7. They concluded that the transition from one convection mechanism to another, represented by the Richardson number  $Ri$  for  $Pr < 1$  and  $Ri / Pr^{1/3}$  for  $Pr > 1$  fluids. Prasad and Koseff [1996] studied the combination forced and natural convection heat transfer for one sided deep lid-driven cavity flow and concluded that the heat transfer coefficient was insensitive to Richardson number for a finite range parameter from 0.1 to 1000 and heat transfer was independent on the depth of the cavity.

Shi and Khodadadi [2005] presented a computational study of periodic laminar flow and heat transfer in a one sided lid-driven square cavity due to an oscillating thin fin. The lid moved from left to right and a thin fin was positioned normal to the right stationary wall. The length of the fin varied sinusoidally with its mean length. They concluded that the amplitude of fluctuations of the kinetic energy and temperature were very intense near the fin. Zoran et al. [2006] used a combination of BEM ( Boundary Element Method) and FEM ( Finite Element Method) in a numerical algorithm for the solution of the velocity-vorticity formulation of Navier-Stokes equations in order to increase the accuracy of computation of boundary vorticities and for computation of internal velocities in 3D lid driven cavity. Their results showed that pair of vortices appeared near the centerline of the cavity and moved out towards

the lower corners as the Reynolds number increased from 100 to 1000 and additional two vortices appeared in the top corners at  $Re=1000$ . Al-Amiri et al. [2007] studied the effect of bottom sinusoidal wavy surface on mixed convection heat transfer in a one sided lid-driven cavity and used the increasing of both the amplitude of wavy surface (0-0.075) and Reynolds number  $Re=500$  of lid-driven wall to follow increasing the average Nusselt number. They concluded that optimum heat transfer happened for two undulations with elevation in the amplitude of the wavy surface at low Richardson number  $Ri=0.01$  and with increasing of the sliding lid speed, also they considered that the corrugated lid-driven cavity as an effective heat transfer mechanism. Basak et al. [2009] presented a finite element simulations to investigate the influence of linearly heated side wall(s) or cooled right wall on mixed convection for one sided lid-driven flows in a square cavity. They concluded that multiple circulation cells appeared inside the cavity with the increase of  $Pr$  for  $Re=10$  and  $Gr=10^5$  in the case of linearly heated side walls.

Lioua et al. [2011] presented three dimensional analyses of laminar mixed convection and entropy generation in a cubic one sided lid-driven cavity. They founded that direction of lid was an effective parameter on both entropy generation and heat and fluid flow for low values of Richardson number  $Ri=0.01$ , but it became insignificant at high Richardson number  $Ri=10$  with fixed Reynolds number at  $Re=100$  and entropy generation rate was lowered near the constant walls. Oztop and Dagtekin [2004] studied numerically a mixed convection in a vertical two-sided lid driven differentially heated square cavity. They concluded that the fluid flow and heat transfer effected by both  $Ri$  and directions of moving walls, where at  $Ri<1$ , the direction effect of the moving walls on the heat transfer was the same when they are moving in opposite direction, while this effect reduced when walls has been in the same direction and at  $Ri>1$ , multi-cells has generated adjacent of the moving walls in addition to the cell at the center of the cavity. Alinia et al. [2011] studied a numerical study of mixed convection in an inclined two sided lid driven cavity filled with nanofluid using two-phase mixture model. They showed that addition of nanoparticles enhanced heat transfer in the cavity remarkably and caused significant changes in the flow pattern of a fluid from natural convection to forced convection regime.

Beya and Lili [2008] presented a numerical simulations of the three-dimensional fluid flow in a two-sided non-facing lid-driven cubical cavity. They concluded that the two-dimensional, two-sided non-facing lid-driven approximation presented the flow in the vertical mid-plane of the three-dimensional two-sided non-facing lid. Wahba [2008] presented a numerical simulations for incompressible flow in two-sided non-facing lid driven square cavities, where the flow had driven by the upper wall that moved to the right direction, while the left wall moved to the downward direction with equal speed. He concluded that due to moves walls, two primary vortices and two secondary vortices had generated in the diagonal line of symmetry of the cavity in low values of Reynolds numbers.

Perumal and Dass [2011] presented a Lattice Boltzmann method to obtain multiple stable solutions for two and four sided non-facing lid driven in square cavity with equal speeds. They concluded and proved that for Reynolds numbers  $Re<1071$ , the symmetric solutions were observed and the streamlines are diagonally symmetric. The symmetric state, where two separate primary vortices were formed apparently adjacent to each of the moving walls. It was evident that for these relatively low Reynolds numbers two primary and two secondary vortices were formed at this range of Reynolds numbers. As the Reynolds number increases the secondary vortices grow bigger and asymmetric flow patterns obtained only above specific critical Reynolds numbers  $Re=1073$ , and they took a sample of  $Re=2000$  and founded that one symmetric state and two asymmetric states. For the four-sided lid-driven square cavity flow, the symmetric state happened for  $Re<127$ , Additional asymmetric flow patterns obtained above the critical Reynolds number of  $Re=129$  and a post-critical Reynolds number of 300 was

chosen that gave one symmetric state and two asymmetric states were identified for the four-sided lid-driven cavity flow at this Reynolds number.

