Tikrit Journal of Pure Science Vol. 13 No.(1) 2008, 182 - 184

Weakly regular and pre weakly regular sets in Intuitionistic fuzzy special topological spaces

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Abstract:

Our goal in this paper is to give definitions of new kind of separation axioms called Weakly regular, Pre-weakly regular, Weakly T_3-Spaces in Intuitionistic fuzzy special topological spaces and study some relationships among them.

Preliminaries:

First, we shall defined an Intuitionistic fuzzy special set (IFSS for short) A which is an object having the form $\langle x, A_1, A_2 \rangle$, where A_1 and A_2 are su set of a nonempty fixed set X , satisfying the following $A_1 \cap A_2 = \phi$.The set A_1 is called the set of member of A , while A_2 is called the set of non-member of A [2].

Every subset A of a non- empty set X is IFSS having the form $\langle x, A, A^c \rangle$.

Let X be a non-empty set and let A,B be IFSS's where A=$\langle x, A_1, A_2 \rangle$, $B = \langle x, B_1, B_2 \rangle$ and let $\{A_i : i \in J\}$be an arbitrary family of IFSS's in X where $A_i = \langle x, A_1^i, A_2^i \rangle$ then:-

$(i)\ A \subseteq B \Leftrightarrow A_1 \subseteq B_1 \wedge A_2 \supseteq B_2;$

$(ii)\ A = B \Leftrightarrow A \subseteq B \wedge B \subseteq A;$

$(iii)\ \overline{A} = \langle x, A_2, A_1 \rangle;$

$(iv)\ \bigcup A_i = \langle x, \cup A_1^i, \cap A_2^i \rangle, \bigcap A_i = \langle x, \cap A_1^i, \cup A_2^i \rangle;$

$(v)\ \tilde{\phi} = \langle x, \phi, X \rangle, \tilde{X} = \langle x, X, \phi \rangle.$

An Intuitionistic fuzzy special topology (IFST for short) on non-empty set X is a family Ψ of IFSS's in X containing $\tilde{\phi}, \tilde{X}$ and closed under finite intersection and arbitrary union[2]. The pair $(X, \Psi)$is called an Intuitionistic fuzzy special topological space ,in this case any IFSS in Ψ is know as Intuitionistic fuzzy open set (IFSOS for short) in X .

The interior and closure of an IFSS A are defined by;

$$IntA = \cup\{G : G \in \Psi, G \subseteq A\},$$

$$ClA = \cap\{K : \overline{K} \in \Psi, A \subseteq K\}$$

A subset A of Intuitionistic fuzzy special topological spaces(X, Ψ)is called a pre-open set if $A \subseteq IntClA$. The union of any family of pre-open sets is pre-open set, while the intersection of two pre-open set need not to be a pre-open set[3].

Now let (X, Ψ)be IFST's, then it's said to be satisfy T_o separation axiom iff the following condition is satisfied;-

$\forall x \forall y, x, y \in X \quad and\ x \neq y\ \exists A \in \Psi\ s.t\ (x \in A \wedge y \notin A) \vee (y \in A \wedge x \notin A)$
.

it's satisfy T_1 separation axiom iff the following condition is satisfied;-

$\forall x \forall y, x, y \in X \quad and\ x \neq y\ \exists U, V \in \Psi\ \ s.t\ (x \in U \wedge y \notin U) \wedge (y \in V \wedge x \notin V)$

it's satisfy T_2 separation axiom iff the following condition is satisfied;-

$\forall x \forall y, x, y \in X \quad and\ x \neq y\ \exists U, V \in \Psi\ \ s.t\ (x \in U \wedge y \in V)\ and\ U \cap V = \tilde{\phi}$

Note that: $x \in A = \langle x, A_1, A_2 \rangle \Leftrightarrow x \in A_1 \wedge x \notin A_2$.

The following implication are hold but the converse is not true []

$T_2 \Rightarrow T_1 \Rightarrow T_0$

We say that (X, Ψ) is regular IFSTS iff its satisfy the following condition:-

$$\forall x \forall C, x \in X \wedge C \subseteq X, C\ is\ closed\ \ subset\ x \notin C$$
$$\exists U, V \in \Psi\ \ s.t\ (x \in U \wedge C \subseteq V)\ and\ U \cap V = \tilde{\phi}$$

Weakly Regular and weakly Pre-Regular sets Intuitionistic fuzzy special topological spaces:

In the definition of regular Intuitionistic fuzzy special topological spaces we saw the problem that appear in $U \cap V = \tilde{\phi} = \langle x, \phi, X \rangle$.Some times $U \cap V \neq \tilde{\phi}$ but equal to $\langle x, \phi, A \rangle$ where $A \subset X$. So we define $\Phi^* = \langle x, \phi, K \rangle$where $K \subseteq X$. So we define An Intuitionistic fuzzy special topological space(X, Ψ)is weakly in the following way :

$$\forall x \forall C, x \in X \wedge C \subseteq X, C\ is\ closed\ \ subset\ x \notin C$$
$$\exists U, V \in \Psi\ \ st\ (x \in U \wedge C \subseteq V)\ and\ U \cap V = \Phi^*$$

Form this definition of weakly regular (W.R.S for short) we see that every regular is weakly regular but the converse is not true in general ,the following example shows the case.

