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NON-LINEAR FREE VIBRATION ANALYSIS OF COMPOSITE LAMINATED PLATES

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ABSTRACT :-

Nonlinear free vibration analysis of composite laminated composite thin plate is studied in present work. Numerical solution using Ansys finite element program (Mechanical APDL 15.0) is used to investigate the effect of strain nonlinearity on the natural frequency of these plates. Design parameter is changed to investigate their effect on natural frequency of the plate such as lamination, aspect ratio, lamination angle and number of plies. An experimental work is programed to manufacture and test the mechanical properties and natural frequency of the plate for two boundary conditions (CCCC) and (CFCF) to compare the results with those obtained from numerical analysis. Numerical results are close to those obtained experimentally; maximum discrepancy is 4.04% for symmetric (CFCF) boundary condition and 5.08 % for an-symmetric (CFCF) boundary condition .

KEYWORDS: Composite Material, Nonlinear, Natural Frequency, Ansys, Free Vibration

التحليل اللاخطي للاهتزاز الحر لصفائح طبقية مركبة

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الخلاصه :-

تم دراسة التحليل اللاخطي للاهتزاز الحر للصفائح الطبقية المركبة الرقيقة ويتم الحل العددي باستخدام برنامج Ansys للعناصر المحددة (Mechanical APDL 15.0) لدراسة تاثير الانفعال اللاخطي على التردد الطبيعي للالواح. تم تغير خصائص التصميم للعينات لغرض دراسة تاثيرها على التردد الطبيعي للالواح مثلا الطبقات ، نسبة الطول الى العرض ، زاوية الطبقات حيث تم تصنيع هذه النماذج عمليا لاجراء الاختبارات للحصول على الخواص الميكانيكية والتردد الطبيعي للعينات تحت شروط تثبيت معينة (مثبتة من كل جهات) و(مثبتة من جهتين) لمقارنة النتائج العملية مع نتائج الحل العددي . وقد وجد ان النتائج النظرية قريبة جدآ الى النتائج العملية، اكبر فرق هو 4.04% للعينات المتناظرة و 5.08% للعينات الغير متناظرة المثبتة من جهتين .

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NOMENCLATURE :-

Symbol	Description
ε_j	strain components
γ_{xy}	Shear strain in XY-axis
u , v ,w	The displacements field of the higher order
$\{\varepsilon_L\}$	Linear strain vector
$\{\varepsilon_{NL}\}$	Non-Linear strain vector
$\{\delta_n\}$	Displacement vector
[K]	element stiffness matrix
[M]	Inertia matrix
F	Force
X,Y,Z	Cartesian coordinate system

INTRODUCTION

A practical interest of nonlinear elasticity problems has prompted several theoretical and computational investigations of finite element theories which classifies the nonlinear problems as physically (or materially) nonlinear and geometrically non-linear .In the physically nonlinear problems the nonlinearity enters through the constitutive (stress-strain) relation, nonlinearity is ascribed to large-deflection problems in which the deformed configuration must be used to write the equilibrium equations, and to problems related to structural stability, This nonlinearity is introduced into the theory of elasticity through the equilibrium equations and by the inclusion in the strain – displacement relation. The study of finite element methods for geometrically nonlinear problems is relatively recent, Akkaram Srikanth and Nicholas (2001), S.Badrinarayanan and N.Zabaras (1993).

A.R.Setoodeh, G.Karami, 2004 employed a three-dimensional elasticity based layer-wise finite element method (FEM) to study the static, free vibration and buckling responses of general laminated thick composite plates. Y.J. Yana, L. Chengb, Z.Y. Wua, L.H. Yamb, 2007 presented a general summary and review of state-of-the-art and development of vibration-based structural damage detection. M.K. Singha, Rupesh Daripa, 2007 Here, the large amplitude free flexural vibration behavior of symmetrically laminated composite skew plates is investigated using the finite element method. The geometric non-linearity based on Von Kármán's assumptions is introduced. The variation of nonlinear frequency ratios with amplitudes is brought out considering different parameters such as skew angle, fiber orientation and boundary condition. Metin AYDOGDU, Taner TIMARCI, 2007 developed Numerical results for vibration frequencies of ant symmetric angle-ply laminated thin square composite plates having different boundary conditions. The effects of various parameters such as Fiber orientation, number of layers, and boundary conditions upon the natural frequencies are studied. N.V. Swamy Naidu, P.K. Sinha, 2007 The nonlinear free vibration behavior of laminated composite shells subjected to hydrothermal environments is investigated using the finite element method. Nabil Hassan Hadi, Kayser Aziz Ameen, 2011 have been developed a dynamic analysis to investigate and characterize embedded delamination on the dynamic response of composite laminated structures. Adnan N. Jameel, Salam Ahmed Abed, 2012 presented an application of a Higher Order Shear Deformation Theory (HOST 12) to problem of free vibration of simply supported symmetric and ant symmetric angle-ply composite laminated plates. Gerard C. Pardoen, 2014 presented the results of a study on the effect of prescribed delamination on the natural frequencies of laminated beam specimens. Experimental modal analysis was used to measure the effect of delamination length on the first four frequencies of the simply supported test specimens.

