A WIDE RANGE FORMULA FOR COMPRESSION FACTOR FUNCTION OF REDUCED PROPERTIES

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ABSTRACT

This paper presents an effort aimed at developing a new formula to calculate compression factor within acceptable error range. A formula has been generalized to permit calculation of the compression factor as a function of reduced properties: reduced temperature T_R and reduced pressure P_R. The new correlations show improvement in temperature and pressure range which offers flexibility for estimating compression factor for a wide range of reduced temperature; T_R=1-5 and reduced pressure ;P_R=0.5- 6.5.

This correlation provides accurate and computationally reliable prediction of the compression factor (Z) with an average absolute error (AAE) 4.6 % for 169 data points.

Keywords: compression factor, reduced temperature, reduced pressure, formula, deviation

الخلاصة

البحث الحالي يمثل محاولة لايجاد علاقة لحساب معمل الانضغاطية وبنسبة خطأ معقولة. تم تطبيق العلاقة لحساب معامل الانضغاطية لمدى واسع بالاعتماد على الخواص التناسبية : درجة الحرارة التناسبية (T_R) والضغط التناسبي (P_R) . العلاقة الجديدة حسنت المدى التطبيقي لدرجة الحرارة والضغط والذي ادى لحساب معمل الانضغاطية وفي مدى واسع لدرجة الحرارة التناسبية (T_R=1−5) والضغط التناسبي (P_R=0.5− 6.5).

الموديل الجديد يوفر التنبؤ الدقيق والموثوق لحساب معمل الانضغاطية وبمعدل نسبة خطأ مطلق (4.6 %) الخاصة في (169) نقطة عملية.

NOMENCLATURE

AAE average absolute error, %
k ratio of specific heats, cp/cv
M molecular weight, kg/kgmol
n isentropic index
P_d discharge pressure, kPa
P_{pr} pseudo-reduced pressure
P_s suction pressure, kPa
R_u universal gas constant, 8.314 kJ/(Kg mol K^{-1}); $R= R_u/M$
T temperature, K
T_d discharge temperature, K
T_{pr} pseudo-reduced temperature
T_R reduced temperature
T_s suction temperature, K
Z observed compression ratio (observed compressibility factor)
Zc calculated compression ratio (calculated compressibility factor)
ρ density, kg m^{-3}

INTRODUCTION

P-V-T measurements for gases under pressure started in the later of the 19^{th} century; upsurge measurement occurred for natural gasses, led by the work of Sage et al.(1940) on their API projects .Out of this study came the compressibility factor (compression factor Z) chart made by Standing and Katz (1945) with the advice assistance of G.G Brown and D. Holocomb .The basis for the correlation of the data used with this chart is Van der Waal theorem of corresponding of stage; substance should have similar properties at corresponding conditions normalized to some basic properties like the critical temperature and pressure , The use of reduced pressure P_r (actual pressure divided by critical pressure) , and reduced temperature T_r (actual temperature divided by critical temperature (absolute)),to determine compressibility factors brought reasonable but not exact correlations (Katz,1959).The original Standing and Kats chart covered pressure to P_R=15 or about 10000 psi (68.9 Mpa).The chart was extended to P_R=30 for the Handbook (Katz,1959).

Standing and Katz1 (1942) presented a generalized compressibility factor chart. The chart represents compressibility factors of sweet and dry natural gas as a function of pseudo-reduced pressure (P_{pr}) and pseudo reduced temperature (T_{pr}). This chart is generally reliable for sweet natural gases with minor amounts of non-hydrocarbons. It is one of the most widely accepted correlations in the oil and gas industry. The Standing and Katz (SK) chart was developed using data for binary mixtures of methane with propane, ethane, butane, and natural gases having a wide range of composition. None of the gas mixtures had a molecular weight in excess of 40. For low molecular weight gases, it was found that the *Z*-factor estimated from the SK chart has an error in the order of 2 to 3%. However, for gas mixtures whose molecular weight is greater than 40, the SK chart provides inaccurate *Z*-factors. Methods for estimating gas compressibility factors normally are used when a reservoir fluid-depletion study is not available. This practice is acceptable for retrograde gases if the gas condensate is lean; however, if the gas is rich, the reserves may be seriously underestimated if the two phase compressibility factor is not used. The compressibility factor chart (SK) is applicable to most gases encountered in petroleum reservoirs and provides satisfactory prediction for all engineering computations. The calculated volume of gases containing only minor amounts of non-hydrocarbon can be accurate within 97%. Standing and Katz correlation for compressibility factor is valid only for dry-gas systems. Corrective methods are introduced to account for the presence of high molecular weight gases

($C7^+$) and to extend the range of application of the SK chart (Sutton, 1985). Retrograde gas condensate reservoirs experience liquid fallout during depletion below the dew point. The two-phase compressibility factor accounts for the formation of a liquid phase. After several decades of existence, the SK chart is still widely used as a practical source of the *Z*-factor for natural gas.5. Several empirical correlations for calculating the *Z*-factor have been developed. These correlations are the following (Adel et al ,2001):
(1) Pappey
(2) Hall-Yarborough
(3) Dranshuk-Abou-Kassem
(4) Dranchuk-Purvis-Robinson
(5) Hankinson-Thomas-Philips
(6) Brill and Beggs
TaKacs (Adel et al ,2001) used 180 values of the *Z*-factor from the SK chart to review the accuracy of the first five correlations. He concluded that the Dranchuk-Abou-Kassem correlation is the best one to fit the SK chart. For the six correlations the average absolute errors are within ranges 6- 28% (Adel et al ,2001).

Compression factor (Compressibility factor) is a useful thermodynamic property for modifying the ideal gas law to account for the real gas behavior (Smith, 2005). In general, deviation from ideal behavior becomes more significant the closer a gas is to a phase change, the lower the temperature or the larger the pressure. Compressibility factor values are usually obtained by calculation from equations of state (EOS), such as the Virial equation which take compound specific empirical constants as input. For a gas that is a mixture of two or more pure gases (air or natural gas, for example), a gas composition is required before compressibility can be calculated. A good knowledge of the compression factor Z is necessary for the reliable determination of the mass flow of the natural gas transferred through pipelines .Because of the very large mass flow, it is desirable that compression factor can be measured or calculated with a good accuracy. Alternatively, the compressibility factor for specific gases can be read from generalized compressibility charts (Cengel, 1988) that plot Z as a function of pressure at constant temperature see **Fig.1**.