Example2.1:

Let

$X = \{a, b, c\}, A = \langle x, \{a\}, \{c\} \rangle, B = \langle x, \{c\}, \{b\} \rangle, C = \langle x, \{b\}, \{a\} \rangle$

defined

$\psi = \{\tilde{\phi}, \tilde{X}, A, B, C\} \cup \{all\ \ pair\ wise\ union\ \} \cup$ {all pair wise intersection}we can see this space W.R but not regular.

Definition2.2

Intuitionistic fuzzy special topological space $(X, \Psi)$is said to be Pre-weakly regular space(P.W.R.S for short) if

$$\forall x \forall C, x \in X \wedge C \subseteq X, C\ is\ closed\ \ subset\ x \notin C$$
$$\exists U, V \in PoX\ \ \ st\ (x \in U \wedge C \subseteq V)\ and\ U \cap V = \Phi^*$$

Remark2.3
Every W.R.S is P.W.R.S , but the converse is not true in general , the following example show the case.

Example2.4
Let
$X=\{1,2,3\}, A=\langle x,\{1,2\},\{3\}\rangle, B=\langle x,\{1\},\{2,3\}\rangle, C=\langle x,\{2\},\{1,3\}\rangle, D=\langle x,\{1,3\},\{2\}\rangle$
$E=\langle x,\{2,3\},\{1\}\rangle$
Let
$\psi=\{\tilde{\phi},\tilde{X},A\}, then\ Po(X)=\{\tilde{\phi},\tilde{X},A,B,C,D,E\}$
so (X,Ψ) is P.W.R.S but not W.R.S.

Remarks2.5
1- If (X,Ψ) is W.R.S, then it's not to be T_2 or T_1 space. see Example2.1.
2- If (X,Ψ) is T_2-space, then it's need not be W.R.S, the following example show the case.

Example2.6
$X=\{a,b,c\}, A=\langle x,\{a,b\},\{c\}\rangle, B=\langle x,\{b,c\},\{a\}\rangle, C=\langle x,\{a,c\},\{b\}\rangle.$
Let
$\psi=\{\tilde{\phi},\tilde{X},A,B,C\}\cup\{all\ \ pair\ wise\ union\ \}\cup$ {all pair wise intersection}.
We see that (X,Ψ) is T_2 but not W.R.S.
By Remark 2.5 we can see that W.R.S and both T_2 and T_1 spaces are independent notions.
Before we give the next proposition we need the following notation;
Let X be any set for each x in X we denote the singleton containing x by $\{x\}=\langle x,\{x\},\{x\}^c\rangle$ and the complement of $\{x\}=\langle x,\{x\},\{x\}^c\rangle$ by $\overline{\{x\}}=\langle x,\{x\}^c,\{x\}\rangle$.

Proposition2.7
Let (X,Ψ) be IFSTS then (X,Ψ) is T_o iff for each x, y in X such that $x\neq y$ then
Cl $\{x\}=Cl\langle x,\{x\},\{x\}^c\rangle$
$\neq Cl\{y\}=Cl\langle y,\{y\},\{y\}^c\rangle$.

Proof:
$\Leftarrow$
Let x, y in X such that $x\neq y$ since
$Cl\ \{x\}=Cl\ \langle x,\{x\},\{x\}^c\rangle$
$\neq Cl\ \{y\}=Cl\ \langle y,\{y\},\{y\}^c\rangle$ then there exist z in X such that either
$(z\in Cl\{y\}\wedge z\notin Cl\{x\})\vee(z\in Cl\{x\}\wedge z\notin Cl\{y\})$
if
$(z\in Cl\{x\}\wedge z\notin Cl\{y\})$.Now x ix not in Cl{y} because if x in Cl{y} then
$Cl\{x\}\subseteq Cl(Cl\{y\})=Cl\{y\}\ so\ z\in Cl\{x\}\subseteq Cl\{y\}$
which is contradiction, then let V= $\overline{Cl\{y\}}$ so V is IFSOS containing x but not containing y . The same argument use for the other case.
$\Rightarrow$
Suppose that (X,Ψ) is be T_o space and x ,y in X such that $x\neq y$ then ; $\exists G=\langle x,G_1,G_2\rangle$ IFSOS such that x in G and y not in G. So $\overline{G}=\langle y,G_2,G_1\rangle$ IFSCS such that x is not in $\overline{G}=\langle y,G_2,G_1\rangle$ and y in $\overline{G}=\langle y,G_2,G_1\rangle$. Now from definition Cl{y} which is the intersection of all IFSCS which containing {y} then y in Cl{y} and x is not in Cl{y} since x is not in $\overline{G}=\langle y,G_2,G_1\rangle$ i.e.
Cl $\{x\}=Cl\langle x,\{x\},\{x\}^c\rangle$
$\neq Cl\{y\}=Cl\langle y,\{y\},\{y\}^c\rangle$.