In this paper the dynamic (free vibration) behavior of general composite plates without damage and with delamination using two boundary conditions either fixed –fixed and fixed-free. Manufacturing the laminated and delaminated composite plate with orientation symmetry (0/90/90/0) and asymmetry (0/90/0/90).Measuring the natural frequency experimentally by building a system for this purpose using ANSYS software to achieve by analytical solution and experimental work and to illustrate the mode shapes of the composite structure in case of laminated and delaminated.

EXPERIMENTAL WORK

Test specimens design and preparation

The method that is used in the present work for manufacturing the laminated composite plates is hand lay-up. To manufacture the composite laminated plated, square flat panels were fabricated from this material hand lay-up of unidirectional (FRPC), using a (30 cm * 30 cm) wood open mold with two X-ray photo sheets covered with wax matrix to avoid abrasive and insure flattening. The x-ray photo-sheet were placed on the bottom of wood mold and pre- measured unsaturated resin (polyester) and hardeners are then thoroughly mixed together, and for ensuring complete air removal and wet out, it should cover the base surface completely especially at the end edges .

The catalyzed resin was applied to the fiber layer with using brushes and rollers the fiber layer would be saturated by resin. Small blade was used to take the bubble out of the fibers. The X-ray photo sheet will cover the composite and with rolling over layer to insure complete air removal. The pressure applied through the formation of the specimen is (two steel block) to get rid of the excess resin and remove air bubble. The panels were cured in furnace with temperature (70 C^0) for (3 hour) period of time. Gap of 3x3 cm is constructed in the middle of the composite layer plate, using aluminum foil .

Mechanical properties of lamina

Each laminate was oriented in longitudinal, transverse and (45) angle relative to designated (0) direction. Test specimens were cut from the panels using water-cooled slow velocity cutting saw, Tensile test specimen of different sizes including standard geometry according to ASTM (D 3039) (1986). The specimens were striped with (0, 90, 45) angle to determine the engineering parameters E_1, E_2, G_{12} and ν_{12} of an orthotropic material can be determined experimentally using the glass polyester composite with volume fraction 0.33.The mechanical properties shown in **(Table 1)** .

Experimental modal analysis

This is the final stage of the specimen's preparation process where the composite sheets are extracted from the mould in the form of square plates (25 cm × 25 cm) and with a specific thickness (4mm). The sheet is cut by a cutter machine to the specific specimen's dimensions according to the standard for required test.

The various boundary conditions considered in this work as shown in **(Fig.1)**.

1. Clamped-Clamped- Free - Free.
2. Clamped - Clamped - Clamped - Clamped.

Free Vibration

This article is concerned with practical experimental techniques for measuring the natural frequency of the laminated and delaminated plate. It begins with fixing the plate then connected to the electronics equipment's. After that connected the oscilloscope is used to issue of data acquisition output.

NUMERICAL ANALYSIS USING FINITE ELEMENT SOFTWARE ANSYS15.0

Nonlinear analysis studied in present work is based on nonlinear strain field as shown is equation (1).Dad Lukkassen and Anetta Meidell (2003).

$$\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\
\varepsilon_{yy} &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\
\varepsilon_{zz} &= \frac{\partial w}{\partial z} \\
\varepsilon_{xy} &= \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) = \frac{1}{2}\gamma_{xy} \\
\varepsilon_{xz} &= \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \frac{1}{2}\gamma_{xz} \\
\varepsilon_{yz} &= \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}\gamma_{yz}
\end{aligned} \tag{1}$$

Using finite element technique to discretize our model and solving nonlinear equation of motion for static and free vibration analysis as shown below. Marco Amabili (2008).

$$([K_L] + [K_{NL}])\{\delta\} = \{F\} \tag{2}$$

$$[M]\{\ddot{\delta}\} + ([K_L] + [K_{NL}])\{\delta\} = 0 \tag{3}$$

ANSYS is a general purpose finite element modeling package for numerically solving a wide variety of engineering problems. These problems include static/dynamic structural analysis (both linear and non-linear), heat transfer and fluid problems, as well as electro-magnetic problems. User's manual of FEA/ANSYS/Version/11, C. D. Josepn (2003) and T. A. Stolarski, Y. Nakasone and S. Yoshimoto (2006).

