

EFFECT THE RADIAL STRAIN ON THE PERFORMANCE EVALUATION OF AN ELECTRO OPTIC MODULATOR

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ABSTRACT

This research presents a mathematical model that established for evaluating the effect of the radial strain on the performance of an electrooptic modulator using MATLAB simulations. It can be used the variable values for the forces applied with in specific area for the crystal's arm for an electro optic modulator and this is under the condition to the value of visibility factor can be improved and using this technique it will evaluate the effect of radial strain. So, this model leads to enhancement the performance an electro optic modulator using MATLAB simulations. Finally, visibility factor of an electro optic modulator versus radial strain can be evaluated for different values of force and which considers important factor on the performance of an electro optic modulator for the modern communication system.

Key words :-Electro-Optic Modulator (EOM), radial strain sensor.

تأثير الاجهاد القطري على تقييم اداء المضمن الاليكترو بصري

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الخلاصة :-

هذا البحث يقدم موديل رياضي أنشأ لتقييم تأثير الإجهاد القطري على أداء المضمن الألكترو البصري باستعمال برنامج المحاكاة ماتلاب . و باستعمال قيم مختلفة للقوى مسلطة ضمن مساحة معينة للذراع البلوري للمضمن الألكترو البصري وهذا يمكن ان يحسن قيم معامل مدى الرؤية وباستعمال هذه التقنيه سيتم تقييم تأثير الإجهاد القطري.لذلك هذا الموديل يؤدي الى تحسين كفاءة المضمن الألكترو البصري باستعمال محاكات ماتلاب. وأخيرا باستعمال محاكاة ماتلاب نستطيع أن نبين بعلاقة توضح تأثير الإجهاد القطري لقيم مختلفة للقوى على كفاءة معامل مدى الرؤية للمضمن الألكترو البصري التي تعتبر عامل مهم في أداء المضمن الألكترو البصري في نظم الاتصالات الحديثة.

LIST OF SYMBOLS

V_B	Bias voltage
V_{Ref}	Reference voltage
P_{out}	Output power
P_{in}	Input power
$V\pi$	Bias voltage
E_{out}	Output electric field
E_{in}	Input electric field
λ	Free space wavelength of the laser
A_O	Area of crystal arm

1. INTRODUCTION :-

Electro optic effect is an effect when an electric field is applied to a crystal, the ionic constituents move to new locations determined by the field strength and the restoring force. A field applied to an anisotropic electro optic material modifies its refractive indices and thereby its effect on polarized light as shown in **Fig. (1)**. Anisotropy in the optical properties therefore can be due to the unequal restoring force along three mutually perpendicular axes in the crystal. These changes can be described in terms of the modification of the index ellipsoid. The linear electrooptic effect or pockel effect is the change in the indices of the ordinary and extraordinary rays is proportional to an applied electric field. This effect exists only in crystals that do not possess inversion symmetry [E.Foutekouv 2005, Jose Antonio Ibarra Fuste2013]. There is a very large variety of useful electro optic materials, covering a wide range values for the electro optic tensors, refractive indices, response time and etc. However, most commercial devices use crystals for example, Potassium Dihydrogen Phosphate (KH₂PO₄) also known as (KDP), Barium Tantalate (BaTiO₃), Lithium Tantalate (LiTaO₃), Lithium Niobate (LiNbO₃) and some liquid crystals [L.Thylen 1998, M. Suzuki 2011]. In this research electro optic effect of a particular crystal, Lithium Niobate will be studied.