However, a few studies are performed in the third category, and the researchers took a large care of important studies, for the stability test of the flow in the cavity, but they are did not present non-facing lid driven with differentially heated cavity, In this study, the mixed convection in two sided non-facing lid driven heated square cavities are carried out.

## 2. PROBLEM DESCRIPTION :-

Figure (1) illustrates the boundary conditions for a two sided non-facing lid driven square cavity that filled with laminar and incompressible fluid, the non-facing lids have different constant temperatures, and the other non-facing walls that which in opposite side to the non-facing lids are insulated and stationary.

Two cases were considered in this problem. Case A, the up wall is moving to the left, while the right wall is moving down. Case B, the left wall is moving up, while bottom wall is moving to the right. For each case, the moving walls have different locations for hot and cold temperatures with the same lids speed and gravitational force direction acts on vertical direction.

## 3- MATHEMATICAL MODELING :-

For the flow which is in two-dimensional, steady state, laminar and incompressible fluid in a non-facing lid-driven square cavities is studied in this study. The governing equations that used in ANSYS 12.1 for solving this problem are:

**Continuity Equation:**

### 3. Mathematical modeling

For the flow which is in two-dimensional, steady state, laminar and incompressible fluid in a non-facing lid-driven square cavities is studied in this study. The governing equations that used in ANSYS 12.1 for solving this problem are:

**Continuity Equation:**

$$\frac{\partial(\rho V_x)}{\partial x} + \frac{\partial(\rho V_y)}{\partial y} = 0 \quad (1)$$

**Momentum Equation:**

$$\frac{\partial(\rho V_x V_x)}{\partial x} + \frac{\partial(\rho V_y V_x)}{\partial y} = \rho g_x - \frac{\partial P}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial V_x}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial V_x}{\partial y} \right) \quad (2)$$

$$\frac{\partial(\rho V_x V_y)}{\partial x} + \frac{\partial(\rho V_y V_y)}{\partial y} = \rho g_y - \frac{\partial P}{\partial y} + \frac{\partial}{\partial x} \left( \mu \frac{\partial V_y}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial V_y}{\partial y} \right) \quad (3)$$

**Energy Equation:**

$$\frac{\partial}{\partial x} \left( \rho V_x C_p T \right) + \frac{\partial}{\partial y} \left( \rho V_y C_p T \right) = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) \quad (4)$$

Where  $V_x$  and  $V_y$  are the velocity components in the x and y direction respectively;  $P$  is the pressure and  $T$  is the temperature.

Boundary conditions are isothermal ( $T_h, T_c$ ), and constant velocity ( $U_p = V_p = U$ ) on the non-facing lids, while adiabatic ( $\frac{\partial T}{\partial x}$ ) and fixed on the opposite non-facing walls.

The stream function is founded from:

$$V_x = \frac{\partial \psi}{\partial y}, \quad V_y = -\frac{\partial \psi}{\partial x} \quad (5)$$

A various parameters affects the heat transfer. The dimensionless parameter that chosen for the heat transfer rate in Nusselt number. The heat equation is taken from ANSYS 12.1 help as shown:

12.1 help as shown:

$$\{q\}^T \{\eta\} = h(T_h - T_c) \quad (6)$$

and the local Nusselt number is calculated by:

$$Nu = \frac{h.L_C}{k} \quad (7)$$

#### **4. NUMERICAL SOLUTION OF GOVERNING EQUATIONS :-**

By using unstructured quadratic element the grid system is created, and higher grid is concentrate adjacent in the four corners of the cavity. The governing equations are solved by alternatively by the finite element approach. Employing the Tri-Diagonal Matrix Algorithm (TDMA) in the ANSYS 12.1 program, the differential equations ( continuity, momentum and energy equations ) are solved. The algebraic equations are solved by using Successive under Relaxation of 0.5, in order to obtained a stable convergence for the solution of momentum and energy equations. A several independent grids (80x80, 100x100,120x120) are tested to ensure the accuracy of the numerical results and to obtained an optimum grid density. The better grid test is (120x120) that carried out with Richardson number 10 and Grashof number of  $10^6$  as illustrate in Fig.(2-1). This grid dimension has shown a negligible deviation in Nusselt number ( $1 \times 10^{-4}$  %) as shown in Fig.(2-2). The convergence criterion of  $10^{-8}$  for each variable, where  $\phi (V_x, V_y, T \dots)$  is the dependent variable in the partial differential equations and in order to obtain a stable convergence for the solutions of momentum and energy equations, an under-relaxation parameter of 0.5 was used, this is give a fine enough to obtain accurate results.

The fluid flow in the cavity is air with  $Pr=0.717$ , the moving wall is represented by Reynold number, and the buoyancy effect is represented by Grashof number, both are gives a ratio of Richardson number which characterized the heat transfer regime in mixed convection and that varied from (0.1-10), also hot and cold temperatures are 295,293 respectively. The characteristic length of the cavity is taken (0.15x0.15), that using by Prasad and Koseff [1996] which deals with an experimental study for combined forced and natural convections heat transfer in deep lid-driven cavity flow, and this size is used in present work as a one of cases that employed in the different depth wise aspect ratio in the above-mentioned search.