Element type

The elements used in this work to achieve the numerical results for nonlinear natural frequency by Ansys program are:

1. Shell 181 element description

It is suitable for analyzing thin to moderately-thick shell structures. It is a four-node element with six freedom degrees at each node: rotations about the x, y, and z-axes and translations in the x, y, and z directions. The precision in modeling composite shells are governed by the first-order shear-deformation theory (usually referred to as Mindlin-Reissner shell theory, as shown in **(fig.2)**.

2 . Targe 170 element description

Targe170 is used to represent various 3-D target surfaces for the associated contact elements (CONTA173 and CONTA174), as shown in **(fig.3).**

3 . Conta 174 element description

Conta174 is an 8-node element that is intended for general rigid-flexible and flexible-flexible contact analysis, as shown in **(fig.4)**.

ANSYS Modeling

1. Meshing : The finite elements mesh, the number of node is a four-node element with six degrees of freedom at each node, as shown in (Fig. 5) .

1. Boundary condition : In this work, two case of boundary condition are exist. The first one is fixed two edges and free two edges, while the other are fixed all as shown in the (Fig.6) .

2. Delamination model : The technique to model the delamination using contact elements is referred to as de-bonding. The interfacial separation is defined in terms of contact gap or penetration and tangential slip distance. The computation of contact and tangential slip is based on the type of contact element and the location of contact detection point. The cohesive zone model can only be used for bonded contact (KEYOPT (12) = 2, 3, 4, 5, or 6) with the augmented Lagrangian method (KEYOPT (2) = 0) or the pure penalty method (KEYOPT (2) = 1). See CONTA174 - 3-D 8-Node Surface-to-Surface Contact for details .

From all the methods mention above, the Contact Elements are used in this work to treat the delamination composite materials. (Fig.7) shows the model of delamination.

RESULTS AND DISCUSSION :-

Numerical solution linear and nonlinear using Ansys and experimental work are matched to analyze free vibration of laminated plate for different boundary condition with and without delamination.

In this work, the laminate square plate is assumed to have h= 4mm, a=b =20 cm, while the mechanical properties are obtained experimentally as mentioned in **(Table.1)**.

The comparison between results of Ansys for linear and nonlinear with experimental natural frequencies for symmetric and an-symmetric cross ply laminated plate (0/90/90/0) and (0/90/0/90) for (CCCC and CFCF) boundary conditions are shown in **(Tables 2 & 3)** respectively. The results of nonlinear analysis for natural frequency are close to those of linear one which shows that maximum decrement where 1.047 for symmetric (CCCC) boundary condition and 0.206 for an-symmetric (CCCC) boundary condition.

From the below tables, it can be deduced that the maximum discrepancy between nonlinear analysis and experimental work are 4.04% for symmetric (CFCF) boundary condition and 5.08 % for an-symmetric (CFCF) boundary condition, as shown in **(Table 2)** and **(Table 3)**.

From the results, it is found that the clamped for all side plate (CCCC) vibrates with natural frequency higher than that of the two clamped and other side free (CFCF) because of the high stiffness due to boundary.

(fig.8) and **(fig.9)** show the mode shapes of free vibrated symmetric and untisymmetric composite plates for (CCCC) and (CFCF) boundary condition .

Similar results are presented for nonlinear natural frequency with experimental work for symmetric and an-symmetric laminated plate with delamination for two boundary condition which shows that the maximum discrepancy between nonlinear and experimental natural frequency is 10.921% for (CCCC) boundary condition as shown in **(Tables 4 & 5)**.From the results, it is found that the clamped for all side plate (CCCC) vibrates with natural frequency higher than that of the two clamped and other side free (CFCF) because of the high stiffness due to boundary, the result of second and third mode of (CCCC) boundary condition are the same because of repeated mode for clamped square plate.

(Fig.10) and **(Fig.11)** show the mode shapes of free vibrated plates for (CCCC) and (CFCF) boundary condition.

