# Effect of Curvature on Stability of Fluid Flow in Tube under Gravity


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## Abstract

In this paper a model of laminar flow for incompressible fluid in curved tube under earth gravity effect was done. The basic differential equations which govern the flow were defined. The stability of this model was analyzed. It was found that the model is unstable under all conditions and the earth gravity effect negatively on the stability of the model .


## 1.Model and Basic Differential Equations

### 1.1 Introduction.

Mathematical Modeling is a study of real phenomena using mathematical tools. A hypothesis of ideal and abstract study of model must not go far away from the problem which we study not to loss it's character. Mathematical modeling art devoted in a study of maximum factors which effect on the problem using simple tools.[1]

Al-Obaidi [2] studied the effect of curvature on stability of fluid flow in tube. In this paper, we shall follow the same study with effect of earth gravity which neglected by [2].

The model of study represents laminar fluid flow in tube with constant diameter. We choose the cylindrical coordinates system such that: the coordinate $r$ represent a change of distance from the tube centre to the edge (the change of radius) , the coordinate $\theta$ represent the change of flow angle and the coordinate $z$ represent the change along the tube. The tube exposes to vertical curvature (figure 1)

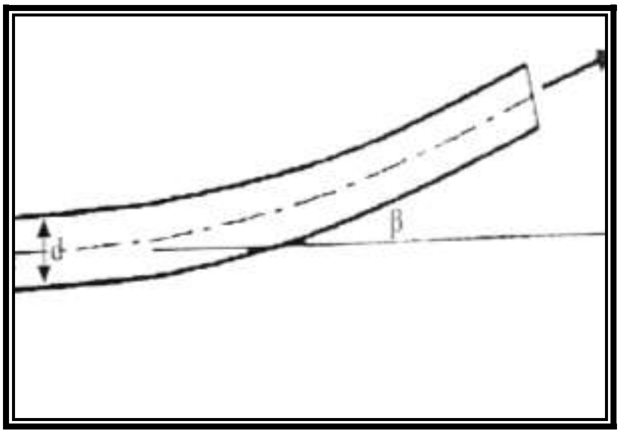


Figure (1) .. the model of study

$R$ - tube radius.
$\beta$ - curvature angle.
$u$ - flow speed.

### 1.2 Equation of Continuity.

The continuity equation of flow given by [3][4]

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

And when it changes to the cylindrical coordinates becomes:

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\partial r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial v_z}{\partial z}=0 \quad \ldots(1.1)$$

And for incompressible flow becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(v_\theta)+\frac{\partial v_z}{\partial z}=0 \;\ldots\ldots(1.2)$$

Using the hypothesis of solution for this model $v_\theta = v_z = 0$ equation (1.2) becomes:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru)=0 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.3)$$

### 1.3 Equation of Motion.

The equation of motion in cylindrical coordinates given as:[3][4]

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right]=F_r-\frac{\partial P}{\partial r}+\mu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right]\;\ldots.(1.4)$$

From (1.3) we obtain:

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}=0 \;\ldots\ldots\ldots\ldots(1.5)$$

Substituting (1.5) in (1.4) we obtain:

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right]=F_r-\frac{\partial P}{\partial r}\;\ldots\ldots\ldots\ldots\ldots(1.6)$$

And using Euler equation of motion (figure 2) we have[5]:

