The Iraqi Journal For Mechanical And Material Engineering, Vol.10, No.1, 2010

Free Vibration of Curved Beam with Varying Curvature and Taper Ratio

By
Mohamed J. Al – Robaiy , Mohammed A. AL-Shjary and Mohamed Y. AL-Janaby
College of Engineering,
Babylon University

Abstract

In this study free vibration of uniform curved beam with varying curvature and taper thickness between the root and tip of beam are investigated by using the finite element method using Ansys(9.0). nine models from curved beam all models have four cases have different in taper ratio for the same curvature are studied. The results obtained from this study are compared with the results of other investigators in existing literature for the fundamental natural frequency and its found from this results the natural frequency increase with increasing the taper ratio and curvature for the servel models from curved beams. Finally the effect of the taper ratio and curvature on the natural frequency are shown in graphics.

Keywords: Free vibration, curved beam, finite element, curvature, taper ratio.

الاهتزاز الحر لعتبة منحنية بتغير التكور ونسبة السمك

محمد جواد عبيد ، محمد علي صيهود ، محمد يوسف جبار

ألخلاصه

في هذه الدراسة تمت دراسة الاهتزاز الحر لعتبة منحنية منتظمة بتغير التكور ونسبة السمك بين الرأس والقاعدة باستخدام طريقة العناصر المحددة بواسطة برنامج (ANSYS 9.0). درست تسعة موديلات كل موديل يمتلك أربعة حالات لها نسبة سمك مختلفة لنفس التكور.تمت مقارنة النتائج التي تم الحصول عليها مع نتائج دراسات أخرى ووجد من هذه النتائج إن الترددات الطبيعة تزداد بزيادة نسبة السمك والتكور للموديلات المختلفة من العينة المنحنية. أخيرا تأثير نسبة السمك والتكور على الترددات الطبيعة بينت بالمخططات.

Many investigators have studied the vibrations of curved beams Sabir and Aswell [2] have discussed the natural frequency analysis of circular arches deformed in a plane. The finite elements developed by using different types of shape functions were employed in their analysis. Petyt and Fleisch [3] have studied the free vibration of a curved beam under various boundary conditions. The coupled twist- bending vibrations of complete, incomplete and transversely supported rings have been investigated by Rao [4]. Sabuncu [5] has also investigated the vibration analysis of thin curved beams. He used several types of shape functions to develop different curved beam finite elements and pointed out the effect of displacement functions on the natural frequencies by comparing the results. Exact solutions for the free vibrations of curved non-uniform beam by Suzuki and Takahashi [6] gives an exact. Fourier series solution to beams with the same boundary conditions at both ends. Sabuncn and Erim [7] they presented the vibration analysis of a tapered curved beam linear and non- linear vibrations of cross- sections were considered in their analysis. Laura and Bambill [8] have studied the free vibration of a non- uniform elliptical ring by using Rayleigh- Ritz method. Rossi [9] has investigated the in –plane vibration of non- circular arcs having non- uniform cross- section with a tip mass. Free and forced in- plane vibrations of circular arches with variable cross- section and various boundary conditions have investigated by Tong et al [10]. Lee and Hsiao [11] have studied free vibration analysis of curved non- uniform beams. They presented effect of taper ratio, center angle and arc length on the natural frequencies of the beams. Oh et al. [12] have examined free vibration of non- circular a riches with non- uniform cross- section. Free vibration analysis of circular arches with varying cross- section using differential quadrate method (DQM) has studied by Karami and Malek Zadeh [13]. Hasan et al. [14] have studied stability analysis of non- uniform cross- section thin curved beams under uniformly distributed dynamic loads by using FEM. The effects of opening analysis vibrations of cross- section, static and dynamic load parameters on the stability regions are shown in graphics .Kress et al. [15] are studied complex- shaped beams have varying thickness and the centerline by using FEM. Gao [16] developed the refined theory of straight beams a refined theory of rectangular curved beams is derived by using Paplcovich- Neuber (P-N) solution in polar coordinate system. Zhu and Meguid [17] have studied vibration analysis of a new curved beam element by study the dynamic characteristics of a finite element by conducting vibration analysis using our newly developed three- node locking- free curved element.

2- Models of curved beams

The geometry of the curved uniform beam is shown in Fig.(1). The curved beams in this study have uniform rectangular cross- sections as shown in Fig.(2). The variation of cross – section of the linear tapered curved beam is represented by mathematical expressions as given in Eq. (1) ***[14].*** The cross- sections have three different configurations, which for simplicity, are denoted by C1, C2 and C3. The explanation of these cross- sections is as follows: ***[2].***

a: Uniform ($t_R = t_t$, $b_R = b_t$, Fig. 2a).

b: Symmetric tapered with constant width ($t_R \neq t_t$, $b_R = b_t$, Fig. 2b).

$$t = \frac{(L - R\theta)}{L}(t_R - t_t) + t_t \quad , \quad b = \frac{(L - R\theta)}{L}(b_R - b_t) + b_t \qquad \text{.... (1)}$$

The buckling and natural frequency parameters are represented by mathematical expressions to be used in numerical analysis as follows: ***[2].***

$$\lambda = \sqrt{\omega_1^2 \frac{\rho A_{Root} R^4}{EI_{Root}}} \quad \text{…. (2)}$$

Where

$$A_{Root} = t_R \, b_R \ , \qquad I_{Root} = \frac{b_R \, t_R^3}{12} \quad \text{…. (3)}$$