(Table 6) shows the effect of boundary condition on fundamental natural frequencies (ω_{NL}) for symmetric and un-symmetric cross-ply glass-epoxy, which shows that increasing number of plies causing an increase in fundamental natural frequencies of un-symmetric cross-ply laminate (9.19 %), while for Symmetric laminate, increasing number of plies have little effect on the natural frequency because of the symmetry of the layer material about the mid plane of the laminate for every layer, the coupling stiffness are zero for the symmetric laminate .

In this analysis, a square symmetric cross-ply laminated plate (0/90/90/0) with different boundary conditions is taken to calculate the nonlinear natural frequency. From **(Table 7)**, it is found that the clamped plate vibrates with natural frequency higher than the other boundaries because of the high stiffness due to boundary. In other hand, the natural frequency for the SFSF plate is minimum due to low stiffness .

For a square plate with different lamination angles and boundary conditions as shown in **(fig.5)**, it is obvious that the natural frequencies of unsymmetrical laminated plate (0/45/0/45) and (0/60/0/60) is greater than of symmetric laminate for the same orientations (0/45/45/0) and (0/60/60/0) for (SSSS) boundary condition. In these cases ,the (CCCC) ,(CSCS)and (CFCF) boundary conditions, the natural frequencies of symmetric laminated plate (0/45/45/0) and (0/60/60/0) are greater than of unsymmetrical laminate for the same orientations (0/45/0/45) and (0/60/0/60), as shown in **(Table 8)** .
The increasing aspect ratio decreases the natural frequency, where the frequency starts to converge with higher aspect as shown in **(fig.12)**. The fundamental natural frequency (ω_{NL}) of all fixed boundary conditions (CCCC) cross-ply symmetric plate (0/90/90/0) decreases as aspect ratio increases .

CONCLUSIONS :-

Through numerical simulation and experimental investigation on the laminated and delaminated composite plate, the following conclusions can be deduced:-

1. The results of nonlinear analysis for natural frequency are close to those of linear one which shows that maximum decrement where 1.047 for symmetric (CCCC) boundary condition and 0.206 for an-symmetric (CCCC) boundary condition.
2. Increasing number of plies causing an increase in fundamental natural frequency of un-symmetric cross ply laminate 0.0842%.
3. The natural frequency decrease when the aspect ratio increase from (0.5 to 3.5) 0.656%
4. The results show a good agreement between nonlinear analysis and experimental work, which shows maximum discrepancy for symmetric (CFCF) 4.04% and for unsymmetrical (CFCF) 5.08%.
5. For composite laminated plate with delamination, the maximum discrepancy between nonlinear natural frequency and experimental work 9.57% for (CCCC) boundary condition .

(Table 1) Mechanical Properties of the laminate

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	υ_{12}	V_f
19.6	5.7	2.2	0.32	0.33

(Table 2) linear, nonlinear FEM (Ansys) and experimental natural frequency for symmetric cross ply laminated plate (0/90/90/0) with different boundary condition.

	Boundary condition	ω				
		FEM (Ansys) ω_L Linear	FEM (Ansys) ω_{NL} Nonlinear	Increment Between. Lin. And non Lin.	ω Exp.	Discrepancy % (nonlinear and exp.)
First mode	CCCC	395.33	395.461	0.131	386	2.392397733
Second mode		693.5	693.309	0.191	684	1.342691354
Third mode		921.43	922.477	1.047	928	0.598714114
First mode	CFCF	310.39	310.565	0.175	298	4.045851915
Second mode		330.83	331.026	0.196	338	2.106783153
Third mode		442.08	442.31	0.23	434	1.878772806

Table (3) linear, nonlinear FEM (Ansys) and experimental natural frequency for unsymmetrical cross -plies laminated plate (0/90/0/90) with different boundary condition.

	Boundary condition	ω				
		FEM (Ansys) ω_L Linear	FEM (Ansys) ω_{NL} Nonlinear	Increment Between. Lin. And non Lin.	ω Exp.	Discrepancy % (nonlinear and exp.)
First mode	CCCC	385.28	385.338	0.058	370	3.980401621
Second mode		795.17	795.376	0.206	820	3.095894269
Third mode		795.17	795.376	0.206	820	3.095894269
First mode	CFCF	254.22	254.254	0.034	252	0.88651506
Second mode		279	279.039	0.039	284	1.777887679
Third mode		433.5	433.561	0.061	408	5.895594853

(Table 4) nonlinear FEM (Ansys) and Experimental natural frequency for symmetric cross ply laminated plate (0/90/90/0) with different boundary condition with delamination.

