# THE SOLUTION OF LAMINATED COMPOSITE SPHERICAL SHELLS BY THE APPLICATION OF CHEBYSHEV SERIES

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**ABSTRACT**

The solution of Composite Spherical Shells subjected to different external loading and boundary condition are investigation and analysis by the application of the classical composite-material theory. In this research are used Chebyshev series in matrix form to reformulate the differential equations of equilibrium of a composite spherical shell . Two problem are solved by using Chebyshev theory, the first problem are solved laminated spherical shell under uniform external pressure with open ( $\square_o=10^o$) , the results obtained for maximum stress is (5.298-5.563 N/m) and maximum moment is (1.189-1.99 N.m/m).The results are compared with available published results and confirmed mach well. The second problem is solved for a supported shells under unite edge line load with different ($\square_o=30^o$ and $80^o$ ), the results is seen the stress for laminated composite shells concreted near the pole or near the equator and the bending is localized around the edges.

**Keywords:** Spherical shell, laminated, composite materials , **c**hebyshev series

## حل القشريات الكروية ذات الصفائح المركبة بتطبيق متسلسلات شبيشيف

**الخلاصة :**

حل القشريات الكروية المركبة المعرضة الى احمال خارجية مختلفة وظروف محددة تم تخمينها وتحليلها بواسطة التطبيقات النظرية الكلاسكية للمواد المركبة . في هذا البحث استخدمت متسلسلات شيبشيف في شكل مصفوفة لأعادة تشكيل معادلات تفاضلية منتظمة لقشريات كروية مركبة . مسئلتين تم حلها بأستخدام نظرية شيبشيف ، المسألة الاولى حل قشريات صفائحية كروية تحت ضغط خارجي منتظم وبزاوية منفتحة ($\square_o= 10^0$) ، النتائج المستحصلة لأقصى جهد كانت (5.298-5.563 نيوتن/متر) واقصى عزم (1.189-1.199 نيوتن.متر/متر). النتائج تم مقارنتها مع نتائج منشورة وأكدت تقارب الحل. المسألة الثانية حل قشريات مسندة تحت حمل لحافة مستوية بزاوية مختلفة ($\square_o= 30^0$ و $80^0$ ) .أظهرت النتائج ان الاجهادات للقشريات ذات الصفائح المركبة تتمركز قرب القطب او العمود، وتمركز الانحناء حول الحافات.

## 1. INTRODUCTION

Laminated composite shells are increasingly being used in various engineering applications including aerospace, mechanical, marine and automotive engineering. Spherical shells form an important class of structural configurations in aerospace as well as ground structures, as they offer high strength-to-weight and stiffness-to-weight ratios. The most method to solve generally laminated composite shells having complex geometries, arbitrary loadings and boundary conditions, is the finite element method. The advantages and analysis complications of composite materials stimulated researchers to develop convenient shell theories and solution techniques for composite shells., Noor and Peters (1988) presented static and dynamic analyses of anisotropic shells using conical shell frustum elements. Grafton and Strome (1963) investigated axisymmetric shells using doubly curved finite shell elements.

However, analytical techniques are more suitable for preliminary design requirements. There exist a few analytical solutions for non-cylindrical laminated shells of revolution. Lestingiand and pandovani (1973) , Pandovan and Lestingi (1974) investigated the influence of material anisotropy on shells of revolution using a multisegment numerical integration technique. Tutuncu and Ozturk (1997) investigated bending stresses in composite spherical shells under axsymmetric edge-loads. Their analysis is confined to a certain class of laminated shells; namely, balanced-symmetric laminates. Krishnamurthy K.S. el al. (2003), the authors extended their work on the impact response of a laminated composite cylindrical shell as well as a full cylinder by incorporating the classical Fourier series method into the finite element formulation and also predicted impact-induced damage deploying the semiempirical damage prediction. Topal U.(2006) used first-order shear deformation theory for Mode-Frequency Analysis of Laminated Spherical Shell . Nguyen-Van, N. Mai-Duy and T. Tran-Cong (2007) analyzed laminated plate/shell structures based on the first order shear deformation theory. Oktem A.S. and Reaz A. Chaudhuri (2008) used Higher-order theory based boundary-discontinuous Fourier analysis of simply supported thick cross-ply doubly curved panels.

Alwar et al. (1990,1991) suggested the use of Chebyshev series in the solution of shell problems. Their works were confined to specially orthotropic laminated spherical shells, and the solution procedure was rather complicated and uneasy to apply to different shell problems.

In the present work a variety of problems of generally laminated axisymunctric spherical shells are analyzed using the proposed matrix formulation of Chebyrshev series. stress results are obtained and compared with the available published results.

