The Iraqi Journal For Mechanical And Material Engineering, Vol.15, No4, 2015

STATIC AND DYNAMIC ANALYSIS OF ASTIFFENED CYLINDER UNDER THE ACTION OF DIFFERENT LOAD CONDITIONS

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ABSTRACT : -

In this paper, the effect of different types of stiffeners on the cylindrical shell structure is presented. The stiffened shell models are examined under the action of static and dynamic excitations. Finite element models for the different stiffened cylindrical steel shell have been created by considering the helix angles and height of stiffeners, and a cantilever supported models are used. Additionally, the models are assumed to have a constant mass which is (4 kg) . All structure models have examined under action of static, harmonic and transient loads. Also the dynamic properties (natural frequencies) are computed for each FEMs. A concentrated load of the magnitude of (100 N) in three directions (x, y and z) is used in these structural analyses (static, harmonic and transient). This force is applying to the upper edge which is opposite the clamping edge . From these analyses, the best helix angle occurs at stiffener angle (90^o), and the best height of stiffener that the modal analysis gives is (2 cm) and that gives by other analysis is (6.25 cm) . Finally, a comparison between experimental work of **[M. Bagheri. and A. A. Jafari(2006)]** and numerical part of the current paper has been occurred with a small percentage error between them.

التحليل الاستاتيكي والديناميكي لاسطوانة مقواة تحت تاثير احمال مختلفة

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الخلاصة :-

في هذا البحث تم استعراض تأثير انواع مختلفة من عناصر التقوية على هيكل اسطواني رقيق. حيث تم اختبار النماذج المقواة سكونياً وحركيا. باستخدام طريقة العناصر المحددة تم بناء نماذج اسطوانية الشكل مقوات من الحديد الصلب مع الاخذ بنظر الاعتبار زاوية ميلان وارتفاع عناصر التقوية، و قد تم تثبيت النماذج من جهة وتركها حرة من الجهة الاخرى بالإضافة الى ثبوت الكتلة لكل النماذج المختلفة حيث كان مقدار الكتلة هو (4 كغم).

ان كل النماذج تم اختبارها تحت تأثير الاحمال السكونية ، الترددية و الاحمال الانتقالية بالإضافة الى حساب الترددات الطبيعية لكل نموذج من نماذج العناصر المحددة، و الحمل المسلط هو عبارة عن حمل مركز ذو قيمة مقدارها (100 نت) سلط بثلاث اتجاهات (x, y and z) و قد سُلِط هذا الحمل عند الحافة العليا المقابلة لحافة التثبيت.

من خلال تلك التحليلات التي تم استخدامها تبين ان افضل زاوية ميلان لعناصر التقوية هي (90 درجة) لكن افضل ارتفاع يعطيه التحليل الحر كان مقداره (2 سم) و (6.25 سم) تم الحصول عليه من التحليلات الاخرى.

اخيراً تم اجراء مقارنة بين الجزء العملي للباحثين **[M. Bagheri. and A. A. Jafari(2006)]** والجزء النظري لهذا البحث للتأكد من صحة النتائج مع ظهور نسبة خطأ صغيرة بين البحثين.

KEYWORDS: Modal Analysis, Static Analysis, Transient Analysis, Harmonic Analysis, Von Mises Stresses, Finite Element Models, Stiffeners.

INTRODUCTION :-

The using of stiffened shell structural in most branches of structural engineering began in the nineteenth century with the application of flat or curved steel plates for hull of ships and subsequently with the development of steel bridges and aircraft structures. The stiffened form provides higher stiffness and carrying capacity for a given structural weight. Though the stiffened shell proved very efficient in cost and material economy, its analysis, however posed a formidable challenge. For this reason the analysis of stiffened shell has attracted many research workers, [**Tahseen Al−Qahwaji (2004)]**.

B. Gangadhara Prusty and S.K. Satsangi (2001), presented a modified approach of a curved shear flexible element. The modified approach can be considered as an alternative to the degenerated theory of shell analysis. A new approach of distributing the stiffness values of stiffener to shell nodes using a three-noded curved beam element has been tried and found to be satisfactory.

Meixia Chen et. al. (2013), developed wave Based Method (WBM) which can be recognized as a semi-analytical and semi-numerical method to analyze the free vibration characteristics of ring stiffened cylindrical shells with intermediate large frame ribs for arbitrary boundary conditions. Boundary conditions and continuity conditions between different substructures are used to form the final matrix whose size is much smaller than the matrix formed in finite element method. Numerical calculations of WBM model show good agreement with the results calculated by finite element method.

D. Tounsi et. al.(2014), presented a dynamic analysis of a stiffened cylindrical shell using the Dynamic Stiffness Method, also known as the Continuous Element Method. This approach is based on the determination of the dynamic stiffness matrix of an unmeshed structure. A finite element model was used in order to validate the numerical results obtained from the method.

In this work, the static and dynamic analysis of stiffened cylindrical shell with a constant mass(4-kg) are investigated and depending on the variation in the stiffeners parameters such as; helix angle and height of stiffeners. The models are built by using finite element approach by (ANSYS program V. 11), where the natural frequency and stresses of the structures are obtained from the structural analyses that analyses offered (static, free vibration and forced vibration).

GEOMETRICAL AND MATERIAL PROPERTIES OF STIFFENED SHELL :-

The cylindrical shell have a stiffened shell with different helix angles and height of stiffeners . The angle of stiffeners ranges from 0^o to 90^o increasing by 22.5^o (i.e. 0^o ,22.5^o,45^o, 67.5^o and 90^o) and the height of stiffeners have a variable values of (2cm & 6.25cm).

This modeling is started with arbitrary stiffener dimension and modified to keep the mass of cylindrical stiffened shell at 4kg.

The material properties depending on**[M. Bagheri. And A. A. Jafari(2006)]** and geometrical dimensions of stiffened cylindrical shell is shown in table(1). A cantilever supporting is used and a concentrated load in three directions (F_x, F_y, F_z)is applied at the upper free edge of cylinder. Two paths (path A is a longitudinal one and path B is a circumferential path) are used to obtain the results along these paths. The finite element of stiffened shell geometry is shown in figure(1).

In this paper, two types of elements have been used to build the stiffened cylindrical shell model. The first one is SHELL281 element for modeling the cylinder body and SHELL181 element for modeling the stiffeners.

THEORETICAL CONSIDERATION :-

The theoretical part includes the evaluation of natural frequencies and Von Mises stresses for different models by using finite element method:

Modal Analysis:

If any elastic structure is disturbed in an appropriate manner initially at t=0, the structure can be made to oscillate harmonically. This oscillatory motion is a characteristic property of the structure and it depends on the distribution of mass and stiffness in the structure .The oscillatory motion occurs at certain frequencies known as natural frequencies or characteristic values and it follows well deformation patterns known as mode shapes or characteristic modes. The analysis of free vibration is usually called modal analysis which is used to determine the vibration characteristics (natural frequencies and mode shapes) of a structure while it is being designed, it also can be starting point for another, more detailed, dynamic analysis, such as a transient dynamic analysis and a harmonic response analysis .The natural frequencies and mode shapes of a structure are very important parameters in the design of a structure for dynamic loading conditions. The basic equation solved by the modal analysis is, [**Hussain A.Dawood (2005)]**

$$[M]\ddot{\bar{\delta}} + [K]\bar{\delta} = 0 \qquad (1)$$

Von-Misses Stress

The von Mises stress is an equivalent or effective stress at which yielding is predicted to occur in ductile materials. In most textbooks for machine design, such a stress is derived using principal axes in terms of the principal stresses σ_1, σ_2 and σ_3 as

$$\acute{\sigma} = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2} \qquad (2)$$

and some of textbooks for machine design began to show that the Von Mises stress with respect to non-principal axes can also be expressed as**, [Ing-Chang Jong And William Springer(2009)]**

$$\acute{\sigma} = \frac{1}{\sqrt{2}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\right]^{1/2} \qquad (3)$$

The von Mises stresses were calculated under the influence of static and dynamic load effects.