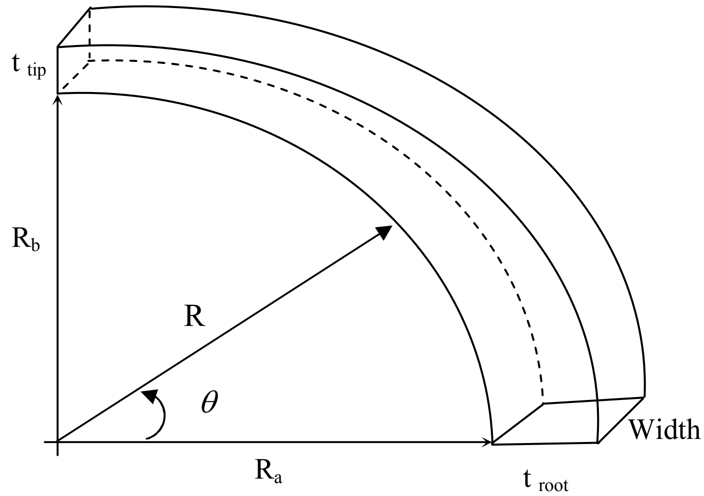
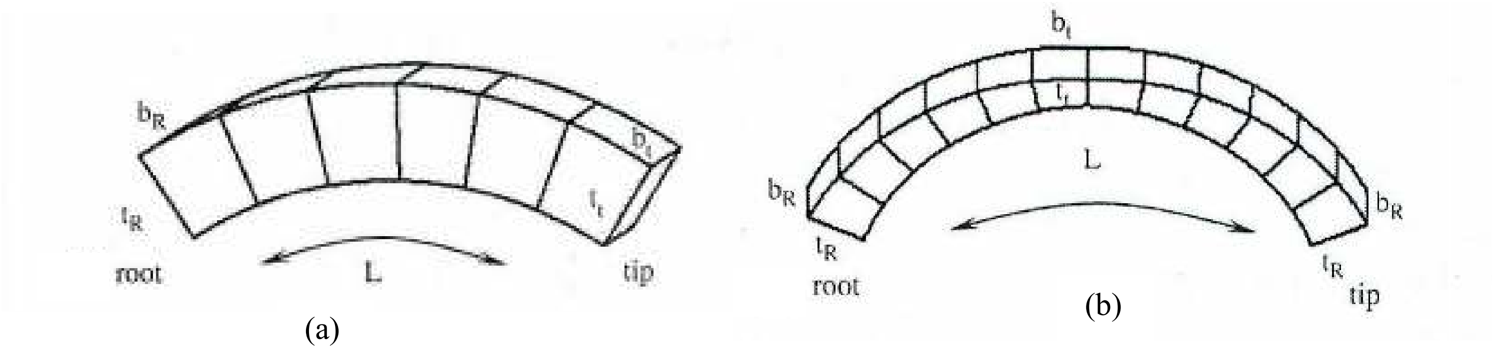
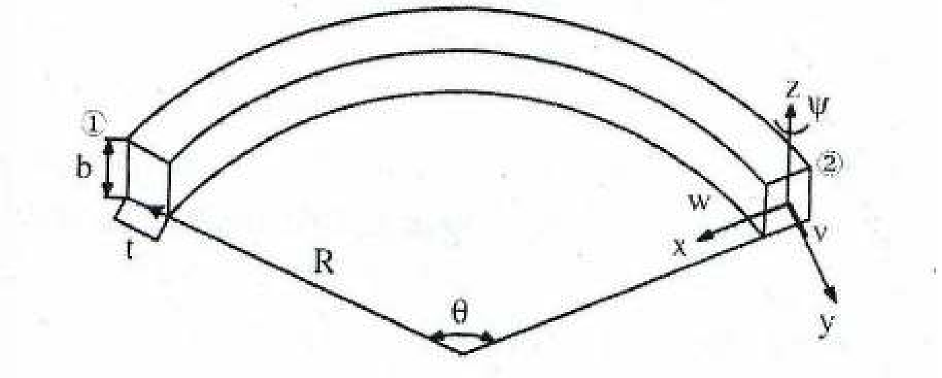


Fig.(1) Geometry and Coordinate System of a Curved Uniform Beam



Fig(2) : Cross-Section of Curved Beams : (a) Uniform ($t_R = t_t$, $b_R = b_t$)
(b) Symmetric Taper with Constant width ($t_R \neq t_t$, $b_R = b_t$)



Fig(3) : Six Degree of Freedom Finite Element Model

3- *Theoretical Analysis*

The following two shape functions are used in the analysis to represent radial and circumferential deflections (Fig. 3), respectively [2],

$$W(\theta)=\sum_{i=1}^{n} N_i(\theta) w(\theta) \quad \text{.... (4)}$$

$$V(\theta)=\sum_{i=1}^{n} N_i(\theta) v(\theta) \quad \text{.... (5)}$$

The deflection vector of the elemental finite element is

$$q^T=[v_1\ w_1 \psi_1\ v_2\ w_2\ \psi_2\ ], \quad \text{.... (6)}$$

Where

$$\psi=\frac{dw}{dy}-\frac{v}{R} \quad \text{.... (7)}$$

After applied the boundary conditions of clamped- free to calculate the interpolation functions (N_i) to describe the distribution of displacement.

The potential energy of the curved beam element is:

$$U=\frac{1}{2}\left[\int_0^{-1} EI_{xx}\left(w''-\frac{v'}{R}\right)^2+EA\left(\frac{w}{R}+v'\right)^2\right]dy, \quad \text{.... (8)}$$

Eq (8) can be written in matrix form as

$$U=\frac{1}{2} q^T\, k_e\, q \quad \text{.... (9)}$$

Where, k_e , elastic stiffness matrix.

$$k_e=\int_0^L E\, I(x)\, N_i''\ N_j''\ dx \quad \text{.... (10)}$$

The kinetic energy of the curved beam element is

$$T=\frac{1}{2}\int_0^1 \rho A\left(\dot{w}+\dot{v}\right)^2 dy. \quad \text{.... (11)}$$

Eq. (11) can be written in matrix form as

$$T=\frac{1}{2}\dot{q}^T\, m_e\, \dot{q}. \quad \text{.... (12)}$$

Where, m_e , mass matrix

$$m_e=\int_0^L m(x)\, N_i N_j\, dx \quad \text{.... (13)}$$

Thus, for a finite element, elastic stiffness matrix k_e and mass matrix m_e are obtained, respectively.

Mass and stiffness matrices of each beam element are used to form global mass and stiffness matrices. The dynamic response of a beam for a conservative system can be formulated by means of Lagrange's equation of motion in which the external forces are expressed in terms of time- dependent potentials, and then performing the required operations the entire system leads to the governing matrix equation of motion:

$$M\,\ddot{q} + K_e\,q = 0 \qquad \text{…. (14)}$$

After simplified the eq.(12), give

$$\left[K_e - \omega^2 M\right]q = 0 \qquad \text{…. (15)}$$

This equation represent the solution of the problem to get on the natural frequency.

4- Results and Discussions

Nine models are studied to represent the variation of the curvature and taper ratio. All models have four cases which different taper ratio and the same curvature. The models involves a uniform cross-sectioned curved beam and with both ends clamped.

Table(1) shows the geometry of the models (radius and thickness) of the carved beam and the results of the finite element analyses for the first three modes of the natural frequencies. the table shows the effect of the curvature ratio and the taper ratio on the natural frequency for all models.

Table(1) Natural frequency of curved beams having different geometry .

