



Effect of Non – Uniform Permeability on the Operational Characteristics of Misaligned Self – Lubricated Journal Bearings with Modified Boundary Conditions

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ABSTRACT:

Effect of non uniform permeability on the operational characteristics of finite width misaligned self lubricated journal bearing is investigated throughout this work. A modified boundary conditions for the oil film pressure is obtained by applying integral momentum equation to the oil film region in the bearing clearance. The performance characteristics of a misaligned self lubricated journal bearing with slenderness ratio of unity are obtained for various eccentricity ratios and misalignment angle. The bearing permeability is changed in radial and circumferential directions. Using self lubricated bearing with non – uniform permeability enhances the performance of such bearing. An decreases in maximum pressure and load carrying capacity, attitude angle, and coefficient of friction. Oil film extent increases with increasing the oil supply pressure.

Key words: Self Lubricated Bearing, Misaligned Bearing, Non-Uniform Permeability, Improved Boundary Condition.

الخلاصة :

INTRODUCTION:

Self – lubricated bearings are that type of hydrodynamic journal bearing made by powder technology and impregnated with suitable oil. It is well known that journal bearings (in general) operates in misaligned condition, (i.e the bearing and the journal center lines are not parallel). Misalignment can vary in magnitude and direction. The most important type cases to be considered are vertical misalignment , twisting or horizontal misalignment and combination of these can also occur. Theoretical analysis of porous bearings operating under hydrodynamic lubrication condition have been performed by many workers, Morgan and Cameron (1957); Reason and Dyer (1973); Prakash.J and Vijs (1974); Cusano (1979).

Kaneko and Obara (1990) show experimentally that even if hydrodynamic lubrication condition existed initially the oil film extent in the bearing clearance will decrease with running time as a result of oil loss, eventually resulting in mixed or boundary lubrication conditions. The effect of different boundary conditions on the mechanism of lubrication for the porous bearing were investigated by many workers; Kaneko, Ohkawa and Hashimoto (1994) and Kaneko and Hashimoto (1995). Kaneko, Hashimoto and Hiroki (1997) used an improved boundary condition rather than Reynolds' boundary condition to analyze the oil film pressure distribution in porous journal bearings. The circumferential condition for oil film pressure is obtained by applying an integral momentum equation to the oil film pressure distribution.

It was found that studying the performance of fluid film bearing with misalignment is important step toward reliable design of the bearing. Working characteristics of the journal bearings with misaligned shaft has been carried out by Pinkus and Bupara (1979), Jang and Chang (1987), Safar et. al. (1988) and Jin and Chong (1987). The effect of misalignment on the performance of planetary gear journal bearings has been studied by Vijayaraghavan and Brew (2005).

The effect of using non uniform permeability matrix on the performance of porous bearings was studied by Yong – Xi et. al., (1985) and Kaneko and Doi (1989). They are found that changing the porous matrix permeability lead to considerable enhancement to the steady state characteristics of the self lubricated bearings.

In this study the effect of changing the porous matrix permeability in specified direction (mainly circumferential and radial directions) together with applying the modified boundary conditions on the performance of the self lubricated bearing with misaligned journal has been carried out theoretically.

NUMERICAL ANALYSIS:

Pressure Distribution in Oil Film:

A schematic diagram of a porous journal bearing with the coordinate system used in the analysis are shown in fig.(1). Two independent angles (γ_1 and γ_2) are used to describe the fluid film gap. The journal rotates with rotational speed (ω_j) about the journal center (O_j). the governing equation for the pressure distribution in the oil film is given by the modified Reynolds' equation including the slip velocity effect can be written as Kaneko and Doi (1989);

$$\frac{\partial}{\partial \theta} \left(h^{\wedge 3} (1 + \zeta_{10}) \frac{\partial P^{\wedge}}{\partial \theta} \right) + \left(\frac{D_i}{L} \right)^2 \frac{\partial}{\partial Z^{\wedge}} \left(h^{\wedge 3} (1 + \zeta_1) \frac{\partial P^{\wedge}}{\partial Z^{\wedge}} \right) = 6 \frac{\partial}{\partial \theta} \left(h^{\wedge 3} (1 + \zeta_{00}) \right) - 12 \Phi_r \left(\frac{\partial P^{\wedge *}}{\partial r^{\wedge}} \Big|_{r^{\wedge}=1} \right) \quad (1)$$

where,

$$\left. \begin{aligned} (\zeta_{0\theta}) &= \left(\frac{s_{\theta}}{(h^{\wedge} + s_{\theta})} \right) \\ (\zeta_{1\theta}) &= \left(3(h^{\wedge} s_{\theta} + 2\alpha^2 s_{\theta}^2) / \{h^{\wedge} (h^{\wedge} + s_{\theta})\} \right) \\ (\zeta_1) &= \left(3(h^{\wedge} s + 2\alpha^2 s^2) / \{h^{\wedge} (h^{\wedge} + s)\} \right) \end{aligned} \right\} \quad (2)$$