Static Analysis

A static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia and damping effects, such as those caused by time-varying loads. A static analysis can, however, include steady inertia loads (such as gravity and rotational velocity), and time-varying loads that can be approximated as static equivalent loads (such as the static equivalent wind and seismic loads commonly defined in many building codes), [**ANSYS (2012)]** .

The basic equation solved by a static analysis is

$$[K]\ \bar{\delta} = \bar{F}(t) \qquad (4)$$

Dynamic Analysis

A dynamic system is often subjected to some type of external force or excitation, called the forcing or the exciting function. This excitation is usually time-dependent. It may be harmonic, non-harmonic but periodic, non-periodic, or random in nature. The response of a system to a harmonic excitation is called harmonic response. The non-periodic excitation may have a long or short duration. The response of a dynamic to suddenly applied non-periodic excitations is called transient response, [**Singiresu S. Rao (2000)].**

A. Harmonic Analysis

Harmonic analysis is a very important technique used to determine the steady-state response of a linear structure subjected to loads that vary sinusoidally (harmonically) with time. The idea is to calculate the structure's response at several frequencies and obtain a graph of some response quantity versus frequency. Peak responses are then identified on the graph and stresses reviewed at those peak frequencies. This analysis technique calculates only the steady state, forced vibrations of a structure, **[Wisam Auday Hussain, (2005)]**.

The basic equation of motion for damped system solved by a harmonic analysis is of the form

$$[\mathbf{M}]\{\ddot{\boldsymbol{\delta}}\} + [\mathbf{C}]\{\dot{\boldsymbol{\delta}}\} + [\mathbf{K}]\{\boldsymbol{\delta}\} = \{\mathbf{F}_{\circ}\}\sin\bar{\omega}t \tag{5}$$

The complete solution of this equation consist of the complementary solution $\boldsymbol{\delta}_c(t)$ and the particular solution $\boldsymbol{\delta}_p(t)$. The total response is obtained by summing the complementary solution (transient response) and the particular solution (steady-state response), that is,

$$\boldsymbol{\delta}(t) = e^{-\xi\omega t}(A\cos\omega_D t + B\sin\omega_D t) + \frac{\delta_{st}\sin(\bar{\omega}t-\theta)}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \tag{6}$$

It must be warned that the constants of integration A and B should be evaluated from initial conditions using the total response given by eq. (6) and not from just the transient component of the response given in eq. (5), [**Mario P. (1985)]**

B. Transient Analysis

Transient dynamic analysis (sometimes called time-history analysis) is a technique used to determine the dynamic response of a structure under the action of any general time-dependent loads. This type of analysis can be used to determine the time-varying displacements, strains, stresses, and forces in a structure**, [Reham Ali Nema (2014)]**

Consider a system subjected to a unit impulse at t=0 . For an under-damped system, the solution of the equation of motion, [**Muhannad AL-Waily (2004)]**:

$$[\mathbf{m}]\{\ddot{\boldsymbol{\delta}}\} + [\mathbf{c}]\{\dot{\boldsymbol{\delta}}\} + [\mathbf{k}]\{\boldsymbol{\delta}\} = 0 \tag{7}$$

is given as follows:

$$\boldsymbol{\delta}(t) = e^{-\xi\omega t}\left\{\boldsymbol{\delta}(0)\cos\omega_d t + \frac{(\dot{\boldsymbol{\delta}}(0)+\xi\omega x(0))}{\omega_d}\sin\omega_d t\right\} \tag{8}$$

where, $\omega_d = \omega\sqrt{1-\xi^2}$, for under-damping vibration.

If the mass is at rest before the unit impulse is applied ($\boldsymbol{\delta} = \dot{\boldsymbol{\delta}} = 0$ for t <0 or at t =0), we obtain, from the impulse-momentum relation,

$$\text{Impulse} = f = 1 = [\mathbf{m}]\{\dot{\boldsymbol{\delta}}\}(t=0) - [\mathbf{m}]\{\boldsymbol{\delta}\}(t=0-) = [\mathbf{m}]\{\dot{\boldsymbol{\delta}}\}(0) \tag{9}$$

Thus the initial conditions are given by:

$$\delta(t=0)=\delta(0)=0$$
$$\dot{\delta}(t=0)=\dot{\delta}(0)=\frac{1}{[m]} \qquad (10)$$

Assuming that at τ, the force $F(\tau)$ acts on the system for a short period of time $\Delta\tau$, the impulse acting at t= τ is given by $F(\tau).\Delta\tau$. At any time t, the elapsed time since the impulse $t-\tau$, so response of the system at t due to this impulse alone is given by

$$\delta(t)=\underset{\sim}{F}.\,g(t-\tau)=\frac{\underset{\sim}{F}}{m\omega_d}e^{-\xi\omega(t-\tau)}\sin(\omega_d(t-\tau)) \qquad (11)$$

by solving this equation, for zero initial conditions we obtain

$$\delta(t)=\frac{1}{m\omega_d}\int_0^t F(\tau)e^{-\xi\omega(t-\tau)}\sin\omega_d(t-\tau)d\tau \qquad (12)$$

VERIFICATION CASE STUDY :-

To verify this case study, steel cylindrical stiffened shell as shown in figure (2) has been taken from **[M. Bagheri. And A. A. Jafari]** with dimensions and material properties illustrated in table (2). In this paper, this model has been solved numerically by FE approach using ANSYS program . The comparison between numerical present work and experimental part of **[M. Bagheri. And A. A. Jafari(2006)]** for first five mode of natural frequencies is shown in figure (3). It was found that the maximum difference between which is estimated by 3% .

RESULTS AND DISCUSSION :-

The numerical results include the natural frequencies and Von Mises stresses results. The Von Mises stresses are obtained for different FEMs under various effect of loads. ANSYS program is used to extract the results These loads are static, harmonic, and transient .

The structures that created have many stiffeners arrangement [height(h=2cm & h=6.25cm) and helix angle(ɵ=0^o, 22.5^o, 45^o,67.5^o and 90^o) of stiffeners].

Modal Analysis Results

The structures are examined under effect of modal analysis to estimate the first five natural frequencies with different stiffener configurations.

Figure (4) shows the effect of helix angles on the natural frequencies for two height (2cm and 6.25cm). The natural frequencies of FEMs increased with increasing stiffeners orientation to have the maximum value at (90^o) for height of stiffeners=2cm ,but the natural frequencies of models are increased with increasing the stiffener orientation to reach the maximum value at (67.5^o) for height of stiffeners=6.25cm.

The minimum natural frequencies occurred at height of stiffeners=6.25 cm with helix angle =0^o which have values (for mode1=65.09 Hz, mode2=3.207 Hz, mode3=73.207 Hz, mode4=73.683 Hz and for mode5=884.85Hz), but the maximum values happened at angle of (90^o) of height (2 cm) which have the magnitude (for mode1=283.73 Hz, mode2=283.73 Hz, mode3=301.83 Hz, mode4=327.44 Hz and for mode5=347.44 Hz).

Von-Mises Stress Results

The values of the Von Mises stresses in this work are evaluated. The applied force is static and dynamic (harmonic and transient), each one contains three individual components (x, y and z-component) with the magnitude (100 N) located at the upper free edge.

1.Static Load Effect

The models in the present work are examined statically under effect of the combined concentrated static load in x, y and z-directions at the upper free edge. The Von Mises stresses in the stiffened shell are calculated for different stiffeners configuration. These values are computed along two paths, longitudinal and circumferential paths(A and B respectively).