	Boundary condition	ω_{NL}		
		FEM (Ansys) Nonlinear	Exp.	Discrepancy % (Nonlinear and exp.)
First mode	CCCC	392.914	350	10.921
Second mode		681.275	670	1.654985
Third mode		889.449	882	0.837485
First mode	CFCF	201.048	202	0.473519
Second mode		231.499	253	9.2877
Third mode		421.381	416	1.276992

(Table 5) nonlinear FEM (Ansys) and Experimental natural frequency for unsymmetrical cross ply laminated plate (0/90/0/90) with different boundary condition with delamination.

	Boundary condition	ω_{NL}		
		FEM (Ansys) Nonlinear	Exp.	Discrepancy %
First mode	CCCC	383.443	360	6.1138
Second mode		774.962	780	0.6500
Third mode		774.962	780	0.6500
First mode	CFCF	254.035	245	3.5565
Second mode		278.777	272	2.4309
Third mode		419.825	398	5.1985

(Table 6) fundamental nonlinear natural frequencies (ω_{NL}) for cross-ply glass-polyester laminated plate with different boundary condition**.**

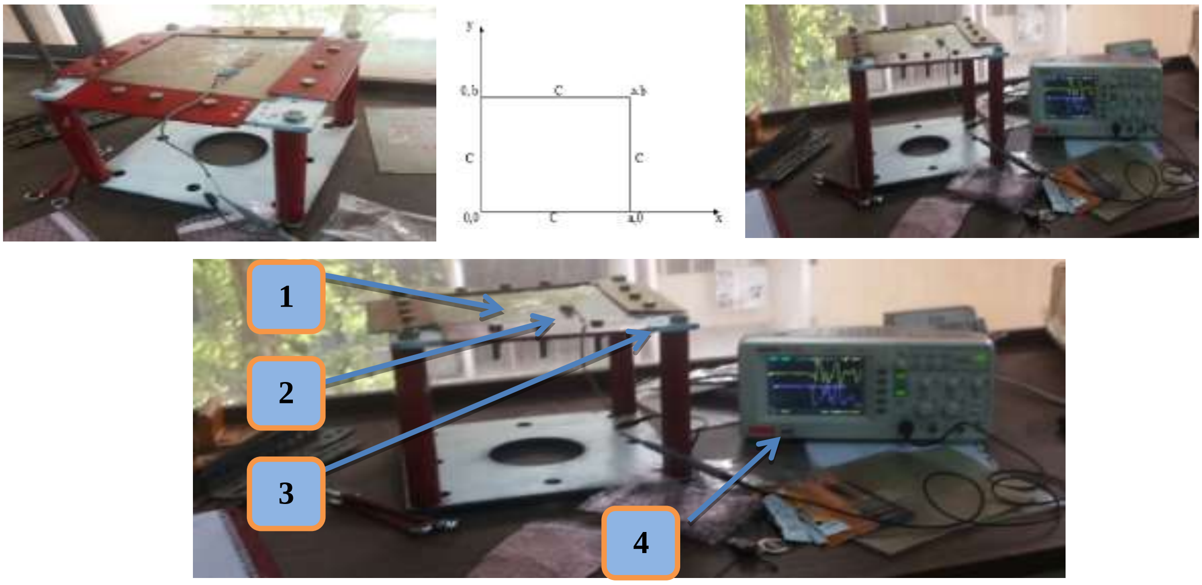
	boundary condition	FEM (Ansys) ω_{NL}			
		Un-symmetric Layers		Symmetric Layers	
		0/90	0/90/0/90	0/90/90/0	0/0/90/90/0/0
First mode	CCCC	352.88	385.338	395.461	391.43
Second mode		728.13	795.376	693.309	654.39
Third mode		728.13	795.376	922.477	935.3
First mode	CFCF	230.41	254.254	310.565	289.1
Second mode		257.41	279.039	331.026	306.14
Third mode		404.62	433.561	442.31	398.50
First mode	CSCS	285.69	306.09	346.55	319.17
Second mode		530.46	574.11	534.11	469.24
Third mode		694.63	755.27	897.36	802.45

Table (7) fundamental nonlinear natural frequencies (ω_{NL}) for symmetric and unsymmetrical cross-ply laminated plate with different boundary condition.