Terms $(\zeta_{0\theta}), (\zeta_{1\theta})$ and (ζ_1) result from the tangential velocity slip, where,

$$(s_\theta) = (\Phi_\theta c / r_i)^{1/2} / \alpha$$

$$(s_r) = (\Phi_r c / r_i)^{1/2} / \alpha$$

$$(\Phi_\theta) = (k_\theta r_i / c^3) \quad \left. \vphantom{(\Phi_\theta)} \right\} \quad (3)$$

$$(\Phi_r) = (k_r r_i / c^3)$$

$$(\Phi) = (k r_i / c^3)$$

The slip coefficient (α) is a dimensionless parameter depending on the material parameter which characterizes the structure of a permeable material within the boundary regions. Its value for laminar channel flow has been estimated by Beavers and Joseph (1967) to be 0.1. The non-dimensional fluid film gap given by Jin et. al. (1987) is adopted in this work:

$$h^* = 1 + \varepsilon \cos \theta - \zeta \sigma_1 \cos \theta + \zeta \sigma_2 \sin \theta \quad (4)$$

Where,

$$\sigma_1 = 2 \left(\frac{R}{C} \right) \left(\frac{L}{D} \right) \tan \gamma_1 \quad (5)$$

$$\sigma_2 = 2 \left(\frac{R}{C} \right) \left(\frac{L}{D} \right) \tan \gamma_2 \quad (6)$$

The two independent misalignment angles (γ_1 and γ_2) are measured from $\xi=0$; the gap geometry depends on (θ and ξ).

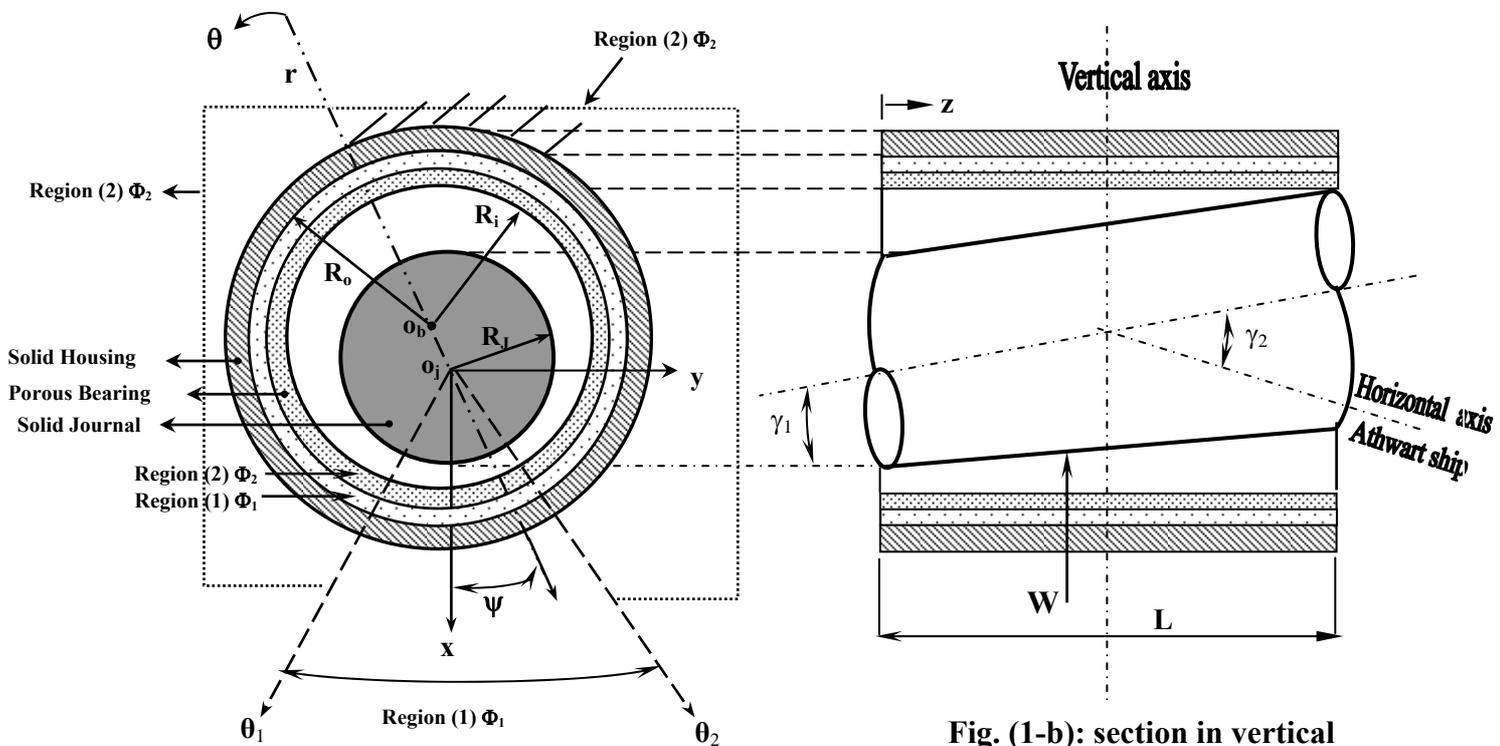


Fig. (1-a): radial and circumferential non-uniform permeability

Fig. (1-b): section in vertical longitudinal plane

Fig. (1): Geometrical configuration of the misaligned Porous Journal Bearing.

