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Improved Distance Spectrum of Asymmetric Turbo Codes

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ABSTRACT:

In this work, the performance of a class of asymmetric turbo codes, which are composed of mixed types of non-identical, recursive systematic convolutional codes has been investigated.
Internal pilot insertion technique is utilized to further improve the *BER* performance of turbo code especially at high *SNR*. The minimum distance of turbo codes and the multiplicity of low weight codewords can be improved by inserting pilot bits within the sequence of information bits entering the turbo encoder. This technique was shown to be useful on the symmetric and asymmetric turbo coded signals.

الملخص:

تم التحري في هذا البحث عن اداء نوع من الترميز التوربيني غير المتناظر والذي يتركب من انواع مختلفة من المرمزات التلفيفية التكرارية. لغرض تحسين اداء المرمز التوربيني من حيث معدل الخطأ بالبت وخاصة عند نسب قدرة اشارة الى ضوضاء عالية تم الاستفادة من تقنية اقحام المسيطر الداخلي. يمكن تحسين المسافة الدنيا للمرمز التوربيني وتحسين التعددية للبيانات المرمزة ذات الوزن القليل عن طريق اقحام اشارات المسيطر ضمنيا داخل البيانات الداخلة للمرمز. هذه التقنية اثبتت نجاحها للمرمزات التوربينة المتناظرة وغير المتناظرة.

1. Introduction

Turbo codes are interesting coding techniques for noisy channel. They have attracting attention since introduced in 1993 [Berrou *et.al* 93]. Their performance allows transmitting the signal with very low signal-to-noise power ratio (*SNR*) which is one of the most crucial demands in mobile systems.
Since turbo codes represent a parallel concatenation of two or multiple convolutional codes linked by interleaver, many literatures are concentrating on the design characteristic of each constituent encoder as well as the type and size of interleaver which is important to achieve a good performance for turbo codes [Siddiqi *et.al* 06] [Benedetto *et.al* 96b].

The bit error rate (*BER*) curve of a turbo code can be divided into two regions. The first region is called "waterfull region", which appears at low *SNR's* and has a steep slop of *BER* for a sufficiently long frame length. The "error-floor" is the second region, which appears at higher *SNR's*. In this region, performance has a shallower slop cause by small weight codewords [Costello *et.al* 01].

Most researches on turbo codes assume identical components convolutional codes, known a symmetric turbo codes. The *BER* performance of symmetric codes is either good in waterfull region or in error-floor region, but mostly not in both [Berrou *et.al* 93].

In this work, the performance of asymmetric turbo codes is investigated. These codes are composed of mixed types of non-identical, recursive systematic convolutional codes (different generator polynomials with possibly different constraint length). The idea is to use one constituent code that is a weak code, while the other constituent code is a strong code. The idea of *weak* and *strong* codes is based on how the individual codes perform as conventional convolutional codes in a turbo configuration. The weak constituent code should help mostly at small *SNR's*, while the strong codes should help mostly at larger *SNR's*. Weak constituent codes produce better extrinsic estimates for the information bits when the a-priori inputs are less reliable, which helps in the initial stages of iterative decoding. Strong constituent codes, on the other hand, give good extrinsic estimates for the information bits when the a-priori inputs are more reliable, which helps an iterative decoder to converge to the maximum likelihood solution for high *SNR's*.

To further improve the *BER* performance of turbo code especially at high *SNR's*, a new technique is proposed, which relays on the study of the distance spectrum of turbo codes. The minimum distance of turbo codes and the multiplicity of low weight codewords can be improved by inserting pilot bits within the sequence of information bits entering the turbo encoder. To avoid reduction in bandwidth efficiency the pilots could be punctured [Ahmed *et.al* 08]