## 2. MATHEMATICAL ANALYSIS

### 2.1 Equilibrium Equations

The equations of a bending-resistant spherical shell under pressure load are given by (Alwar,1991 and Trimosbmkoi , 1959)

$$\frac{\partial N_s}{\partial s}+\frac{\cot\phi}{R}\left(N_s-N_\theta\right)+\frac{1}{R\sin\phi}\,\frac{\partial N_{s\theta}}{\partial\theta}+\frac{Q_s}{R}=0$$

$$\frac{\partial N_{s\theta}}{\partial s}+\frac{2\cot\phi}{R}N_{s\theta}+\frac{1}{R\sin\phi}\,\frac{\partial N_\theta}{\partial\theta}+\frac{Q_\theta}{R}=0$$

(1)

$$\frac{\left(N_s+N_\theta\right)}{R}-\frac{\cot\phi}{R}Q_s-\frac{\partial Q_s}{\partial s}-\frac{1}{R\sin\phi}\,\frac{\partial Q_\theta}{\partial\theta}=0$$

$$\frac{\partial M_s}{\partial s}+\frac{\cot\phi}{R}\left(M_s-M_\theta\right)+\frac{1}{R\sin\phi}\,\frac{\partial M_{s\phi}}{\partial\theta}-Q_s=0$$

The shell geometry and stress resultants are depicted in **Fig. 1 .**

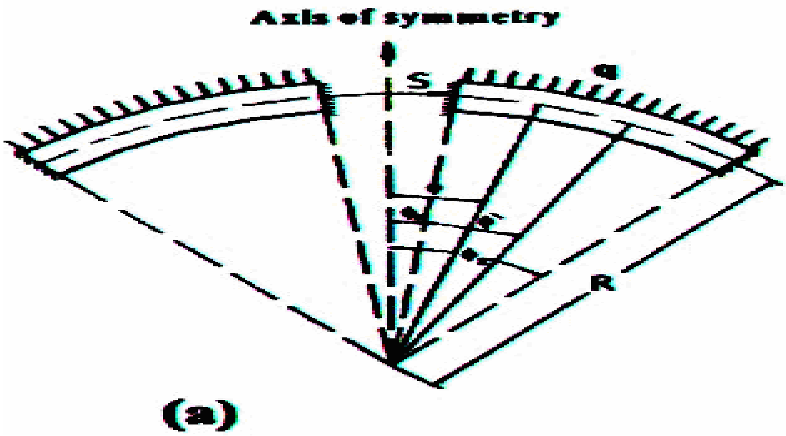


**Fig. 1 .** Spherical Shell (a) Geometry (b) Stress resultants (Harry,1967)

## 2.2 Constitutive Relations:

The shell constitutive relations between the stress resultants and the strain and curvature components according to the classical lamination theory are given by (Jores,1975):

$N_s = A_{11}\,\varepsilon_s + A_{12}\,\varepsilon_\theta + A_{16}\,\gamma_{s\theta} + B_{11}\,k_s + B_{12}\,k_\theta + B_{16}\,k_{s\theta}$

$N_\theta = A_{12}\,\varepsilon_s + A_{22}\,\varepsilon_\theta + A_{26}\,\gamma_{s\theta} + B_{12}\,k_s + B_{22}\,k_\theta + B_{26}\,k_{s\theta}$

$N_{s\theta}=N_{\theta s}=A_{16}\,\varepsilon_s + A_{26}\,\varepsilon_\theta + A_{66}\,\gamma_{s\theta} + B_{16}\,k_s + B_{26}\,k_\theta + B_{66}\,k_{s\theta}$

(2)

$M_s = B_{11}\,\varepsilon_s + B_{12}\,\varepsilon_\theta + B_{16}\,\gamma_{s\theta} + D_{11}\,k_s + D_{12}\,k_\theta + D_{16}\,k_{s\theta}$

$M_{\theta} = M_{12}\ \varepsilon_s + M_{22}\ \varepsilon_{\theta} + M_{26}\ \gamma_{s\theta} + D_{12}\ k_s + D_{22}\ k_{\theta} + D_{26}\ k_{s\theta}$

$M_{s\theta}=M_{\theta s}=B_{16}\ \varepsilon_s + B_{26}\ \varepsilon_{\theta} + B_{66}\ \gamma_{s\theta} + D_{16}\ k_s + D_{26}\ k_{\theta} + D_{66}\ k_{s\theta}$

The definition of extensional, coupling, and bending stiffness coefficients $A_{ij}$, $B_{ij}$, and $D_{ij}$ respectively (i,j=1,2,6) is give by ( Jores , 1975)