One of the main objectives of the current work is to find a simplified generalized formula to calculate the compressibility factors.

THEORY

Many gas compression calculations performed by hand-held analyzers and PLCs (programmable logic controllers) use P-V data based on ideal gas models. For ideal gases, the pressure, temperature, and density are related by:

$$P = \frac{\rho R_u T}{M} \qquad (1)$$

(All pressures and temperatures must be in absolute units.) If the compression process for an ideal gas is isentropic, the following equation relates suction and discharge pressure and temperature:

$$\frac{T_d}{T_s} = \left(\frac{P_d}{P_s}\right)^{\frac{k-1}{k}} \qquad (2)$$

where *k* is the ratio of specific heats, *cp/cv*.

Ideal gas models work for most gases at low pressures. However, in the range of conditions encountered in EOR (Enhanced Oil Recovery) applications, gases do not always behave ideally. A compressibility factor, *Z*, can be added to the ideal gas law as a correction, such that:

$$Z = \frac{PM}{\rho R_u T} \tag{3}$$

Alternately, a more complicated EOS can be used to model gas behavior. Analyzers and PLCs often use the Redlich-Kwong EOS for its simplicity and speed of calculation.
The compression process in a reciprocating compressor is often treated as isentropic. The isentropic compression equation is often inaccurate when *k* is used in the exponent, but its form is useful for flow and power calculations. Therefore, *k* is often replaced by a variable called *n* (isentropic index), as in eq.(4) below. The value of *n* is calculated using the P-V card from an analyzer, or from pressure and temperature values taken from entropy charts for isentropic compression.

$$\frac{T_d}{T_s} = \left(\frac{P_d}{P_s}\right)^{\frac{n-1}{n}} \tag{4}$$

From a thermodynamic perspective, any EOS that can generate values for Z can also be used to directly calculate the entropies and enthalpies required to model isentropic compression (Span and Wagner (1996) and Ref Prop software created by NIST(2002)) . However, for most EOS models, such calculations require time-intensive iterative processes that may be beyond the capability of a PLC or portable analyzer. Furthermore , many programs written for these devices already use equations that require values of Z and n. This paper will therefore present simplified correlation for Z.

The compressibility factor is a ubiquitous concept in measurement. It arises in many industry practices and standards. Unfortunately the mathematical methods and data associated with it obscure some of the basic ideas behind it. The purpose of this paper is to suggest a formula for correlating compressibility factor Z with relative critical properties

FORMULA DEVELOPMENT

A multiple regression analysis is required to develop a functional relationship between compression factor Z, with the corresponding reduced temperature T_R and reduced pressure P_R. Applying a computer program; (Statistica software program version 5) to the data obtain by Wilcox et. al.(2009) , Cengel (1988) and Yuns (1990) lead to the development of a general formula for the compression factor predictions, the equation is ;

$$Z = C_1 + (C_2+C_3T_R+C_4T_R^{2}+C_5T_R^{-6})(C_6+C_7 P_R+C_8P_R^{4}+C_9P_R/T_R+C_{10}(T_R/P_R)^{C11}) \tag{5}$$

This equation is valid for reduced temperature range;1-5 and reduced pressure 0.5-6.5 where C_1 to C_{11} of eq.(5) are listed in **Table 1** .
The procedure of applying Statistica software program version 5 to fit eq.(5) is as follow:

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To choose a topic, point to it and click the left-mouse-button, or press TAB to highlight the underlined topic and press ENTER .
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Quasi-Newton Method
Simplex Procedure
Hooke-Jeeves Pattern Moves
Rosenbrock Pattern Search

Hessian Matrix and Standard Errors

Click on the following topics for overviews of the common nonlinear regression models and evaluation of the fit of the data to the nonlinear model.
Common Nonlinear Regression Models

5) Evaluating the Fit of the Model

Nonlinear Estimation Procedures - Quasi-Newton Method
As you may remember, the slope of a function at a particular point can be computed as the first-order derivative of the function (at that point). The "slope of the slope" is the second-order derivative, which tells us how fast the slope is changing at the respective point, and in which direction. The quasi-Newton method will, at each step ,evaluate the function at different points in order to estimate the first-order derivatives and second-order derivatives. It will then use this information to follow a path towards the minimum of the loss function .

RESULTS AND DISCUSION

From open literature ,Hashim et. al. (2009) developed compressibility factor correlation function of reduced temperature and pressure. This correlation offers flexibility for estimating Z for a wide range of reduced temperatures (T_R=1-5) and reduced pressure (P_R= 0.5- 6.5). The present work has an improvement for this correlation in percentage error and in number of correlation constants.

The average absolute error of the proposed correlation (eq.(5)) is 4.6 % for 169 data points while the corresponding Hashim et al is 4.9 of 180 data points with significant local errors (more than 20%). Limited works are available in literate of correlating simple formula of the compressibility factor Z with critical prosperities. These suggested correlations become more difficult and complicated for the wide range of relative critical properties.
Fig.2 to Fig.12 shows the graphical comparisons of the observed compressibility factor Z and calculated compressibility factor Z from eq.(5) for all reduced temperature ranges (1, 1.05, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.8, 2, 2.5,3.5 and 5) and reduced pressures range (0.5-6.5). **Table 2** shows the average percentage error of the observed compressibility factor Z with the corresponding calculated one for each range of reduced temperature and reduced pressure (that are summarized in **Fig.2 to Fig.12**).
Table 2 have a significant errors of applying proposed formula eq.(5) for ; T_r <1.2 and P_r range 0.5-6.5.
The agreement in **Table 2** is reasonably good except in the case of No.1 and 2 for values of T_R= 1 and 1.05. These significant errors in cases No.1 and 2 are due to that gases temperature values approximately near critical temperature values (i.e. near the saturation line and phase change (gas to liquid phase)). The agreement between the present correlation (eq.(5)) and the measured values of the compressibility factor Z is excellent despite some small discrepancy at P_R=0.5. These cases are presented in **Figs. 2,3 and 4**, while more accurate data are available for these figures for P_R > 3. In **Fig. 5 to Fig.12**, the results show a tendency to decrease deviation between measured compressibility factor Z and the corresponding calculated Z_c. In other words, more ideal behavior for the gas at higher temperature values (i.e. higher T_R values). **Fig. 7,8 and 9** have a smaller error ranges for compressibility factor prediction for reduced temperature; T_R values 1.5, 1.6 and 2.0.