2. Performance Bound of Proposed Asymmetric Turbo Code:

The weight distribution for the codewords produced by the turbo decoder depends on how the codewords from one of the simple component encoders are teamed with codewords from the other encoder. Generally, distance functions of convolutional codes can be written as a sum of weights joint with all possible paths leaving the all zero-path at time *t*=0 and merge again at any time *t*. So, the input redundancy weight enumerating function (*IRWEF*) of the code is defined in [Benedetto *et.al* 96a] as

$$A^C(W,Z)=\sum_{w,j} A_{w,j} W^w Z^j \qquad (1)$$

Where $A_{w,j}$ denotes the number of codewords of weight $d=w+j$ generated by input information sequences of weight *w* and parity sequences of weight *j*. *W* and *Z* are dummy variables denoting the systematic and non-systematic output weight, respectively. The conditional weight enumerating function (*CWEF*) A_w^C of the parity check bits generated by the code *C* corresponding to the input words of weight *w* can be obtained from *IRWEF* as:

$$A_w^C(Z)=\sum_j A_{w,j} Z^j=\frac{1}{w!}\frac{\partial^w A^C(W,Z)}{\partial W^w}\Big|_{W=0} \qquad (2)$$

Deriving the weight distribution for turbo codes is not tractable for a particular interleaving scheme. In [Benedetto *et.al* 96a] the idea of forming an average weight function over all possible interleaving schemes has been devised. They had introduced the notation of *uniform interleaver* of length *N*, defined as a probabilistic device that maps a given input sequence of length *N* and weight *w* into all distinct $\binom{N}{w}$ permutations of it with equal probability of $1/\binom{N}{w}$. Making use of the properties of a uniform interleaver, the average conditional weight enumerate function of all possible turbo codes with respect to the whole class of interleavers for turbo code system can be evaluated as given below:

$$A_w^{TC}(Z) = \frac{A_w^{C_1}(Z) \cdot A_w^{C_2}(Z)}{\binom{N}{w}} \quad (3)$$

and A are the weight enumerating functions for *RSC1* and *RSC2* encoders, respectively. Equation (3) represents an average turbo code with given constituent codes at block size, *N* over all possible interleavers. Here codewords produced by both encoders are independent of each other, because and A are assumed as individual components [Benedetto *et.al* 96b]. The average bit-error probability of the proposed asymmetric turbo code system over *AWGN* channel is evaluated by

$$P_{\text{bit}} \leqslant \sum_j \sum_w \frac{w}{N} A_w^{TC}(Z) P_2(j) \quad (4)$$

where $P_2(j)$ is the pair-wise error probability between the all-zero codeword and codeword with minimum Hamming weight, *d*.

The upper bound may transform into the following approximation [Yuan *et.al* 99] and [Benedetto *et.al* 96a]

$$P_{\text{bit}} = \frac{1}{2} \sum_{d=d_{\min}} A_d . Q\left(\sqrt{d.R_c.\frac{E_b}{N_o}} \right) \quad (5)$$

d_{min} is the minimum Hamming distance, R_c is the code rate, $\frac{E_b}{N_o}$ is the ratio of the bit energy to noise power spectral density and A_d is the error coefficient which determines the contribution of the codewords with the same weight *d* to the bit error probability [Yuan *et.al* 99]:

$$A_d = \sum_{d=w+j} \frac{w}{N} A_{w,j} \quad (6)$$

3. Proposed Asymmetric Turbo Codes:

The proposed scheme consists of two non-identical constituent recursive systematic convolutional (*RSC*) encoders with S-random [Pollara *et.al* 95] interleaver preceding the second constituent encoder as shown in Fig.(1).

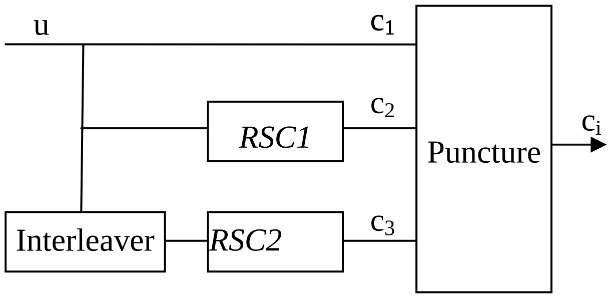


Fig.(1) Proposed Asymmetric Turbo Encoder.

Different combinations of a proposed generator polynomial $g(D)$ for *RSC1* and *RSC2* are presented in Table (1). The values of $g(D)$ is in octal (*FB*,*FF*), where *FB* and *FF* is the feedback and feed-forward polynomial respectively. The corresponding decoding scheme is shown in Fig.(2).