Figures (5 and 6) show the variations of Von Mises stresses with longitudinal and circumferential paths (path A and B) for different models. From these figures, it is found that the maximum Von Mises stresses occurred at the point which the load is applied (upper free edge) as shown in figure (10).

The maximum von Mises stresses have been drawn with helix angle with different height of stiffeners as shown in figure (10). At 90^o of stiffener orientation, the stress have the minimum value (best value), whereas it's value obtained (15.482MPa). This value appears when the structure have height of stiffeners (6.25cm). But, the maximum value (178.51MPa) is appeared at 45^o of stiffener orientation with height of stiffeners (2cm).

2. Dynamic Load Effect

The dynamic loads that used in the current work are harmonic and transient loads. The benefit of this study is to know the von Mises stresses of the stiffened shell structure under effect of these combined concentrated load at upper free edge. The stiffened shell structure of various stiffener parameters that exposure to these dynamics load are solved numerically by using ANSYS program. The value of the damping ratio that used in this analysis is computed experimentally.

A. Harmonic Load

The stiffened shell models are examined under effect of harmonic load with the magnitude of (100 sinωt N). This load is applied at free upper edge in the X, Y and Z directions as a concentrated load. The von Mises stresses are extracted numerically under influence of these loads.

The load excitation includes a frequency domain within the range from (0 Hz) to the value above the fifth mode of the natural frequency (600 Hz).

The von Mises stresses have been drawn with the frequency for different models as shown in figure (7). The results that extracted have been drawn to point which the load is applied load on these point.

In figure (10) the maximum value of Von Mises stresses due to harmonic load have been drawn with helix angle for different models. Generally, it is found that the minimum value occurs at (90^o) of helix angle. The minimum value of Von Mises stresses is (29.19 MPa) for height (6.25cm), so this model is the best one due to high stiffness.

B. Transient Load

This load is applied to various models with period (0.3 sec.) as a concentrated load. The Von-Mises stresses that extracted are shown in figures (8 and 9).

The maximum value of von Mises stresses due to transient load is shown in figure (10). These values have been drawn with helix angle of each stiffeners. It is found that the minimum value of stresses under transient load effect occur at helix angle (90^o). This stiffener have the height of (6.25 cm), whereas it's magnitude (14.12MPa).

CONCLUSIONS

According to the obtained results, the following conclusions have been obtained:

1. In the modal analysis, when the height of the stiffeners is small, frequency is increased with increasing of the helix angle.
2. It is found that for von Mises approach, the values of stresses decrease when the height of stiffeners as large as possible.
3. For harmonic analysis, the location of peak value of the response coincide with the first natural frequency because of that the frequency of the external loading is equal to the natural frequency of the structure.

Table 1: Geometrical and Material Properties for Current Work.

Parameters	symbols	value
Radius of cylinder (cm)	R	19.0985
Length of cylinder (cm)	L	60
Thickness of cylinder (mm)	Tc	0.5
Load (N)	F_x, F_y, F_z	$F_x = F_y = F_z = 100$
Modulus of elasticity (GPa)	E	201
Mass density (Kg/m³)	ρ	7823
Poisson's ratio	υ	0.3

Table (2) Geometrical and Material Properties of Reference **[[M. Bagheri. and A. A. Jafari(2006)]**.

Characteristics (dimensions)	Dimension Values M1 Model
Number of Rings (N)	4
Shell Radius R (m)	0.0825
Shell Thickness h (m)	0.0025
Shell Length L (m)	0.2475
Ring Depth dr (m)	0.0037
Ring Width br (m)	0.002
Modulus of Elasticity E (Gpa)	201
Mass Density ρ $\left(\frac{kg}{m^3}\right)$	7823
Poisson's Ratio υ	0.3
Stiffening Type	External

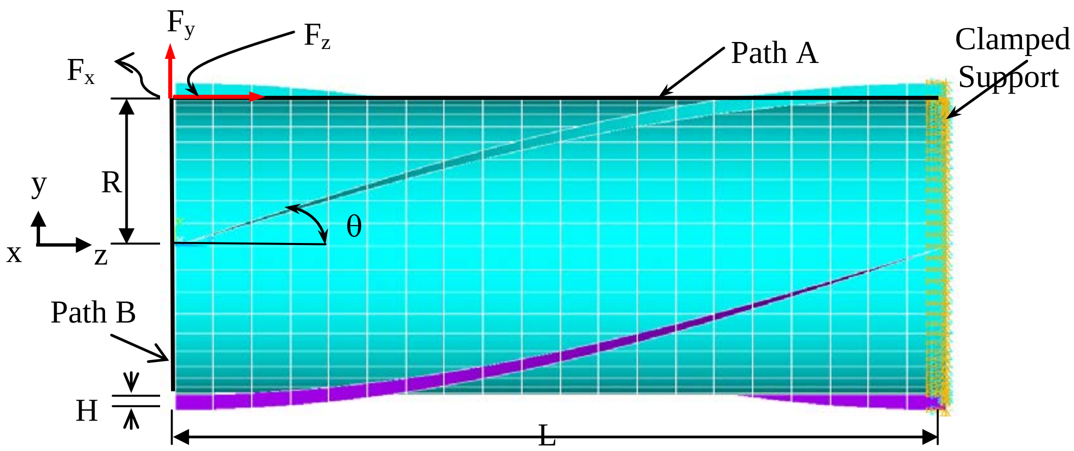


Fig.1: Finite Element Shell Geometry.

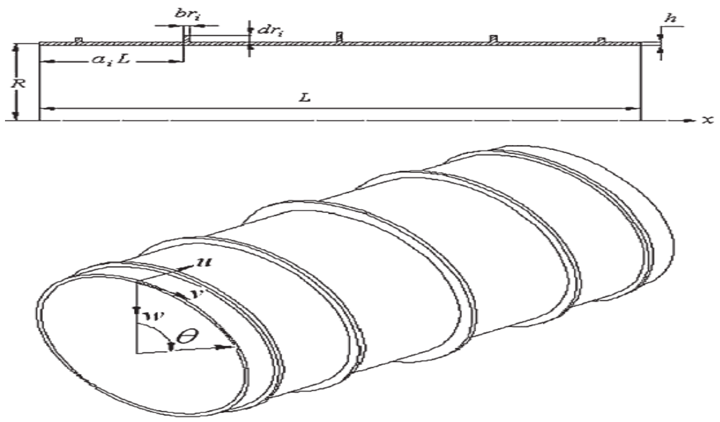


Fig.(2) Geometry of Reference **[M. Bagheri. and A. A. Jafari(2006)]**.

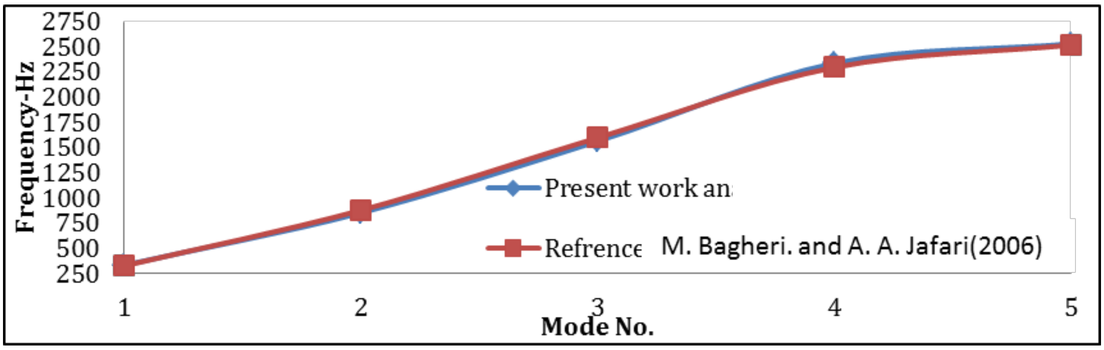


Fig.(3) Comparison between the results.