**2.3 Strain-Displacement Relations**

The shell strain-displacement relations for small displacements are given by (Harry,1967)

$$\varepsilon_s = \frac{\partial u}{\partial s} + \frac{w}{R} \qquad \varepsilon_\theta = \cot\phi \frac{u}{R} + \frac{1}{R\sin\phi}\frac{\partial v}{\partial \theta} + \frac{w}{R}$$

$$\gamma_{s\theta} = \frac{\partial v}{\partial s} + \frac{1}{R\sin\phi}\frac{\partial u}{\partial \theta} - v\frac{\cot\phi}{R} \tag{3}$$

$$k_s = \frac{1}{R}\frac{\partial u}{\partial s} - \frac{\partial^2 w}{\partial s^2} \qquad k_\theta = \frac{\cot\phi}{R}\left(\frac{u}{R} - \frac{\partial w}{\partial s}\right) + \frac{1}{R^2\sin^2\phi}\left(\frac{\partial v}{\partial \theta} - \frac{\partial^2 w}{\partial \theta^2}\right)$$

$$k_{s\theta} = \frac{1}{R\sin\phi}\left(\frac{\partial v}{\partial s} - 2\frac{\partial^2 w}{\partial s\,\partial\theta}\right) + \frac{1}{R^2\sin\phi}\left(\frac{\partial u}{\partial\theta} - 2v\cot\phi + 2\cot\phi\frac{\partial w}{\partial\theta}\right)$$

Substituting the strain-displacement relations (3) into the stress resultant-strain relations (2), and eliminating $Q_s$ and $Q_\theta$ from eqs. (1) we end up with three equilibrium equations for generally laminated arbitrarily loaded spherical shells. In case of axisymmetric generally laminated shell problems we have $\frac{\partial}{\partial\theta} = 0$ and $v \neq 0$. Using the non-dimensional coordinate $\xi = (s - R\phi_o)/L$ and $L = R\,(\phi-\phi_o)$, the system of equations reduces to:

$$\left(\frac{Q_{17}\cot\phi}{R} + \frac{Q_{20}\cot\phi}{R^2\sin\phi}\right)\frac{1}{L}\frac{dv}{d\xi} + \left(\frac{Q_{16}}{R^2} + \frac{Q_{17}\cot^2\phi}{R^2} + \frac{2Q_{24}\cot^2\phi}{R^3\sin\phi}\right)v - \tag{4}$$

$$R_1\frac{1}{L^3}\frac{d^3w}{d\xi^3} - \frac{R_1\cot\phi}{R}\frac{1}{L^2}\frac{d^2w}{d\xi^2} + \left(\frac{R_4}{R} + \frac{R_2\cot^2\phi}{R^2}\right)\frac{1}{L}\frac{dw}{d\xi} + \frac{R_7\cot\phi}{R^2}w = 0$$

The trigonometric function terms appearing in eq. (4) will be designated as:

$$F_1 = \frac{1}{\sin\phi}\,, \qquad F_2 = \frac{1}{\sin^2\phi}\,, \qquad F_3 = \frac{1}{\sin^3\phi}\,, \qquad F_4 = \sin\phi\,,$$

$$F_5 = \frac{\cot\phi}{\sin\phi}\ ,\ F_6 = \frac{\cot\phi}{\sin^2\phi}\ ,\ F_7 = \frac{\cot\phi}{\sin^3\phi}\ ,\ F_8 = \cot^2\phi\ , \tag{5}$$

$$F_9 = \frac{\cot^2\phi}{\sin\phi} \qquad F_{10} = \frac{\cot^3\phi}{\sin\phi}$$

### 2.4 Boundary Conditions

**-** Pole conditions

$$u = v = \frac{dw}{d\xi} = 0 \text{ and } k_s = 0 \text{ i.e.} \frac{1}{R}\frac{\partial u}{\partial \xi} - \frac{\partial^2 w}{\partial \xi} - \frac{\partial^2 w}{\partial \xi^2} = 0 \tag{6}$$

**-** Clamped-edge conditions

$$u = w = v = \frac{dw}{d\xi} = 0 \tag{7}$$

**-** Simply supported edge conditions, free to move in the horizontal direction

Vertical displacement (- u cos □ + w sin □) = v = $M_s$ = 0

Horizontal force ($N_s$ sin □ + $Q_s$ cos □) = H (8)

### 3. CHEBYSHEV SERIES REPRESENTATION

Any continuous function f(□) in the interval $0 \le \xi \le 1$ can be written in Chebyshcv series as given by (Alwar and Narasinthan,1990):

$$f(\xi) = \sum_{r=0}^{\infty}{}^{+} a_r T_r(\xi) f \tag{9}$$

Where:

+ sign means that the 1st term is halved.

$a_r$ … are constants to be determined so as to obtain the best possible fit.