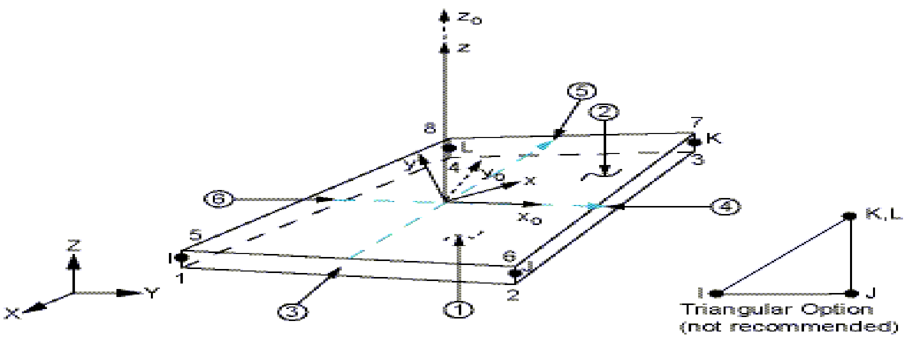
	Boundary Conditions	ω_{NL}	
		0/90/90/0	0/90/0/90
First mode	SSSS	198.53	196.23
Second mode		444.05	516.43
Third mode		602.26	517.47
First mode	CCCC	395.461	385.338
Second mode		693.309	795.376
Third mode		922.477	795.376
First mode	CSCS	346.55	306.09
Second mode		534.11	574.11
Third mode		897.36	755.27
First mode	SFSF	54.665	54.171
Second mode		173.90	137.30
Third mode		241.70	216.64
First mode	CFCF	310.565	254.254
Second mode		331.026	279.039
Third mode		442.31	433.561

(Table 8) Nonlinear fundamental natural frequency of various lamination angles for different boundary conditions.

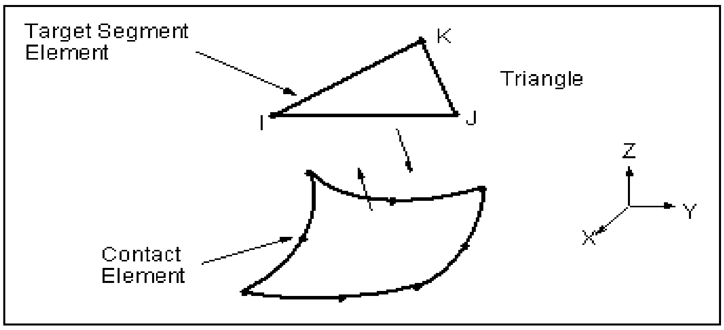
Angle Ply Orientations		ω_{NL}				
		Type of boundary conditions				
		SSSS	CCCC	CSCS	SFSF	CFCF
First mode	0/45/45/0	202.41	394.30	352.56	79.504	313.96
Second mode		430.28	665.45	526.65	138.11	337.69
Third mode		615.87	935.79	897.34	322.40	448.73
First mode	0/45/0/45	207.53	382.07	329.53	85.203	270.62
Second mode		464.97	698.98	544.05	153.20	305.47
Third mode		577.31	857.93	826.44	348.89	452.13
First mode	0/60/60/0	201.42	394.55	349.67	83.073	311.23
Second mode		437.54	678.21	530.70	138.06	334.33
Third mode		609.66	928.00	905.05	336.59	447.02
First mode	0/60/0/60	204.39	381.91	317.53	79.050	258.27
Second mode		486.34	737.19	557.13	153.14	292.40
Third mode		553.00	827.42	787.63	366.39	446.48



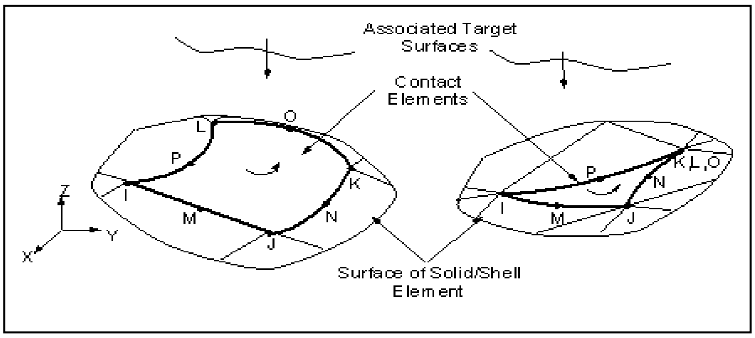
(Fig.1) free vibration techniques with all edges are clamped (CCCC).
1. Tested plate. 2. Piezoelectric. 3. Testing structure with CCCC boundary condition . 4. Oscilloscope.