Table(1): Proposed symmetric and asymmetric systems

RSC1	*RSC2*
(7,5)	(7,5)
(7,5)	(5,7)
(5,7)	(5,7)
(5,7)	(7,5)
(13,15)	(7,5)
(13,15)	(13,15)

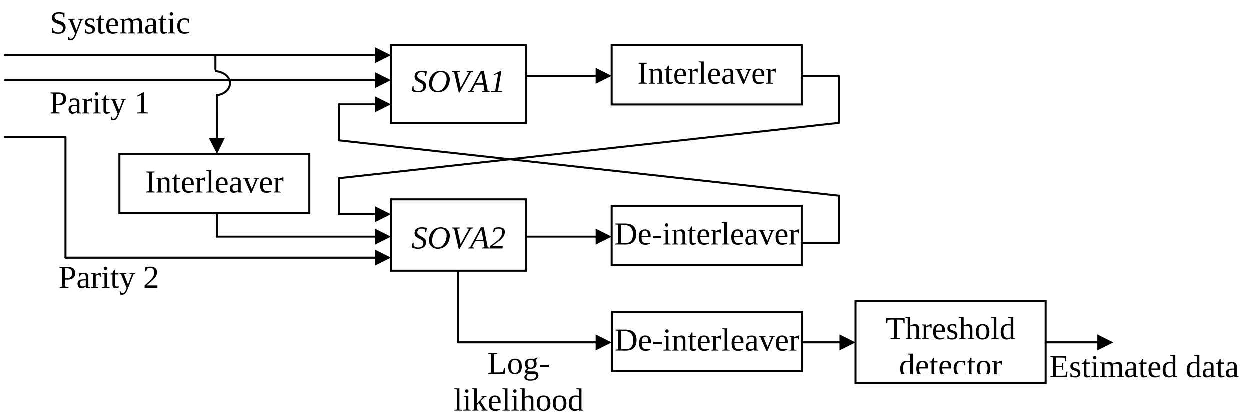


Fig.(2) Asymmetric Turbo Decoder.

4. Simulation Results of Asymmetric Turbo Codes:

For simplicity, the soft-output Viterbi algorithm (*SOVA*) [Hagenauer *et.al* 89] with maximum number of iteration as 5 is considered to implement turbo decoding. The complexity of *SOVA* decoder is approximately half of the maximum a-posteriori (*MAP*) [Bahl *et.al* 74] decoder, but suffers a degradation of about 0.5 dB at *BER* of 10^{-4} as compared to the *MAP*.

To select the best combination of *g(D)*, simulations were carried out for frame size, *N*=56 with *RSC* constraint length, *K*=3 and 4 and code rate R_c=1/2. Figures (3a) and (3b) show the *BER* performance of (7,5;7,5), (5,7;5,7), (5,7;7,5) and (7,5;5,7) half rate turbo codes at low and high E_b/N_o, respectively.