Models	Cases	Geometry of the curved beam				Natural frequency for first three modes		
		R_a (m)	R_b (m)	t_a (m)	t_b (m)	W1 Hz	W2 Hz	W3 Hz
1	Cb1	6	5	0.03	0.03	2.895	14.855	21.495
	Cb2	6	5	0.03	0.025	3.07	15.175	20.245
	Cb3	6	5	0.03	0.02	3.3	15.56	18.875
	Cb4	6	5	0.03	0.015	3.628	16.05	17.337
2	Cb5	6	4	0.03	0.03	3.39	18.83	26.59
	Cb6	6	4	0.03	0.025	3.57	19.204	24.744
	Cb7	6	4	0.03	0.02	3.825	19.67	22.75
	Cb8	6	4	0.03	0.015	4.18	20.26	20.534

3	Cb9	6	3	0.03	0.03	3.84	23.7	32.57
	Cb10	6	3	0.03	0.025	4.018	24.09	29.7
	Cb11	6	3	0.03	0.02	4.257	24.6	26.6
	Cb12	6	3	0.03	0.015	4.566	23.23	25.3
4	Cb13	6	2	0.03	0.03	3.686	18.127	26.71
	Cb14	6	2	0.03	0.025	3.911	18.51	26.33
	Cb15	6	2	0.03	0.02	4.21	18.98	23.81
	Cb16	6	2	0.03	0.015	4.62	19.58	22.07
5	Cb17	5	4	0.03	0.03	4.51	23.7	33.9
	Cb18	5	4	0.03	0.025	4.76	24.2	31.8
	Cb19	5	4	0.03	0.02	5.11	24.79	29.5
	Cb20	5	4	0.03	0.015	5.607	25.5	26.94
6	Cb21	5	3	0.03	0.03	5.368	31.629	43.9
	Cb22	5	3	0.03	0.025	5.63	32.18	40.41
	Cb23	5	3	0.03	0.02	5.98	32.87	36.66
	Cb24	5	3	0.03	0.015	6.5	32.5	33.8
7	Cb25	4	4	0.03	0.03	4.76	22.74	34.2
	Cb26	4	4	0.03	0.025	5.04	23.18	32.75
	Cb27	4	4	0.03	0.02	5.43	23.79	31.35
	Cb28	4	4	0.03	0.015	5.95	24.55	29.27
8	Cb29	4	3	0.03	0.03	6.17	30.6	44.65
	Cb30	4	3	0.03	0.025	6.54	31.2	42.22
	Cb31	4	3	0.03	0.02	7.02	31.93	39.56
	Cb32	4	3	0.03	0.015	7.714	32.9	36.55
9	Cb33	3	2	0.03	0.03	7.89	43.51	61.02
	Cb34	3	2	0.03	0.025	8.302	44.3	56.8
	Cb35	3	2	0.03	0.02	8.86	45.3	52.22
	Cb36	3	2	0.03	0.015	9.67	46.5	47.15

Table(2) The first and second natural frequency (rad/sec) of a curved uniform beam simply support at root. (Thickness ratio t_r/t_t=1) and width =0.3 m, E= 207 Gpa, ν=0.3, ρ=7800 kg/m^3).

			First mode		Second mode	
θ Rad	R (m)	L (m)	Present work	Ref. [11]	Present work	Ref. [11]
0.2	30	6	38.2	38.9	76.8	77.9
0.5	12	6	38.95	39.44	77.5	78.12
1	6	6	39.92	41.05	77.92	78.96

Table(2) shows the first and second natural frequency, of a simply supported uniform curved beam, compared with those givens in the existing literature ***[11]***. The comparison shown that give good consistent.

Fig(4) shows the effect the taper ratio for the curvature (R_a/R_b=6/5) on the natural frequency for first four models. As seen from this fig. the natural frequency increase with increasing of taper ratio because the beam is stiffer because increase in the potential energy of it.

Fig(5) shows the natural frequency increase because the ratio (R_a/R_b=6/4) this case decreased in the length of beam causes increasing in the stiffness of beam and natural frequency increased and shown that in fig.(6-12) where ($R_a/R_{b)}$ increasing causes decreased in the length of beam and increased in the natural frequency for all taper ratio in this models.

Fig.(13-21) the influence of the thickness ratio or taper ratio (t_a/t_b) on the first three modes of the natural frequency on the curved beam with curvature ratio (R_a/R_b) is shown. It is found that the natural frequency of the carved beams with the same curvature ratio increase at the taper ratio is increased.

Fig.(22-33) shown that the curvature ratio has significant influence n the first three modes of the natural frequency of the curved beam with various taper ratios. It is found that the natural frequency of the carved beams with the same curvature ratio increases as the taper ratio increased.

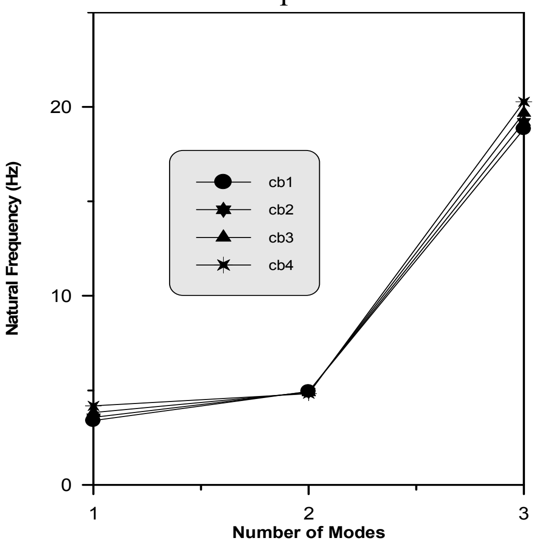


Fig.(4): The first three modes of natural frequency of carved beam for model(1)

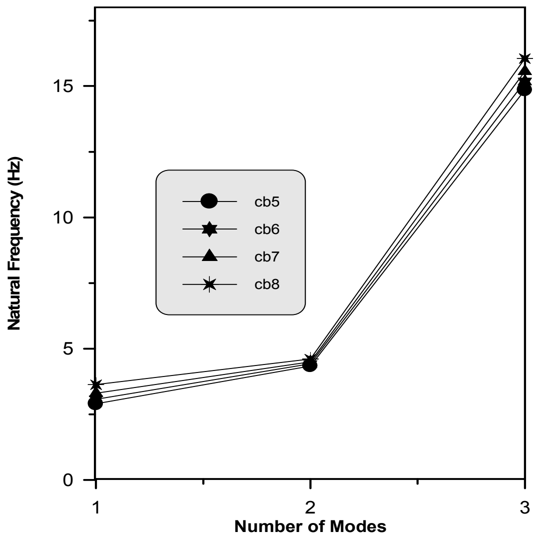


Fig.(5): The first three modes of natural frequency of carved beam for model(1)