The phase shift induced by the electro-optic modulator is analyzed by an optical interferometer. The interferometer is an optical device which utilizes the effect of two beam interference; hence it is a sensitive tool to measure any phase shift between the two beams. By monitoring the intensity of the interference, the half wave voltage can be determined since the phase shift from 0 to π , the constructive interference undergoes to destructive interference or vice versa. The phase change, in the case, is due to the voltage applied to each arm [I. P. Kaminow 1966, J. Cartledge 2009]. The scheme is presented in **Fig. (2a and 2 b)**. In electro-optic modulator (EOM), the incoming light is split into two waveguide under the influence of conducting electrodes. The electro – optic effect induces a change in the refractive index of each interferometer arm and phase – modulates the light propagating

into that arm according to the electric voltage applied to each electrode . Mathematically [G. Pecere 2011]:

$$E_{out} = E_{in} \left((1-\alpha)e^{-j\frac{\pi V_1}{V_{\pi 1}}} + \alpha e^{-j\frac{\pi V_2}{V_{\pi 2}}} \right) \quad (1)$$

Where again, (α) is the interferometry splitting ratio, 1/2 in the ideal case. The photo detected power (P_d) will be, with the substitution $\theta_1 = \frac{\pi V_1}{V_{\pi 1}}$ and $\theta_2 = \frac{\pi V_2}{V_{\pi 2}}$

$$\begin{aligned} P_d &= \frac{1}{2} |E_{out}|^2 \\ &= \frac{1}{2} E_{in}^2 \{ [(1-\alpha)\cos\theta_1 + \alpha\cos\theta_2]^2 \\ &\quad + [-(1-\alpha)\sin\theta_1 - \alpha\sin\theta_2]^2 \} \end{aligned} \quad (2)$$

This device has been reported in the literature as an electro-optic modulator (EOM) for high digital bit-rate and RF transmission over optical fiber communication systems. The main operation of electro-optic modulators (EOM) is based on the linear electro-optical effect (pockels effect) where the refractive index of a medium is modified on proportion to the strength of the applied electric field. The electro-optic modulator divided into two types as follows [G. Pecere 2011, Jose Antonio Ibarra Fuste 2013].

a. Symmetric Configuration

In this section the modulating signal and the bias voltage are applied to only one of the interferometry branches, either to the same or to different branches. In this type using the above condition in Eq. (2), the transfer function (3) is as shown in **Fig. (3)**.

$$F = \frac{P_{out}}{P_{in}} \quad (3)$$

b. Push-Pull Configuration

This configuration is obtained by applying data and bias voltage in one arm (V_1) and inverted bias voltage in the other arm (V_2), i.e.

$$V_1 = -V_2 \quad (4)$$

This increases the relative phase shift in one arm and decreases it in the other arm . Since phase changes are equal in magnitude but opposite in sign in each arm a chirp free intensity modulation is obtained, as shown in **Fig.(4)**. Following from Eq.(2), using $\alpha = 1$ and the condition in the Eq.(4), the transfer function is shown in **Fig.(4)**.

2. CONDITION FOR HIGH VISIBILITY FACTOR INTERFERENCE

Superposition principle is a basic property of all kinds of waves and is a key element to understand the condition for interference. When two waves are superimposed, the resulting amplitude distribution is the addition of the instantaneous amplitude of these two waves.

Consider an incident light beam with an electric field (\mathbf{E}) which oscillates at a frequency (ω_t) and has travelled the path(\mathbf{r}) [N. Tien 2007, S.K. Saha, 2011].

$$\mathbf{E} = E_0 e^{i(\mathbf{k}_r - \omega_t)} \quad (5)$$

Where (E_0) is the amplitude and (\mathbf{k}) is the wave number equals to $2\pi/\lambda$. If the two beams from path A and B interfere with each other, the intensity is

$$I \propto |\mathbf{E}_A + \mathbf{E}_B|^2 \quad (6)$$

Constructive interference corresponds to the cosine term equals to (1) whereas destructive interference is equals to (-1). Hence,

$$I_{max} = I_{OA} + I_{OB} + 2\sqrt{I_{OA}I_{OB}} \text{ For constructive interference}$$

$$I_{min} = I_{OA} + I_{OB} - 2\sqrt{I_{OA}I_{OB}} \text{ For destructive interference}$$

The visibility factor f is defined as follows [1]:

$$f = \frac{2\sqrt{I_{OA}I_{OB}}}{I_{OA} + I_{OB}} \quad (7)$$

A full visibility factor f of (1) happens when $I_{min} = 0$. In general, the following conditions must be met for two waves to be interfered:

- (a) Same polarization. Two waves with orthogonal polarization cannot interfere with each other.
- (b) Same frequency. The two waves must oscillate at the same frequency to cancel the frequency dependent term in Eq. (6) as derived.
- (c) Constant phase relationship. The phase difference must be constant at any given point in the superposition region. Otherwise, no stable interference pattern can be observed.

