

HEALTH MONITORING ANALYSIS OF CRACKED PLATE BASED ON STATIC DEFLECTION AND NATURAL FREQUENCIES

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ABSTRACT

This research presents a SHM (Structural Health Monitoring) techniques that are based on static and modal analyses of cracked plate structure. Finite element models (FEMs) for the different cracked steel plate have been created by considering the length and the orientations of the crack, and a cantilever supported plates (parallel to the crack) are used. The finite element models are examined under the action of static concentrated lateral load. It is found that the maximum deflections of cracked plate increases when there is an increase in the crack length. The highest value of the maximum deflection is at (0.12 m) of half-crack length. The reason of this behavior is attributed to the decrease in material's stiffness causing structure deformation. However, this deflection decreases when the orientation of crack increases until it reaches the minimum value at 90° of crack angle. Modal analysis is also performed to each FEM to extract the values of the natural frequencies. It is noticed that the values of natural frequencies decrease with an increasing of crack length, and they increase with the increasing of crack orientation angle. An experimental test has been done to check the validity of the intact plate. The results obtained is the fundamental natural frequency. A comparison between the natural frequency and finite element result is made an error of less than (10 %) is found. Finally, SHM analysis is applied in order to predict the early behavior of the plates in accordance with the results above. The extrapolation for certain symbols of the results is performed to derive numerical formulas of the natural frequencies and static deflections. The natural frequencies and static deflections' formulas are used as a function for the half-crack length, however, there is a formula for each orientation angle.

Structural Health Monitoring, Length Keywords: Cracked Plate, Finite Element, Extrapolation, Half, Predicting Static Deflections, Predicting Natural Frequencies

تحليل مراقبة اهلية الصفيحة الحاوية على الشقوق اعتمادا على التشوه الميكانيكي في الحالة الساكنة والتردد الطبيعي

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الخلاصة:

قدم هذا البحث تقنيات SHM (مراقبة سلامة الهياكل) بالاعتماد على التحليلات السكونية و الاهتزاز الحر لهيكل صفيحة محتواة على شق. و باستخدام طريقة العناصر المحددة تم انشاء نماذج مختلفة لصفائح من الحديد الصلب و المحتواة على شق مع الاخذ بنظر الاعتبار طول و زاوية ميلان الشق و قد تُبنت الصفائح من جهة واحدة و تركت حرة من الجهات الاخرى.

إن النماذج التي تم بنائها اختبرت تحت تأثير حمل سكوني مركّز، مسلط عرضياً. حيث اظهرت النتائج ان التشوهات العظمى (maximum deflections) تزداد مع زيادة طول الشق ونحصل على اعلى قيمة لها عند نصف طول شق مقداره (0.12 م) والسبب في هذا يعزى الى نقصان صلابة المادة مسبباً تشوهاها. لكن، هذه التشوهات تتناقص عند زيادة زاوية ميلان الشق لتكون اقل قيمة لها عندما يكون الشق بزواوية (90°). ايضاً، لقد تم تحليل كل نموذج باستخدام طريقة العناصر المحددة حركياً لإيجاد الترددات الطبيعية للهياكل. وقد لوحظ أن قيم الترددات الطبيعية تتناقص مع زيادة طول الشق، في حين تزداد مع زيادة زاوية ميلان الشق. و تم اجراء اختبارات عملية لصفحة خالية من الشق و مثبتة من جهة واحدة و بطريقة الاهتزاز الحر لإيجاد مقدار التردد الطبيعي ومقارنتها مع نتائج طريقة العناصر المحددة. وأخيراً، تم تطبيق تحليل SHM من أجل التنبؤ بسلوك الصفيحة في وقت مبكر وفقاً للنتائج المذكورة أعلاه حيث اظهرت عملية الاستقراء لبعض النتائج لاشتقاق الصيغ العددية للترددات الطبيعية والتشوهات السكونية معادلات خاصة بها. حيث ان لكل زاوية ميلان شق صيغ خاصة و هي دالة لنصف طول الشق.

INTRODUCTION :

Structural Systems in civil, mechanical and aerospace engineering, or any other, are susceptible to sudden damage, deterioration and aging. Therefore, a health monitoring system that is able to detect and identify any damage in real time in its earliest stage is essential to maintain the structural stability, integrity and to maximize the life span of the structure as much as possible. Therefore, the process of implementing a damage detection strategy for aerospace, civil and mechanical engineering infrastructure is referred to as **Structural Health Monitoring (SHM)**, **Hoon Sohn et. al (2004)**. The health monitoring is studied by several researchers in recent years, where **Meneghetti and Maggiore (1994)** derived a sensitivity formulation for locating a crack in a beam from frequency shifts. Also, a method based on changes in uniform load surface (ULS) curvature was developed by **D. Wu and S. S. Law (2004)** for damage localization in two-dimensional plate structures. An improved damage quantification methodology for a plate structure was presented by **Choi S., et al. (2006)**. The methodology utilizes the relationship between the stiffness loss and the fractional changes of the modal parameters due to damage. To improve the damage quantification performance, the methodology is derived by eliminating erroneous assumptions in the existing mode shape-based methods and adopting additional modal information, i.e., natural frequencies. The validity of the proposed method is demonstrated using numerical data from a simply-supported plate structure. In addition, the effect of fundamental natural frequency on the cracked composite plate with three different boundary condition (SSSS, SSCC and SSFF) was studied by **Muhannad Al-Waily (2012)**. The results show that a sensing of the fundamental natural frequency of cracked plate where is decreased as plate elasticity decreases, but, **Rainah Ismail (2012)** studied a vibration analysis for a thin isotropic plate containing an arbitrarily orientated surface crack. It is found that the vibration characteristics and nonlinear characteristics of the cracked plate structure can be greatly affected by the orientation of the crack in the plate. The above researchers focused on modal parameters to detect the defect in structures. But, the present paper represents a static and modal analyses for elastic plate models to extract the behaviours of these models under effect of these analyses. Therefore, the basic items which is assumed to be as an objective for this work is the prediction of the plates' behaviours' by using Structural Health Monitoring's techniques to predict the damage history. This has been verified by calculation the variation of the natural frequencies and the static deflections of the cracked thin plates.