Pressure Distribution in Porous Matrix :

The governing equation for the pressure distribution in the porous matrix with non uniform permeability can be expressed as: Kaneko and Doi (1989);

$$\frac{1}{r^{\wedge}} \frac{\partial}{\partial r^{\wedge}} \left(r^{\wedge} \Phi_r \frac{\partial P^{\wedge*}}{\partial r^{\wedge}} \right) + \frac{1}{r^{\wedge 2}} \frac{\partial}{\partial \theta} \left(\Phi_{\theta} \frac{\partial P^{\wedge*}}{\partial \theta} \right) + \left(\frac{D_i}{L} \right)^2 \frac{\partial}{\partial Z^{\wedge}} \left(\Phi_z \frac{\partial P^{\wedge*}}{\partial Z^{\wedge}} \right) = 0 \quad (7)$$

BOUNDARY CONDITIONS:

The following boundary conditions have been used with the above governing equation to solve the problem;

$$\left. \begin{aligned} P^{\wedge}(\theta_1, z^{\wedge}) &= P^{\wedge*}(r^{\wedge}, \theta_1, z^{\wedge}) = 0 \\ P^{\wedge}(\theta_2, z^{\wedge}) &= P^{\wedge*}(r^{\wedge}, \theta_2, z^{\wedge}) = 0 \\ P^{\wedge}(\theta, \pm 1) &= P^{\wedge*}(r^{\wedge}, \theta, \pm 1) = 0 \\ P^{\wedge}(\theta, z) &= P^{\wedge*}(r^{\wedge}, \theta, z) \text{ at } (r^{\wedge})=1 \end{aligned} \right\} \quad (8)$$

The outer surface of porous matrix consist of two parts as shown in figure(1). The first is the part press – fitted inside the solid housing where the pressure is evaluated from the condition that the permeability of the housing adjacent to the porous matrix is zero i.e.

$$\Phi_{\theta}, \Phi_z, \Phi_r = 0 \text{ or } \frac{\partial P^{\wedge*}}{\partial r^{\wedge}} \text{ at } (r^{\wedge}) \geq (r_o/r_i) \text{ and } 0.5 \leq |Z^{\wedge}| \leq 1 \quad (9)$$

The second is that exposed to the circumferential groove in the housing, where the pressure is given by;

$$(P^{\wedge*}) = P_s^{\wedge} = \left(\frac{c_2^2 P_s}{r_i^2 \eta \omega_j} \right) \text{ at } (r^{\wedge}) = r_o \text{ and } |Z^{\wedge}| \leq 0.5$$

The following boundary conditions are used to determine the oil film extent. The leading edge (θ_1) of the oil film can be determine by the boundary condition used by Kaneko et. al. (1997);

$$\left(M_{\theta_1} - M_{\theta_2} - M_{\theta_c} - M_{\theta_b} \right) = 0 \quad (10)$$

where;

$M_{\theta_1}, M_{\theta_2}, M_{\theta_c}$ and M_{θ_b} are the circumferential momentum flow rates across the control surfaces of the oil films, as shown in figure (2). The momentum flow rates are given as follows;

$$\left. \begin{aligned}
 (M_{\theta_1}) &= 2 \int_0^{L/2} \int_0^{(h_{\theta_1})} \rho \left[(u_{\theta}|_{\theta_1})^2 \right] dy dz \\
 (M_{\theta_2}) &= 2 \int_0^{L/2} \int_0^{(h_{\theta_2})} \rho \left[(u_{\theta}|_{\theta_2})^2 \right] dy dz \\
 (M_{\theta_c}) &= 2(r) \int_{\theta_1}^{\theta_2} \int_0^{(h)} \rho \left[(u_{\theta} * u_z) \Big|_{z=L/2} \right] dy d\theta \\
 (M_{\theta_b}) &= 2(r) \int_0^{L/2} \int_{(\theta_1)}^{(\theta_2)} \rho \left[(u_{\theta_m}) * (u_r^*) \Big|_{r=(r)} \right] d\theta dz
 \end{aligned} \right\} \quad (11)$$

The calculation of the circumferential momentum flow rates are the same as that in Basim, and Lekaa (2008).

The velocity components (u_{θ}) and (u_z) represent the components of the oil velocity in circumferential and axial directions in the oil film, while (u_r^*) represents the radial velocity component of the oil inside the porous matrix. The values of (θ_1) and (θ_2) are assumed to be constant in z – direction.

On the other hand the oil film extent at the trailing edge (θ_2) can be obtained by ensuring the continuity of the bulk flow a cross the boundary line at (θ_2) .

$$(q_{\theta_p} / q_{\theta_c}) = 0 \quad (12)$$

where q_{θ_p} and q_{θ_c} are the flow rates a cross the trailing boundary line due to the Poiseuilles' and Couettes' flows respectively. Equation (12) can be rewritten as;

$$(q_{\theta_p} / q_{\theta_c}) = \left(\left(\frac{(1 + \zeta_{1\theta})}{6(1 + \zeta_{0\theta})} h^{\wedge 2} \int_0^1 \frac{\partial P^{\wedge}}{\partial \theta} dz^{\wedge} \right) \Big|_{\theta=\theta_2} \right) \quad (13)$$

Knowing the values of (θ_1) and (θ_2) the angular extent of the oil film (β) is expressed in the form;

$$(\beta) = (\theta_2) - (\theta_1) \quad (14)$$

BEARING PARAMETERS:

Knowing the pressure distribution the dimensionless film force components along and perpendicular to the line of centers can be obtained, respectively as;

$$\left(\hat{W}_R \right) = - \int_0^1 \int_{\theta_1}^{\theta_2} (P^{\wedge}(\theta, z) \cos \theta) d\theta dz^{\wedge} \quad (15)$$

$$\left(\hat{W}_T \right) = \int_0^1 \int_{\theta_1}^{\theta_2} (P^{\wedge}(\theta, z) \sin \theta) d\theta dz^{\wedge} \quad (16)$$