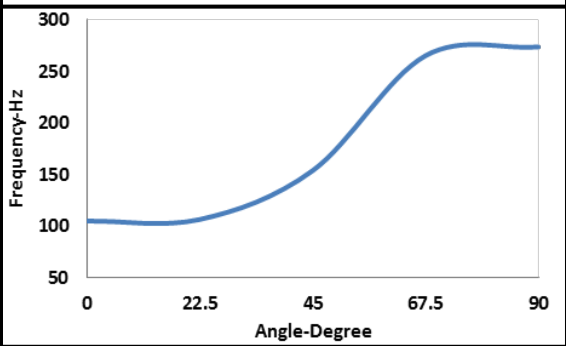
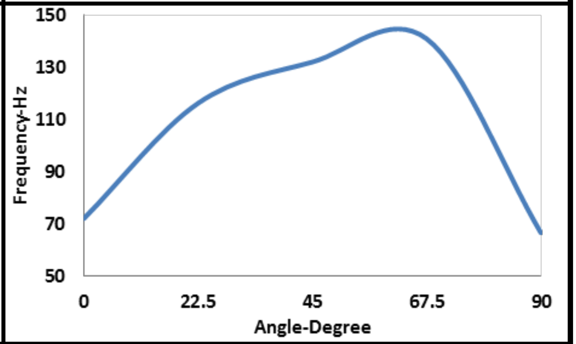
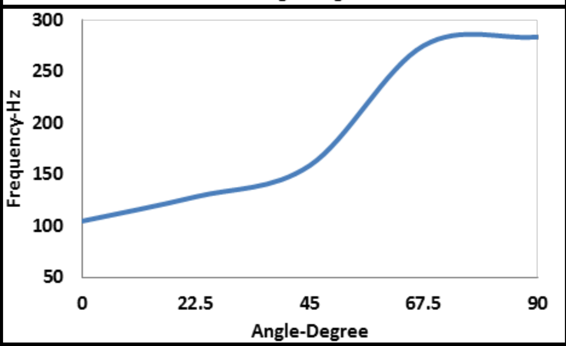
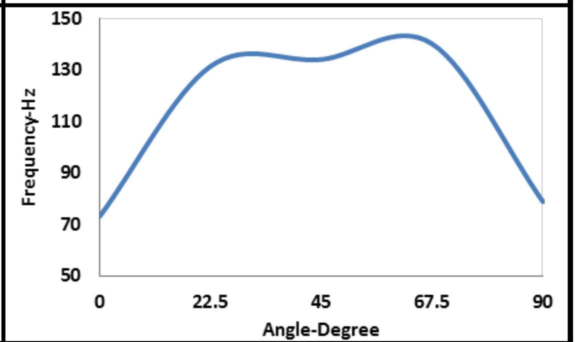
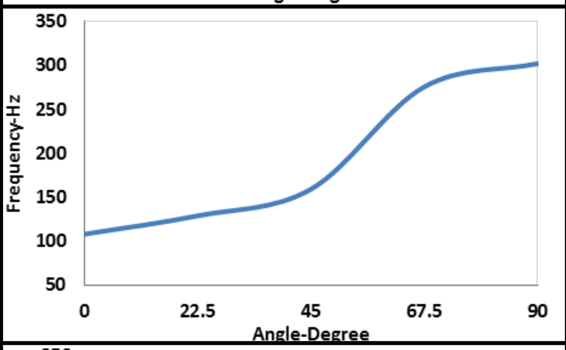
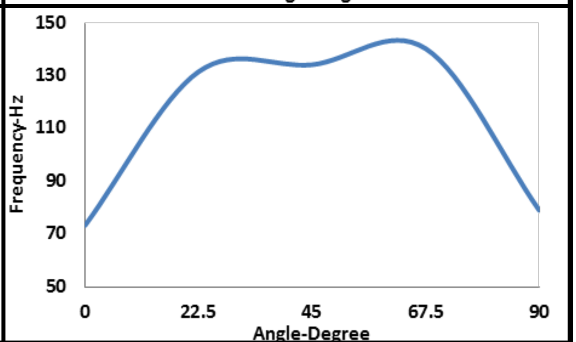
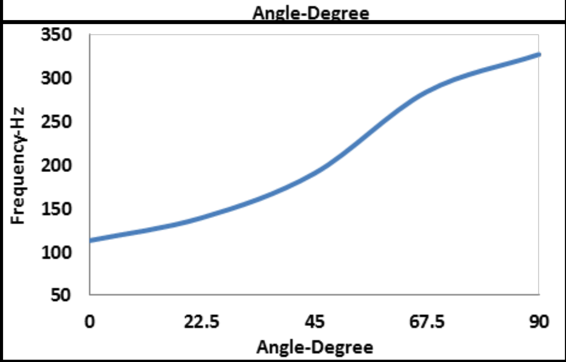
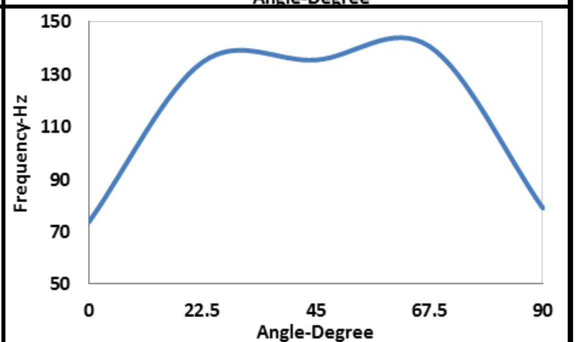
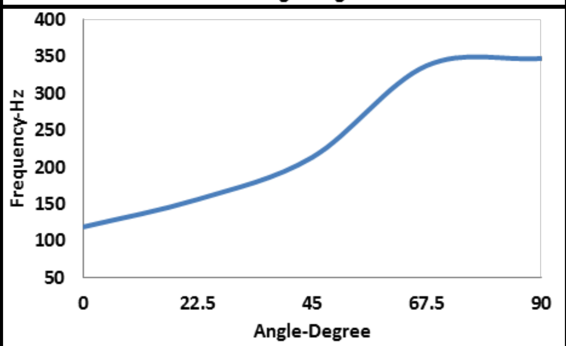
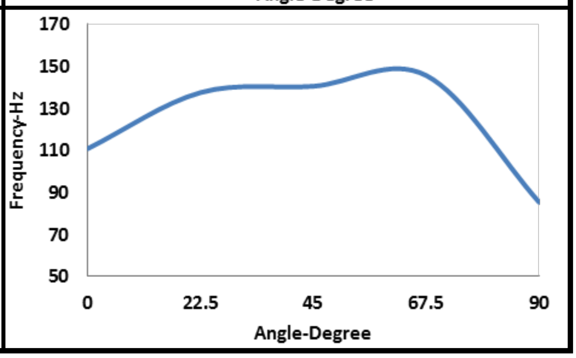
	Height of stiffeners=2cm	Height of stiffeners=6.25cm
Mode-1		
Mode-2		
Mode-3		
Mode-4		
Mode-5		

Fig. (4) Variations of Natural Frequency with Helix Angle for Different Models.