ANALYSIS INVESTIGATION :

Statics, as the name implies, is concerned with the study of bodies at rest or, in other words, in equilibrium, under the action of a force system. Actually, a moving body is in equilibrium if the forces acting on it are producing neither acceleration nor deceleration. However, in structural analysis, structural members are generally at rest and therefore in a state of static equilibrium, **Dr. T. H. G. Megson (1996)**.

In addition, the modal analysis is used to determine the vibration characteristics (natural frequencies and mode shapes) of the structure. It also can be a starting point for the harmonic response analysis. The basic equation solved in a typical modal analysis is the classical eigenvalue problem, **Haider Hussein Hamad (2012)**.

The geometric model of this paper is a square thin plate with a central crack with different length and angles as shown in **Fig. (1)**. Isotropic steel plate with mechanical properties shown in **Table (1)** of dimensions (30 cm *30 cm) and (1mm) thickness is used in the present work. The central crack have half crack-length ranges from (0.015 m - 0.12 m) increased each (0.015 m) and also have the angle ranges from (0°- 90°) increased by (15°).

EXPERIMENTAL WORK :

The vibration test involves studying the fundamental natural frequency for the intact plate by analysing the signal data obtained. The plate dimensions used in this test are (b=30 cm, c=30 cm and h=1 mm) with cantilever plate which is supported (CFFF) as a boundary condition. The dimensions of the plate sample used in the vibration test are given below and shown in **Fig. (2)**:

$$c_t = c + 8 \text{ cm (support)} = 30 \text{ cm} + 8 \text{ cm (supported)} = 38 \text{ cm} ,$$

$$b = 30 \text{ cm}, h = 1 \text{ mm}$$

Fig. (3) illustrates the plate sample which is tested to evaluate the fundamental natural frequency.

Fig. (3) consists of the following parts:

1. Rig structure is made of steel plate with thickness of (10 mm). It is used as a support on which the plate sample is fixed also it is used as a table at which the other parts are set.
2. The support parts are made of steel plate of (1.5 mm). It is used to fix the plate specimen on rig structure.
3. Impact hammer of model (086C01-PCB Piezotronics vibration division) is used. The additional information of impact hummer are: sensitivity (11.2 mV/N), measured range (± 444 N), resonant frequency (≥ 15 kHz), non-linearity ($\leq 1\%$).
4. The model of accelerometer (352C68) is also used. The information regarding this accelerometer are: sensitivity (10.2 mV/(m/s²)), measurement range (491 m/s²), resonant frequency (≥ 35 kHz), non-linearity ($\leq 1\%$).
5. The amplifier is used with the model No. (480E09). It amplifies the signal measured by the accelerometer to be displayed by the oscilloscope.
6. Digital storage oscilloscope model (ADS 1202CL+) and serial No.01020200300012 is used. It has a maximum frequency of (200 MHz), also it has a maximum read of sample per second (500 MSa/s), FFT spectrum analysis and two input channels.

RESULTS AND DISCUSSION :

The results include the experimental and numerical results. The numerical results include the static deflections, natural frequencies, predicting static deflections and predicting natural frequencies results.

Experimental Results

The experimental results include the calculation of the first mode of the natural frequency of intact plate structure. Through the analysis of the accelerometer signal which is shown in **Fig.(4)** with sigview software, the natural frequency of the intact plate is evaluated. This software is used to transform the signal obtained from time domain into frequency domain by using **FFT** function as shown in **Fig. (5)**. The comparison between numerical natural frequency of intact plate with experimental result shows that the percentage error between them is estimated by (6%). The natural frequency that evaluated numerically is (9.557 Hz) and its value that computed experimentally is (9.0729 Hz).

Numerical Results

As mentioned above, the numerical results include the static deflections, natural frequencies, predicting static deflections and predicting natural frequencies results.

The predicting process is performed with the aid of extrapolation method. An additional models have been built in order to obtain the natural frequencies and their static deflections results. The purpose of building these models is to find the best fit and to produce the equations as a function of half crack-length in order to predict the maximum static deflections and natural frequencies for the remaining models when the crack propagates. The models that introduced are the intact and cracked plate models with half-crack length of (0.0075, 0.0225 and 0.0375 m). In this work, there is an attempt to predict the first mode of natural frequencies and maximum static deflections for different models. The equations produced are based on the results of models with half-crack length of (0, 0.0075, 0.015, 0.0225, 0.03, 0.0375 and 0.045 m).

A. Static Deflections Results

It is found that the maximum deflections of cracked plate increases when there is an increase in the crack length. The highest value of the maximum deflections is at (0.12 m) of half-crack length. The reason of this behaviour is attributed to the decrease in material's stiffness causing structure deformation. However, this displacement decreases when the orientation of crack increases until it reaches the minimum value at 90° of crack angle as it is shown in **Fig. (6) & (7)**. **Fig. (7)** shows that the deflection of cracked plate decreases when the angular orientation increases because the effect of crack was minimized to be minimum at vertical crack form.

B. Natural Frequencies Results

The natural frequencies of FEMs are computed numerically for the first fourth modes. These values calculated by taking in consideration the effects of crack length and angle of crack as shown in **Fig. (8)**.