The total dimensionless load can be expressed as;

$$\left(\hat{W}\right) = \sqrt{\left(\hat{W}_R\right)^2 + \left(\hat{W}_T\right)^2} \quad (17)$$

The attitude angle (Ψ) can be evaluated as;

$$\left(\Psi\right) = \tan^{-1}\left(\hat{W}_T/\hat{W}_R\right) \quad (18)$$

The coefficient of friction can be evaluated as:

$$F^\wedge = \int_0^1 \int_{\theta_1}^{\theta_2} \frac{h^\wedge}{2} \frac{\partial P^\wedge}{\partial \theta} d\theta dz^\wedge + \int_0^1 \int_{\theta_1}^{\theta_2} \frac{h^\wedge}{2} \frac{\zeta_{1\theta}}{3} \frac{\partial P^\wedge}{\partial \theta} d\theta dz^\wedge + \int_0^1 \int_{\theta_1}^{\theta_2} \frac{(1-\zeta_{0\theta})}{h^\wedge} d\theta dz^\wedge \quad (19)$$

Hence the coefficient of friction can be evaluated as ;

$$\left(\mu^\wedge\right) = \frac{\left(F_r^\wedge\right)}{\left(\hat{W}\right)} \quad (20)$$

METHOD OF SOLUTION:

Pressure distribution in the oil film can be obtained by solving the modified Reynolds' equation which includes the slip velocity effect (equation1). The pressure distribution through the porous matrix can be obtained by solving Darcy's equation including the effect of non uniform permeability (equation7). The permeability of the porous matrix was assumed to have circumferential and radial non – uniform permeability. The above equations are discretized and solved simultaneously with an appropriate boundary conditions. The interested domain was divided into (180) divisions in circumferential direction, (100 divisions for rupture zone and 80 divisions for the effective zone). Twelve divisions in the axial direction and eight divisions in the radial direction.

The governing equations are solved iteratively with successive under relaxation factor. The iterations are continued until the following inequalities are satisfied simultaneously;

$$\left(\frac{\sum \sum \sum \left| P_{i,j,k}^{*(n+1)} - P_{i,j,k}^{*(n)} \right|}{\sum \sum \sum \left| P_{i,j,k}^{*(n)} \right|} < 10^{-5} \right) \quad (21)$$

$$\left(\frac{\sum \sum \left| P_{j,k}^{\wedge(n+1)} - P_{j,k}^{\wedge(n)} \right|}{\sum \sum \left| P_{j,k}^{\wedge(n)} \right|} < 10^{-5} \right) \quad (22)$$

$$\left(\left| M_{\theta_1}^\wedge - M_{\theta_2}^\wedge - M_{\alpha}^\wedge - M_{\theta_b}^\wedge / M_{\theta_1}^\wedge \right| < 10^{-3} \right) \quad (23)$$

$$\left(\left| \frac{q_{\theta_p}}{q_{\theta_c}} \right| \right)_{ii} = \left(\left| \left(\frac{1 + \zeta_{1\theta}}{6(1 + \zeta_{0\theta})} h^{\wedge 2} \int_0^1 \frac{\partial P^{\wedge}}{\partial \theta} dz^{\wedge} \right) \right|_{\theta=\theta_2} \right) < 10^{-3} \quad (24)$$

Always (n) and (n+1) used in above equations denote two consecutive iterations. The points i, j, k represent the grid number in radial, circumferential, and axial directions respectively.

RESULTS AND DISCUSSION :

The computer program prepared to solve the problem of the present work has been verified by comparing the results obtained in this work with that obtained by Kaneko et., al., (1989) and (1997), as shown from figures (1,2,3). These figures shows that the obtained results in this work are in a good agreement with the published results in the previous works.

The effect of changing the permeability, in both circumferential and radial direction with taking the modified boundary conditions into consideration on the maximum oil film pressure of the misaligned bearing has been shown in figure(4). It is clear that the effect of changing the permeability in both directions, (circumferential and radial direction) is decreasing the value of the maximum pressure generated in oil film the reduction in the magnitude of the maximum pressure become higher when the permeability changes in radial direction.

The maximum oil film pressure was found to increase when the combined misalignment has been taken into consideration, as shown in figure (5). This indicates that the misalignment of this type causes a decrease in minimum oil film thickness.

The effect of increasing the supply pressure can be shown in figure(6). The magnitude of the maximum oil film pressure increases with increasing the oil supply pressure which indicates the oil film extent increases in this case.

The attitude angle seems to be slightly decreases when the porous matrix permeability changes in circumferential direction than that obtained when the permeability changes in radial direction as shown in figure(7), while it is highly enhanced (decreases) when compared with uniform permeability which indicates the bearing works more stable in the former case.

The coefficient of friction tends to slightly increases with Sommerfeld number as shown in figure(8). The coefficient of friction seems to be lower when changing the permeability of the porous matrix in circumferential and radial direction. This ia attributed to the increase in oil film extent in this case.

The effect of combined misalignment on the oil film extent of the bearing shown in figure(9). It is clear that the oil film increase with increasing values of Sommerefeld number. It is clear from figure (10) that changing the permeability in both directions (radial and circumferential direction) with taking the modified boundary conditions for the porous bearing cause a decrease in load carrying capacity when compared with that obtained for a porous bearing with uniform permeability. Increasing oil supply pressure of the oil cause to increase the oil film extent as shown in figures (11 and 12).