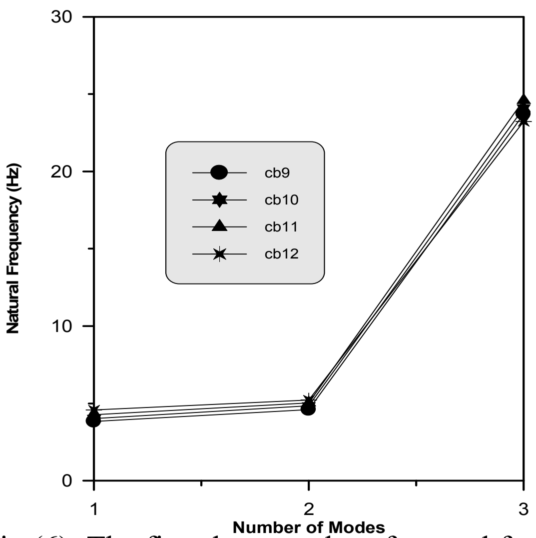


Fig.(6): The first three modes of natural frequency of carved beam for model(1)

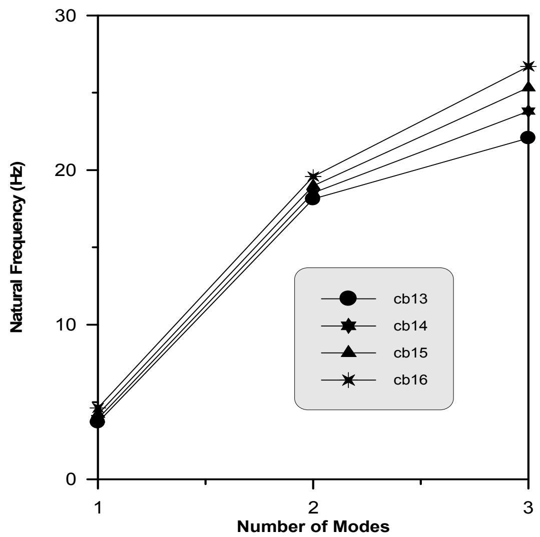


Fig.(7): The first three modes of natural frequency of carved beam for model(1)

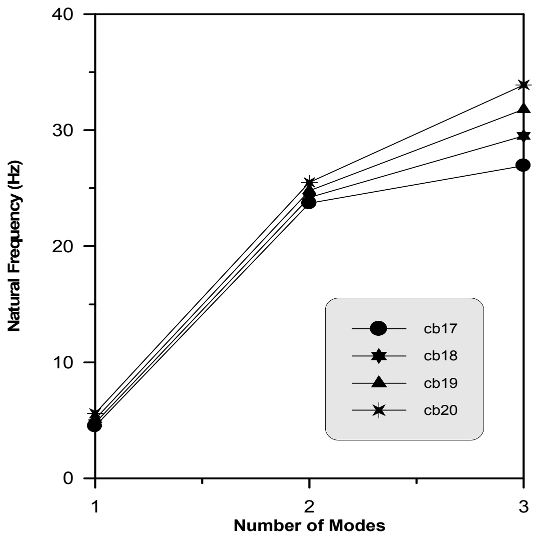


Fig.(8): The first three modes of natural frequency of carved beam for model(1)

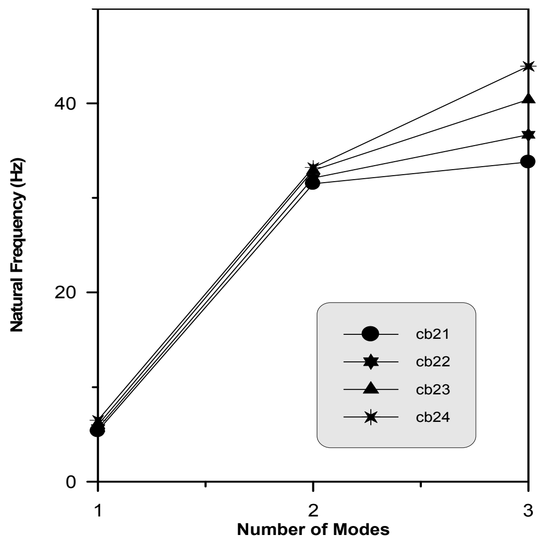


Fig.(9): The first three modes of natural frequency of carved beam for model(1)

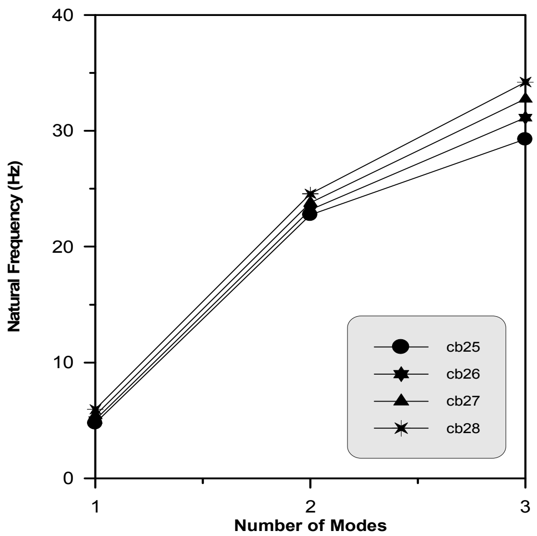


Fig.(10): The first three modes of natural frequency of carved beam for model(1)

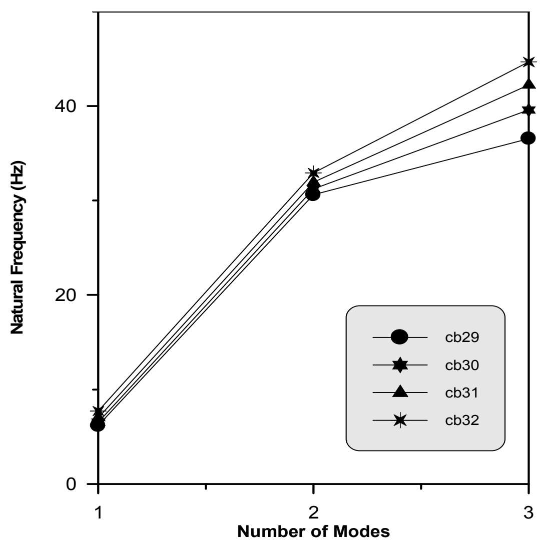


Fig.(11): The first three modes of natural frequency of carved beam for model(1)

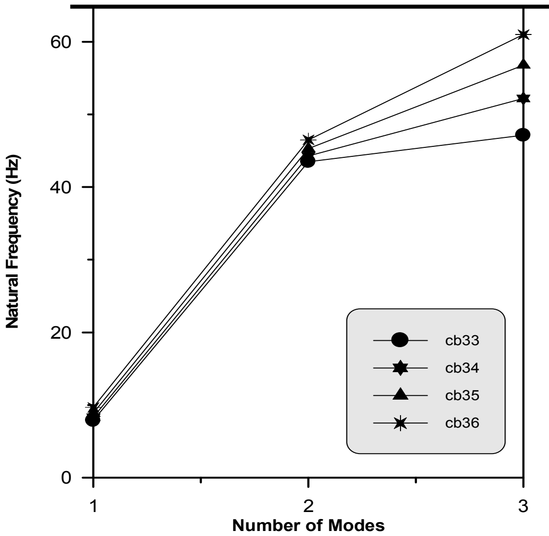


Fig.(12): The first three modes of natural frequency of carved beam for model(1)