The shifted Chebyshev polynomials satisfy the recurrence relations:

$T_{r+1}$ (□) = 2 (2□-1) $T_r$ (□)- $T_{r-1}$ (□) , $1 \le r \le \infty$ (10)

and the orthogonally conditions:

$$\int_0^1 \frac{T_m(\xi)T_n(\xi)}{\sqrt{\xi}\sqrt{1-\xi}}d\xi = \begin{cases} 0 & \text{for} \quad m \neq n \\ \frac{\pi}{2} & \text{for} \quad m = n \neq 0 \\ \pi & \text{for} \quad m = n = 0 \end{cases}$$

For any continuous function f(□) the series expansion (9) is fast converging, and a good approximation is obtained by taking a finite number of terms. Therefore, eq. (9) is approximated by:

$$f(\xi) = \sum_{r=0}^{N} + a_r T_r(\xi) \tag{11}$$

Where, for a known function f(□), the coefficients $a_r$ are given by:

$$a_r = \frac{2}{\pi}\int_0^1 \frac{f(\xi)T_r(\xi)}{\sqrt{\xi}\sqrt{1-\xi}}d\xi \qquad 0 \leq r \leq N \tag{12}$$

The first derivative $f^{\backslash}$(□) is expressed in Chebyshev series as (alwar and Narasinthan 1990 , Alwar and Narasindan 1991):

$$f^{\backslash}(\xi) = \sum_{r=0}^{N-1} {}^{+}a_r^{(1)} T_r(\xi) \tag{13}$$

The coefficients $a_r^{(1)}$ satisfy the recursive relation:

$$a_{r-1}^{(1)} - a_{r+1}^{(1)} = 4ra \quad , \qquad z \leq r \leq N \tag{14}$$

Similarly higher function derivatives can be written as:

$$f^{\backslash\backslash}(\xi) = \sum_{r=0}^{N-2} {}^{+}a_r^{(2)} T_r(\xi) \tag{15}$$

$$f^{m}(\xi) = \sum_{r=0}^{N-m} {}^{+}a_r^{(m)} T_r(\xi)$$

Where;

$$a_{r-1}^{(2)} - a_{r+1}^{(2)} = 4ra_r^{(1)} \quad , \qquad 1 \leq r \leq N-1$$

$$a_{r-1}^{(m)} - a_{r+1}^{(m)} = 4ra_r^{(m-1)} \quad , \qquad 1 \leq r \leq N-(m-1) \tag{16}$$

## 4- FORMULATION OF EQUILIBRIUM EQUATIONS

Expanding u(□), v(□) and w(□) in (N+1)-term Chebyshev series we have a total of 3N+3 unknown coefficients. The trigonometric functions (5) appearing in the equilibrium eqs. (4) are also expanded in Chebyshev series having M+1 terms. The M+I expansion coefficients can be computed easily by forcing the function eqs. (5) to take on their actual values at a number of chosen points in the interval $0 \leq \square \leq 1$. Using matrix formulation for the functions and function derivatives, and applying the rule of matrix multiplication as explained in Alwar and Narasindan 1991 , equilibrium eqs. (4) can be written as a system of algebraic equations in the following matrix form:

$$[A]=\begin{bmatrix}1&0&3&0&5\\0&2&0&4&0\\0&0&3&0&5\\0&0&0&4&0\\0&0&0&0&5\end{bmatrix} \qquad [A01]=\begin{bmatrix}0&1&0&3&0&5\\0&0&2&0&4&0\\0&0&0&3&0&5\\0&0&0&0&4&0\\0&0&0&0&0&5\end{bmatrix} \tag{17}$$

The matrices [A01] to [A04] relate the 1st, 2nd, 3rd and 4th derivative coefficients of a function to the original function coefficients respectively. The first-order-derivative coefficients $\{a_r^{(1)}\}$ in eq. (13) can be written in terms of the original function coefficients $\{a_i\}$ using matrix notation as follows:

$$\left\{a_r^{(1)}\right\}=4[A01]\{a_i\} \qquad ; \quad \begin{matrix} r = 0,1,2,\ldots\ldots\ldots\ldots, N-1 \\ i = 0,1,2,3\ldots\ldots\ldots\ldots, N \end{matrix}$$

where [A01] is of order N * (N+1). It is composed of an N * N matrix designated as [A] matrix and an N * 1 column with zero entries at the left of the matrix [A]. Matrix [A] is an upper triangular matrix. Its elements $a_{ij}$ are defined as.

$$a_{ij}=\begin{cases}0 & i>j \quad or \quad i+j \quad odd\\ j & i\leq j \quad and \quad i+j \quad even\end{cases}$$

From the matrices form (17) for N=5: The matrices [A02], [A03], [A04] and [A0n] are obtained as follows:

$$[A02]=[A]_{-1,-1}\ {}^{-1}[A01] \qquad [A03]=[A]_{-2,-2}\ {}^{-1}[A02]$$

$$[A04]=[A]_{-3,-3}\ {}^{-1}[A03] \qquad [A0n]=[A]_{1-n,1-n}\ {}^{-1}[A0(n-1)] \tag{18}$$

Where,

n the order derivative

$[A]_{1-n,1-n}$ matrix [A] after deleting the last (n-1) rows and (n-1) columns.