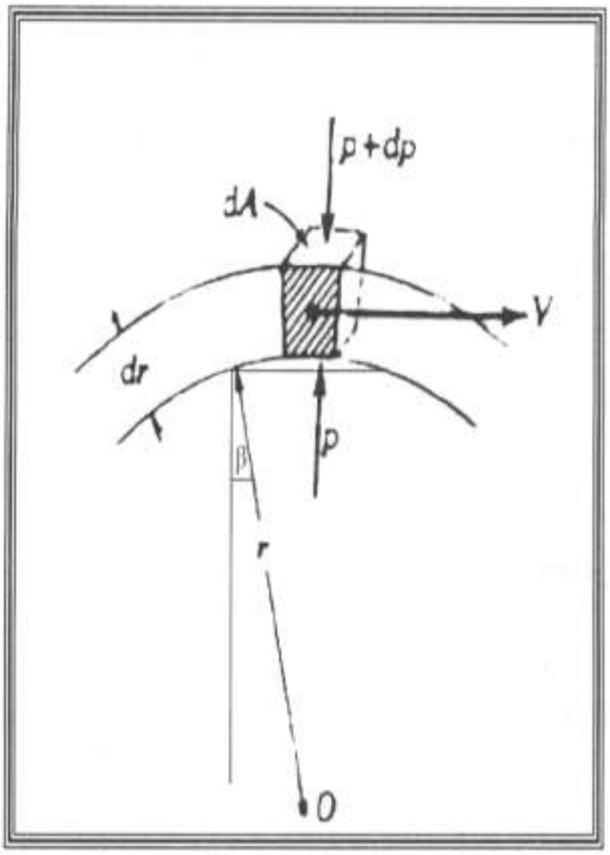


Figure (2)

$$\frac{dP}{dr}=\frac{\rho u^2}{s} \quad \text{……………(1.7)}$$

And representing the curvature radians $s$ using the relation $s=\frac{L}{\beta}$ when $L$ is the tube radiance , the equation (1.7) becomes:

$$\frac{dP}{dr}=\frac{\rho u^2\beta}{L} \quad \text{………………(1.8)}$$

Substituting (1.8) in equation (1.6) and representing the external forces $F_r=\frac{\rho gL\sin\beta}{\pi R^2}$ when $R=\frac{d}{2}$ we obtain:

$$\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}\right]=-\frac{u^2\beta}{L}+\frac{\rho gL\sin\beta}{\pi R^2} \quad \text{….(1.9)}$$

## 1.4 Boundary Conditions.

The boundary conditions which governing our model:

1. $r=0 \quad \rightarrow \quad u=u_\infty$
2. $r=R \quad \rightarrow \quad u=0$
3. $0\le\beta\le{}^{\pi}\!/_{2}$

## 1.5 Dimensionless Equations.

To generalize the model for measuring an effect we find dimensionless equations which governing the model without units for this goal we define a some dimensionless variables as follows:[3]

$$\bar{u}=\frac{u}{u_\infty} \quad , \quad \bar{r}=\frac{r}{d} \quad , \quad \bar{P}=\frac{P-P_0}{Pu_\infty{}^2} \quad , \quad \bar{t}=\frac{tu_\infty}{d} \quad \bar{\beta}=\frac{\beta}{\pi/2}$$

And substituting these variables in equation of continuity (1.3) and motion (1.9) we obtain:

$$\frac{\partial}{\partial\bar{r}}(\bar{r}\bar{u})=0 \quad \text{……………………(1.10)}$$

$$\frac{\partial\bar{u}}{\partial\bar{t}}+\bar{u}\frac{\partial\bar{u}}{\partial\bar{r}}=-a\bar{u}^2+B\sin(\frac{\pi}{2}\bar{\beta}) \quad \text{……(1.11)}$$

Where $a=\frac{\pi d\bar{\beta}}{2L}$ and $B=\frac{4gL}{\pi d\bar{u}_\infty^2}$ and dimensionless boundary conditions becomes:

1. $\bar{r}=0 \quad \rightarrow \quad \bar{u}=1$
2. $\bar{r}=1 \quad \rightarrow \quad \bar{u}=0$

## 2. Stability Analysis

Any system may be under external effects which can led to disturbance, this disturbance can cause existing of the system from its phase to another or it return to its phase.[6]

To analyze stability of this model we suppose that the system effected by disturbance and we suppose that $\bar{u}_1(\bar{r})$ represent the steady state flow and $\bar{u}_2(\bar{r},\bar{t})$ represent the disturbance state which effect to the system.[7]

$$\bar{u}(\bar{r},\bar{t})=\bar{u}_1(\bar{r})+\bar{u}_2(\bar{r},\bar{t}) \quad \text{…………(2.1)}$$

Substituting the equation (2.1) in the dimensionless equation (1.11) we obtain:

$$\frac{\partial}{\partial\bar{t}}(\bar{u}_1+\bar{u}_2)+(\bar{u}_1+\bar{u}_2)\frac{\partial}{\partial\bar{r}}(\bar{u}_1+\bar{u}_2)=-a(\bar{u}_1{}^2+\bar{u}_2{}^2)+B\sin(\frac{\pi}{2}\bar{\beta}) \quad \text{……..(2.2)}$$

$$\frac{\partial\bar{u}_1}{\partial\bar{t}}+\frac{\partial\bar{u}_2}{\partial\bar{t}}+\bar{u}_1\frac{\partial\bar{u}_1}{\partial\bar{r}}+\bar{u}_1\frac{\partial\bar{u}_2}{\partial\bar{r}}+\bar{u}_2\frac{\partial\bar{u}_1}{\partial\bar{r}}+\bar{u}_2\frac{\partial\bar{u}_2}{\partial\bar{r}}= \quad \text{…….(2.3)}$$
$$=-a\bar{u}_1{}^2-a\bar{u}_2{}^2+B\sin(\frac{\pi}{2}\bar{\beta})$$