<table>
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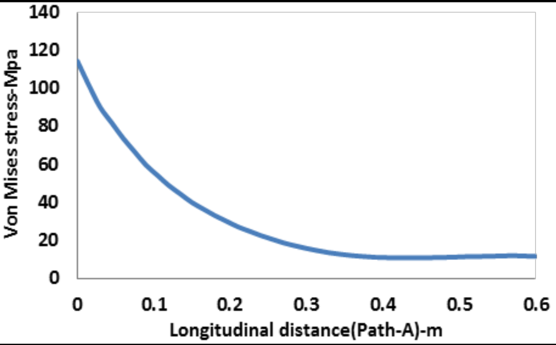
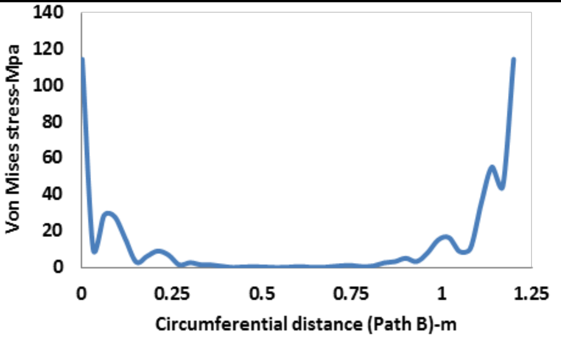
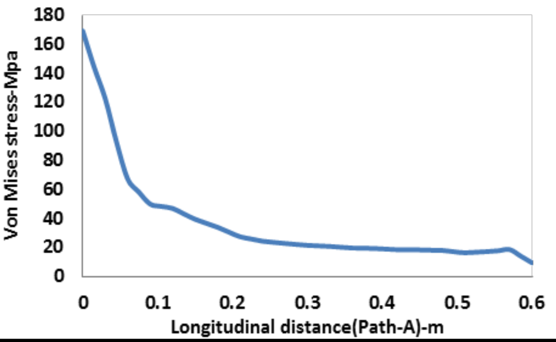
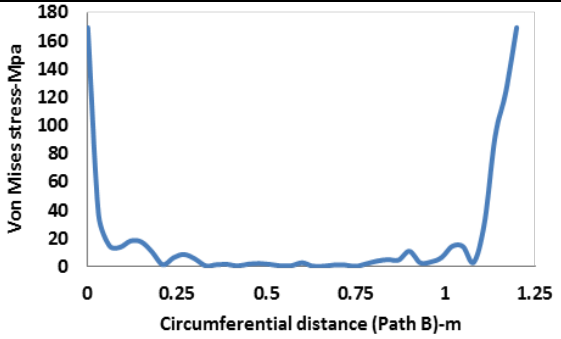
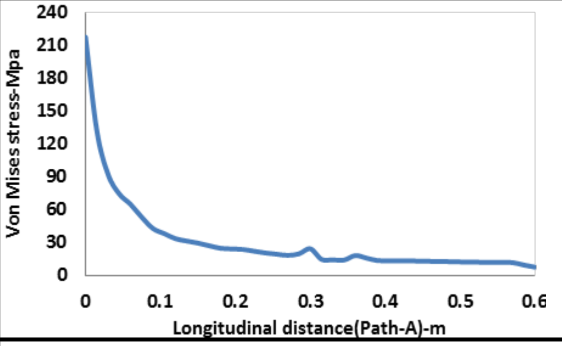
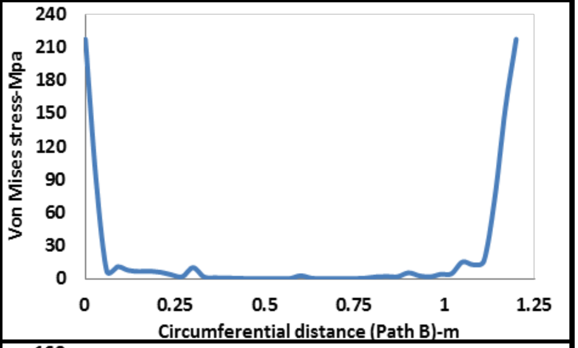
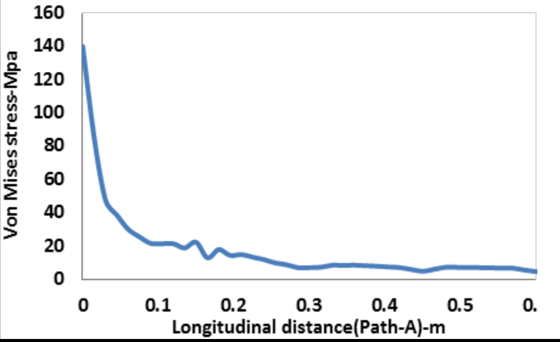
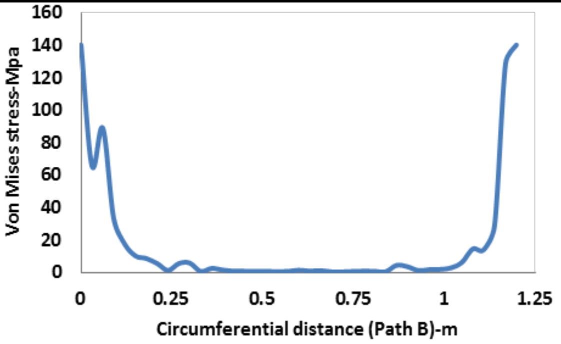
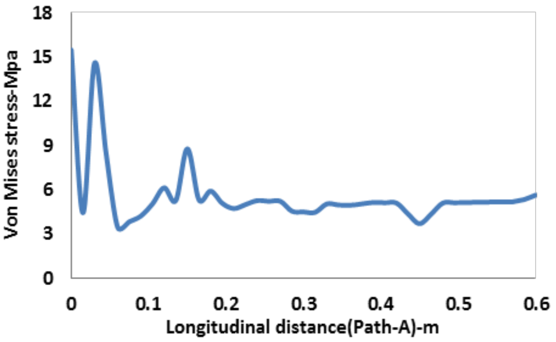
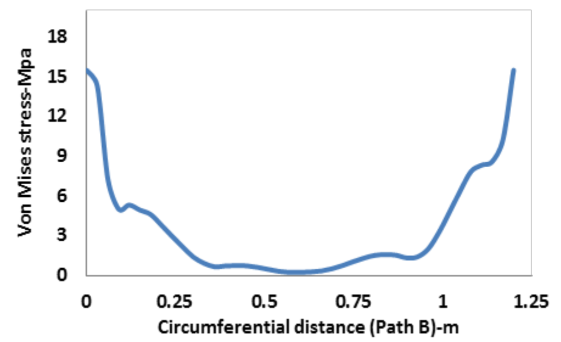
<tr><td>Theta=0°</td><td></td><td></td></tr>
<tr><td>Theta=22.5°</td><td></td><td></td></tr>
<tr><td>Theta=45°</td><td></td><td></td></tr>
<tr><td>Theta=67.5°</td><td></td><td></td></tr>
<tr><td>Theta=90°</td><td></td><td></td></tr>
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Fig. (5) Static-Von Mises Stress for Height of Stiffeners=2cm.