The validation of present computer code Ansys 12.1 has been verified for two-sided non-facing lid driven cavity flow problem by Wahba [2008]. As can be seen from Fig.2-3, there is quite good agreement for streamlines distribution at  $Re= 500$  when compared to those of Wahba [2008].

#### **5. RESULTS AND DISCUSSION:-**

Mixed convection in non-facing two sided lid-driven square cavities are carried out in this present work. Richardson number is the governing parameter that indicate to the ratio between natural and forced convections. By fixing Grashof number at  $Gr= 10^6$  and change Reynold numbers (  $Re = 2000, 1000, 316$ ) that including the wall velocities which takes the same values in both non-facing moving walls, this gives changing in Richardson numbers (

Ri= 0.1, 1, 10 ) in each case. Two cases are performed according to the directions of the moving walls. Each case has different locations of hot and cold temperatures that will be presented in next sections.

**Case A:** The cold wall near to the upper moving lid is moving to the left while the hot wall near the right moving lid is moving downward with equal speeds. The left and bottom walls are stayed stationary and thermally insulated. Streamlines are shows in Figs.3 (a and c) at the upper row and isotherms are shows Figs.3(b and d) at the bottom row for various Richardson numbers.

Figures 3(a and b), shows the streamlines and isotherms for different values of Richardson number. For  $Ri = 0.1$ , the forced convection is generated by moving lids, where shear force is dominant on moves mostly recirculation of the flow in the cavity and appearance a large primary vortex ( $\Psi=4.773 \cdot 10^{-3} \text{ m}^2/\text{s}$ ) rotates in counter clockwise direction which be adjacent to the cold wall and passing the diagonal line of symmetry, and small secondary weak vortex is rotate clockwise and appears near the hot wall at the downward corner of right side of the cavity and it is effect by the strength of the main primary vortex. Isotherms shows that the flow in the center of the cavity is affect the thermal gradient that observe highly steeper on the hot wall and restricted intensity at the downward corner of right side. For this case, the adding buoyancy force is effects in the same direction of the shear force in the right hot wall while it is in opposite to the direction of the main cell that generated from the upper cold wall. At  $Ri=1$ , the forced and free convections are balanced and they are generate the other weak cell in the left bottom corner beside the secondary cell which is be flatter along the hot wall, also they contributes in fewness of the strength of the main primary vortex ( $\Psi= 2.276 \cdot 10^{-3} \text{ m}^2/\text{s}$ ) and gives lower steeper of temperature gradient than previous Richardson number 0.1 along the hot wall as a results of decreasing in heat transfer and evolution of streamlines of the main cell due to buoyancy effects regime. Generally, at Richardson number (0.1,1), the center of cavity is not much affected by the horizontal thermal gradient that which is steeper near the hot wall. For  $Ri=10$ , in this case, the effect of buoyancy force can be seen perceptible and decrease in strength of the main primary vortex ( $\Psi= 9.17 \cdot 10^{-4} \text{ m}^2/\text{s}$ ) which intended to partition to two cells and that seems clearly fills the cavity due to limited center of the secondary cell that located in the upper right corner of the cavity and becomes weaker and smaller, i.e., the secondary cell nearly vanishes for the free convection that becomes more dominate there. Isotherms, the thermal gradient is low near the hot wall due to sovereignty of the free convection and the flow is suspend, as a result for this, the heat transfer observably decreased.

Figures 3(c and d), shows that the streamlines and isotherms for different Richardson number. At  $Ri = 0.1$  two cells are generated also at the same position of the moving walls for non-facing lid-driven of the cavity, except the variation only in locations of hot and cold temperatures (the hot temperature at the upper wall and the cold temperature at the right wall), that perform to generate the main cell ( $\Psi= 4.463 \cdot 10^{-3} \text{ m}^2/\text{s}$ ) near the cold wall and rotated clockwise, and the other weak cell in the upper left corner of the cavity that is rotated counter-clockwise direction. The directions of the moving plates which are causes domination of the forced convection. The main cell is extended beyond the diagonal line of symmetry towards the weak cell that have been restricted in the upper left corner, due to opposite direction of adding buoyancy force with the main cell which is limited diffusion of the steeper temperature gradient near the hot upper wall that seems observed larger steeper than the case which illustrated in Fig.3b. At  $Ri = 1$ , participation forced and free convections in the effects. The main cell ( $\Psi = 1.587 \cdot 10^{-3} \text{ m}^2/\text{s}$ ) is moving to the right of the diagonal line of symmetry of the cavity, due to appearance another clockwise weak cell in the lower left corner. In spite of participation of buoyancy and shear forces together for drawing the heat from the hot wall, but happened a high relative gradient in temperature, because of the opposite direction of adding buoyancy force with the cold main cell, that gives less dissipation of heat transfer. For  $Ri = 10$ , seems the change clearly appearance in center and shape of the main cell ( $\Psi = 3.92 \cdot 10^{-4} \text{ m}^2/\text{s}$ ),

which penetrated towards lower right corner of the cavity, and the secondary cell which be adjacent from the hot wall and becomes more flatter along the upper hot wall. This gives indication for the free convection effect is to be larger, because of weakness of the main cell strength then a permission for heat that is to be lower gradient through the cavity with increasing of Richardson number, because of suspended of the flow and as a result of dominated the natural convection. According to that, the air in the cavity is to be larger cold in this case when compared with other differentially heated cavity orientations. The details to examine local Nusselt number along the hot wall is presented in Figs.4(i and ii) for case (A) that shows the variation of local Nusselt number with different Richardson numbers.