3. MATHEMATICAL MODULE

If an electric field is applied during a certain length to one arm, a phase difference $\Delta\phi$ between the light propagation through the two waveguides will be introduced see **Figs.(5a and 5b)**. This results in an intensity modulation at the electro-optic modulator output, which can be written as equation (8) [J. S. Wilkinson 1998]:

$$\Delta I = \frac{I_1}{I_2} = \frac{1}{2} [(1 - E_1)^2 + E_1^2 + 2E_1(1 - E_1) \cos \Delta\phi] \quad (8)$$

If a laser diode source is used, the visibility factor (f) is:

$$\therefore f = \frac{E_1 E_2}{2E_1^2 + 2E_1 + 1} \quad (9)$$

$$E_2 = \frac{f(2E_1^2 + 2E_1 + 1)}{E_1} \quad (10)$$

Where: I_1 = The intensity of the interfered light wave, I_2 = The intensity of the initial light wave, I_2 = The intensity of the initial light wave, f = visibility factor, E_1 = The electric field of first arm for EOM (V), and E_2 = The electric field of second arm for EOM (V)

$$\begin{aligned} &= E_1^2 + (1 - E_1)(1 - E_1) + (2E_1 - 2E_1^2) \cos \Delta\phi \\ &= E_1^2 + E_1^2 - 2E_1 + 1 + (2E_1 - 2E_1^2) \cos \Delta\phi \\ &= 2E_1^2 - 2E_1 + 1 + (2E_1 - 2E_1^2) \cos \Delta\phi \\ \frac{2I_1 - I_2}{I_2} &= 2E_1[(E_1 - 1) + (1 - E_1) \cos \Delta\phi] \therefore E_2 = (1 - E_1) \\ \therefore \frac{2I_1 - I_2}{I_1} &= 2E_1[(E_1 - 1) + E_2 \cos \Delta\phi] \end{aligned} \quad (11)$$

Substituting Eq.(10), into Eq. (11) is result:

$$\begin{aligned} \frac{2I_1 - I_2}{I_2} &= 2E_1 \left[(E_1 - 1) + \frac{f(2E_1^2 + 2E_1 + 1)}{E_1} \cos \Delta\phi \right] \\ &= 2E_1 \left[\frac{E_1^2 - E_1 + f(2E_1^2 + 2E_1 + 1)}{E_1} \cos \Delta\phi \right] \\ &= \left[\frac{2E_1^3 - 2E_1^2 + 2E_1 f(2E_1^2 + 2E_1 + 1)}{E_1} \cos \Delta\phi \right] \\ I_2 [2E_1^3 - 2E_1^2 + 2E_1 f(2E_1^2 + 2E_1 + 1) \cos \Delta\phi] &= (2I_1 - I_2) E_1 \\ f(4E_1^3 I_2 + 4E_1^2 I_2 + 2E_1 I_2) \cos \Delta\phi &= (2I_1 - I_2) E_1 - 2E_1^3 I_2 + 2E_1^2 I_2 \\ f &= \frac{(2I_1 - I_2) E_1 - 2E_1^3 I_2 + 2E_1^2 I_2}{(4E_1^3 I_2 + 4E_1^2 I_2 + 2E_1 I_2) \cos \Delta\phi} = \frac{2I_1 E_1 - E_1 I_2 - 2E_1^3 I_2 + 2E_1^2 I_2}{(4E_1^3 I_2 + 4E_1^2 I_2 + 2E_1 I_2) \cos \Delta\phi} \end{aligned}$$

The mathematical expression of visibility factor (f) for electro-optic modulator (EOM) is shown in Eq.(12).