The figure shows that the natural frequency of the plate decreases when the central crack length increases. Since, the increase in crack length causes decreasing in the plate stiffness, so the value of natural frequency of the cracked structure will be decreased. Also, it is found that the values of natural frequencies are affected by the value of crack inclination. It is worth to be mentioning that an increase in the crack angle causes an increase in the natural frequency of cracked structures as shown in **Fig. (9)**. Considering the oblique crack of half

length (a) and angle (θ) has two crack components, the first one is the horizontal component parallel to the support and the other is the perpendicular one (parallel to free edges). The horizontal component has more influence when (θ) small and that influence decreases when crack angle increases. This is an explanation of the increase of the natural frequencies when the angle increases.

C. Predicting Static Deflections

The maximum static deflections of the various models have been predicted. The equations of maximum static deflections that are induced in the models are derived as shown in **Table (2)**. These formulae are a function of half crack length and there is an equation for each orientation of the crack. The predicted and numerical results are compared to get the maximum difference estimated by (14.5 %) and this occurs at (90°) of crack orientation as shown in **Fig. (10)**. In addition, the maximum deflections due to static load are predicted as shown in **Fig. (11)**.

D. Predicting Natural Frequencies

It is important to predict the modal parameter (natural frequency) to know the remaining life of a structure in order to enhance the structure and assure a safer life.

Therefore, equations as a function of half crack length are tabulated in **Table (3)** to predict the value of natural frequencies when the crack is growing. In this paper, only the first mode of natural frequencies is predictable. The results obtained from the mathematical equations are compared with numerically obtained results with a maximum discrepancy between them estimated by (3.5 %) as shown in **Fig. (12)**. Generally, it has been noticed that **Fig. (12)** shows an approximate divergence between the two curves (Numerical and Predicted Curves) which occur almost above (0.075 m) of half crack-length. Also, the predicted first mode of natural frequencies of cracked plate with various orientation angle is shown in **Fig. (13)**.

CONCLUSIONS :-

1. The maximum deflection grows when increasing the crack length to reach maximum at ($a = 0.12$ m).
2. The predicted values of maximum deflection gave a minor difference of approximately (14.5 %) when compared to numerical results.
3. A comparison is made between the natural frequency of the intact plate which is tested experimentally and this value which is computed numerically gave a percentage error of (6%).
4. The value of natural frequency of the cracked plate decreases with an increase of the crack length due to the decrease of the stiffness of the plate as the crack length increases.
5. It is found that the value of the natural frequency of the cracked plate increases when increasing the angle of crack and this is as a result of a decrease in the effect of the crack on plate structure.
6. When comparing the predicted and numerical values of the natural frequencies of different models, the largest percentage error is estimated by (3.4%) at zero crack angle and (- 0.22%) at (90°) crack angle on half-crack length (a) which is equal to (0.12 m).

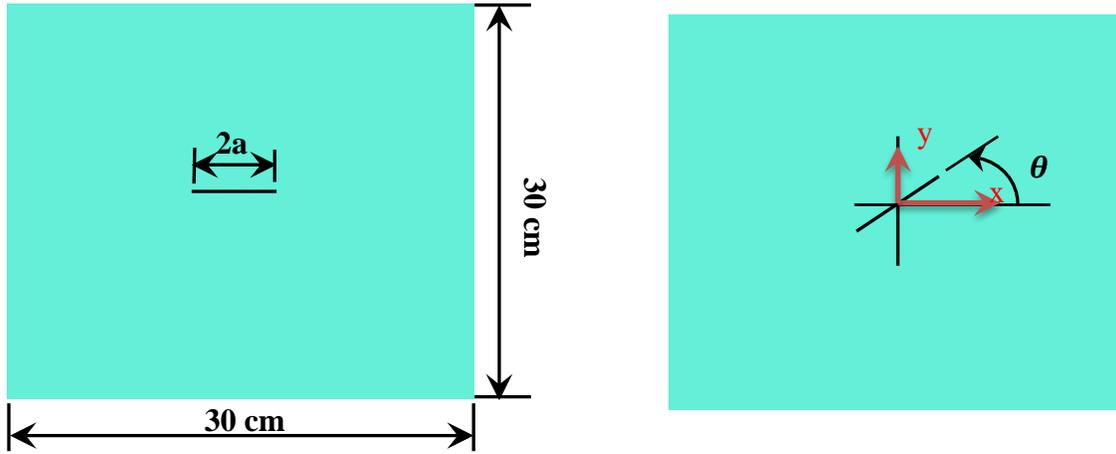


Fig. (1): location and orientations of crack

Table (1): mechanical properties of steel plate, Dr. N. V. Srinivasulu (2012)

Parameters	Value
Modulus of Elasticity (E)	207 Gpa
Density (ρ)	7850 kg/m ³
Poisson's Ratio (ν)	0.29