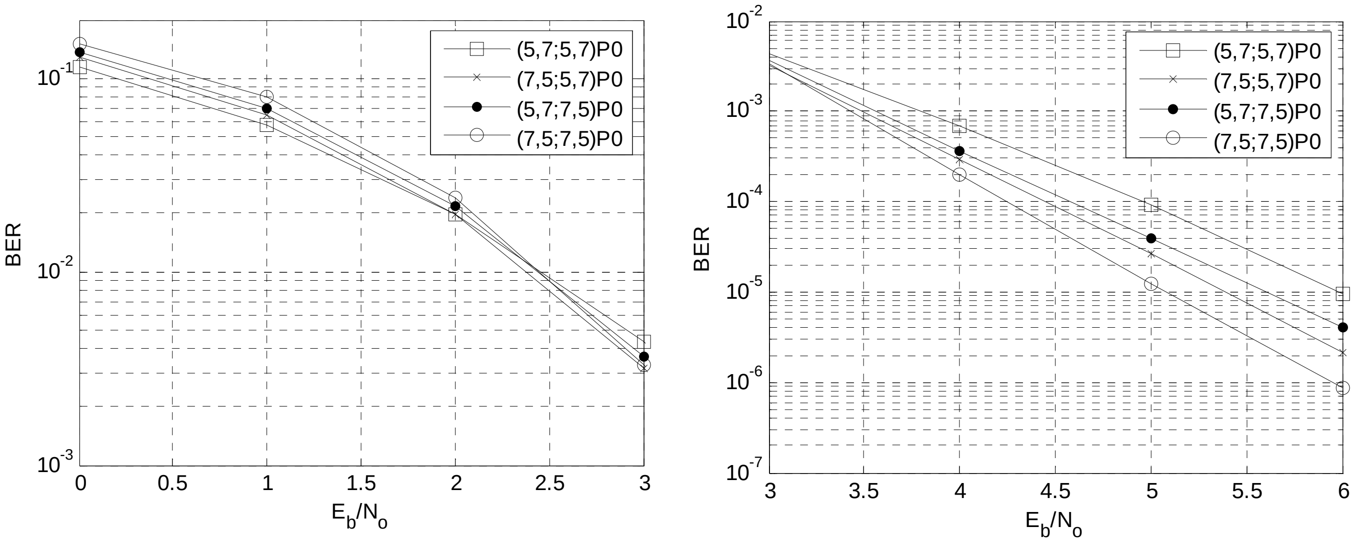
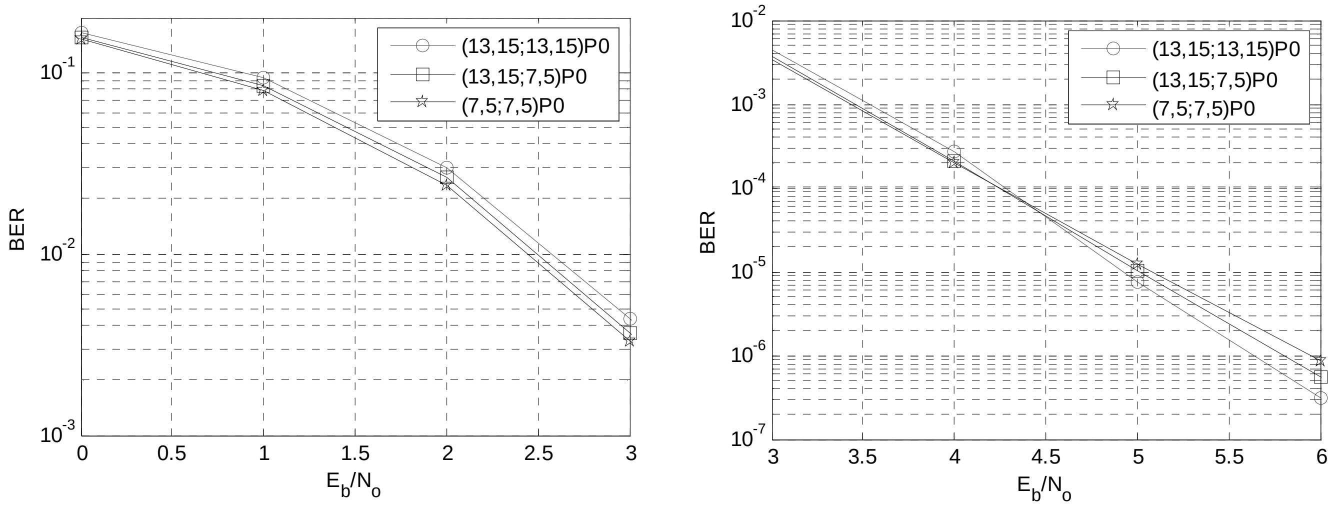


Fig.(3): *BER* performance of different symmetric and asymmetric turbo codes systems at (a) low and (b) high E_b/N_o with *K*=3.

To examine the effect of using different constraint length, Figures (4a) and (4b) show the *BER* performance of (13,15;13,15), (7,5;7,5), (13,15;7,5) half rate turbo codes.