By separating the steady and disturbance states and neglecting the order (2) companies , we obtain:[8]

$$\bar{u}_1\frac{d\bar{u}_1}{d\bar{r}}=-a\bar{u}_1{}^2+B\sin(\frac{\pi}{2}\bar{\beta}) \quad \text{………(2.4)}$$

$$\frac{\partial\bar{u}_2}{\partial\bar{t}}+\bar{u}_1\frac{\partial\bar{u}_2}{\partial\bar{r}}+\bar{u}_2\frac{\partial\bar{u}_1}{\partial\bar{r}}=0 \quad \text{………(2.5)}$$

### 2.1 Steady State Equation

The equation (2.4) represent the steady state of the model and solving this equation we obtain:

$$a\bar{u}_1^2=w-e^{-2a(r+c)} \quad \text{………..(2.٦)}$$

where $w=B\sin(\frac{\pi}{2}\bar{\beta})$

and using the dimensionless boundary conditions we obtain:

$c=\frac{\ln(w-a)}{-2a}$ and substituting this value in equation (2.6) we obtain:

$$\bar{u}_1=\left(\frac{w-e^{-2a(\bar{r}+\frac{\ln(w-a)}{-2a}}}{a}\right)^{1/2} \quad \text{……….(2.7)}$$

We substitute this value in equation (2.4) to obtain

$$\frac{d\bar{u}_1}{d\bar{r}}=\frac{e^{-2a(\bar{r}-\frac{\ln(w-a)}{2a}}}{\left(w-e^{-2a(\bar{r}-\frac{\ln(w-a)}{-2a}}\right)^{1/2}} \;\dots\dots\dots(2.8)$$

**2.2 Disturbance State Equation**

The equation (2.5) represent a disturbance state equation of the model, to solve it we suppose the capacity is constant and then the solution becomes in the form:[9]

$$\bar{u}_2(\bar{r},\bar{t})=Ae^{ik\bar{r}+c\bar{t}} \;\dots\dots(٢.9)$$

Where:

1. $c$ is a wave speed which is a complex $c=c_1+ic_2$, and the negative value of $c_1$ show the stability of the system[9,10]
2. $A$ - amplitude and it is a constant.
3. $k$ - a waves number.

And substituting $\bar{u}_2$ from the equation (2,6) and $\bar{u}_1$ from the equation (2.7) we obtain:

$$cAe^{ik\bar{r}+ct}+\frac{\left(\frac{w-e^{-2a(\bar{r}-\frac{\ln(w-a)}{2a}}}{2a}\right)^{1/2}}{a}.ikAe^{ik\bar{r}+ct}+Ae^{ik\bar{r}+ct}.\left(\frac{w-e^{-2a\bar{r}-2a-\ln w}}{a}\right)^{-1/2}.e^{-2a\bar{r}-2a-\ln w}=0$$

$$cA+\left(\frac{w-e^{-2a\bar{r}-2a-\ln w}}{a}\right)^{1/2}.ikA+A\left(\frac{w-e^{-2a\bar{r}-2a-\ln w}}{a}\right)^{-1/2}.e^{-2a\bar{r}-2a-\ln w}=0$$

$$cA=A.\left(\frac{w-e^{-2a\bar{r}-2a-\ln w}}{a}\right)^{-1/2}.e^{-2a\bar{r}-2a-\ln w}=0 \;\dots\dots\dots\dots\dots\dots\dots\dots(2.10)$$

$$c=\frac{\sqrt{a}}{\sqrt{w-e^{s}}}.e^{s},\; s=-2a\bar{r}-2a-\ln w \;\dots\dots\dots(2.11)$$

This is show that $c$ always a positive value and in result $c_1$ is always a positive value, which mean that the system always unstable under the hypnotic conditions.

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# تأثر الانحناء على استقرارية جريان مائع في أنبوب تحت تأثير الجاذبية


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