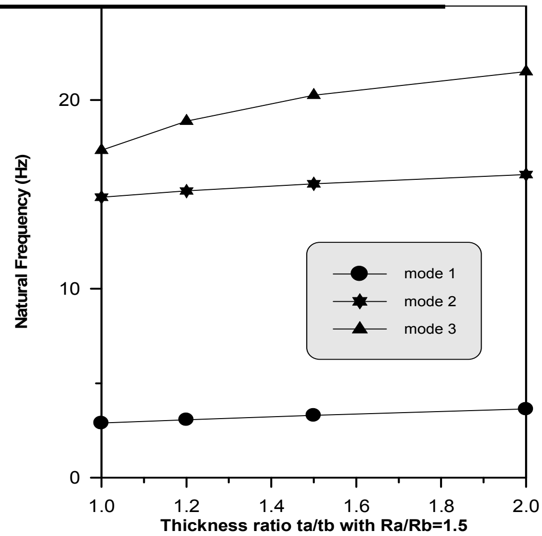


Fig.(13): The effect of the taper ratio on natural frequency of carved beam

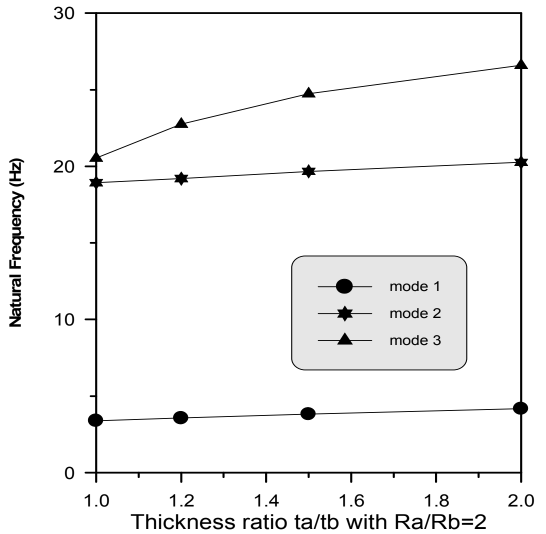


Fig.(14): The effect of the taper ratio on natural frequency of carved beam

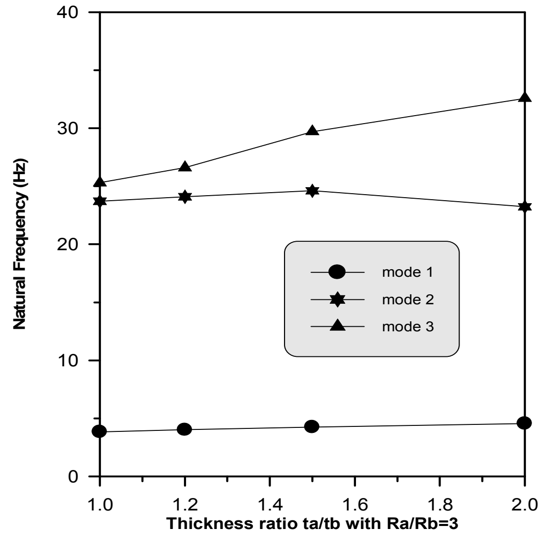


Fig.(15): The effect of the taper ratio on natural frequency of carved beam

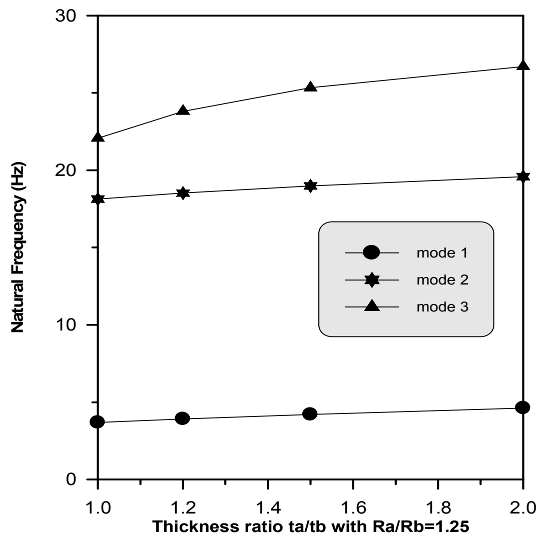


Fig.(16): The effect of the taper ratio on natural frequency of carved beam

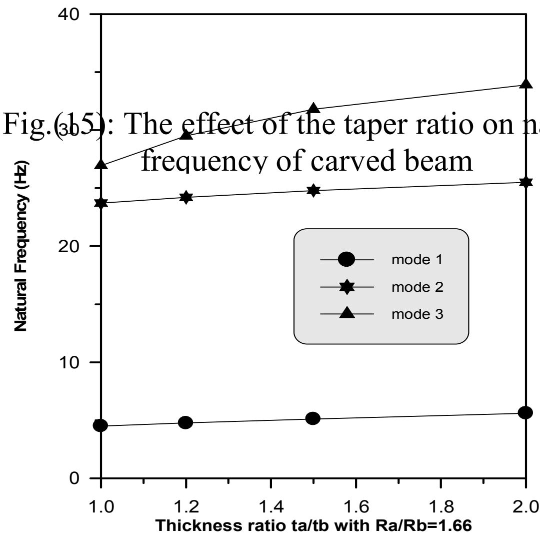


Fig.(17): The effect of the taper ratio on natural frequency of carved beam

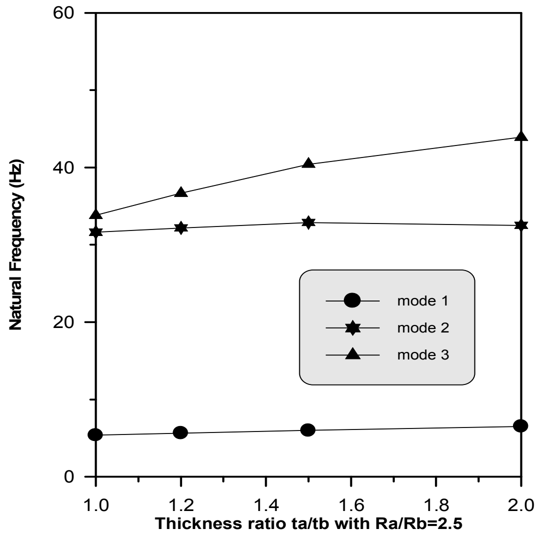


Fig.(18): The effect of the taper ratio on natural frequency of carved beam

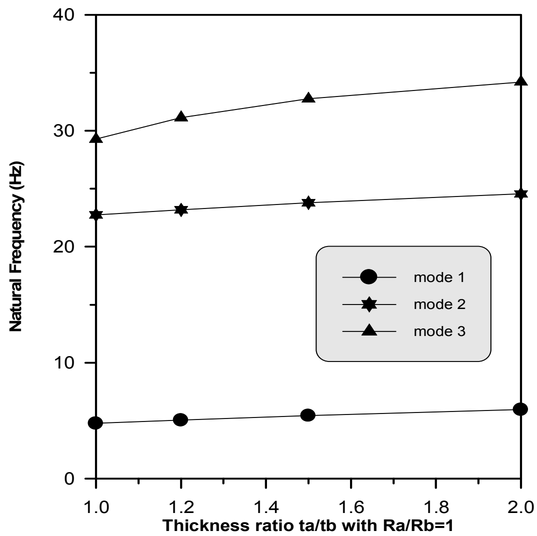


Fig.(19): The effect of the taper ratio on natural frequency of carved beam

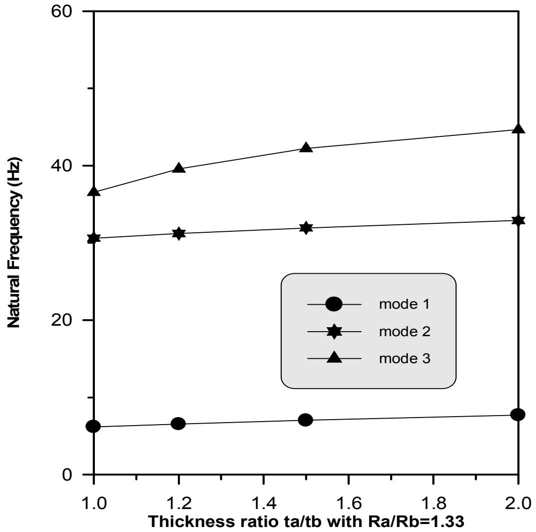


Fig.(20): The effect of the taper ratio on natural frequency of carved beam

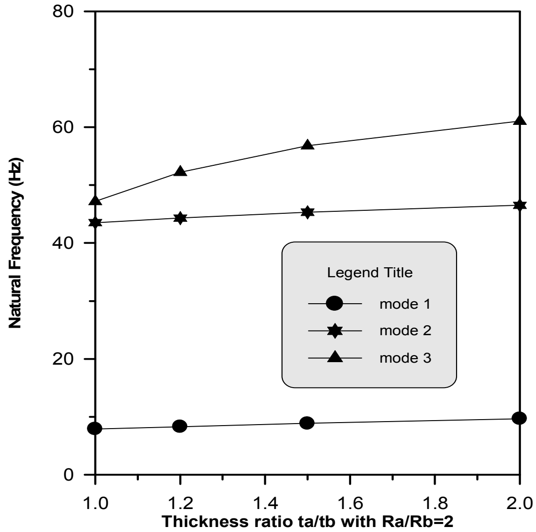


Fig.(21): The effect of the taper ratio on natural frequency of carved beam

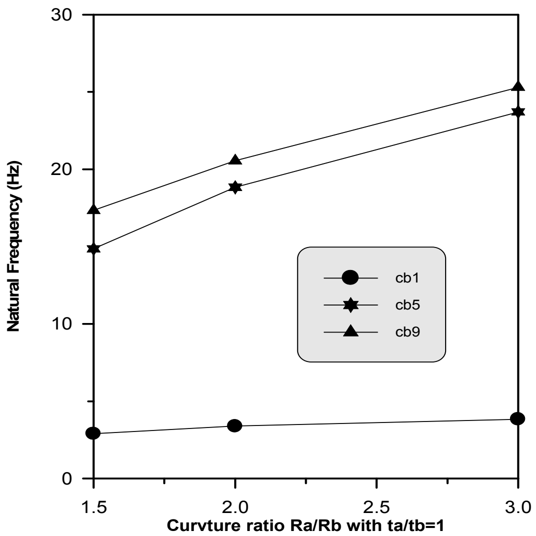


Fig.(22): The effect of the curvature ratio on natural frequency of carved beam

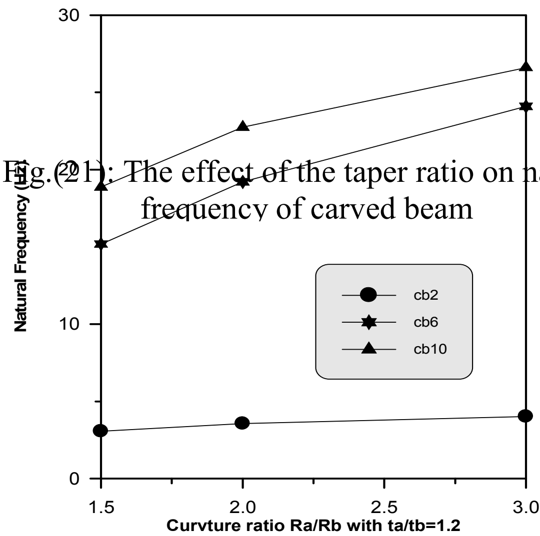


Fig.(23): The effect of the curvature ratio on natural frequency of carved beam

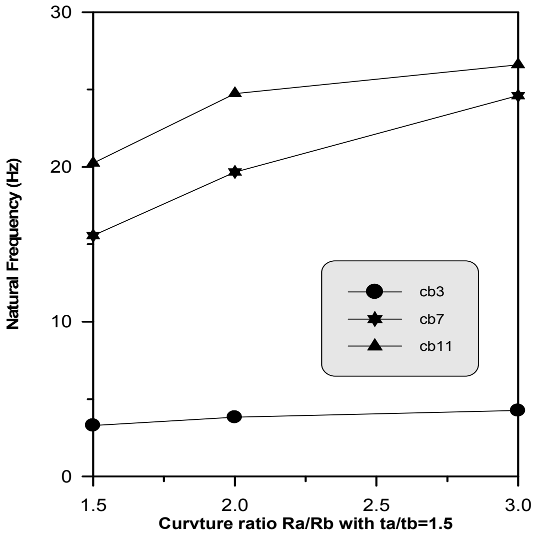


Fig.(24): The effect of the curvature ratio on natural frequency of carved beam

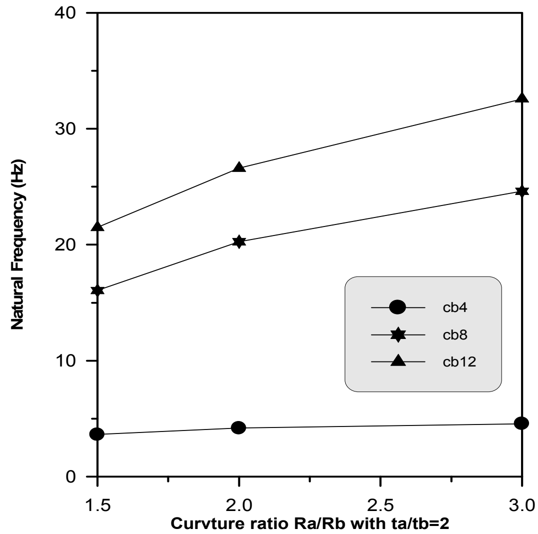


Fig.(25): The effect of the curvature ratio on natural frequency of carved beam