${}^{-1}[\ \ ]$ matrix [A] after deleting the first row.

## 5. RESULTS AND DISCUSSIONS

Two problems of spherical shells under different loads and boundary conditions will be considered.

**Problem 1**

The first problem is a clamped-clamped generally laminated open spherical shell with □$_o$=10$^o$, under uniform external pressure. Convergence and comparison studies for the [90/0] laminated shell is with the results of (Alwar and Narasinthan 1990) presented in **Table 1**. It is clear from the table that especially orthotropic lamination are giving the minimum hoop stress resultant and meridional moment resultant, so that good convergence to the right answer can be obtained by about 16 terms in Chebyshev series.

**Table 1** convergence study for [0/90] spherical shell

| | N=16 | N=22 | Alwar and Narasinthan 1990 |
|---|---|---|---|
| Max.hoop stress resultant $N_\theta$ | 5.2989 | 5.5639 | 5.410 |
| Max.deflection $W=10^4E_2w\ h^3/(pL^4)$ | 0.1502 | 0.1489 | 0.1478 |
| Max.meridional moment resultant $M_s$ | 1.1898 | 1.997 | 1.175 |

Where :

$p=6900N/m^2$ $E_1/E_2=20$ $N_\theta= 10^6\ N_s\ h^3/(pL^4)$

$R/h=30$ $G_{12}/E_2=0.5$ $\nu_{12}=0.28$

$M_\theta=10^6M_s\ h^3/(pL^4)$ $\phi_0=0^0$ , $\phi_1=90^0$

**Problem 2**

The second problem is a generally laminated spherical shell under horizontal edge line load as shown in **Fig. 2** . The material used has the properties: $E_1/E_2=20$, $G_{12}/E_2=0.5$, $v_{12}=0.28$. The radius of the sphere R is 0.15 m, and the thickness h is 0.005 m. The problem is solved for a simply supported shell with different □$_o$=30$^o$ and 80$^o$ with non-dimensional meridional coordinate ($\zeta$ = 0 to 1 ) under unit edge line load.

From **Figs.3,4,5** represent the variation of axial moment, circumferential stress, axial shear stress respectively in the meridional direction ($\zeta$) for different laminations ($\phi_0=30^0$ , $\phi_1=150^0$) and with different laminate schemes (0/45 , 0/90 , 45/-45 ). **Fig.3** It shows the distribution of axial moment for the shell and it seen maximum value of axial moment about (1.51 - 2.35) N.m/m at $\zeta$=0.1 and 0.9, also depend on laminate schemes, also seen that the axial moment is near of the equilibriums profile in range $\zeta$= 0.2 – 0.8 . This indicated the effect of maximum axial moment is localized around the edges ,and diminishes as the edge distance increases.**Fig.4** It shows the maximum value of the circumferential stress at $\zeta$ =0.1 and 0.9 about (0.5-0.75) N/m and depend on laminate schemes.**Fig.5** It seen the distribution of axial shear stress and it is the maximum value about (0.1,-0.1) N/m in the same interval of $\zeta$ = 0.1 and 0.9 . The results from **Fig.4,5** it seen that the laminates exhibit substantial differences in the stress distributions for shells opened near the pole or near the equator.

From **Figs.6,7,8** the plotted shows the variation of axial moment, circumferential stress, axial shear stress respectively in the meridional direction ($\zeta$) for different laminations ($\phi_0$ =$80^0$ , $\phi_1$ =$120^0$) and with different laminate schemes (0/45 , 0/90 , 45/-45 ). **Fig.6** the maximum average value of the axial moment in this condition equal between (3.9-7.4)N.m/m at $\zeta$=0.1 and 0.9 . **Fig.7** shows the distribution of the circumferential stress and it seen the average value between (-13,-9) at $\zeta$ =0.1 and 0.9 .**Fig.8** it seen the axial shear stress with average value (0.5,-0.5) at $\zeta$=0.1 and 0.9. From **Figs.6,7,8** shows that the maximum distribution of axial moment around the edges and the the stress distributions for the shell opened near the pole.