CONCLUSIONS

A formula has been generalized to permit calculation of the compression factor as reduced temperature T_R and reduced pressure P_R. The new correlations offers flexibility for estimating compression factor for a wide range of reduced temperature; T_R=1-5 and reduced pressure; P_R=0.5- 6.5.

This paper presents a simplified model for compressibility factor that is functionally equivalent to the more complex models available today. This correlation provides accurate and computationally reliable prediction of the compression factor (Z) with an average absolute error 4.6 % for 169 data points.

Table 1: Constants of eq.(5)

<table>
<tr><td>C_1</td><td>C_2</td><td colspan="2">C_3</td><td colspan="2">C_4</td><td>C_5</td></tr>
<tr><td>1.0140</td><td>-440.650</td><td colspan="2">258.4585</td><td colspan="2">-26.4235</td><td>-89.4936</td></tr>
<tr><td>C_6</td><td>C_7</td><td>C_8</td><td colspan="2">C_9</td><td>C_{10}</td><td>C_{11}</td></tr>
<tr><td>0.00000035</td><td>-0.00046</td><td>1.0</td><td colspan="2">-1.57841</td><td>1.58061</td><td>-0.99940</td></tr>
</table>

Table 2: Shows the average percentage error of the observed compressibility factor Z with the corresponding calculated one for each range of reduced temperature and reduced pressure that are summarized in **Fig.2**

No.	Fig. no.	T_r	Pr	Number of	AAE%
1	2	1	0.5~6.5	13	9.8
2	3	1.05	0.5~6.5	13	10
3	4	1.1	0.5~6.5	13	7.0
4	5	1.2	0.5~6.5	13	6.6
5	6	1.3	0.5~6.5	13	5.1
6	7	1.4	0.5~6.5	13	3.8
7	8	1.5	0.5~6.5	13	2.5
8	9	1.6	0.5~6.5	13	2.7
9	10	2.0	0.5~6.5	13	1.2
10	11	3.5	0.5~6.5	13	3.9
11	12	5.0	0.5~6.5	13	3.2
	All the data points	1~5	0.5~6.5	169	4.6

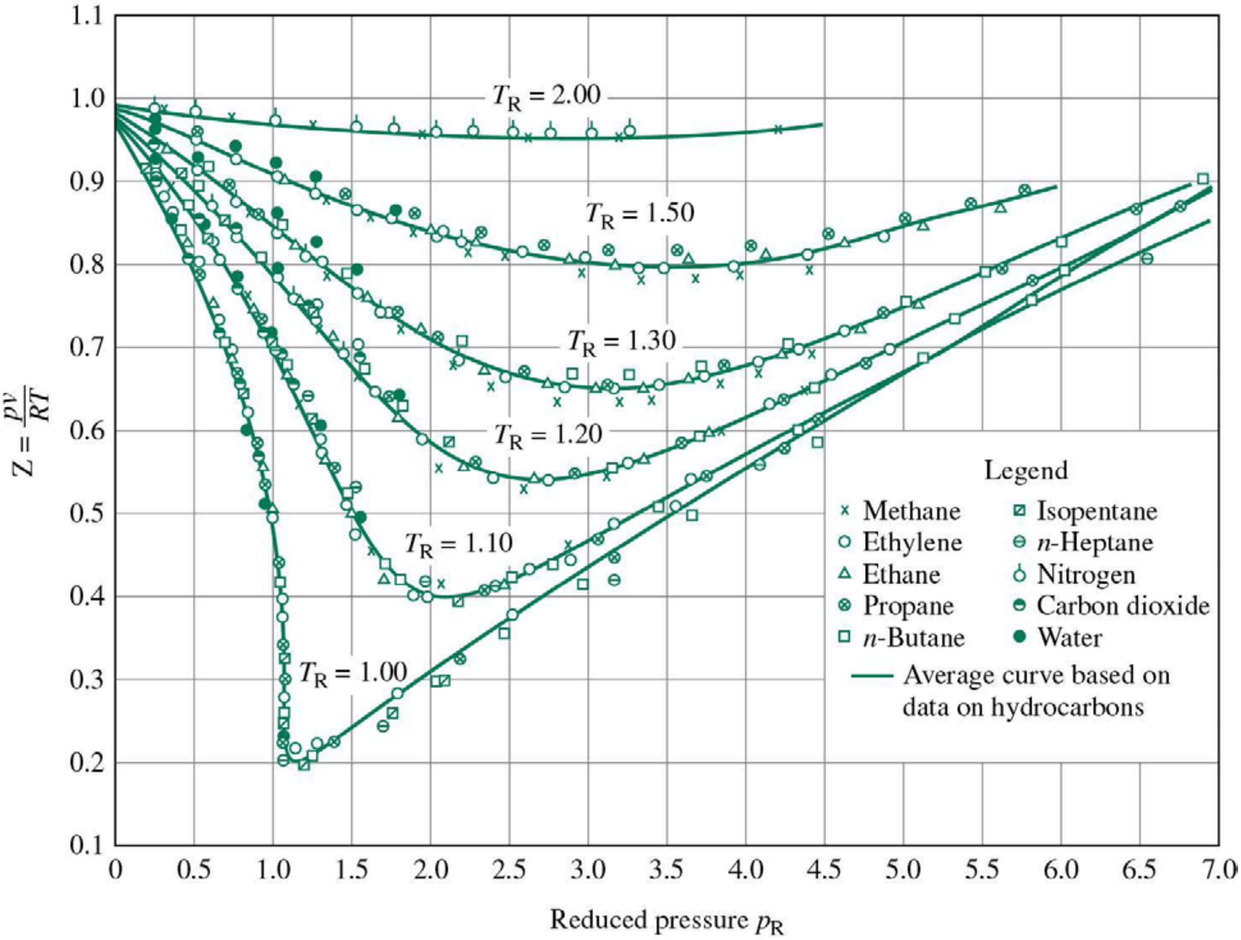

Fig.1 : Generalized compressibility factor chart (Cengel, 1988)