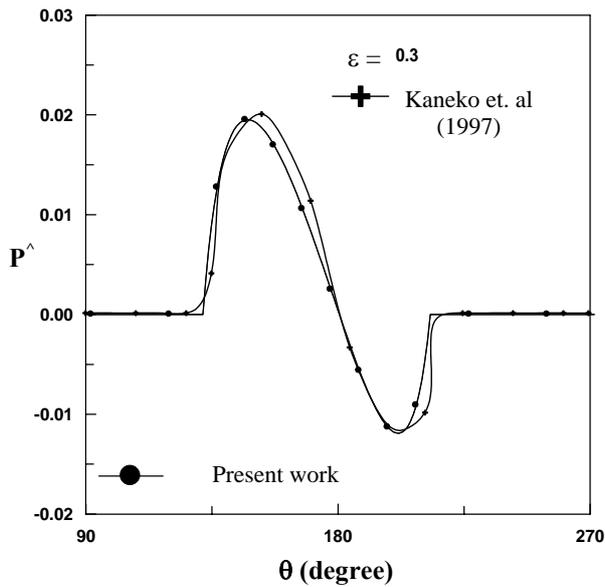


Fig (1): Comparison between present and published result Kaneko et. al. (1997) for pressure distribution as $P^s = 0.1$

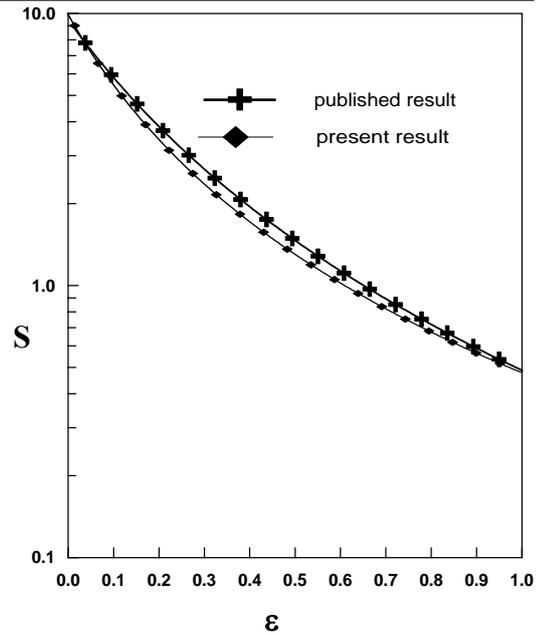


Fig.(2): Comparison between present and published result Kaneko et. al. (1989) at $P_s = 0.1$, $\Phi_1 = 0.01$ and $\Phi_2 = 0.1$ for Sommerfeld number versus eccentricity ratio at non-uniform permeability.

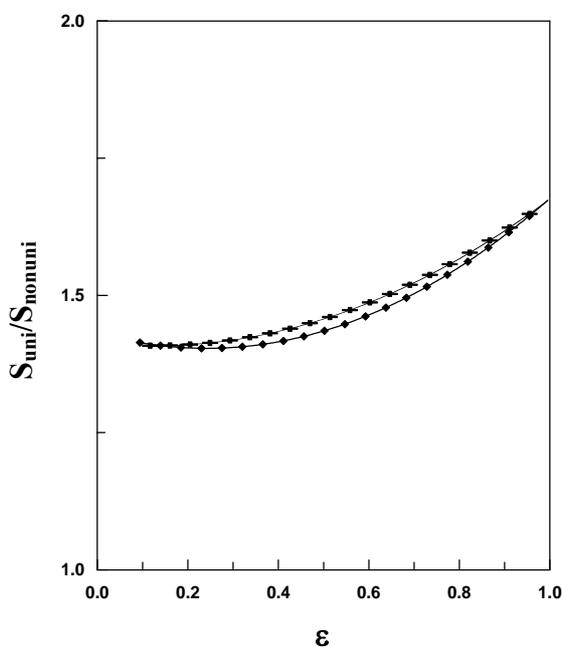


Fig.(3): Comparison between present and published result Kaneko et. al. (1989, for ratio of the Sommerfeld number for uniform type $\Phi = 0.1$ to that non-uniform type $\Phi_1 = 0.01$ and $\Phi_2 = 0.1$ versus eccentricity ratio.

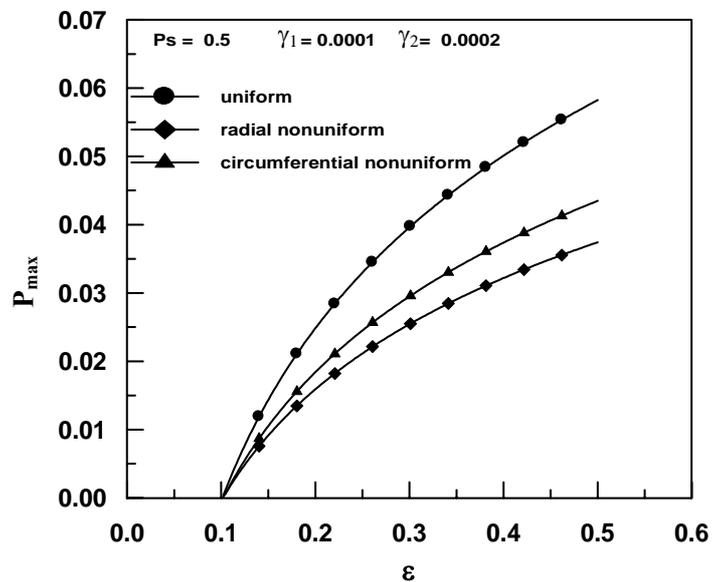


Fig. (4): Max. Pressure versus eccentricity ratio for uniform and nonuniform permeability