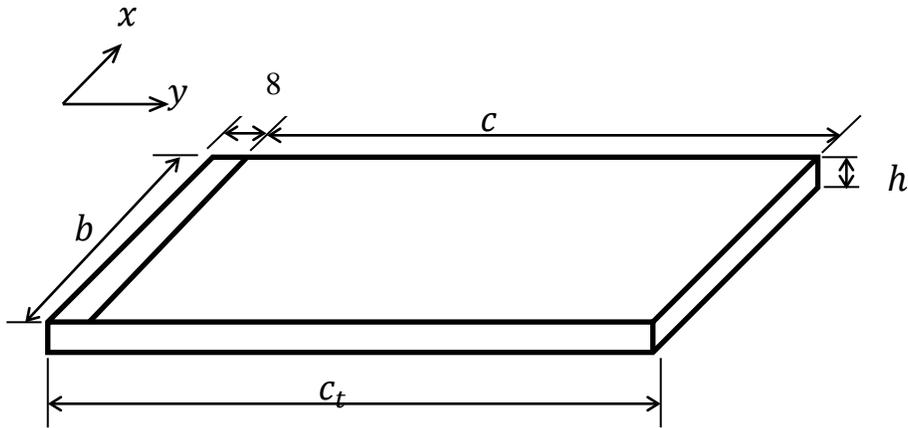


Fig. (2): shape and dimensions of the plate sample used in vibration test



Fig. (3): rig and vibration test of the plate structure

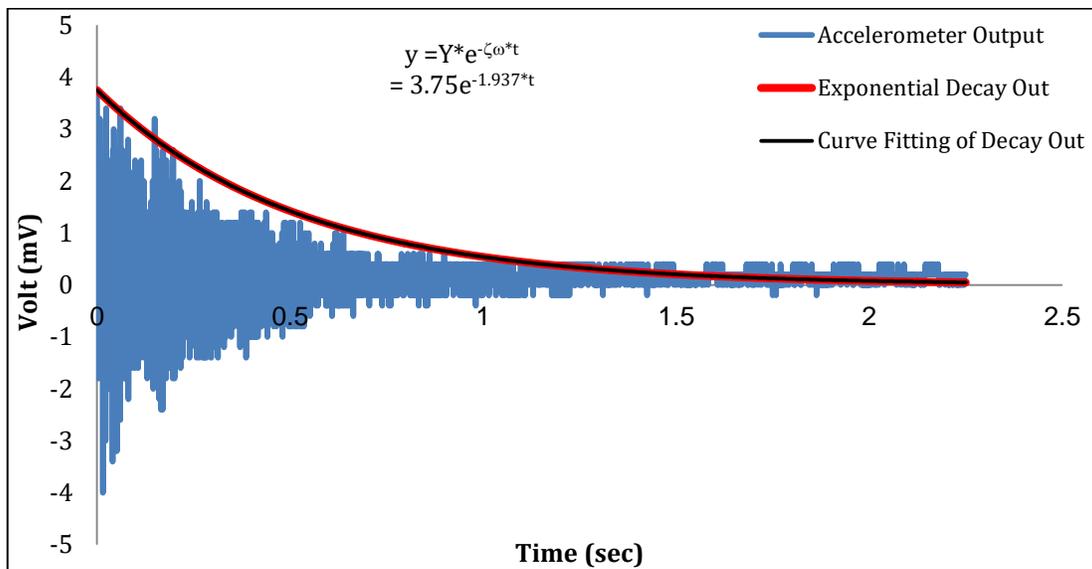


Fig. (4): experimental signal of free vibration

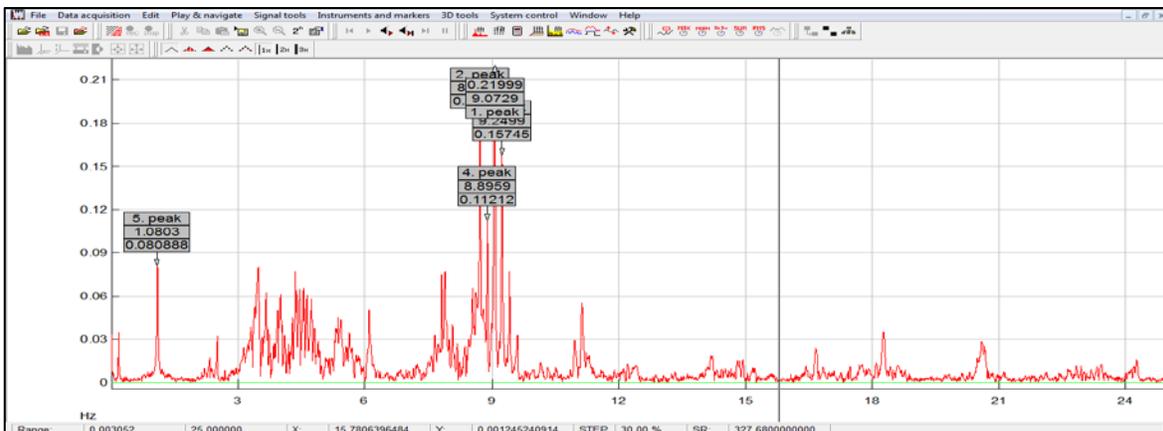


Fig. (5): experimental signal analysed by sigview software

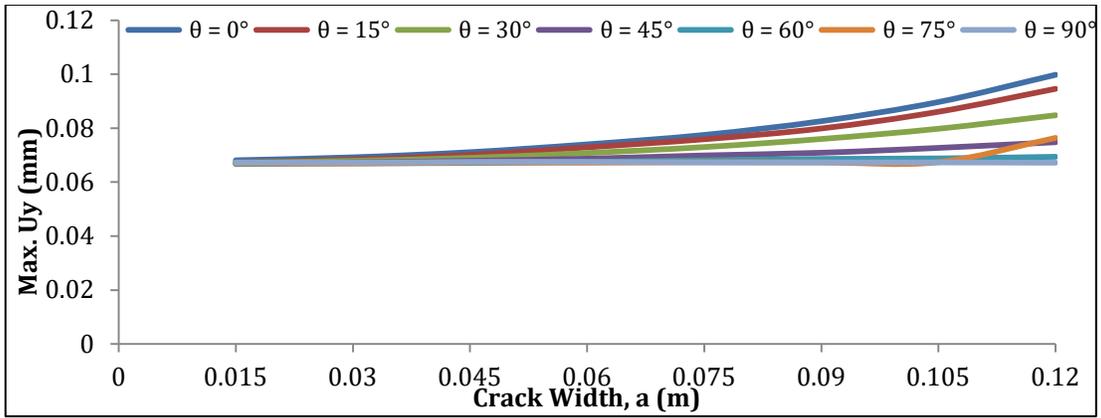


Fig. (6): maximum displacement in y-direction as a function of half-crack length of different crack angle