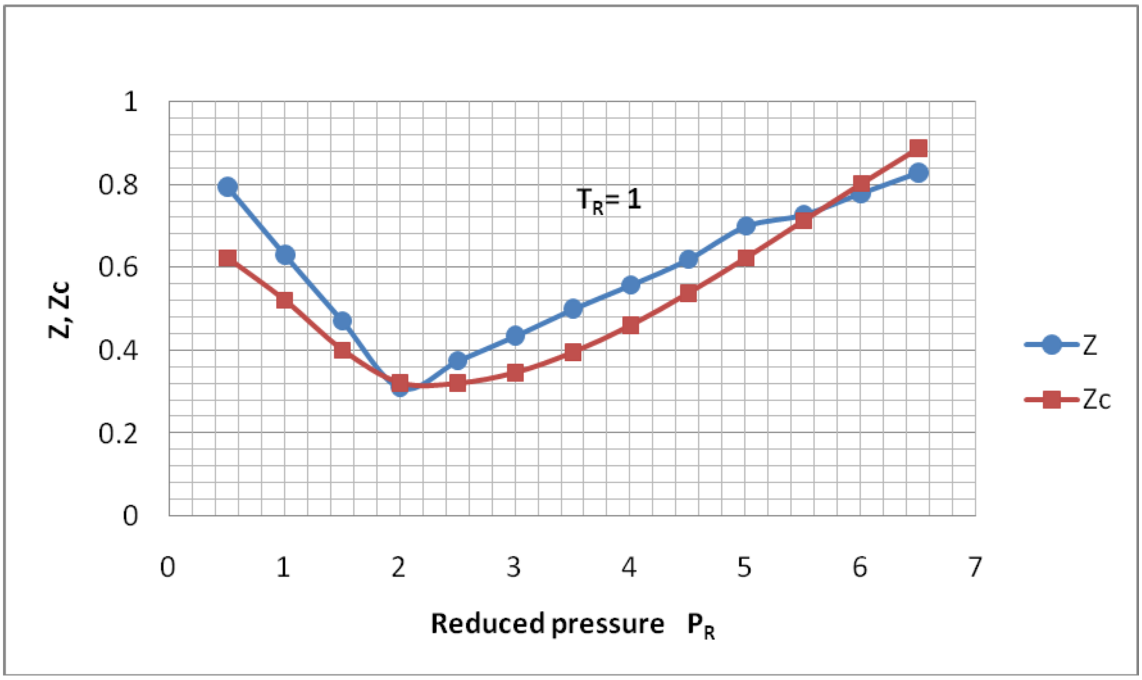

Fig.2 : Comparisons of observed Z and calculated Z_c at T_r=1

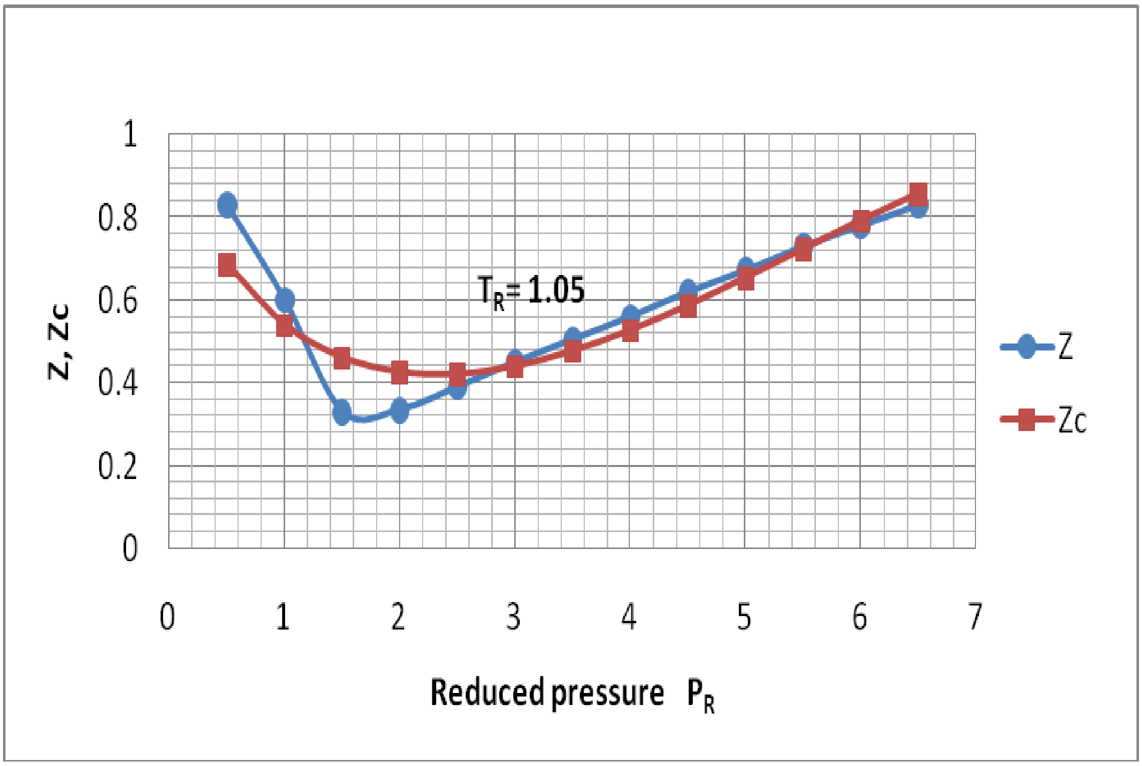

Fig.3 : Comparisons of observed Z and calculated Zc at Tr=1.05

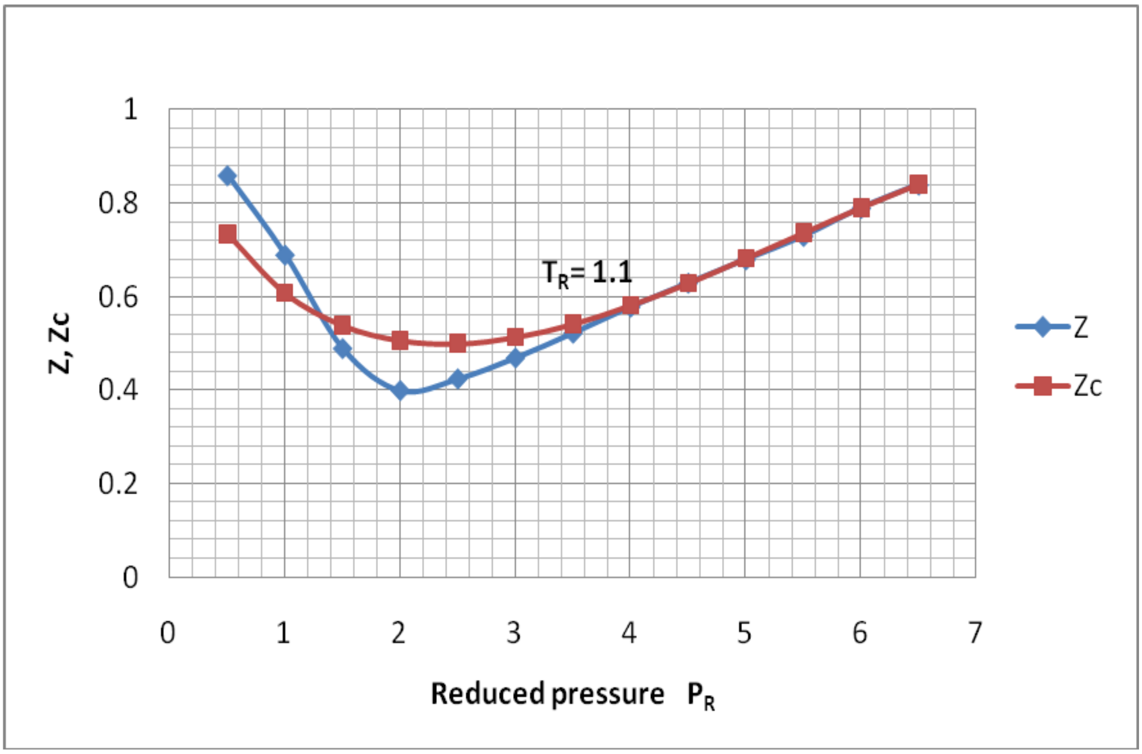