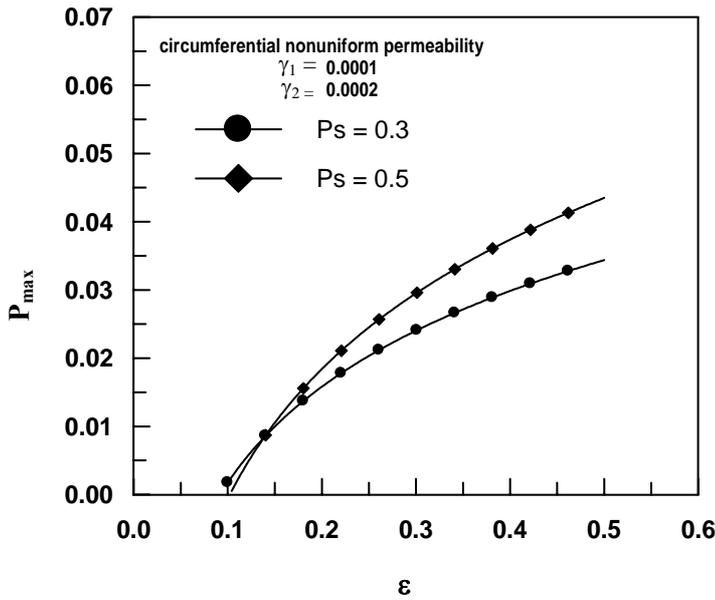


Fig. (6): Max. Pressure versus eccentricity ratio for different supply pressure.

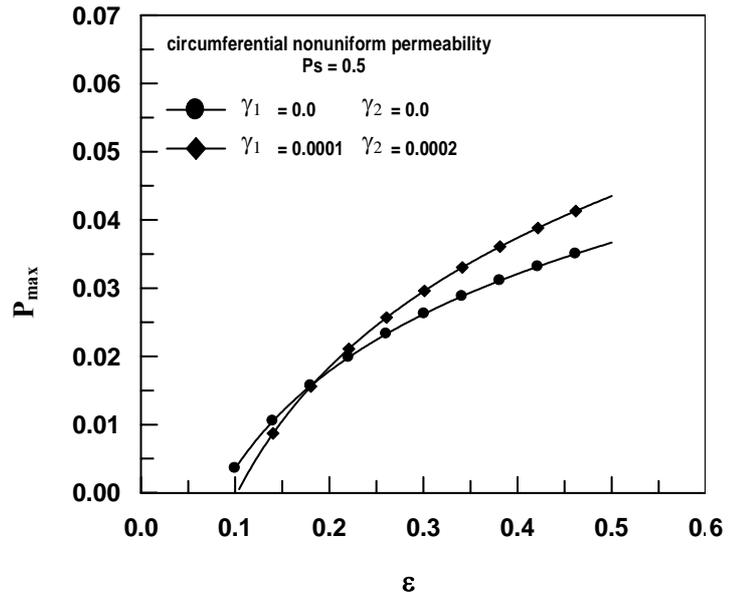


Fig. (5): Max. Pressure versus eccentricity ratio for different misaligned ratios.

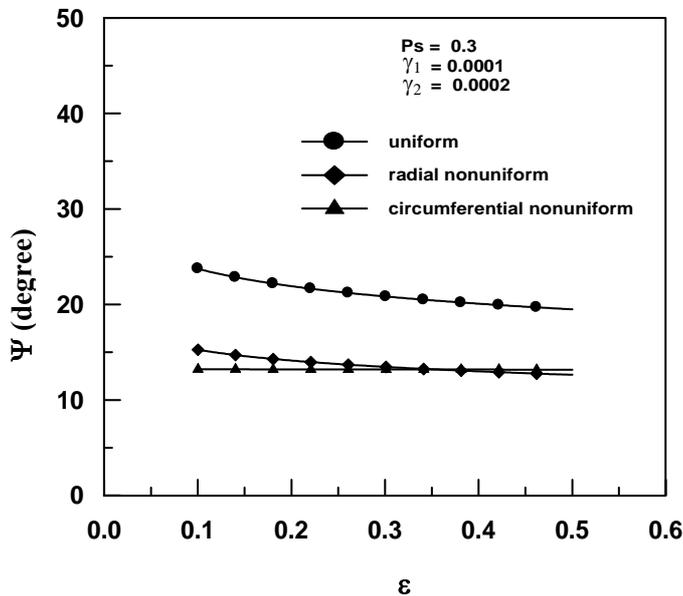


Fig. (7): Attitude angle versus eccentricity ratio for uniform and nonuniform permeability.

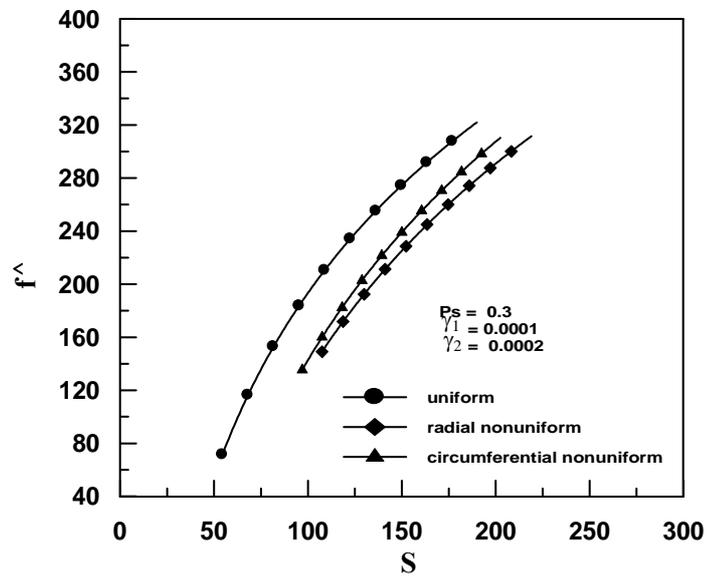


Fig. (8): Coefficient of friction versus Sommerfeld number for uniform and nonuniform permeability .