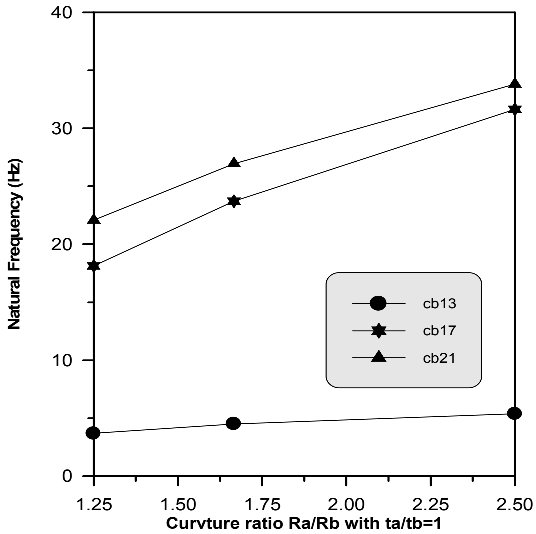


Fig.(26): The effect of the curvature ratio on natural frequency of carved beam

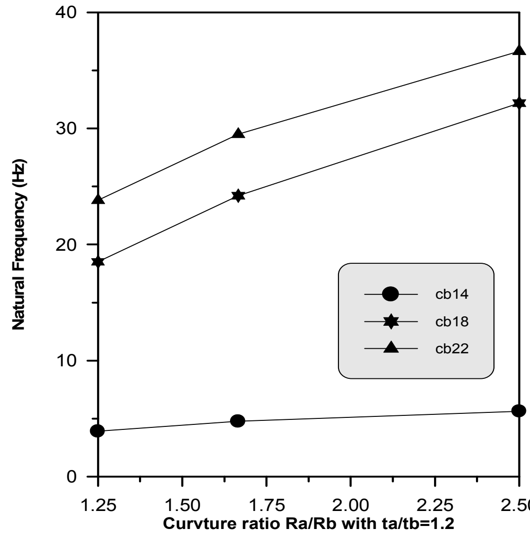


Fig.(27): The effect of the curvature ratio on natural frequency of carved beam

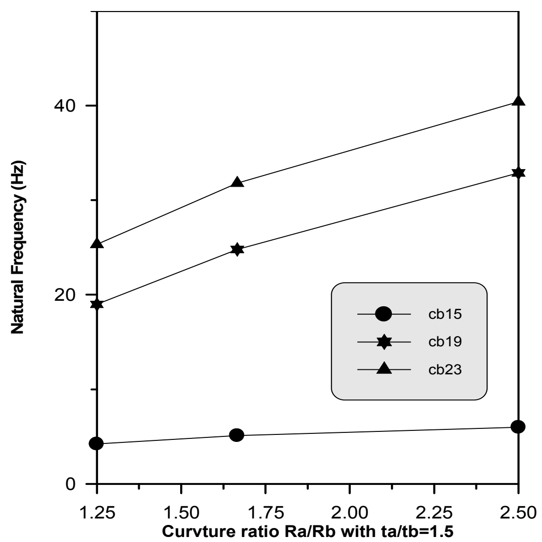


Fig.(28): The effect of the curvature ratio on natural frequency of carved beam

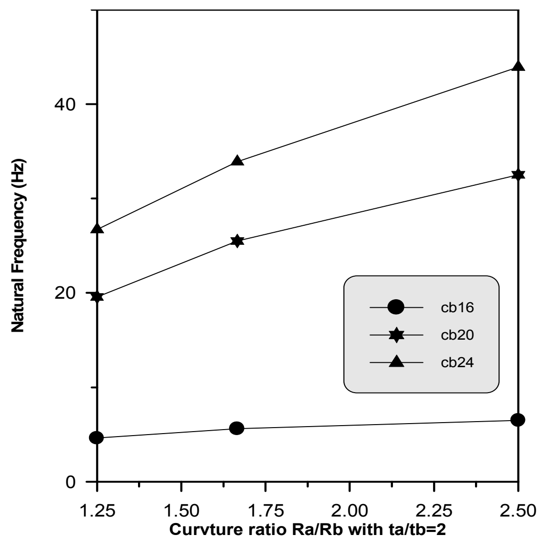


Fig.(29): The effect of the curvature ratio on natural frequency of carved beam

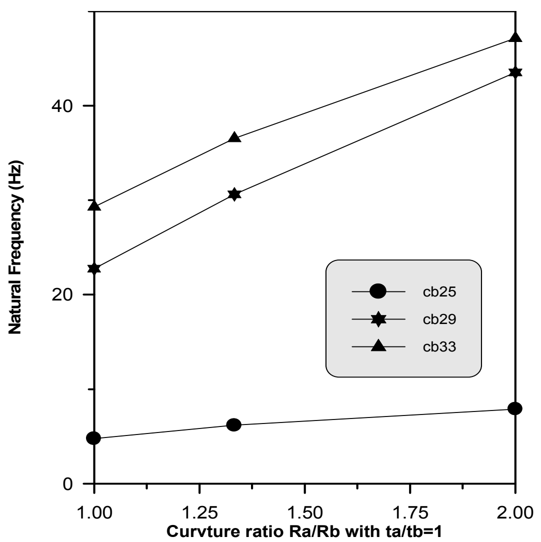


Fig.(30): The effect of the curvature ratio on natural frequency of carved beam

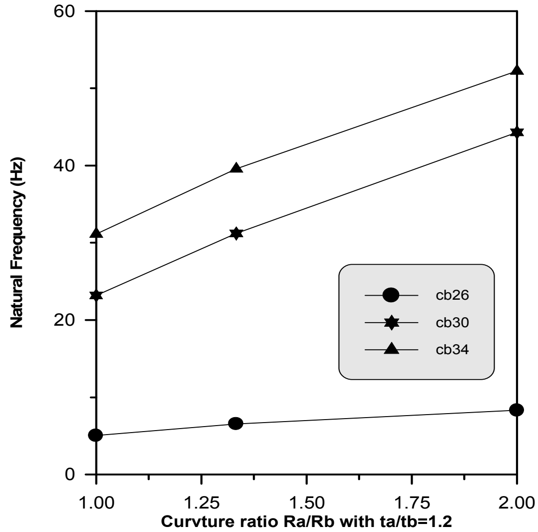


Fig.(31): The effect of the curvature ratio on natural frequency of carved beam

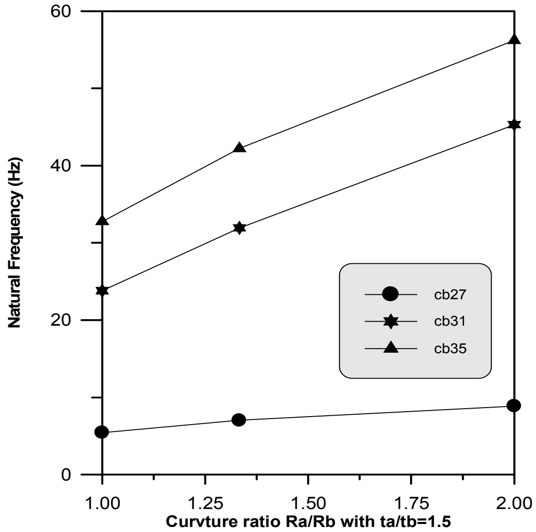


Fig.(32): The effect of the curvature ratio on natural frequency of carved beam

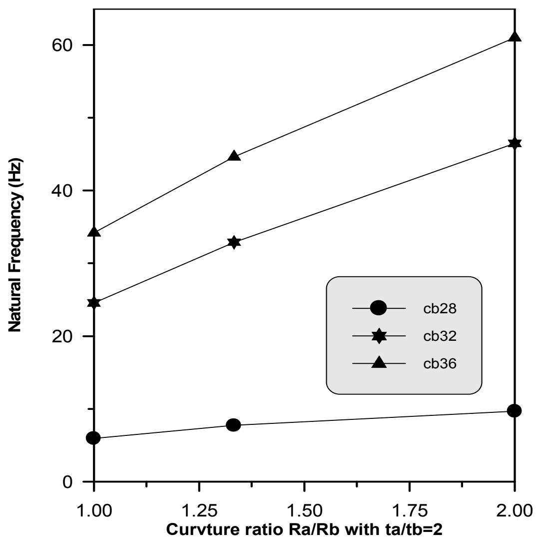


Fig.(33): The effect of the curvature ratio on natural frequency of carved beam

5- Conclusions

In this paper the free vibration of uniform curved beams with varying in the curvature ratio and taper ratio are studied and the following conclusions are drawn;

1-Whene the curvature ratio increases the length of the carved beam decrease this cause increase in the natural frequency at the same taper ratio.

2- When the taper ratio increase the natural frequency increase because the beam is stiffer.

6- References

1- D.G. Ashwell, A.B. Sabir, Limitations of certain curved finite elements when applied to arches, International Journal of mechanical Science 13 (1971) 133-139.