$$f = \frac{2E_1 I_2 - 2E_1^2 I_2 - I_2 + 2I_1}{(4E_1^2 I_2 + 4E_1 I_2 + 2I_2) \cos \Delta\phi} \quad (12)$$

Equation (12) explains the relationship between the visibility factor (f) and the phase difference $\Delta\phi$. The phase difference $\Delta\phi$ is related by equation (13), [D. D. Johnson 2010].

$$\Delta\phi\Delta T = \phi\Delta T \left[\left(\frac{1}{n} \right) \left(\frac{\partial n}{\partial T} \right) \Delta T + \epsilon_r \left[1/\mu - \left(\frac{n^2}{2} \right) [(\rho_z + \rho_r) + \frac{\rho_r}{\mu}] \right] \right] \quad (13)$$

Where: $\phi = \frac{2\pi nL}{\lambda} \text{ rad}$, $K = \frac{2\pi}{\lambda}$, K = wave number, λ =Free space wavelength of the laser in (nm) L = The length of the crystal arm in the modulator in (μm), n = Refractive index (unit less), ΔT = Temperature difference in (k), (unit less), ϵ_r = radial strain (unit less), and ρ_z and ρ_r = pocket coefficient. The final mathematical expression of phase difference with respect radial strain forelectro-optic modulator (EOM) is shown in Eq. (14).

$$\Delta\phi = \phi \left[\left(\frac{1}{n} \right) \left(\frac{\partial n}{\partial T} \right) \Delta T + \epsilon_r \left[1/\mu - \left(\frac{n^2}{2} \right) [(\rho_z + \rho_r) + \frac{\rho_r}{\mu}] \right] \right] \quad (14)$$

$$\epsilon_z = \frac{F}{A_o G} \quad (15)$$

$$A_o = \left(\frac{d}{2} \right)^2 \times \pi$$

$$\epsilon_r = \frac{\Delta d}{d} \quad (16)$$

Where: F = Force (Newton), G =Young's module (100GPa), d = the diameter of the crystal arm (62.5 μm), and Δd = the change in diameter of the arm. This change results from radial strain in (μm). Addition Eq. (14) ,into $\Delta\phi$ for the Eq. (12) is result:

$$f = \frac{2E_1 I_2 - 2E_1^2 I_2 - I_2 + 2I_1}{(4E_1^2 I_2 + 4E_1 I_2 + 2I_2) \cos \phi \left[\left(\frac{1}{n} \right) \left(\frac{\partial n}{\partial T} \right) \Delta T + \epsilon_r \left[1/\mu - \left(\frac{n^2}{2} \right) [(\rho_z + \rho_r) + \frac{\rho_r}{\mu}] \right] \right]} \quad (17)$$

The equation (17) is the fundamental equation for measurement the efficiency of visibility factor which considers the necessary factor in work the optical modulator in the optical communication systems .Also; this equation shows the relationship between the visibility factor (f) , and the radial strain.

4. SIMULTION AND RESULTS

If a force in the form tensile stress is applied on an arm of the EOM, the radial strain will be produce as shown in **Fig. (6)**, and **table (1)** . It can be noticed from the table below that the radial strain and the change of the diameter (Δd) have gradually increased from (-5.5×10^{-2}) and $(3.45 \mu\text{m})$ until it reaches its maximum value -1 and $62.5 \mu\text{m}$ respectively.

The mark of minus sign for the radial strain due to the tensile stress which it applies on an arm of crystal for EOM. Also, when (Δd) reaches its maximum value $62.5 \mu\text{m}$, then a branch of EOM will be an inactive. Meaning that, the interference signal of EOM is removed and this it results from a damaged the arm of EOM. From all of this, it can be suggested a mathematical model and an equation (16), as shown in a previous section, to analysis these results and the damaged can be remove by using this technique, as shown in these **Figs (7, 8, and 9)**.