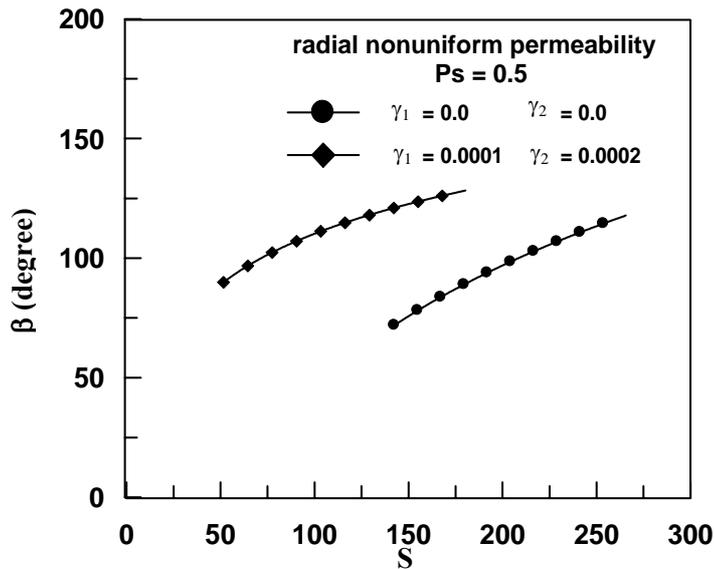


Fig. (9): Oil film extent versus Sommerfeld number for different misaligned ratios.

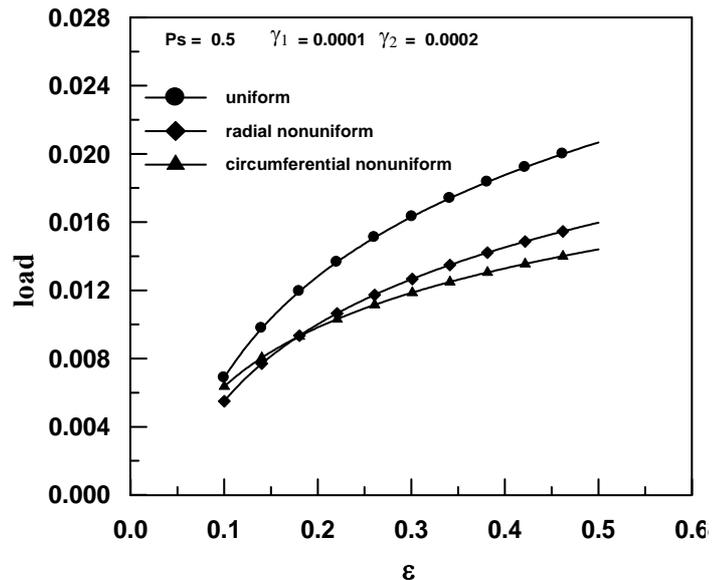


Fig. (10): Load versus Eccentricity ratio for uniform and nonuniform permeability.

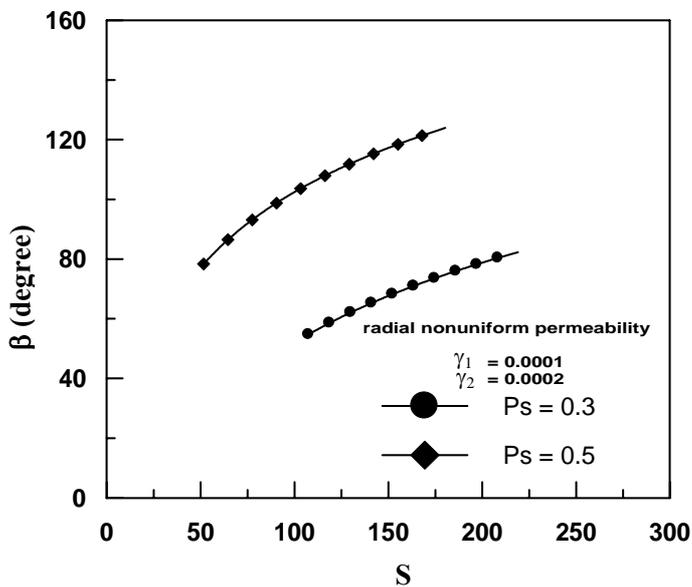


Fig. (11): Oil film extent versus Sommerfeld number for different supply pressure.