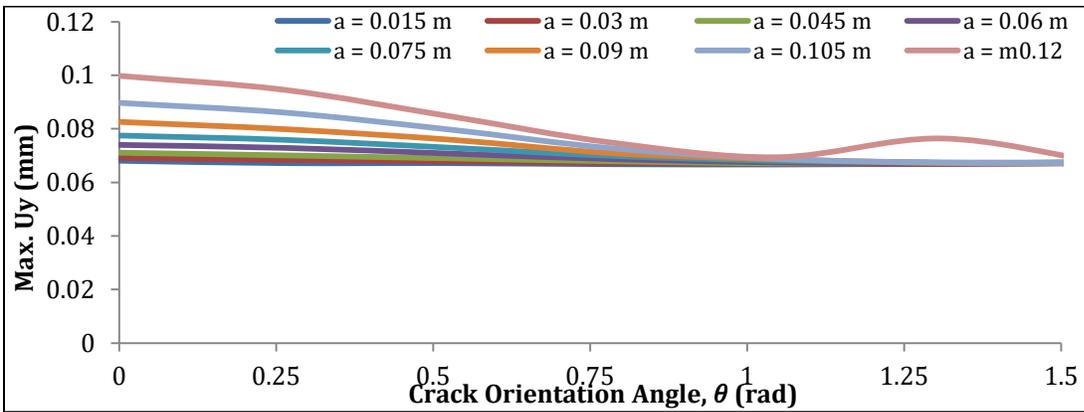


Fig. (7): maximum displacement of cracked plates as a function of crack angle for different crack length

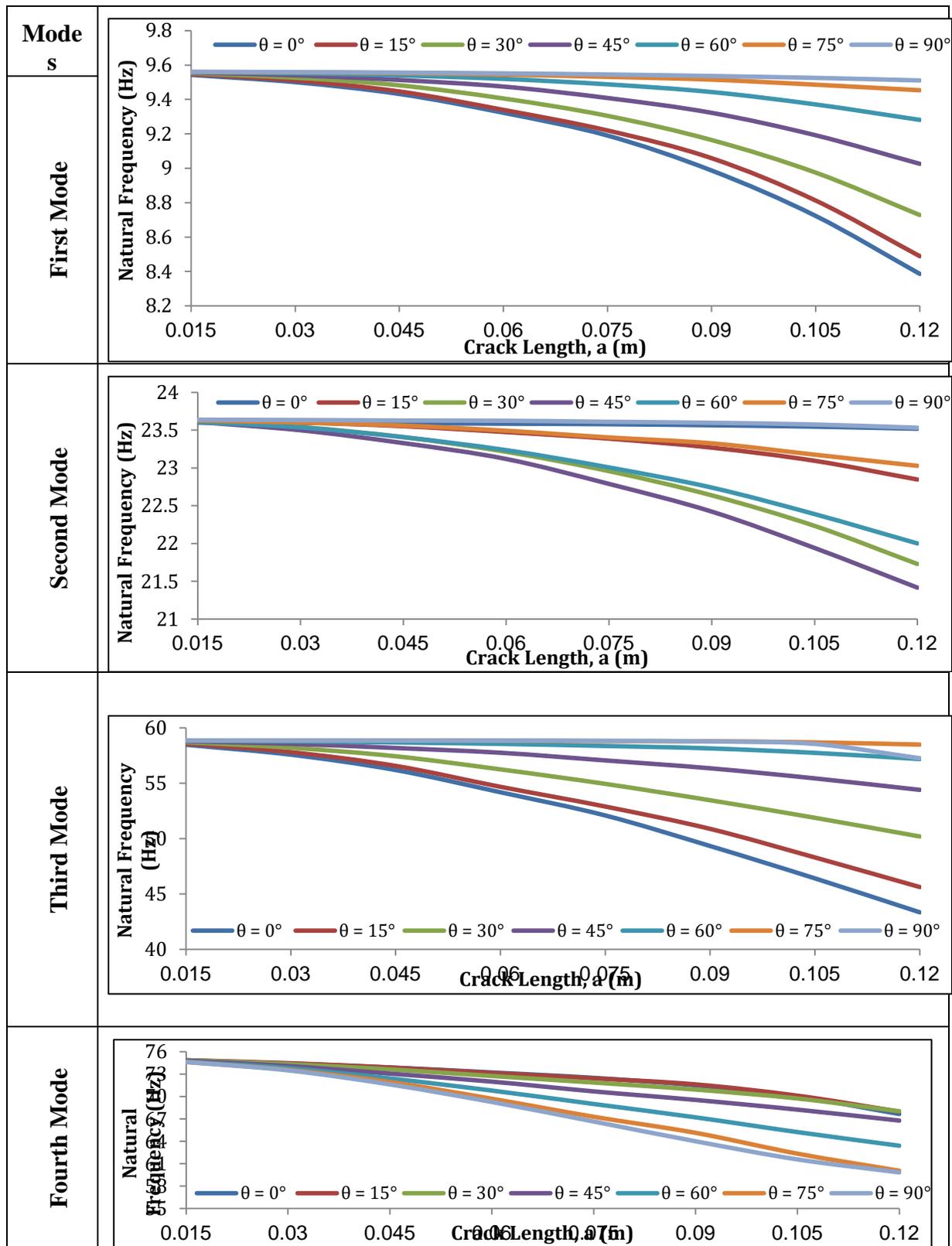


Fig. (8): first fourth modes of natural frequencies for various width and orientation of the cracked plate