Fig.4 : Comparisons of observed Z and calculated Z_c at T_r=1.1

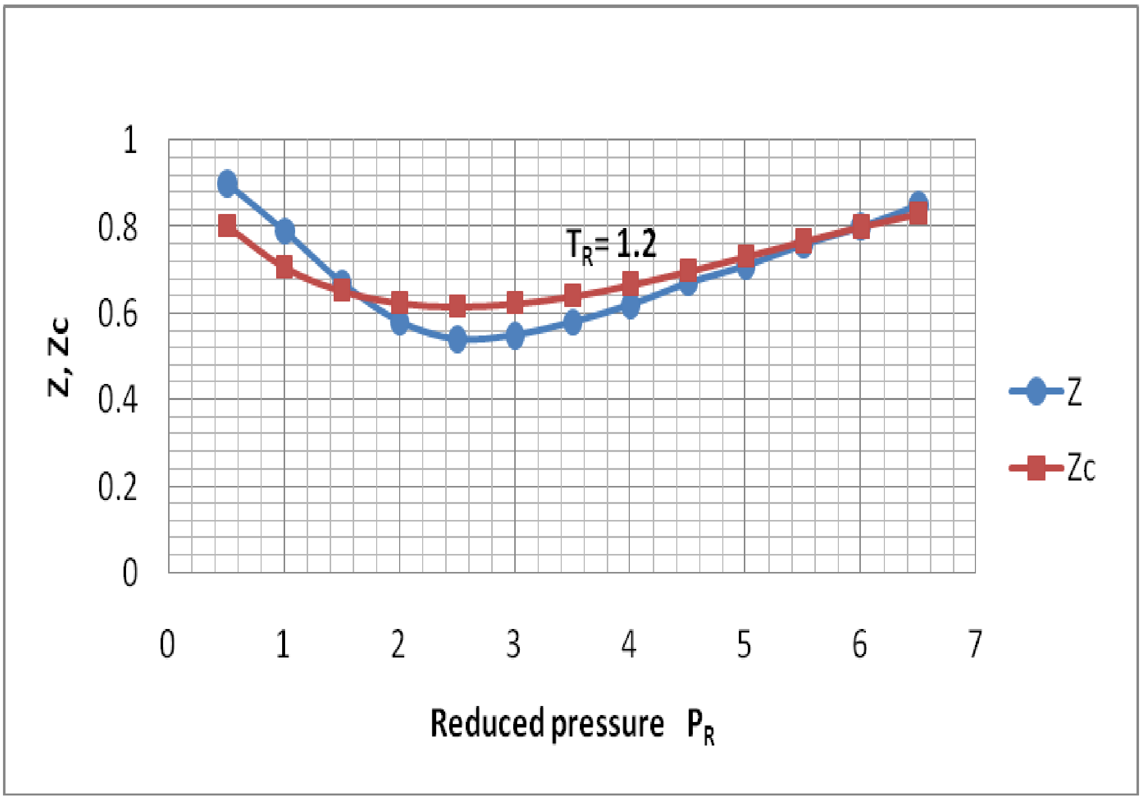

Fig.5 : Comparisons of observed Z and calculated Z_c at T_r=1.2

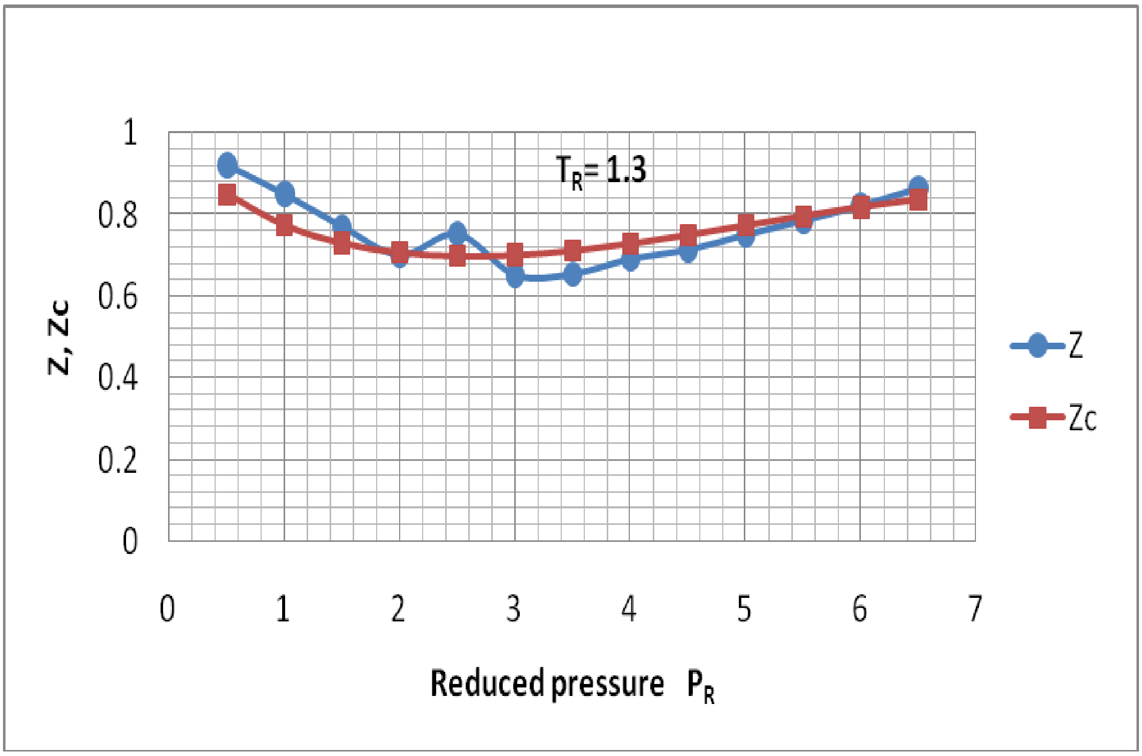