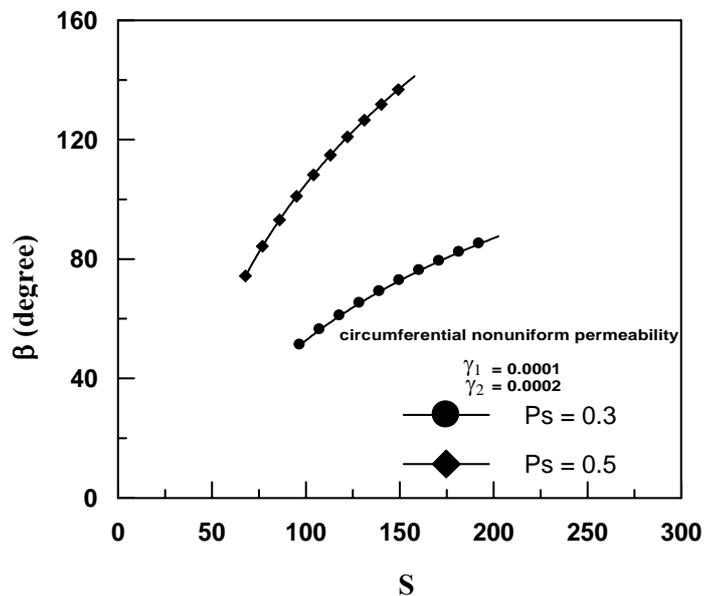


Fig. (12): Oil film extent versus Sommerfeld number for different supply pressure.

CONCLUSIONS:

The effect of using porous bearing with porous matrix of variable permeability in both radial and circumferential directions together with applying

the modified boundary conditions on the performance of self lubricated bearings are highly lighted her in.

Referring to the previous discussion of the results obtained through this work the following remarks can be concluded:

Changing the permeability in circumferential or radial direction with taking the modified boundary conditions into consideration cause:

- 1- the maximum oil film pressure decreases when compared with that obtained from the porous bearing with uniform porous matrix.
- 2- Decrease in load carrying capacity when compared with that obtained self lubricated bearing of uniform porous matrix.
- 3- A decrease in attitude angle when compared with that obtained from the porous bearing of uniform porous matrix.
- 4- The coefficient of friction decreases when compared with that obtained from the porous bearing of uniform permeability.
- 5- The oil film extent increases with increasing the supply pressure of the oil.
- 6- The maximum oil film pressure increases when the combined misalignment has taken into consideration.

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NOMENCLATURE:

The following symbols are used throughout this work.

c	Journal Bearing Clearance (m)
\hat{h}	Dimensionless Film Thickness, ($\hat{h} = h/c$) _{ii}
$k_{\theta,r}$	Permeability of the Porous Matrix in Circumferential and Radial Direction respectively (m^2)
k	Permeability of the Porous Matrix in Axial Direction respectively (m^2)
L	Length of the Bearing (m)
M_{θ_1}	Circumferential Momentum Flow Rate across Oil Film Surface at Inlet End of Oil – Film Region, i.e. at $\theta = \theta_1$
M_{θ_2}	Circumferential Momentum Flow Rate across Oil – Film Surface at Trailing End of Oil –Film Region, i.e. at $\theta = \theta_2$
M_{θ_c}	Circumferential Momentum Flow Rate across Oil–Film Surface at Both Axial Ends($z = \pm L/2$)
M_{θ_b}	Circumferential Momentum Flow Rate across Oil – Film Surface Adjacent to Inner Surface of the Bearing, i.e. ($y=0$)

N_j	Journal Rotational Speed (r.p.m)
P^\wedge	Dimensionless Oil-Film Pressure, $P^\wedge = c^2 P / (r^2 \eta \omega)$
$P^{*\wedge}$	Dimensionless Oil – Film Pressure Inside the Porous Matrix, $P^{*\wedge} = c^2 P^* / (r^2 \eta \omega)$
P_s	Supply Pressure (N/m ²)
r^\wedge	Normalized radial coordinate, $r^\wedge = r / r_i$
R_j	Journal Radius(m)
r_i	Inner Radius(m)
r_o	Outer Radius(m)
(S)	Sommerfeld Number , $S = (R \eta \omega_j L / W) * (r_i / c)^2$
s	Slip parameter
T^\wedge	Dimensionless Frictional Torque, $T^\wedge = T c / \eta \omega_j r_i^3 L$
U_j	Journal Velocity (m/s)
u,v,w	Oil – Film Velocity Components in θ, r, z Directions Respectively(m/s)
u^*, v^*, w^*	Oil Velocity Components inside the Porous Matrix in θ, r, z Directions Respectively (m/s)
W^\wedge	Dimensionless Load Carrying Capacity, $W^\wedge = W c^2 / \eta \omega_j r_i^3 L$
(W_r^\wedge)	Dimensionless Component of Oil – Film Force Along the Line of Centers,
(W_T^\wedge)	Dimensionless Component of Oil – Film Force Perpendicular to the Line of Centers
Z^\wedge	Normalized axial coordinate, $Z^\wedge = z / (L/2)$

Greek Symbols

ε	Eccentricity Ratio
η	Absolute Viscosity of Oil(pa . s)
θ	Angular Coordinate from Maximum Film Thickness Position (Degree)
ρ	Density of oil (kg/m ³)
μ^\wedge	Dimensionless Friction Coefficient $\mu^\wedge = (R/c)\mu$
ρ	Density of oil (kg/m ³)
(Φ)	Permeability parameter (m ²).
ψ	Attitude Angle (degrees)
r, θ , z	Bearing coordinates in radial, circumferential and axial directions.
σ_1 , σ_2	two independent misalignment parameters
ξ	Normalized axial coordinate (z/L)

Subscript

b	Referring to Bearing
j	Referring to Journal

Superscript

\wedge	Dimensionless Quantity
*	Porous Parameter