Figure (4-i) shows the variation of the local Nusselt numbers along the hot wall with different Richardson numbers. The local Nusselt number has higher values in bottom of the hot wall and dwindle gradually with increasing of Richardson number, while along the wall until top, the change observe in the local Nusselt number which vice versa with increasing of Richardson number. This ascribed to effects of movement the center of the secondary vortex near the upper hot wall, that seems observed in Figs.3(a and b). Fig.(4-ii) illustrate that, along the hot wall, the local Nusselt number is diminish with increasing of Richardson number and generally with increasing wall length from bottom to top and this gives indications on taking place heat transfer in the cavity. As compared as and specially at low Richardson number, the local Nusselt number in this figure verse Fig.(4i) is to be larger, due to effects of location of the hot temperature in the upper wall and also moving away of the cold main cell from the hot wall and more approaching to the cold wall.

**Case B:** At this case, the moving walls are to be at the opposite non-facing walls of case (A), which have the hot temperature in the left wall and the cold temperature in the bottom wall as illustrated in Figs.5 ( e and f ), while in Figs.5 (g and h) , the cold temperature in the left wall and the hot temperature in the bottom wall.

Figures 5(e and f) illustrate the streamlines and isotherms respectively for different Richardson number. At  $Ri = 0.1$ , the forced convection is to be dominant, and the main cell ( $\Psi = 4.45 \times 10^{-3} \text{ m}^2/\text{s}$ ) is adjacent near the hot moving wall which rotate clockwise and a weak secondary cell is adjacent near the bottom cold moving wall which rotate counter-clockwise direction. The main cell is appear across on the diagonal line of symmetry of the cavity and press the secondary cell towards bottom right corner, and they are generated by the shear force, while the buoyancy force is to be weakness. Thermally, because of existence for the main cell near the left vertical hot moving wall, the fluid will draw the heat towards the bottom cold moving wall, due to effects of agreement of the aiding buoyancy and shear forces, that gives a concept increase of the heat transfer in the cavity.

With increasing of Richardson number to  $Ri=1$ , free and forced convection are equally important, the main cell is seems weaker ( $\Psi=1.582 \times 10^{-3} \text{ m}^2/\text{s}$ ) and the secondary weak cell tends to flatter along bottom cold wall, beside to generated another weak cell in the upper right corner of the cavity as a result of participated of the aiding buoyancy force and the shear force, which they are gives decreasing in the temperature gradient along the hot wall and decrease of heat transfer, because of fewer strength of the hot main cell and appearance of buoyancy force in the cavity. For  $Ri = 10$ , the effect of buoyancy force is to be overcome the shear force, and the center of the main cell ( $\Psi = 3.91 \times 10^{-4} \text{ m}^2/\text{s}$ ) is moving toward the upper left corner adjacent to the hot moving wall, and converts shape of the cell, which cause sliding of the secondary cell along cold moving wall. Isotherms, shows the temperature gradient is being lower, comparison with the other previous Richardson numbers at the same case, due to less the strength of the main cell and dominated of buoyancy force in the cavity, which causes the flow is to be suspended and lower drawing of the heat from the hot wall.

Generally , the present case is seen as vice versa and upside down of case (A), Figs.3(c and d) for streamlines and temperature gradients ,due to following reasons: firstly, due to move

wall lids in another non-facing side, and secondary, founded the main cell in case(A) nearly from the cold wall, while in case (B), the main cell is founded nearly the hot wall. This gives indications for effects of the locations and directions of moving walls and locations of the hot and cold temperatures on the heat transfer.

Figures 5(g and h) illustrate streamlines in the upper row and isotherms in the down row for different Richardson numbers, these figures are as similar to case (B) Figs.5(e and f) in directions of the moving walls except differently in the locations for hot and cold temperatures ( the cold temperature at the left wall and the hot temperature at the bottom wall), this gives also as observed the main cell adjacent to the hot wall. At  $Ri=0.1$ , the forced convection is to be dominate in the cavity, and two cells are appeared, and the main hot cell ( $\Psi =4.759*10^{-3} \text{ m}^2/\text{s}$ ) is rotate counter-clockwise which exceed on the diagonal line of symmetry, that restricted the secondary cold cell in the upper left corner of the cavity and they are generated due to shear forces which are dominated. Because of this hot main cell that filled cavity, it is try to draw and bear the heat from the hot wall towards the cold wall and seems observed, increasing of the temperature gradient in the bottom hot wall then the evolution of heat diffusion. For  $Ri=1$ , both forced and free convections are comparable. The main cell ( $\Psi =2.268*10^{-3} \text{ m}^2/\text{s}$ ) is rotate counter-clockwise, which is not quit at the diagonal line of symmetry, and it is pressing the secondary cold cell that rotate in clockwise towards the vertical cold wall and flatter on the wall, besides to generate another weak cell that rotate also clockwise in the upper right corner of the cavity, due to effect of the free convection. Because of movement of the buoyancy force with the main hot cell in opposite direction and the temperature gradient will be less. This gives indication on less effect for the moving wall on the streamlines and isotherm, consequently, the temperature gradient is to be less.