Figures (3) and (4) illustrate the idea of utilizing weak and strong codes. Here, asymmetric codes show a *BER* performance which may consider a compromise between week and strong symmetric turbo codes over the whole range of *SNR*.
For further enhancement in performance especially at high *SNR*, the new technique invented by [Ahmed *et.al* 08] is employed. For short constraint length (*K*=3) the modified systems offer an improvement of about 0.2~0.3 dB over unmodified ones at 10^{-6} *BER*. The notation *Pi* refers to system with *i* internally inserted pilots (see Fig.(5)).
To explain the performance of symmetric and asymmetric systems in presence of Gaussian noise, and the effect of internal pilot insertion technique on the distance spectrum of such codes, The partial distance spectrum for the original code (without pilot insertion *P*0) and modified code with two inserted pilots (*P*2) is shown in Fig.(6). For low to moderate weight values, the error coefficients (A_d) given by Equation (6), which belong to asymmetric systems are greater, compared with symmetric ones. This explains the deterioration of performance for asymmetric codes at high *SNR*, where low weigh codewords are the main reason for bad performance.



(a) **(b)**

Fig.(4): ***BER*** **performance of different symmetric and asymmetric turbo codes systems at (a) low and (b) high** E_b/N_o **with** ***K*****=3 and 4.**

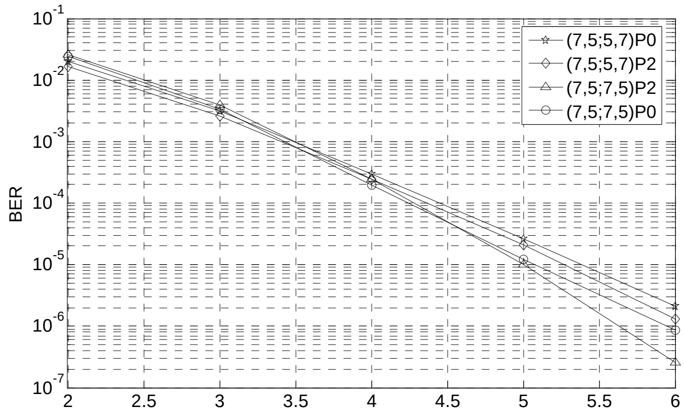


Fig.(5): ***BER*** **performance of different symmetric and asymmetric turbo codes systems, with and without inserted pilots and** ***K*****=3.**

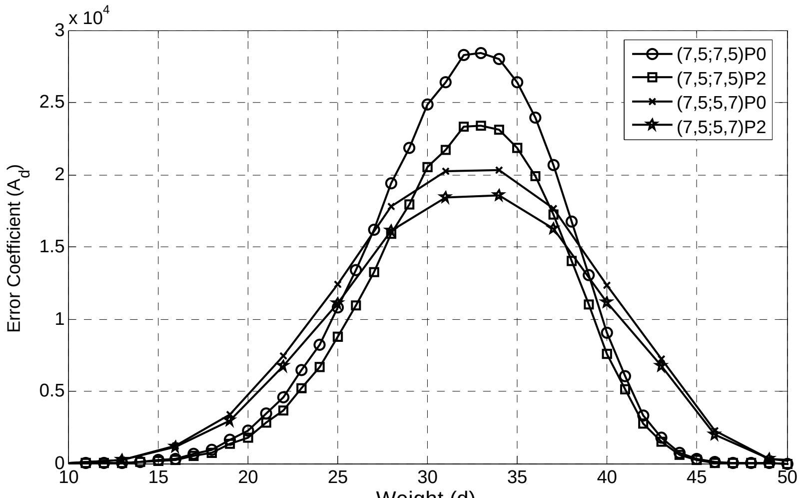


Fig.(6): The partial distance spectrum for symmetric and asymmetric systems with and without inserted pilots.

However, the situation is inverted at the center of the spectrum. Once more, this explain the superiority of performance for asymmetric over the symmetric codes at low to moderate *SNR*, where codewords at the center of the spectrum has the dominant effect on performance at high *BER*. For both, symmetric and asymmetric, the internally pilot inserted systems shows depressed spectrum which clarify the improvement in performance when using this technique.

5. Conclusion:

In this paper, simulations are carried out for systems which merge between two schemes; asymmetric construction of constituent *RSC* codes, and the internal (instead of external) pilot insertion. The results show that the asymmetric codes with internally pilot insertion gives a favorable trade off between the good characteristics of both constituent codes. The resulting code make a compromise between classical, symmetric and asymmetric turbo codes in terms of *BER* performance at waterfull and error-floor regions. The proposed systems need no extra complexity and the coding rate is adaptive under fixed frame length constraint.

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