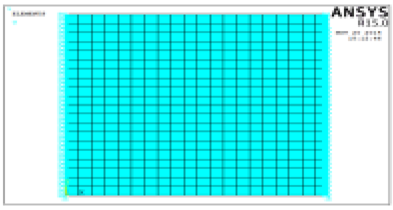
(Fig.2) shell181 Geometry



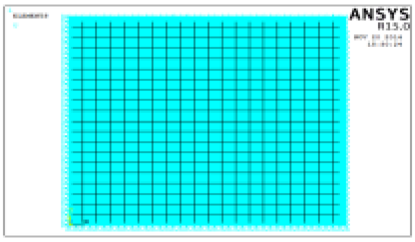
(Fig.3) Targe170 target surface element.



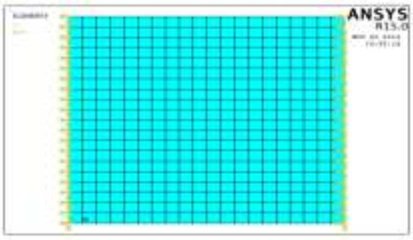
(Fig.4) Conta174 3-D surface-to-surface contact element (8-node).



(Fig.5) the Geometry of the Process



A- All edges are Fixed

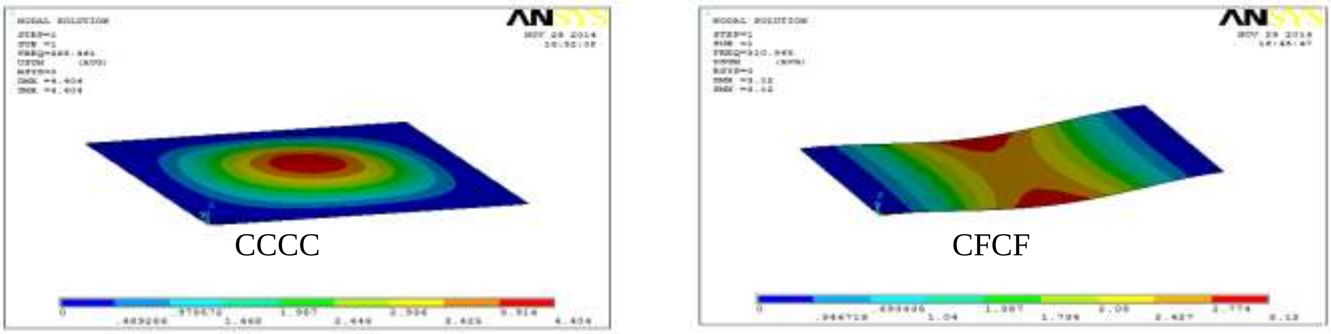


B- two edges are fixed and two edges free

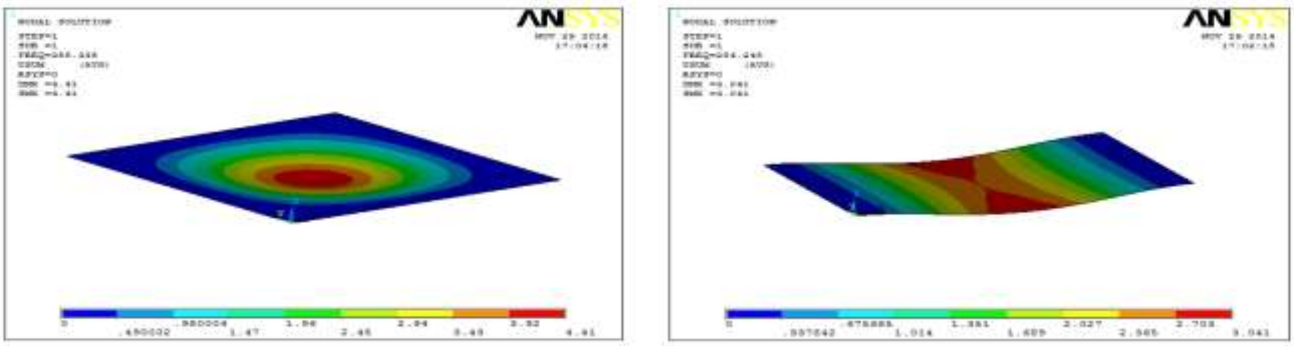
(Fig.6) The boundary conditions of plate without delamination.

A- All edges are Fixed **B-** two edges are fixed and two edges free

(Fig.7) : The boundary conditions of plate with delamination.