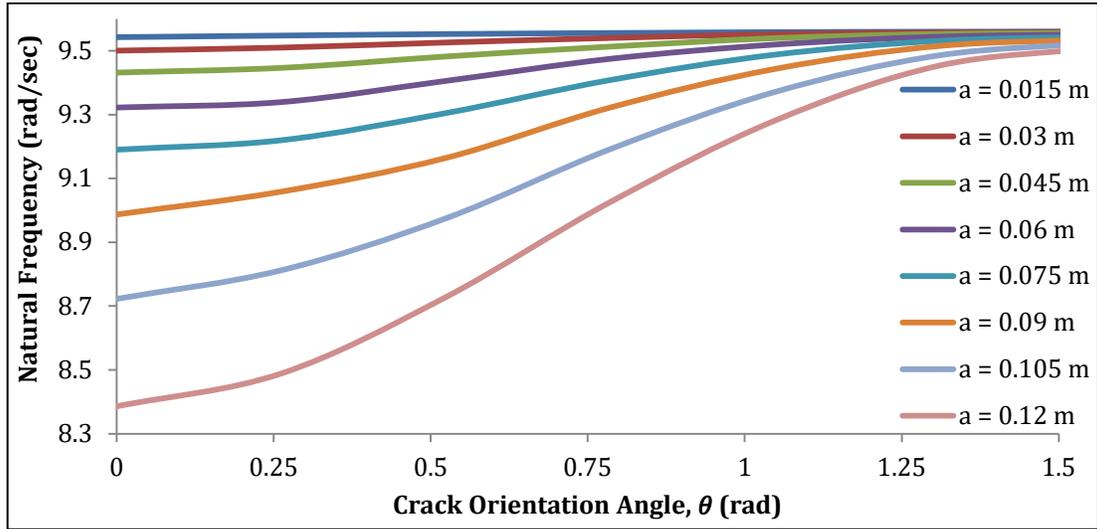


Fig. (9): first mode of natural frequency as a function of crack angle orientation for different crack width

Table (2): formulas of prediction the maximum static deflections for cracked plate models

Crack Angle	The Equations
$\theta = 0^\circ$	$Uy(a) = 0.0019a^2 - 2E-05a + 7E-05$
$\theta = 15^\circ$	$Uy(a) = 0.0017a^2 - 7E-06a + 7E-05$
$\theta = 30^\circ$	$Uy(a) = 0.0009a^2 + 4E-06a + 7E-05$
$\theta = 45^\circ$	$Uy(a) = 0.0006a^2 - 6E-06a + 7E-05$
$\theta = 60^\circ$	$Uy(a) = 0.0001a^2 + 8E-06a + 7E-05$
$\theta = 75^\circ$	$Uy(a) = 0.0011a^2 - 6E-05a + 7E-05$
$\theta = 90^\circ$	$Uy(a) = 0.0009a^2 - 5E-05a + 7E-05$