By Going to  $Ri=10$ , in which case the buoyancy force is dominated regime. The main cell is rotate counter-clockwise ( $\Psi =9.18*10^{-4} \text{ m}^2/\text{s}$ ) which intended to partition to two cells due to the aiding forces on the hot wall, while the secondary cold cell is seems observe stretch along cold wall in the lower left corner, and which is to be vanished when increasing of the Richardson number. Due to partition of the main cell and evolution of streamlines for it, the natural convection is to be strongly presented, then temperature gradient is to be low along the hot wall and decrease in heat transfer.

Generally, seems clearly from Figs.5(g and h) vice versa of case (A) in Figs.3(a and b) for stream lines and isotherms, due to the same causes that illustrated in Figs 5(e and f). This gives indications for effects of the locations and directions for moving walls and hot and cold temperatures distributions on the heat transfer.

Figure (6-i) shows variation of the local Nusselt number along hot wall with different Richardson number. The local Nusselt number is high in the bottom wall and it is decreasing gradually from bottom to top, which is to be very low. Justification for this phenomena that at low Richardson number, the forced convection is dominate in the cavity and the strong circulation is observed near the hot wall which gives increasing in heat transfer, while with increase Richardson number, variations of Nusselt number is low as clearly observed, because of domination for natural convection regime and the fluid is subdued in the cavity due to the buoyance force in opposite direction of the main cell.

Figure (6-ii) shows, as Richardson number increased, the local Nusselt number decreases along the hot wall from bottom to top, and the variation is approximately convergent in the upper of the wall. This is expected due to domination of the free convection, where is the flow motion is approximately suppressed. Consequently, the heat transfer is to be low.

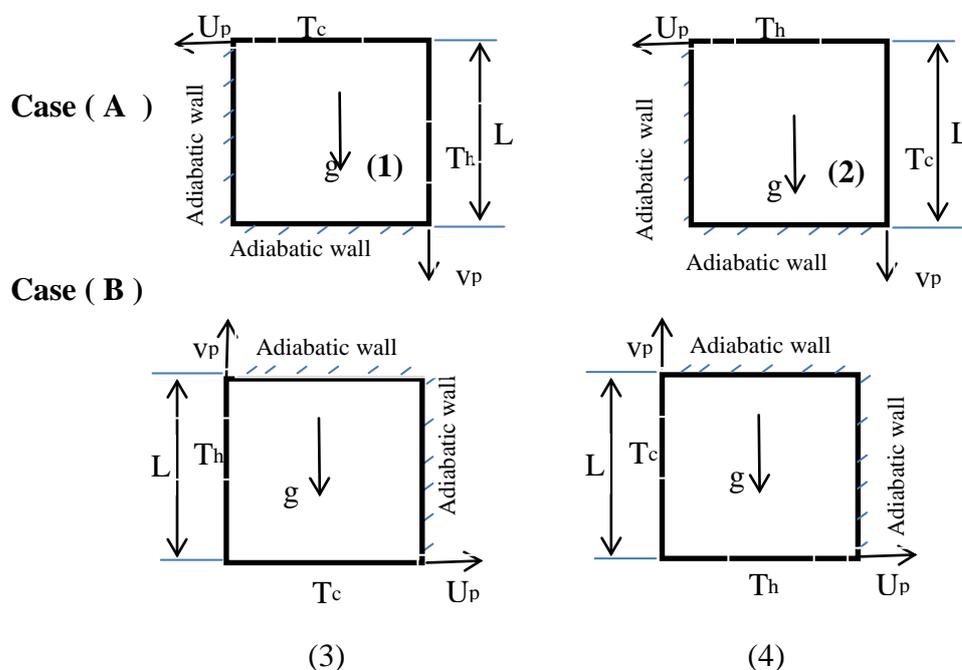
As compared as and specially in this figure with Fig.(6i), also at low Richardson number, the Nusselt number is to be large, due to effects of location of the hot temperature in the bottom wall, and seems observed the most value for different heated cavities for two cases.

**6-CONCLUSIONS :-**

A numerical solution of mixed convection in two – sided non facing lid driven differentially heated square cavities has been studied in the present work. It has been represented for two different cases to be distinguished by the directions of movement of non-facing walls. Richardson number is the governing parameter that which describe the heat transfer regime in mixed convection.

The results can be summarize by the following points:

- 1) The main cell for the case (A) is founded adjacent to the cold wall, while, for case (B) the main cell is founded near the hot wall. However, the heat diffusion for case (A) is found to be less as compared with the case (B) due to founded of the main cell near the cold wall.
- 2) Increasing of  $Ri=10$ , gives less graduated in the local Nusselt number along hot wall from bottom to top, then decreasing in heat transfer, while at low  $Ri=0.1$ , the local Nusselt number is enhanced.
- 3) The non-facing lids in a two sided of the cavity are gives the same results for streamlines and isotherms with the another opposite non-facing lids, but inversely and upside down.
- 4) In Case (B), when the hot temperature in the bottom wall and the cold temperature in the left wall, the local Nusselt number is to be relatively higher and heat transfer is enhance, as compared with the others differential heated cavities.