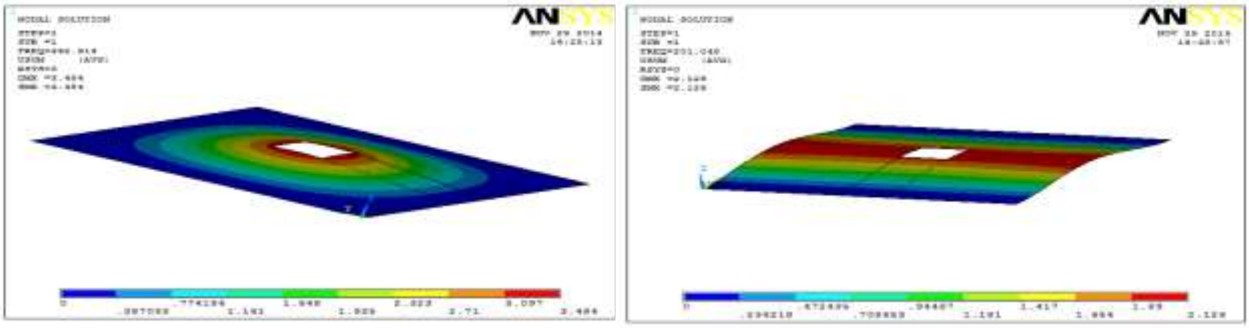


(Fig.8) : fundamental Mode shape for free vibration of symmetric cross ply laminated plates (0/90/90/0) of (CCCC) and (CFCF) plate by Ansys **.**



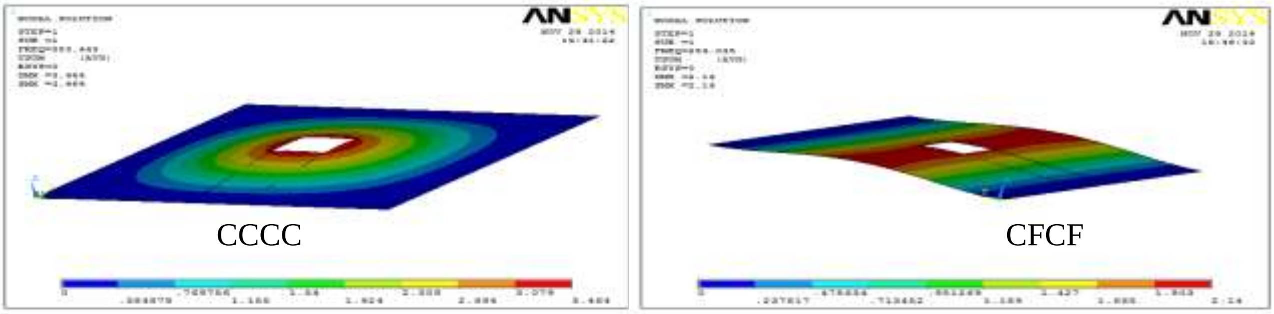
CCCC CFCF

(Fig.9): fundamental Mode shape for free vibration of unsymmetrical cross ply laminated plates (0/90/0/90) of (CCCC) and (CFCF) plate by Ansys**.**

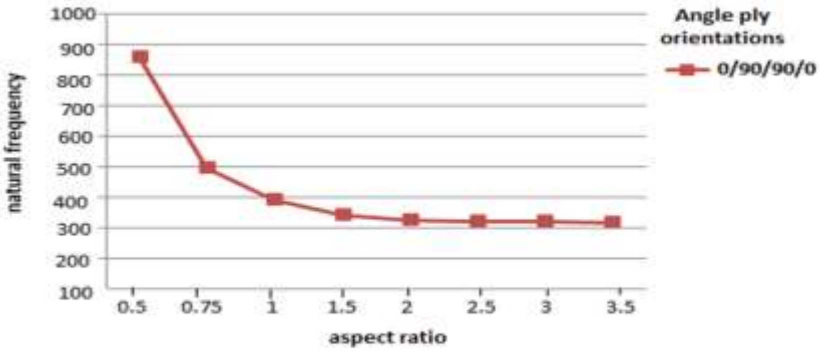


CCCC CFCF

(Fig.10): Fundamental Mode shape for free vibration of symmetric cross ply laminated plates (0/90/90/0) of (CCCC) and (CFCF) plate by Ansys.



(Fig.11): Fundamental Mode shape for free vibration of unsymmetrical cross ply laminated plates (0/90/0/90) of (CCCC) and (CFCF) plate by Ansys**.**



(Fig.12) : Variation of nonlinear fundamental natural frequency with aspect ratio for cross-ply (0/90/90/0) for (CCCC).

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