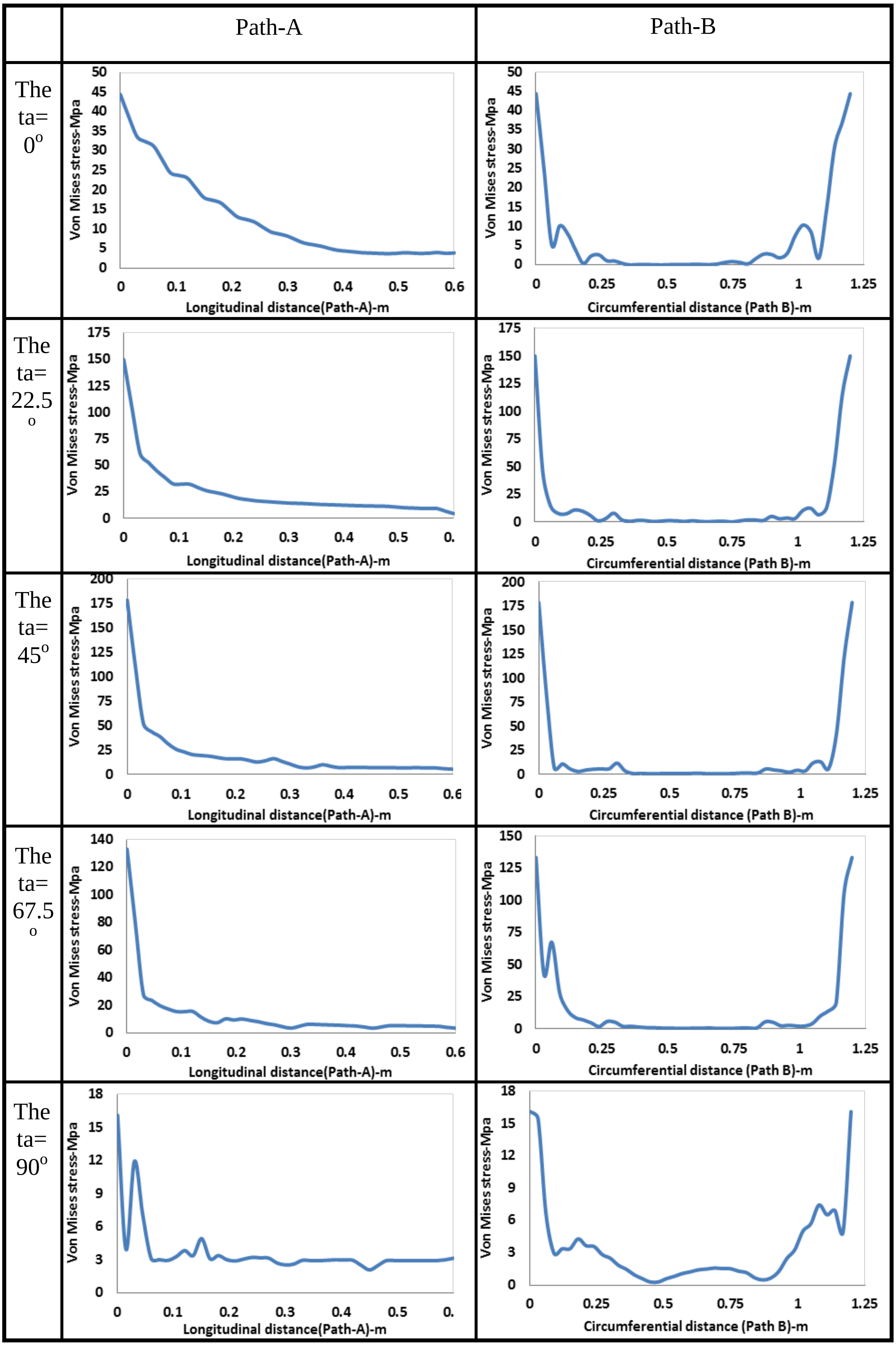
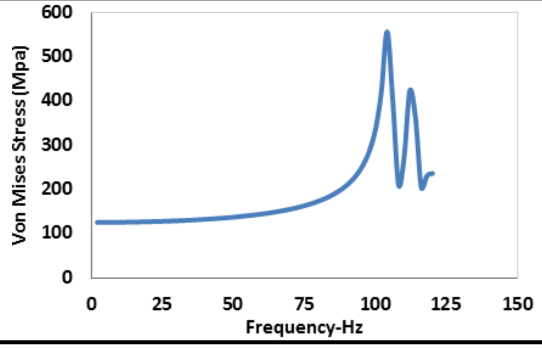
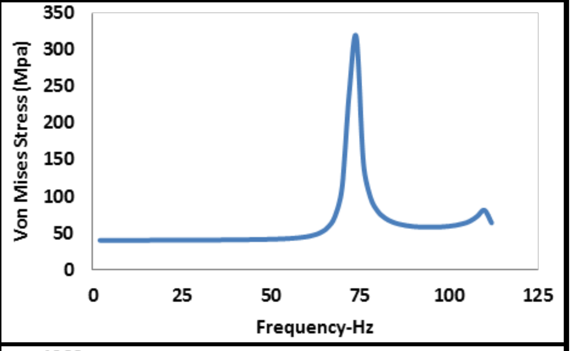


Fig. (6) Static-Von Mises Stress for Height of Stiffeners=6.25cm.