**Fig.( 1 ) Schematic diagram for two cases**

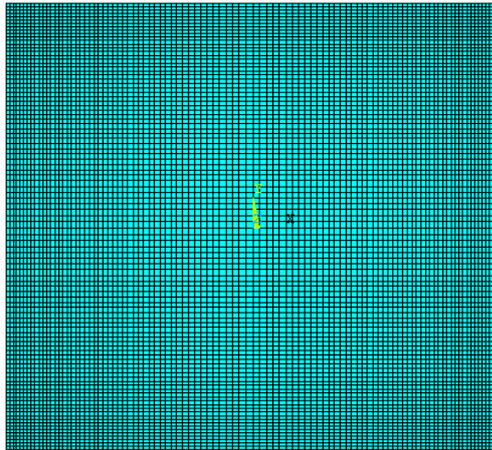


Fig. (2-1) Schematic diagram of the grid system of square cavity.

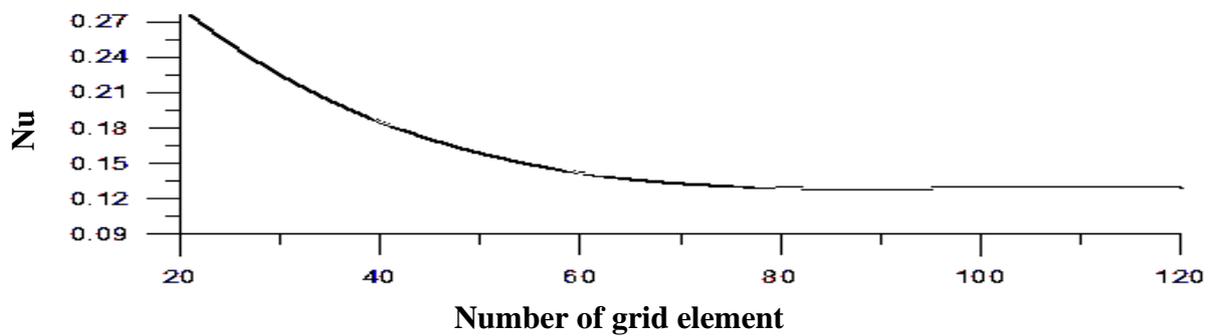


Fig.(2-2) Nusselt number convergence V.S. number of grid element

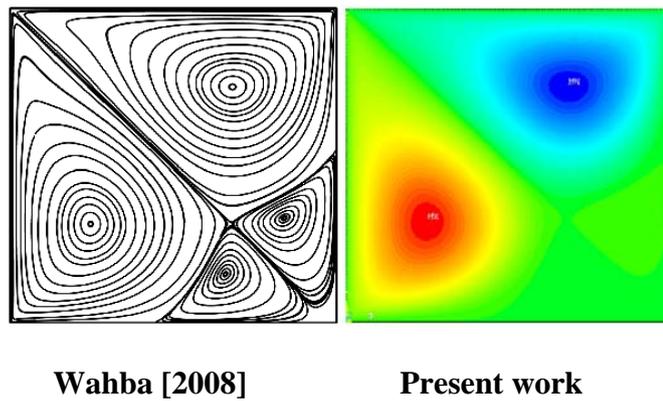


Fig.(2-3) Comparison of the streamlines between the present work and that of Wahba [2008] for validation