Fig.6 : Comparisons of observed Z and calculated Z_c at T_r=1.3

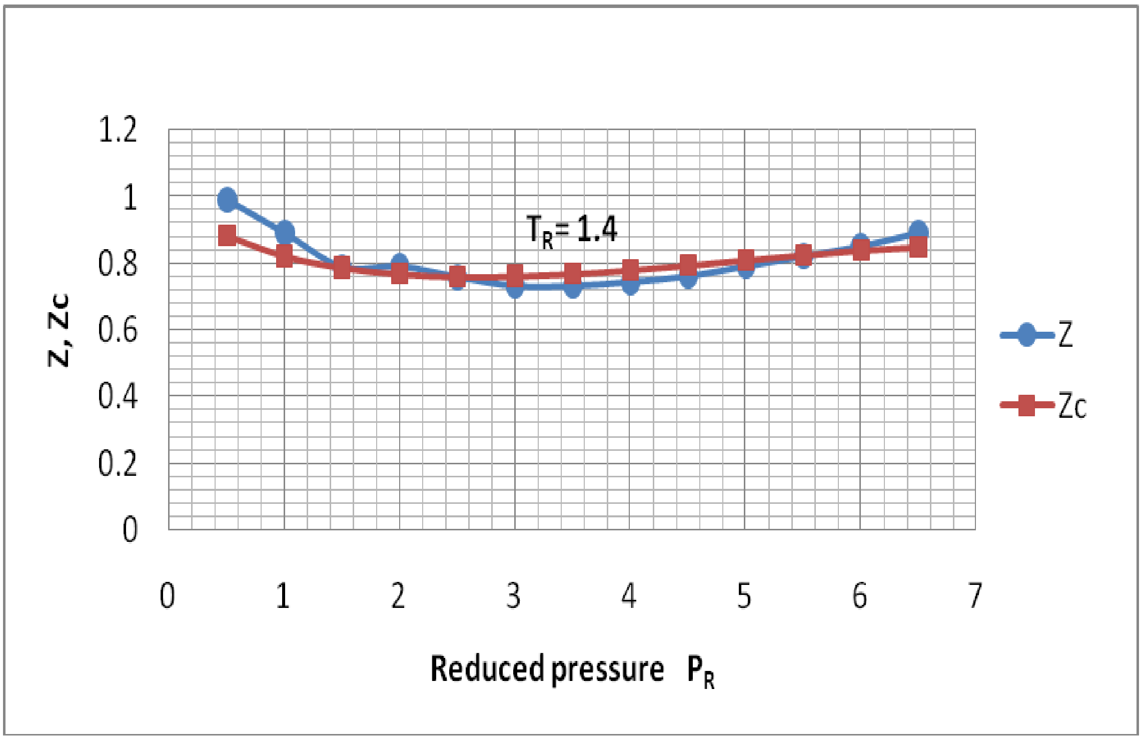

Fig.7 : Comparisons of observed Z and calculated Z_c at T_r=1.4

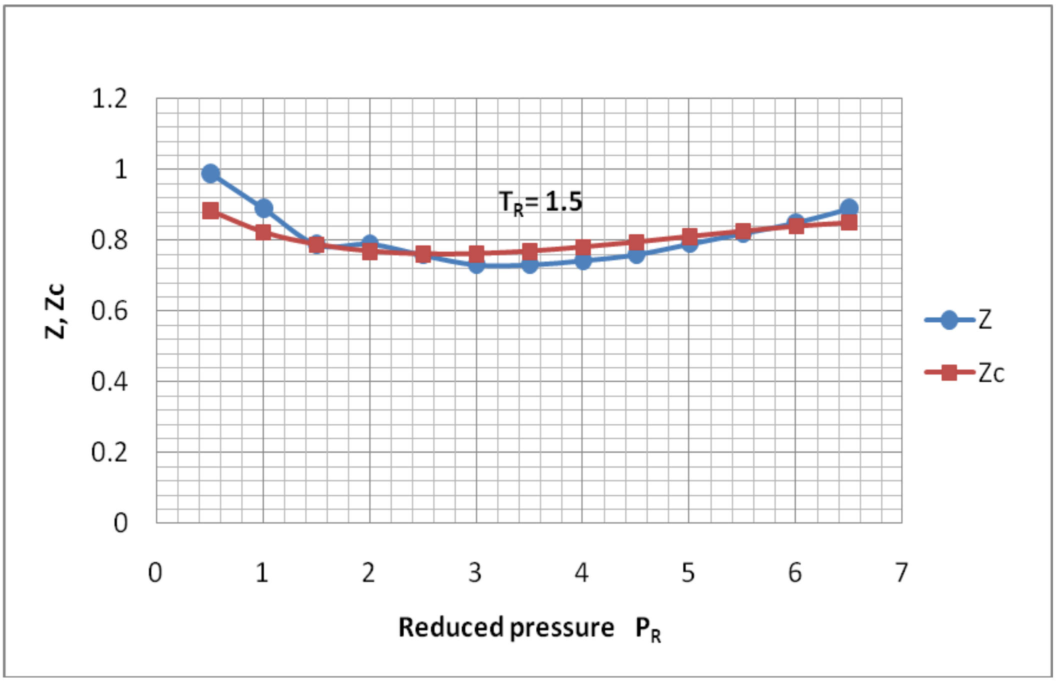

Fig.8 : Comparisons of observed Z and calculated Z_c at T_r=1.5