<table>
<tr><th></th><th>Height of Stiffeners=2cm</th><th>Height of Stiffeners=6.25cm</th></tr>
<tr><td>The ta=0°</td><td>
</td><td>
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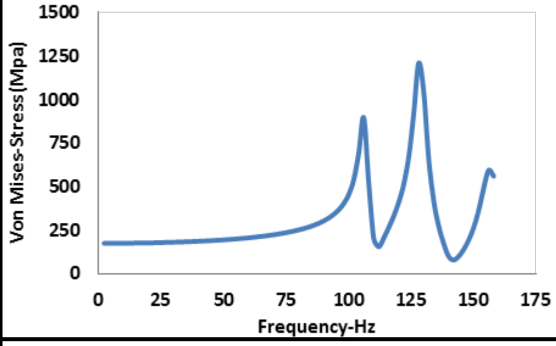
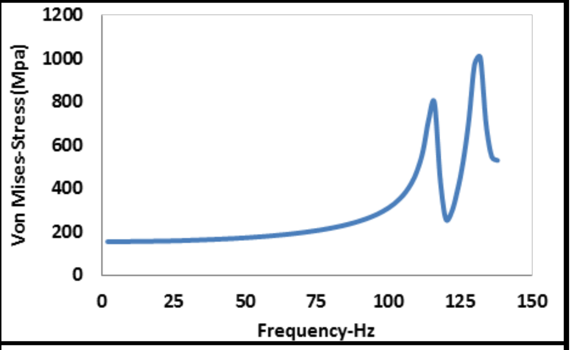
<tr><td>The ta=22.5°</td><td>
</td><td>
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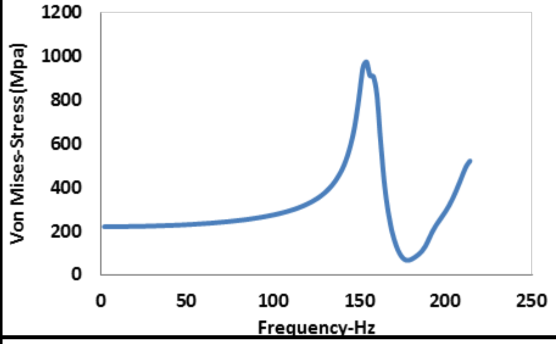
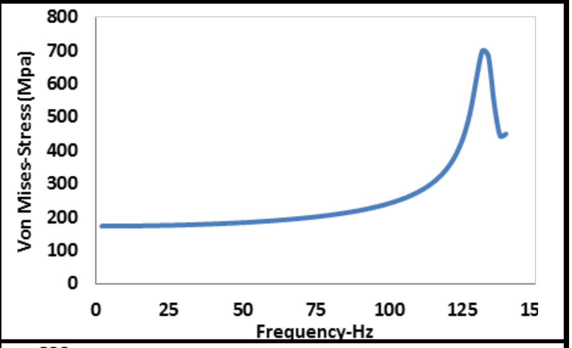
<tr><td>The ta=45°</td><td>
</td><td>
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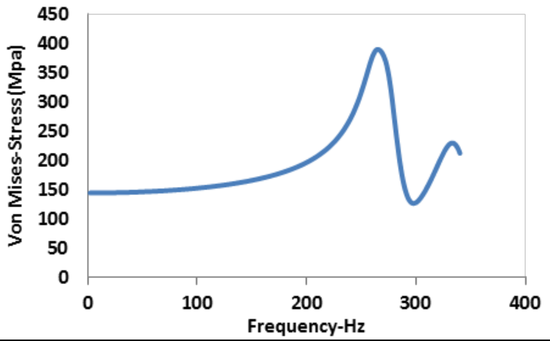
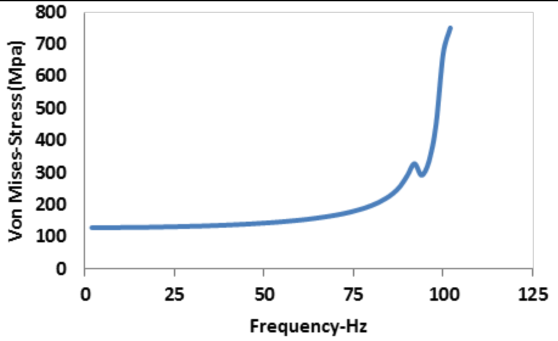
<tr><td>The ta=67.5°</td><td>
</td><td>
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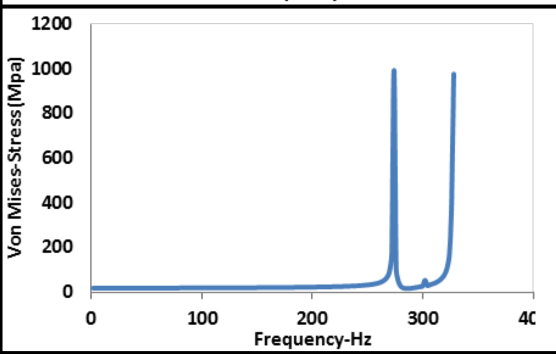
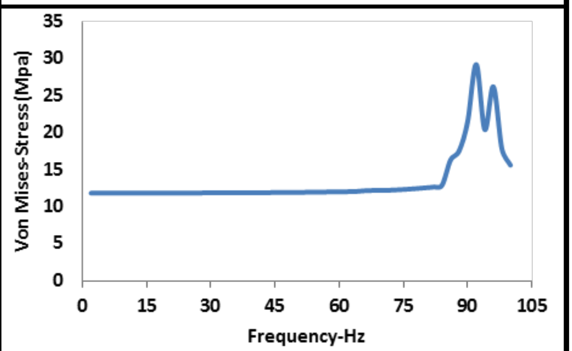
<tr><td>The ta=90°</td><td>
</td><td>
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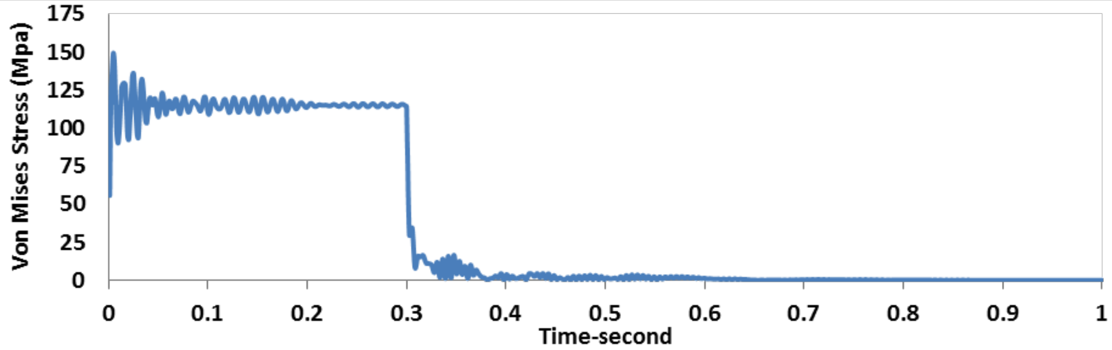
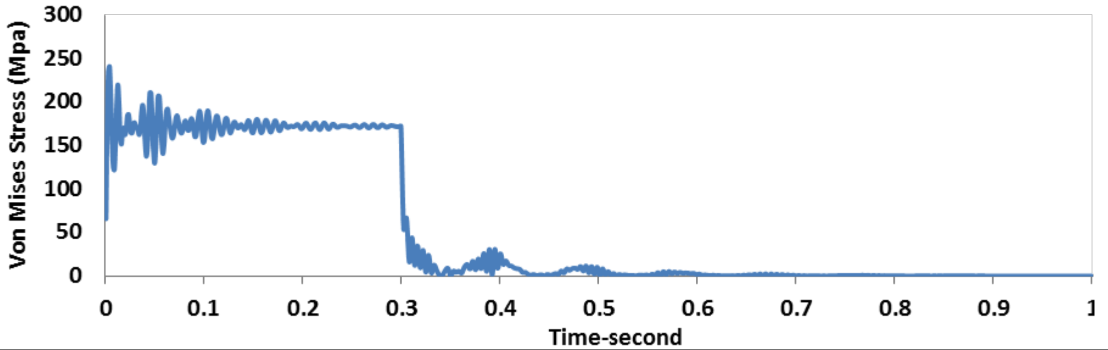
Fig. (7) Harmonic Von Mises Stress for both Height of Stiffeners.

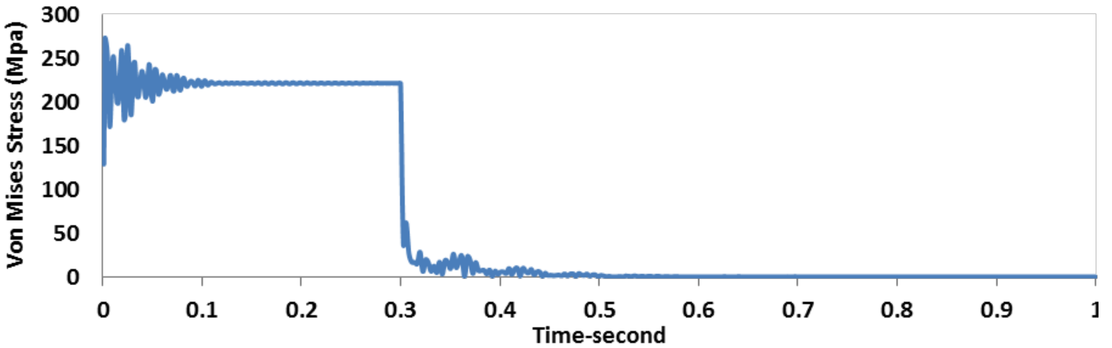
Theta=0°



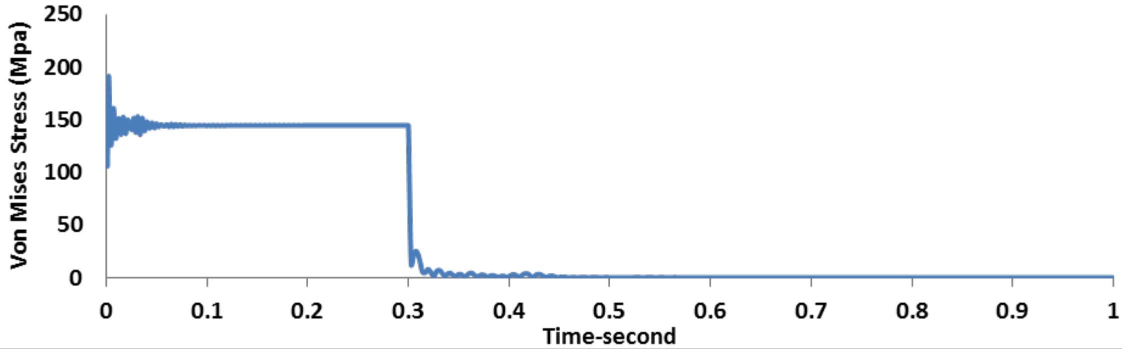
Theta=22.5°



Theta=45°



Theta=67.5°



Theta=90°

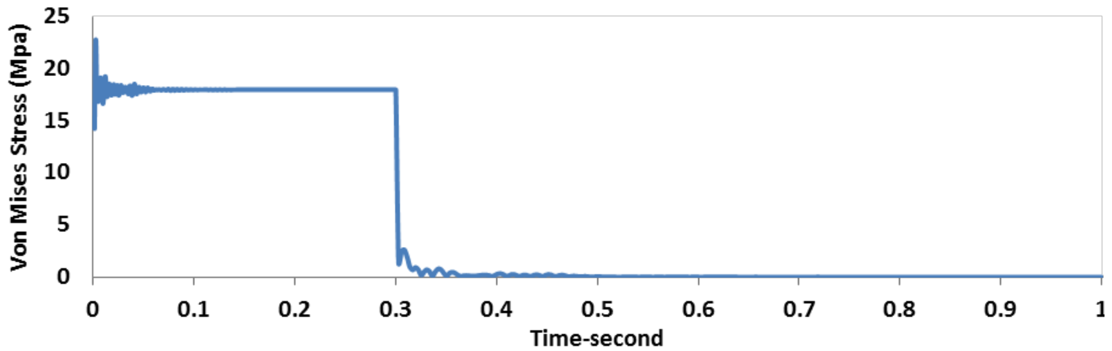


Fig.(8) Transient Von Mises Stress Height of Stiffeners=2cm.