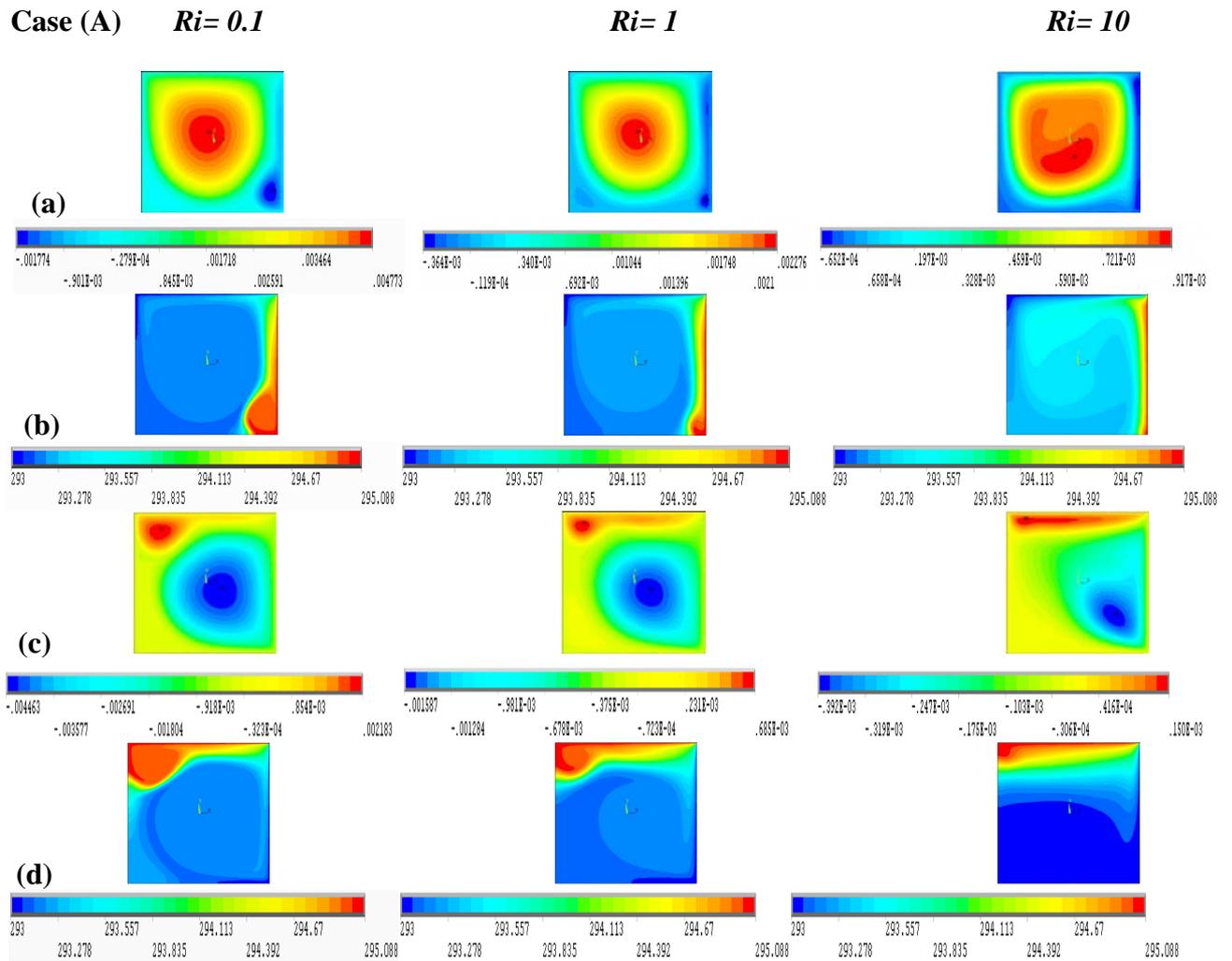


Fig.(3) Streamlines [a,c] and isotherms [b,d] at  $Gr=10^6$ ,  $\Delta T=2^\circ C$   
for Case (A) at different Richardson numbers

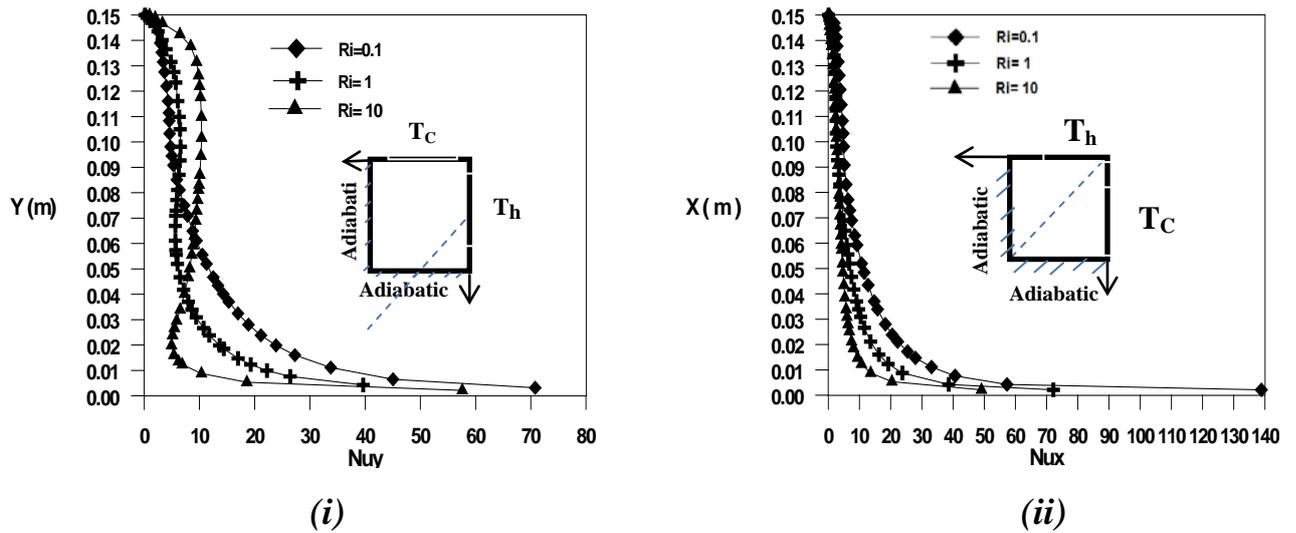


Fig. (4) Variation of local Nusselt number along the hot wall. (i) the hot wall in the right (ii) the hot wall in the top.