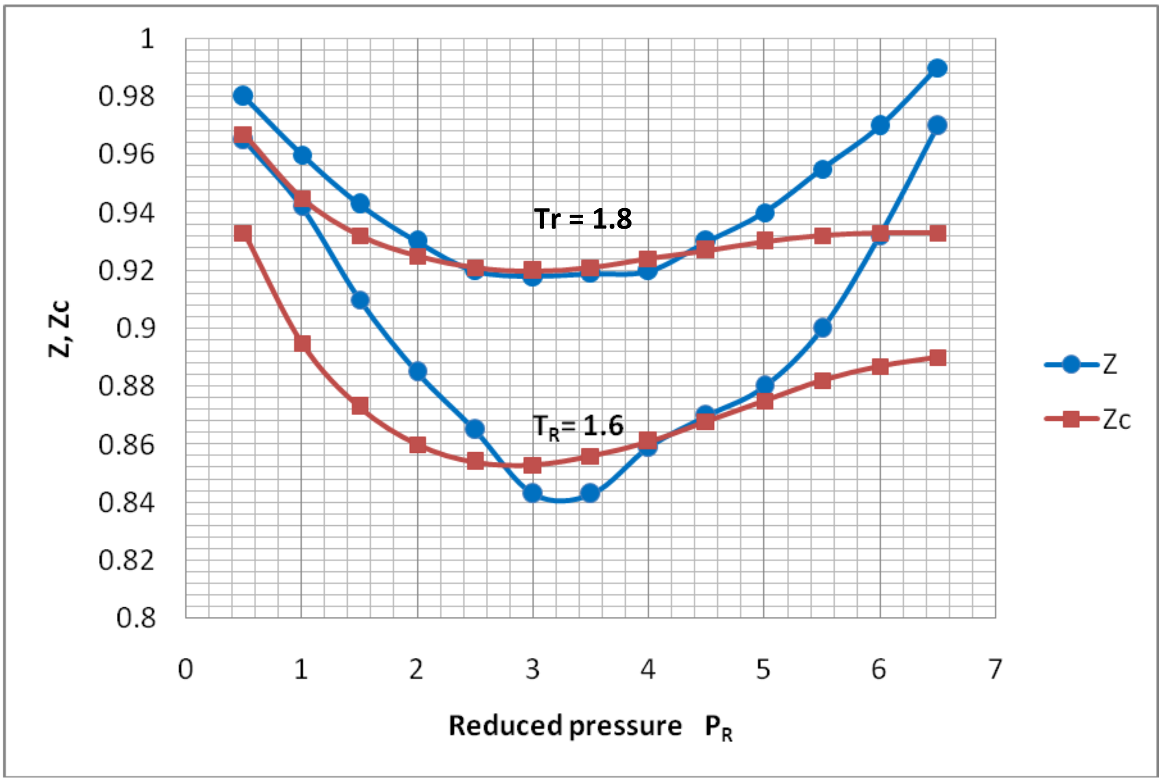

Fig.9 : Comparisons of observed Z and calculated Z_c at T_r=1.6 and 1.8

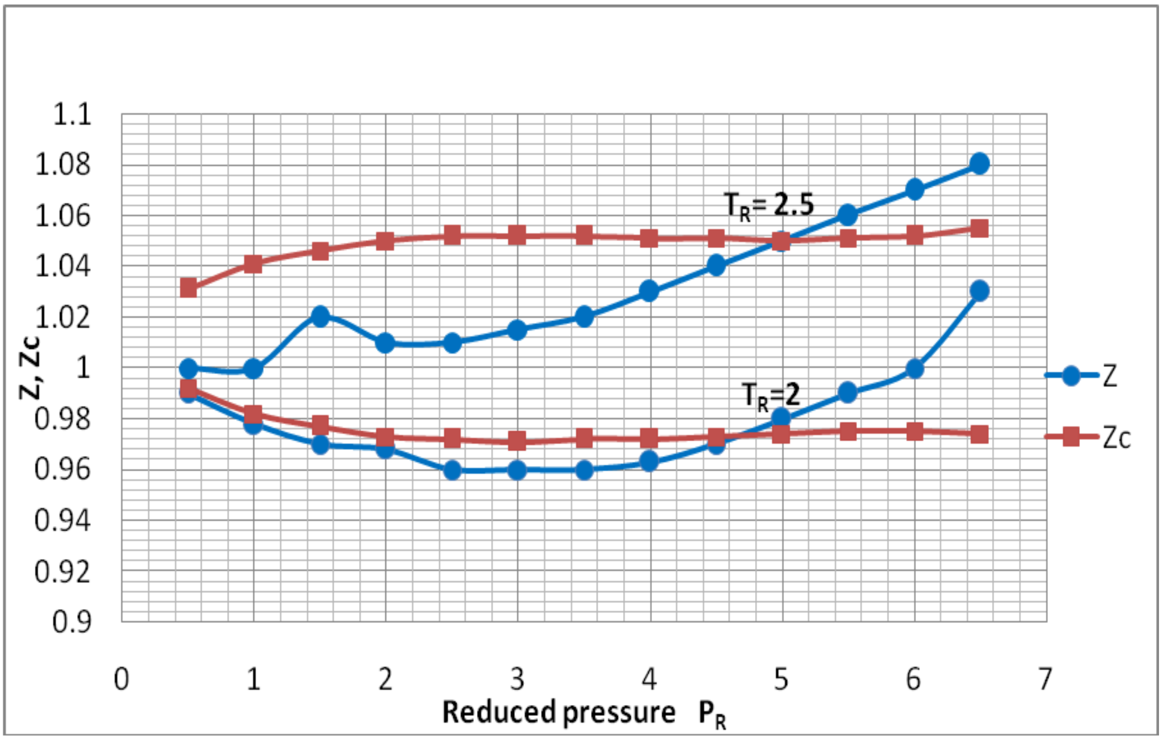

Fig.10 : Comparisons of observed Z and calculated Z_c at T_r=2 and 2.5