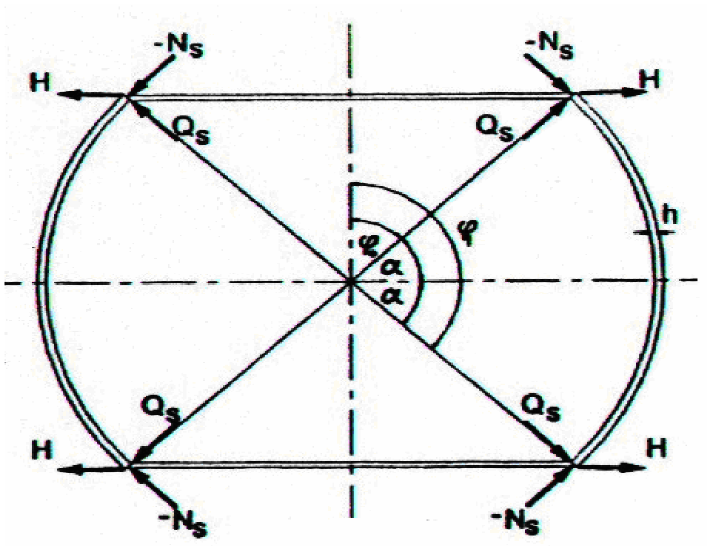


**Fig. 2 .** Spherical shell under uniform edge load

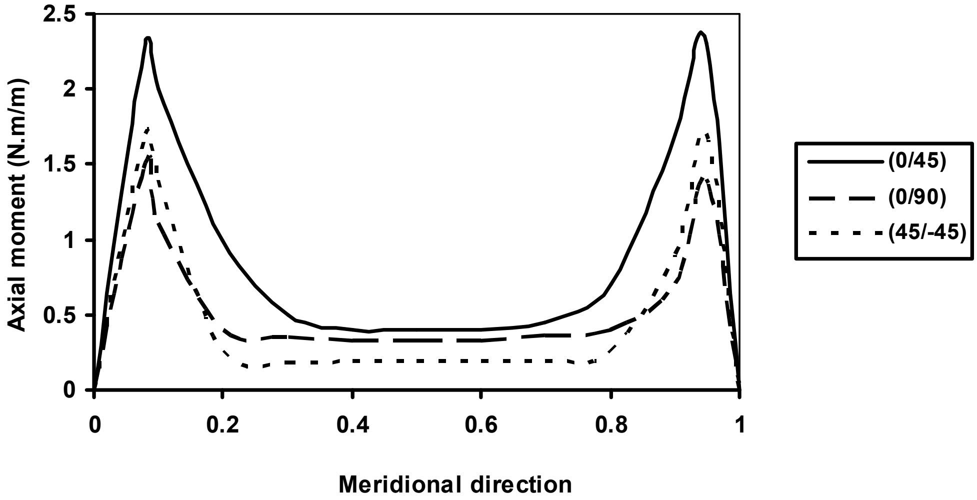


**Fig. 3 .** Variation of axial moment in the meridional direction ($\zeta$) for different laminations ($\phi_0 = 30^0$ , $\phi_1 = 150^0$)

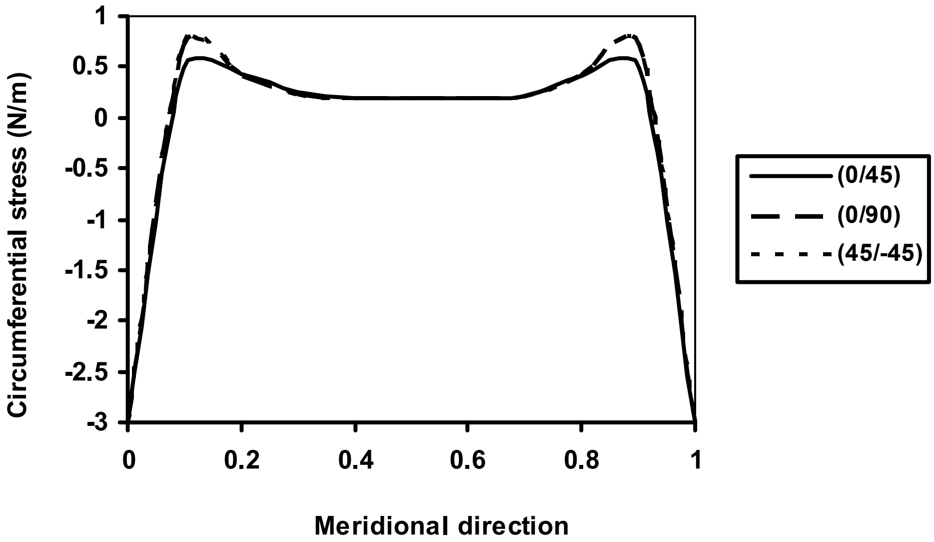


**Fig. 4 .** Variation of circumferential stress in the meridional direction ($\zeta$) for different laminations ($\phi_0 = 30^0$ , $\phi_1 = 150^0$)