In the **Fig. (7)**, when the effect of radial strain is below 0.2 (i.e. The effect of radial strain neglects), the EOM operates with a high of efficiency and this leads to constructive interference .Otherwise, if a radial strain increases until reaching to its value between 0.5 and 0.6, and above of these values (i.e., 0.8, 1, etc) the visibility factor (f) of the EOM is equal 0%, and this means an interference signal removes and this results from destructive interference.

Also, from **Fig. (8)**, the performance of EOM can be improved by reducing the change of diameter ($\Delta d < 10 \mu\text{m}$), which it results from effect of a radial strain. Therefore, the change of diameter (Δd) is limited by decreasing the radial strain. Thus, the **Fig.(9)**, gives details of view on an effecting the load of force on the factor which measures a performance of EOM, and this means the radial strain is limited by decreasing the load of force ($F < 200 \text{ N}$) . Finally, the performance of EOM can be enhanced by controlling on the effect a radial strain and this achieves by using the mathematical model.

5. CONCLUSIONS

In this research, we introduced a mathematical model is an effective way to improve the electro-optic modulator sensitivity when the visibility factor (f) is relatively high and it can improve EOM performance by reducing an effect of the radial strain. We discussed the impact of increasing radial strain effect on the performance of EOM therefore it can be concluded when the radial strain increases depending reaching to its value (-1), this leads to decreasing a performance of EOM and thus it results a damage the arm of EOM. We concluded the performance of EOM can be improved by decreasing the change of the diameter (Δd), (i.e. $\Delta d < 10 \mu\text{m}$), which results from increasing the radial strain therefore it is limited by controlling on the radial strain. Finally, this presented a mathematical model it has ability to suppress the effect of radial strain by evaluating its effects (i.e. $F < 200 \text{ N}$).

Table 1

Force (N)	tensile stress ($\sigma_{ax}=F/A_0$)	axial strain ($\epsilon_z = \sigma_{ax} / G$)	radial strain(ϵ_r)	Δd (μm)
100	$3.25 * 10^4$	0.325	$- 5.52 * 10^{-2}$	3.45
200	$6.5189 * 10^4$	0.65189	$- 1.108 * 10^{-1}$	6.925
300	$9.778 * 10^4$	0.9778	$- 1.662 * 10^{-1}$	10.38
400	$1.3037 * 10^5$	1.3037	$- 2.216 * 10^{-1}$	13.85
500	$1.6297 * 10^5$	1.6297	$- 2.770 * 10^{-1}$	17.3
600	$1.9556 * 10^5$	1.9556	$-3.323 * 10^{-1}$	20.7
700	$2.2816 * 10^5$	2.2816	$- 3.877 * 10^{-1}$	24.23
800	$2.6075 * 10^5$	2.6075	$- 4.432 * 10^{-1}$	27.7
900	$2.933 * 10^5$	2.933	$- 4.981 * 10^{-1}$	31.13
1000	$3.259 * 10^5$	3.259	$- 5.540 * 10^{-1}$	34.6
1200	$3.911 * 10^5$	3.911	$- 6.648 * 10^{-1}$	41.5
1400	$4.563 * 10^5$	4.563	$- 7.757 * 10^{-1}$	48.48
1600	$5.215 * 10^5$	5.215	$- 8.865 * 10^{-1}$	55.4
1850	$6.030 * 10^5$	6.030	- 1	62.5

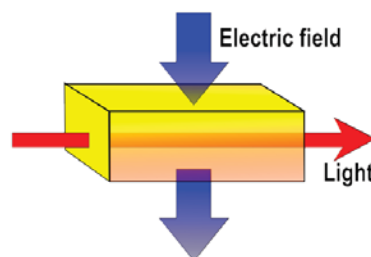
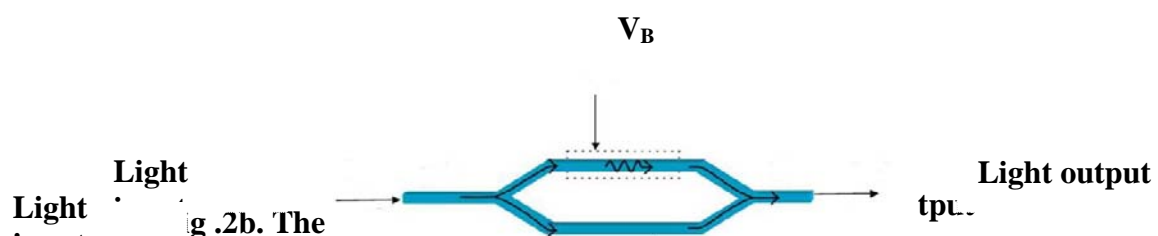


Fig.1. an electric field applied to an electro opticmaterial modifies its refractive indices.