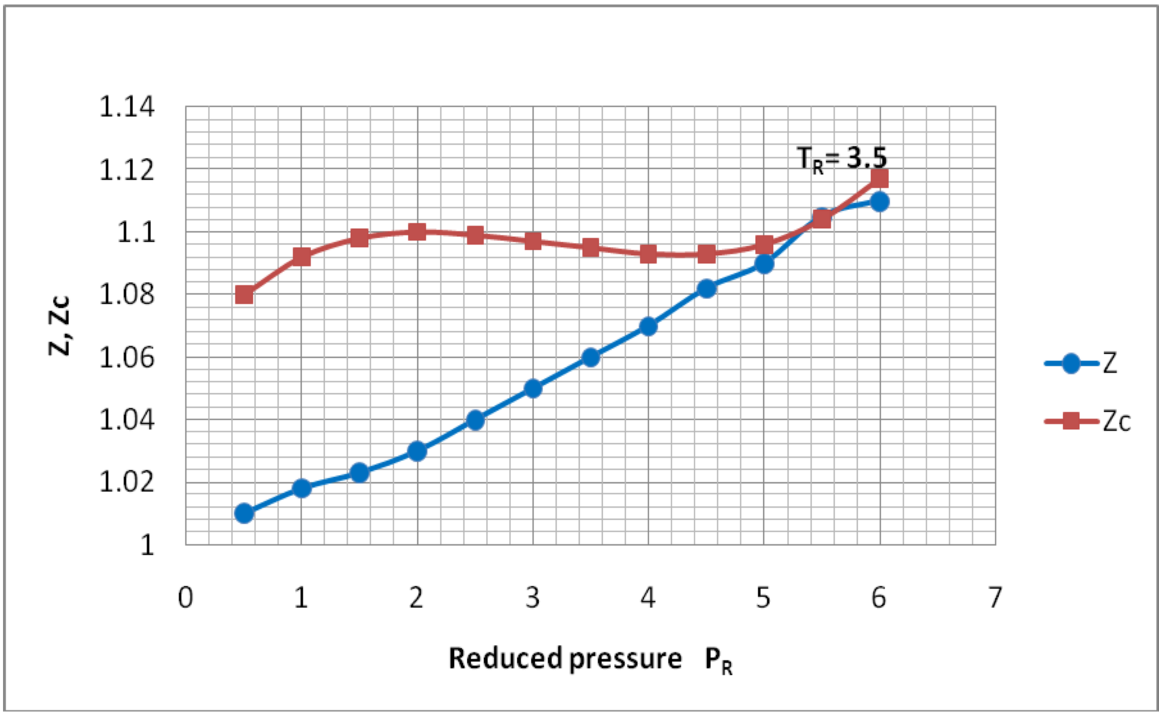

Fig.11 : Comparisons of observed Z and calculated Z_c at T_r=3.5

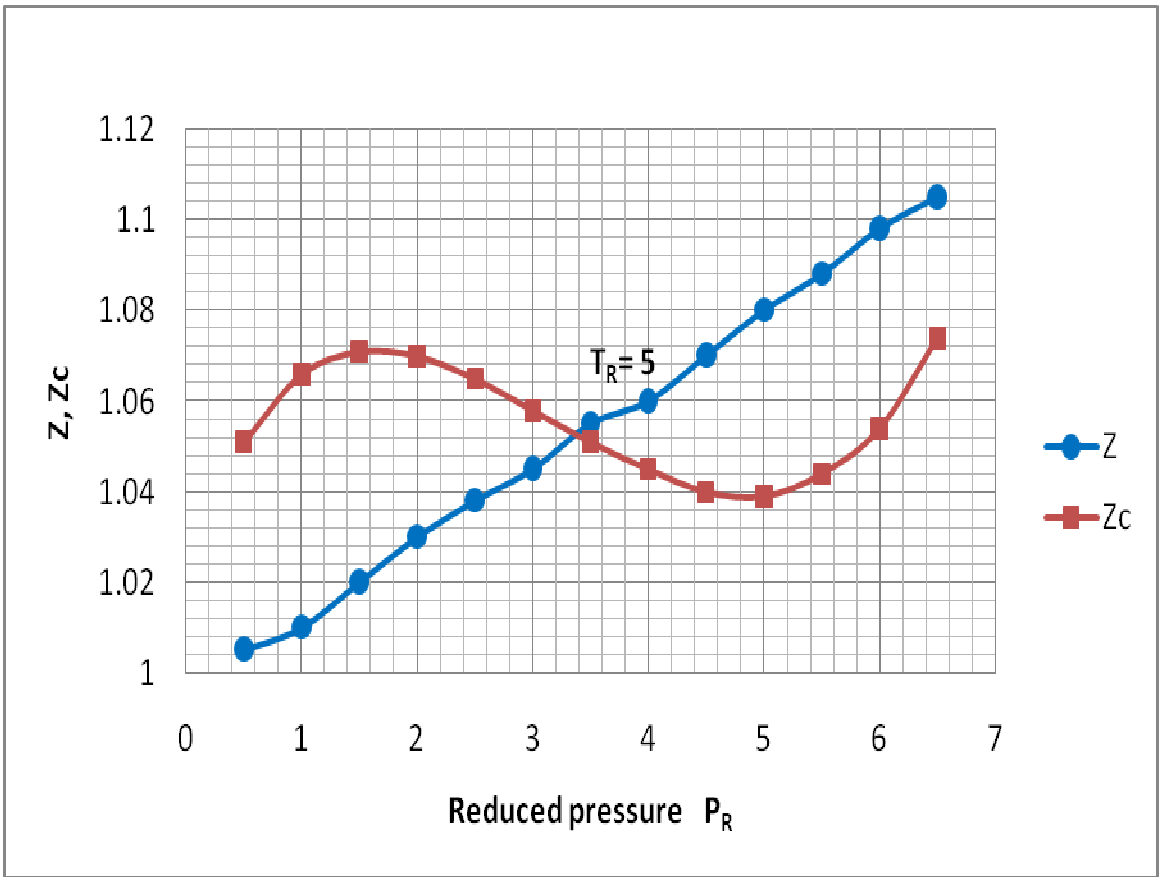

Fig.12 : Comparisons of observed Z and calculated Z_c at T_r=5

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