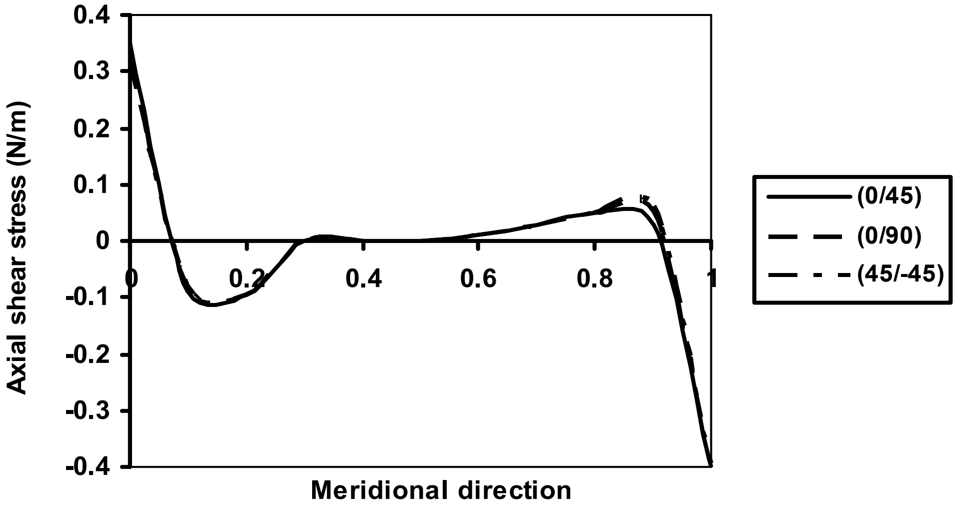


**Fig. 5 .** Variation of axial shear stress in the meridional direction($\zeta$)

for different laminations ($\phi_0 = 30^0$ , $\phi_1 = 150^0$ )

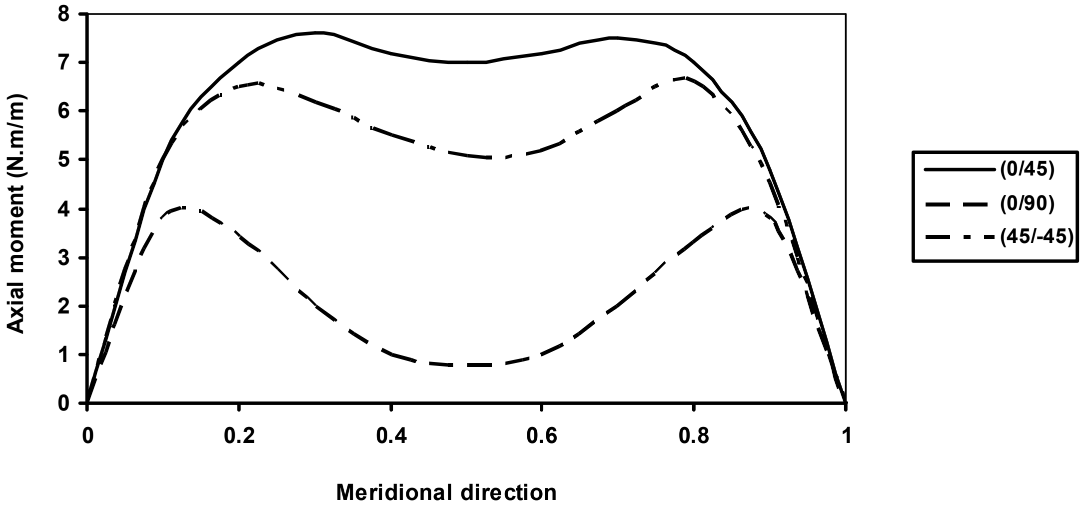


**Fig. 6 .** Variation of axial moment in the meridional direction ($\zeta$)

for different laminations ($\phi_0 = 80^0$ , $\phi_1 = 120^0$ )