2- A.B. Sabir, D. G. Ashwell, A comparison of curved beam finite elements when used in vibration problem, Journal of Sound and Vibration 18 (4) (1971) 555-563.
3- M. Petyt, C.C. Fleischer, Free vibration of a curved beam, Journal of Sound and Vibration 18 (1) (1971) 17-30.
4- S. S. Rao, Effect of transverse shear and rotary inertia on the coupled twist-bending vibrations of circular rings, Journal of Sound and Vibration 16 (4) (1971) 551-566.
5- M. Sabuncu, vibration Characteristics of Rotating Aerofoil Cross- section Bladed- disc Assembly, PHD Thesis, University of Surrey, 1978.
6- Suzuki, K., Takahashi, S, in –plane vibrations of curved bars with varying cross- section. Bull JSME 25, 1100-1107 (1982).
7- M. Sabuncu, S. Erim, In- plane vibration of a tapered curved beam, Proceeding of the Seventh World Congress on the Theory of Machines an Mechanisms, Sevilla, Spain, 1987.
8- Laura, P.A. A., Bambill, E., Filipich, C. P., Rossi, R. E; Anote on free flexural vibrations of a non uniform elliptical ring in its plane. J. Sound Vibration 126 (2), 249-254 (1988).
9- R. E. Rossi, P.A.A. Laura, P. L. Verniere De Irassar, In- plane vibrations of cantilevered non- circular arcs of non- uniform cross- section with a tip mass, Journal of Sound and Vibration 129 (2) (1989) 201-213.
10- X. Tong, N. Mrad, B. Tabarrok, In- plane vibration of circular arches with variable cross- sections, Journal of Sound and Vibration 212 (1) (1998) 121-141.
11- S. Y. Lee and J. Y. Hsiao, Free in- plane vibrations of curved non uniform beams, Acta Mechanics 155 (2002) 173-189.