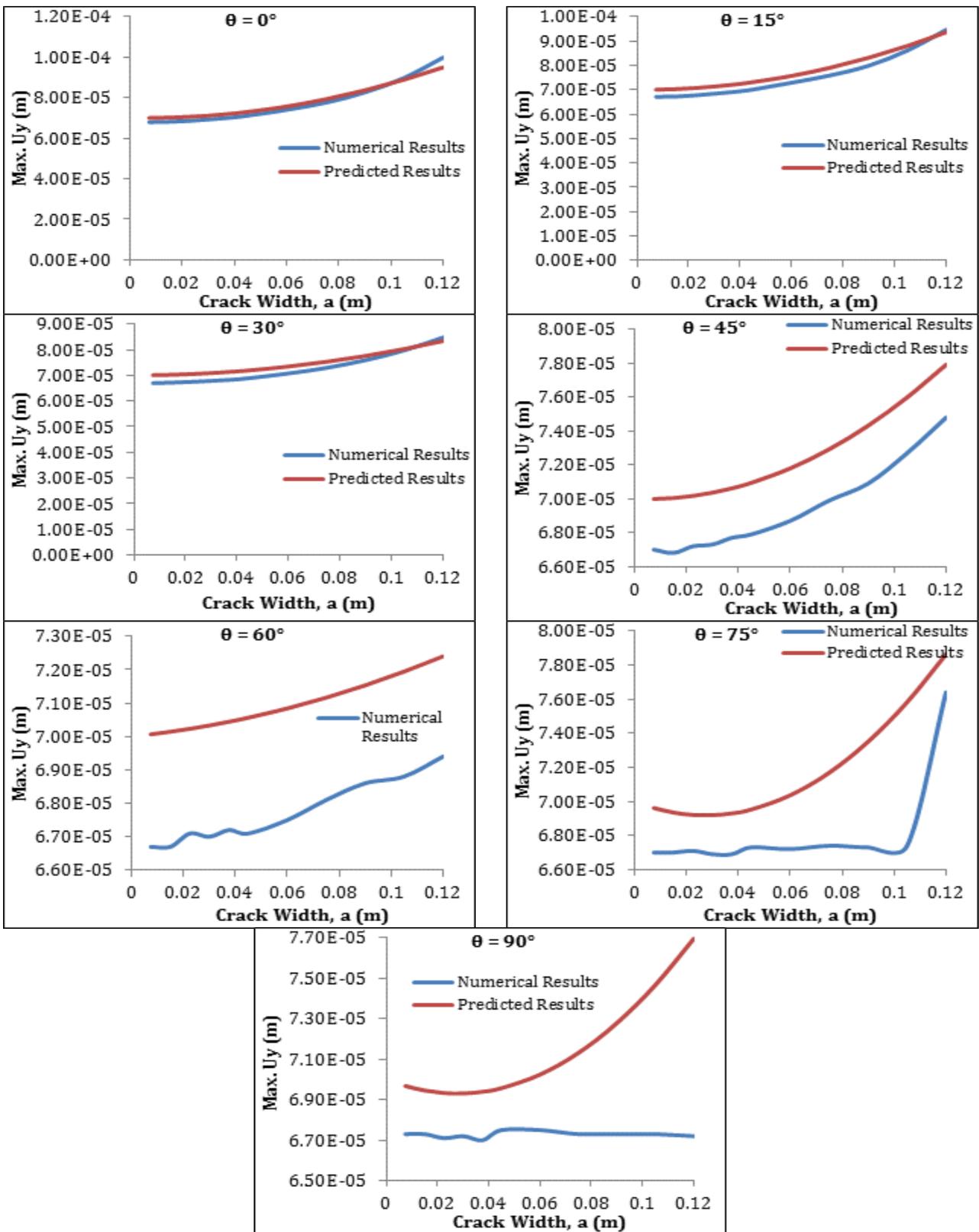


Fig. (10): a comparison between numerical and predicted results of maximum static deflections for various FEMs

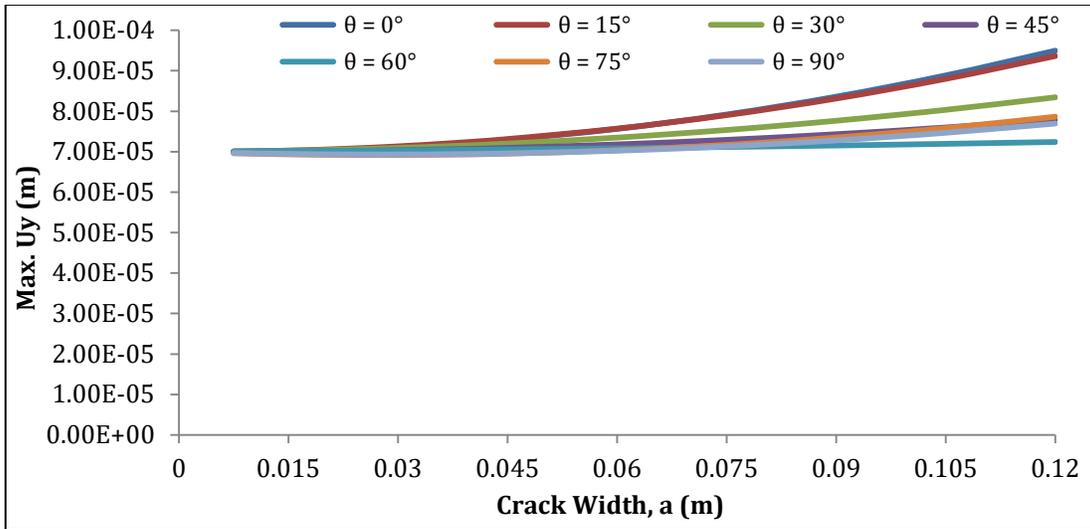


Fig. (11): the maximum static deflection for different cracked plate symbols which has been predicted

Table (3): equations to predict the natural frequencies for different cracked plate

Crack Angle	The Equations
$\theta = 0^\circ$	$f(a) = -61.05a^2 - 0.032a + 9.557$
$\theta = 15^\circ$	$f(a) = -63.15a^2 + 0.408a + 9.557$
$\theta = 30^\circ$	$f(a) = -44.86a^2 + 0.388a + 9.557$
$\theta = 45^\circ$	$f(a) = -31.83a^2 + 0.502a + 9.557$
$\theta = 60^\circ$	$f(a) = -15.85a^2 + 0.342a + 9.557$
$\theta = 75^\circ$	$f(a) = -9.333a^2 + 0.324a + 9.557$
$\theta = 90^\circ$	$f(a) = -6.941a^2 + 0.275a + 9.557$