)(Mpa)

 V_{RF}

Fig .2a. Electro-optic modulator with two voltages on the deferent arms

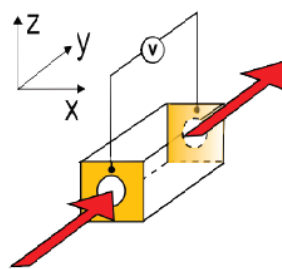


Fig .2b. The electrodes are placed in both ends of crystal

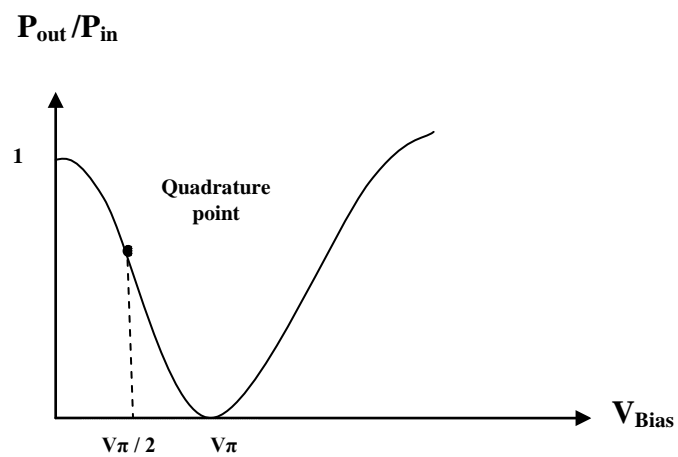
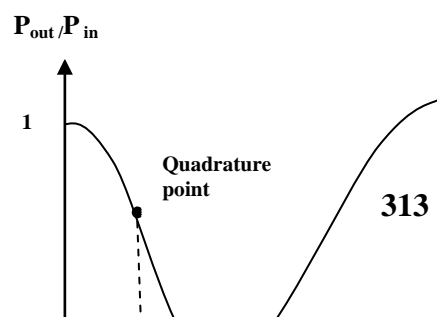


Fig. 3. Transfer function of EOM in the symmetric Configuration [G. Pecere 2011]