12- S. J. Oh, B. K. Lee, I.W. Lee, Free vibration of non- circular arches with non-uniform cross- sections, International Journal of Solids and Structures 37 (2003) 4871-4891.
13- G. Karami, P. Malekzadeh, In- plane free vibration analysis of circular arches with varying cross-sections using different quadrate method, Journal of Sound and Vibration 274 (3-5) (2004) 777-799.
14- Hasan Ozturk, Iso Y. and Mustafa S., In- plane stability analysis of non-uniform cross- sectioned curved beams, Journal of Sound and Vibration 296 (2006) 277-291.
15- G. Kress, M. Sauter and P. Ermanni, Complex- shaped beam finite element, Finite Element in Analysis and Design 43 (2006) 112-126.
16- Y. Gao, M. Z. Wang and B.S. Zhoo, The refined theory of rectangular curved beams, Acta Mechanics 189 (2007) 141-150.
17- Z. H. Zhu and S. A. Meguid, Vibration analysis of a new curved beam element. Journal of Sound and Vibration 309 (2008) 86-95.

Nomenclature

A_{Root}	Cross- sectional area at root of the beam	R	Radius of the curved beam
b_R	Width at the root of the curved beam	t_R	Thickness at the root of the curved beam
b_t	Width at the top of the curved beam	t_t	Thickness at the top of the curved beam
E	Young's modulus	U	Strain energy
k_e	Elastic stiffness matrix of the elemental finite element	v	Circumferential deflection
K_e	Global elastic stiffness matrix	ρ	Mass density
M	Global mass (inertia) matrix	θ	Opening angle of the finite element
m_e	Mass (inertia) matrix of the elemental finite element	ψ	Rotation of tangent
q	Generalized coordinates	ω_1	Fundamental frequency of a curved beam