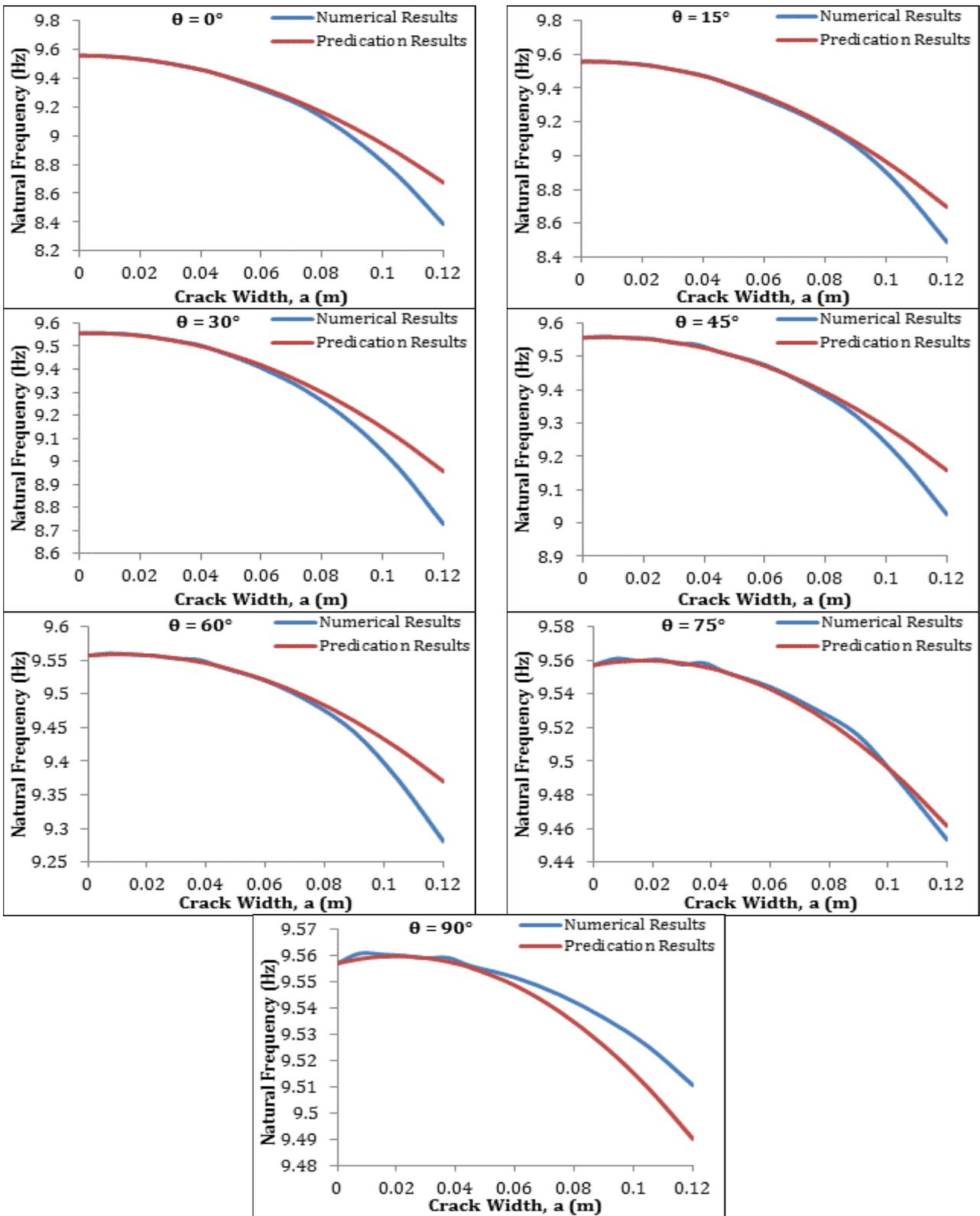


Fig. (12): a comparison between numerical and predicted results for first mode of natural frequencies

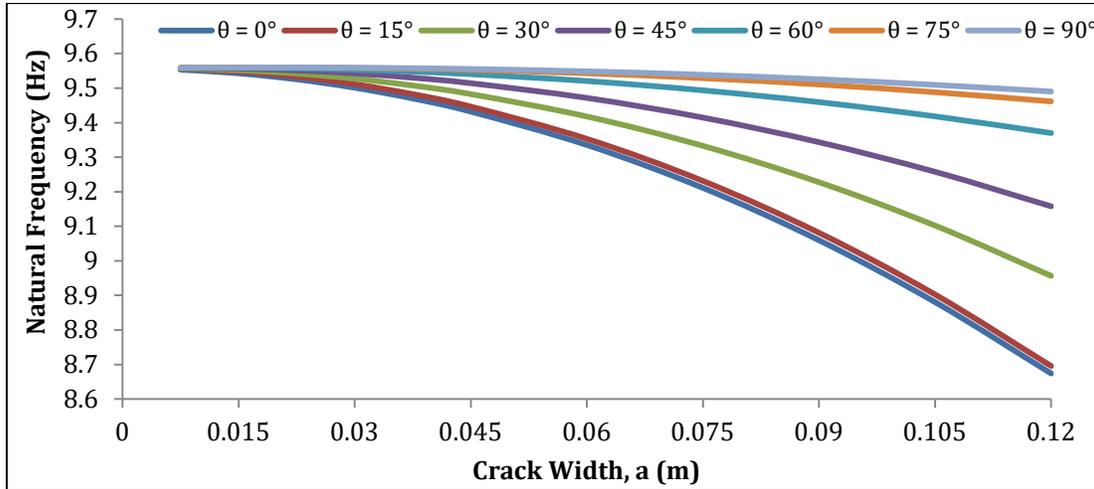


Fig. (13) : first mode of natural frequencies for different cracked plate symbols which has been predicted

REFERENCES

Choi S., S. Park, N. Park and N. Stubbs, 2006, "Improved fault quantification for a plate structure," *Journal of Sound and Vibration*, 297(3-5): 865-879.

Dr. T. H. G. MEGSON, 1996, "Structural And Stress Analysis," published in Great Britain by Arnold.

D. Wu and S. S. Law, 2004, "Damage localization in plate structures from uniform load surface curvature," *Journal of Sound and Vibration*, Vol. 276, No. 1-2, pp. 227-244.

Haider Hussein Hamad, 2012, "Dynamic Response of Solar Concentrating Power system for self-cleaning," Ph. D. Thesis, University of Technology.

Hoon Sohn, Charles R. Farrar, Francois M. Hemez, Devin D. Shunk, Daniel W. Stinemates, Brett R. Nadler and Jerry J. Czarnecki, 2004, "A Review of Structural Health Monitoring Literature: 1996-2001," Los Alamos National Laboratory Report, LA-13976-MS.

Meneghetti, U. and A. Maggiore, 1994, "Crack Detection by Sensitivity Analysis," in *Proc. of the 12th International Modal Analysis Conference*, 1292-1298.

Muhannad L. S. Al-Waily, 2012, "Investigation Of Health Monitoring Of Composite Plate Structures-Using Vibration Analysis," Phd Thesis, Alnahrain University.

Rainah Ismail, 2012, "Vibration Analysis of a Plate With an Arbitrarily Orientated Surface Crack," PhD Thesis, University of Glasgow.

Wisam Auday Hussain, 2005, "Non-linear Dynamic Analysis of Spur Gear, Analytical and Numerical Studies," M. Sc. Thesis, University of Technology.