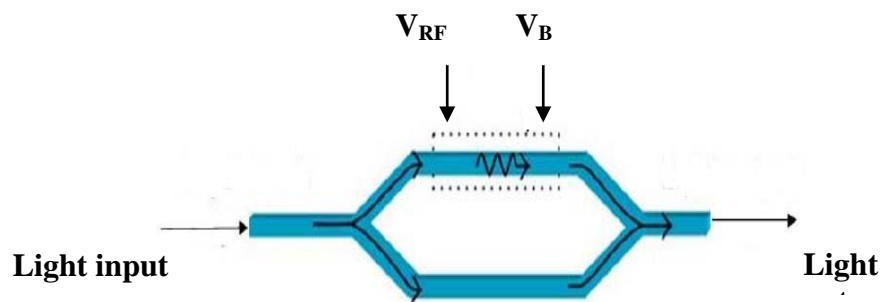


Fig .5a. EOM with two voltages on the one arm

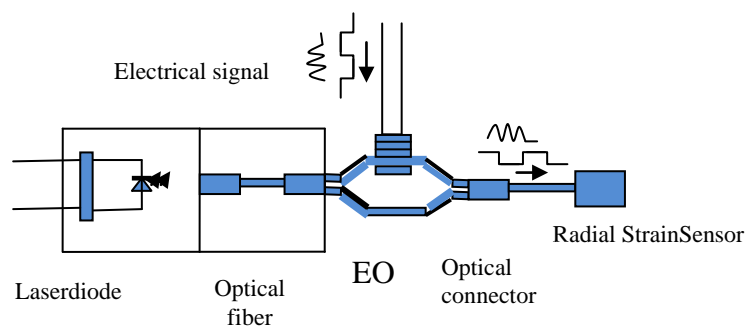


Fig.5b.Illustration visibility factor of EOM with radial strain sensor



Fig.6.Illustration applying force on an arm of the EOM

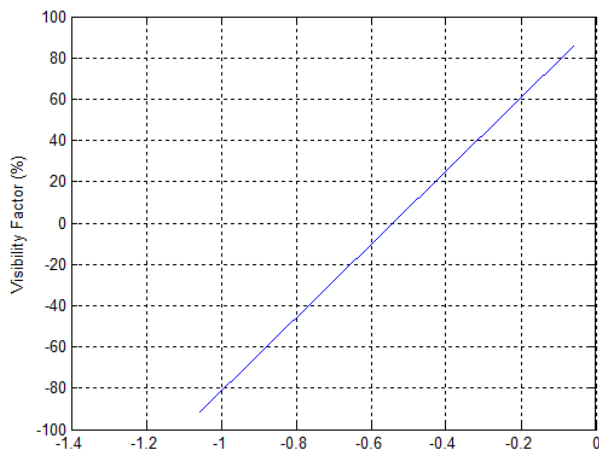


Fig. 7. Illustration the effect of radial strain on the performance of EOM

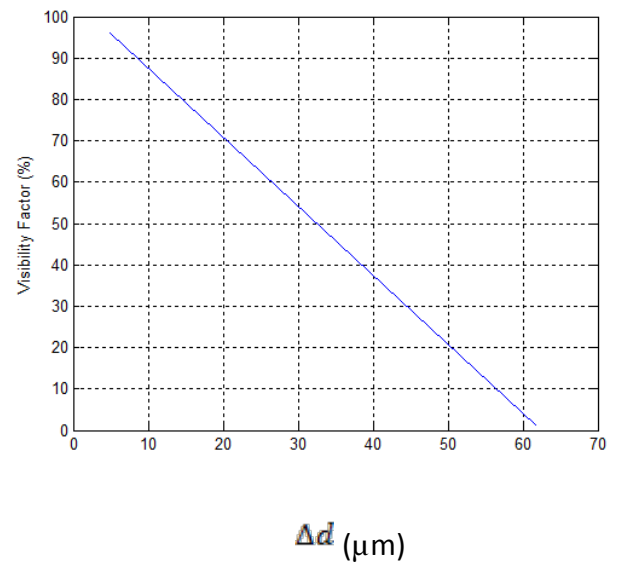


Fig. (8). Illustration the effect of change of diameter (Δd) on the performance of EOM

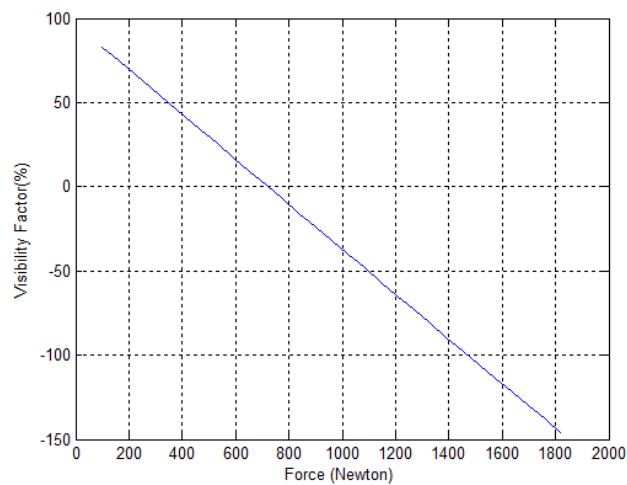


Fig. 9. Illustration the effect of force on the performance of EOM

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