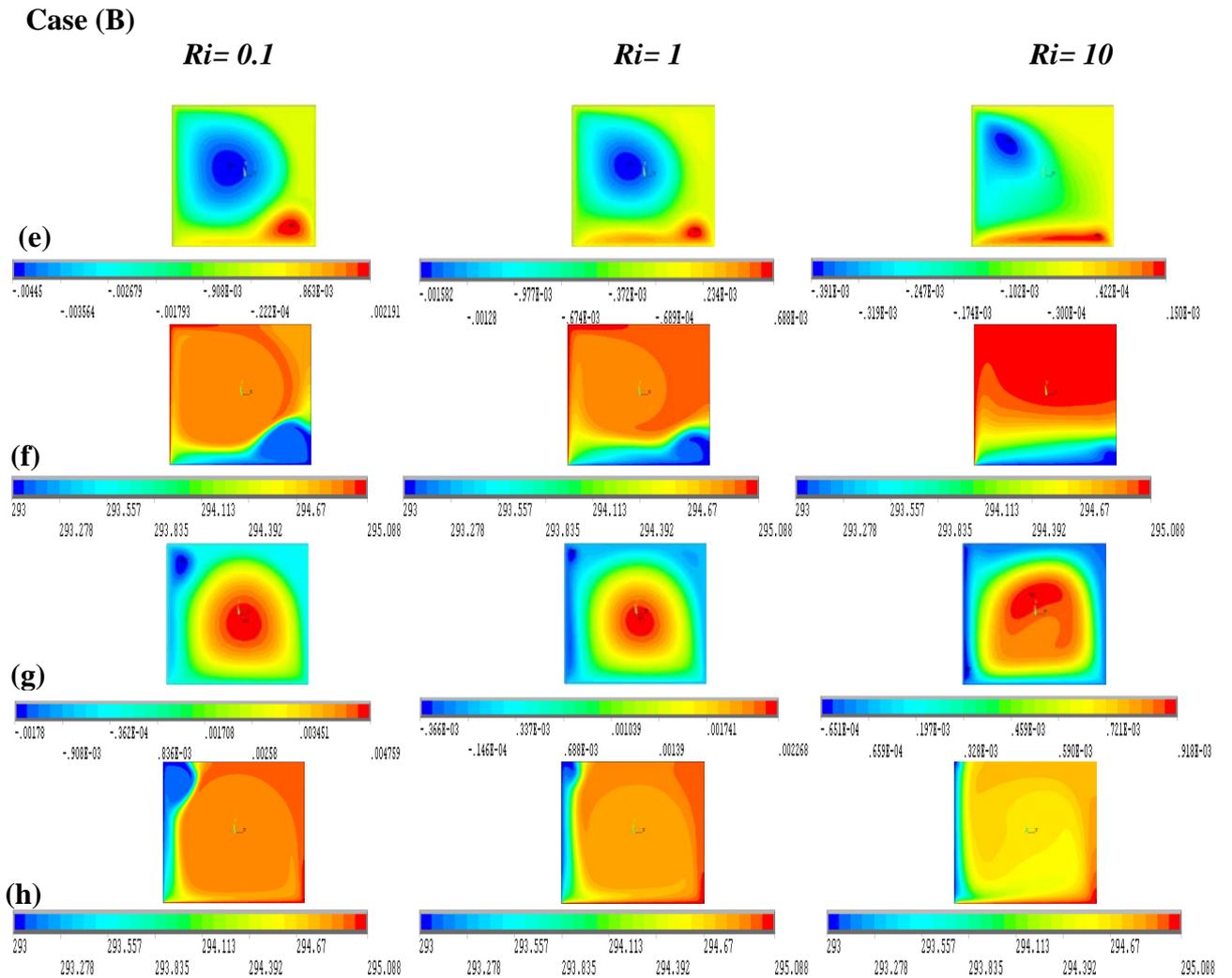
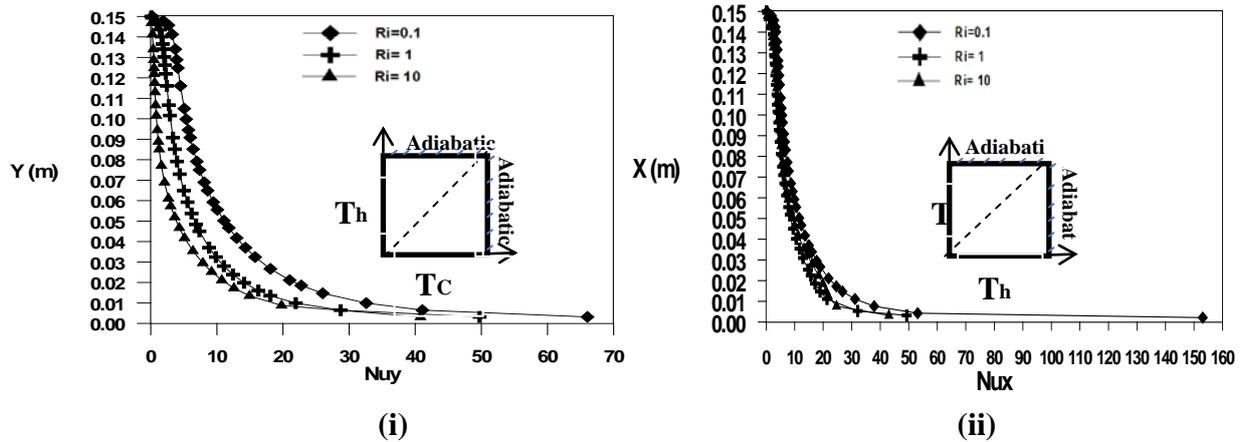


Fig.(5) Streamlines[e,g] and isotherms [f,h] at  $Gr=10^6$ ,  $\Delta T= 2^\circ C$  for case (B) at different Richardson numbers



**Fig. (6) Variation of local Nusselt number along the hot wall for case (B)**  
(i) the hot wall in the left (ii) the hot wall in the bottom.

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