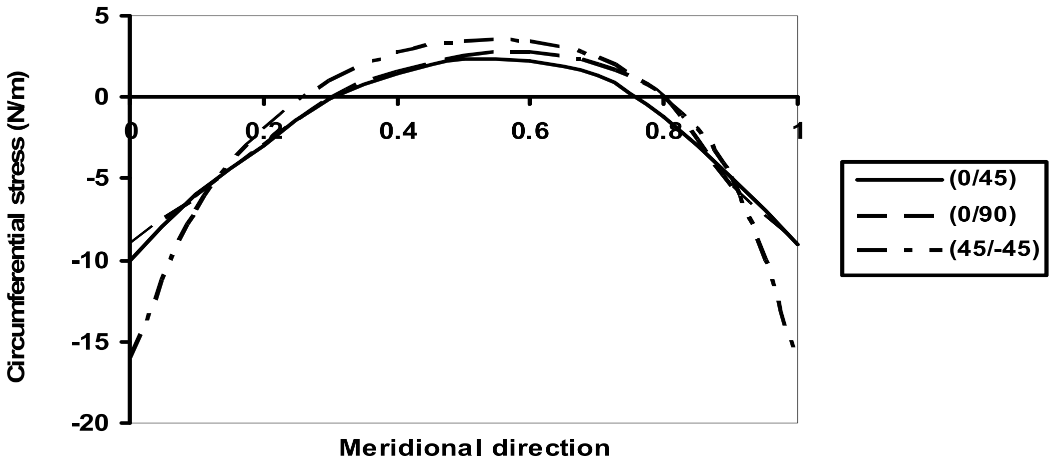


**Fig. 7 .** Variation of circumferential stress in the meridional direction ($\zeta$) for different laminations ($\phi_0 = 80^0$ , $\phi_1 = 120^0$ )

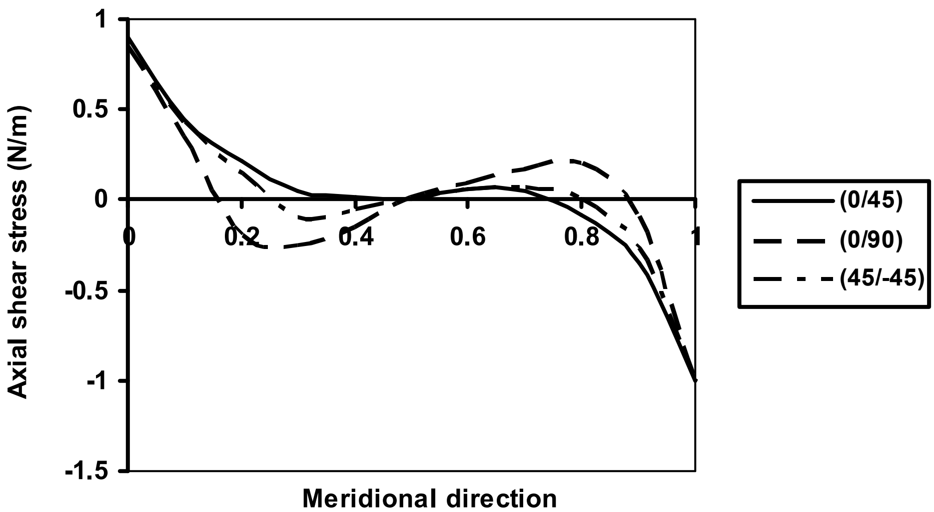


**Fig. 8 .** Variation of axial shear stress in the meridional direction ($\zeta$)

for different laminations ($\phi_0 = 80^0$ , $\phi_1 = 120^0$ )

## 6- CONCLUSION

The method has been presented for the solution of arbitrarily laminated composite spherical shells by expanding displacement functions in Chebyshev series. The method is used to solve a variety of spherical shell problems with different fiber orientations and boundary conditions.

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**List of Symbols**

| | | |
|---|---|---|
| $A_{ij}$, | extensional stiffness coefficient, i , j=1,2,6 | |
| $B_{ij}$, | coupling stiffness coefficient, i , j =1,2,6 | |
| $D_{ij}$ | bending stiffness coefficients , i, j = 1, 2, 6. | |
| E | modulus of elasticity in tension and compression | (N) |
| G | modulus of elasticity in shear | (N) |
| $k_s$, $k_\square$, $k_{s\square}$ | change in curvatures in s, θ coordinates | (m,deg$^0$) |
| H | horizontal force | (N) |
| h | thickness of a shell | (m) |
| $M_s$ , $M_\theta$ | bending moments | (N.m/m) |
| $M_{s\theta}$ | twisting moment | (N.m/m) |
| N | number of terms | |
| $N_s$, $N_\square$, $N_{s\square}$ | In-plane normal and shearing stress resultant | (N/m) |
| p | pressure | (N/m$^2$) |
| R | radius of spherical shell | (m) |
| r,s,θ | polar coordinates | (m,m,deg$^0$) |
| $Q_s$, $Q_\square$ | transverse shear stress | (N/m$^2$) |
| x,y,z | rectangular coordinate | (m) |
| u,v,w | components of displacements | (m) |
| α | semi conical angle | (deg$^0$) |
| $\square_{s\square}$ | shear strains in polar coordinate | |
| $\square_s$, $\square_\square$, | meridional strains | (N/m$^2$) |
| $\phi$, $\phi_i$ | angle of laminated orientation, i=0,1 | (deg$^0$) |
| ζ | non-dimensional meridional coordinate | |
| ν | poisons' ratio | |