Theorem2.8
If an IFSTS (X,Ψ) is both T_o and W.R.S then (X,Ψ) is T_2 and then T_1.

Proof:
Let (X,Ψ) be is both T_o and W.R.S and let x and y be any distinct points of X, then $\exists U=\langle x,U_1,U_2\rangle$ such that x in U and y not in U or y in U and x not in U. suppose that the first case hold , the other case is similar. Since (X,Ψ) is W.R.S and Cl{x}is IFSCS set containing x and y not in it by virtue of prop.2.7. So $U\cap\overline{Cl\{x\}}$ is IFSOS does not contain y , So there exist IFSOS $G=\langle k,G_1,G_2\rangle$ such that $U\cap\overline{Cl\{x\}}$ subset of G and $U\cap G=\tilde{\phi}$ .So (X,Ψ) is T_2 and so is T_1 .

Theorem2.9
An IFSTS (X,Ψ) is W.R.S iff for every point x in X and IFSOS $G=\langle x,G_1,G_2\rangle$ contain x there exist an IFSOS $G^*=\langle x,G_1^*,G_2^*\rangle$ such that x in G^* and $ClG^*\subseteq G$.

Proof:
Suppose That (X,Ψ) is W.R.S and G be IFSOS such that x in G then $F=\overline{G}$ is an IFSCS which dose not contain x, So by Weak regularity , there exist two IFSOS's G_F and G^* such that F subset of G_F and x in G^* and $G^*\cap G_F=\tilde{\phi}$. So $G^*\subseteq\overline{G_F}$ therefore
$ClG^*\subseteq\overline{ClG_F}=\overline{G_F}\subseteq\overline{F}=G$ as required.
Conversely; Suppose That the condition holds and suppose for each x is and each IFSCS F not contain x , then $x\in\overline{F}$ so by hypothesis there exists an IFSOS G^* containing x and $ClG^*\subseteq\overline{F}$ so $F\subseteq\overline{ClG^*}$ and G^* and $\overline{ClG^*}$ are disjoint.

Definition2.10
Let (X,Ψ) and (Y,Ω) be two IFST's and let $f:X\to Y$ be a function. A function f is said to be continuous if the inverse image of every IFSOS in Y is IFSOS in X. and said to be IFSO function(IFSC function if the direct image of every IFSOS in X (IFSCS in X) is IFSOS in Y (IFSCS in Y).

Theorem2.11
Let (X,Ψ) and (Y,Ω) be two IFST's and let $f:X\to Y$ be Continuous, Open and surjective function then if X is W.R.S then Y is W.R.S.

Proof:
Let (X,Ψ) and (Y,Ω) be two IFST's and let $f:X\to Y$ be Continuous, , Open and subjective function and suppose that X is W.R.S , let C be any IFSCS in Y and let y be any element dose not belong to Y. Since f is subjective then there exist, $x\in \overline{f^{-1}(C)} \ni f(x)=y$. Since f is continuous $f^{-1}(\overline{C})=\overline{f^{-1}(C)}\wedge f^{-1}(C)$ is closed. subset in X and x not in it. So by W. Regularity of X , there exists an IFSOS's U and V such that x in U , $f^{-1}(C)\subseteq V$ and $U\cap V=\Phi^{*}$. It is clear that y in f(U) and C subset of f(V) and $f(U)\cap f(V)=\Phi^{*}$

Definition 2.12
An Intuitionistic fuzzy special topological space (X,Ψ) is called weakly T_3(weakly Pre-T_3) if (X,Ψ) is T_o and weakly regular(weakly Pre-regular)space.
Note that in the definition of T_3 spaces in general topology we say that a space is T_3 if it's regular and T_1[4], in above definition we use T_0 instead of T_1 which is weaker condition.

Remark2.13
Every weakly T_3 space is weakly Pre-T_3, but the converse is not true in general.

Example2.14
Let $X=\{a,b,c\}, A=\langle x,\{a,b\},\{c\}\rangle$ defined $\psi=\{\tilde{\phi},\tilde{X},A,\}$ then (X,Ψ) is weakly Pre-T_3 but not weakly T_3.

Remark2.15
Every weakly pre-T_3 space is weakly Pre-regular, but the converse is not true in general. See example 2.4.
Note that definition 2.12 is a generalization of W.R and W.P.R.S in the next paper we try to study it's properties and it's relation with other kind of separation axioms.

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الفضاءات المنتظمة الضعيفة والمنتظمة الضعيفة قبليا في الفضاءات التبولوجية الحدسية الخاصة

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الملخص:

يتناول هذا البحث بديهية جديدة من بديهيات الفصل في الفضاءات التبولوجية الحدسية الخاصة سميت بالفضاءات المنتظمة الضعيفة والمنتظمة الضعيفة قبليا وبعض العلاقات التي تربط بينهما.