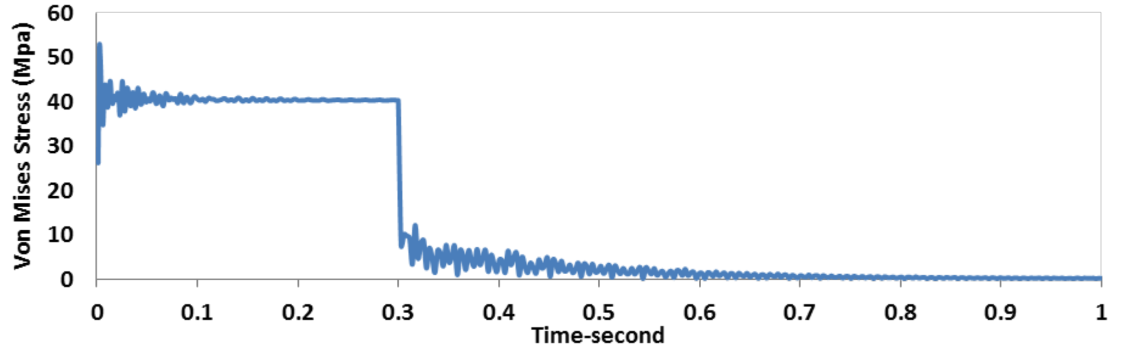
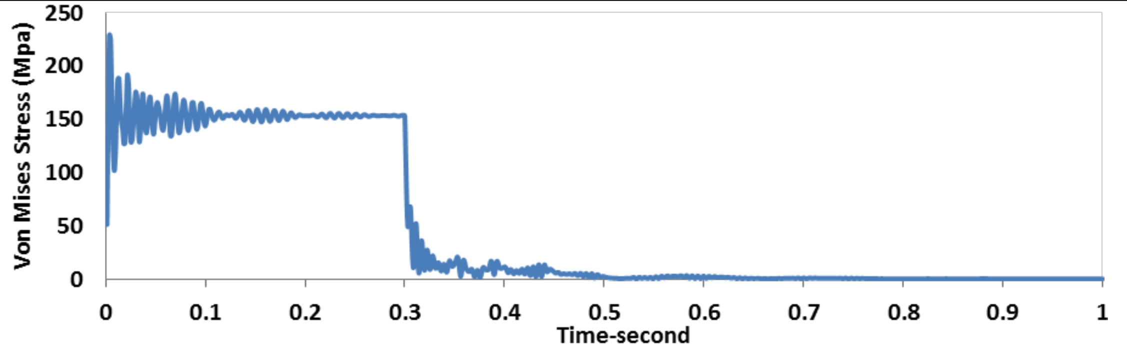
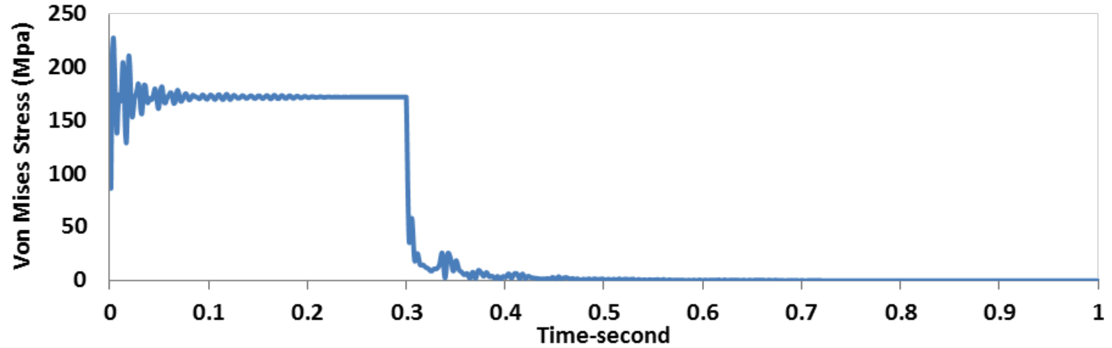
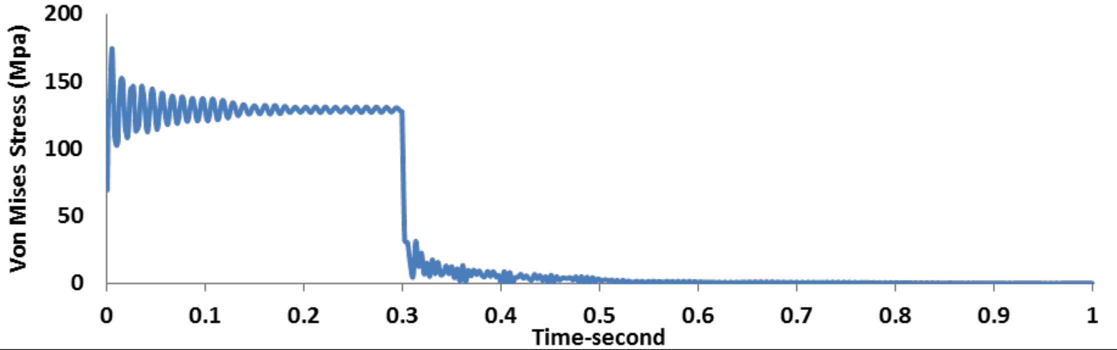
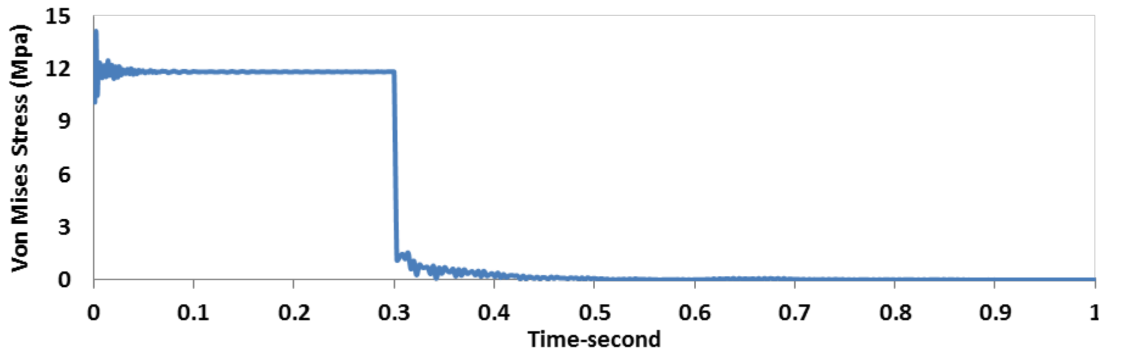
Theta=0^o	
Theta=22.5^o	
Theta=45^o	
Theta=67.5^o	
Theta=90^o	

Fig.(9) Transient Von Mises Stress for Height of Stiffeners=6.25cm.

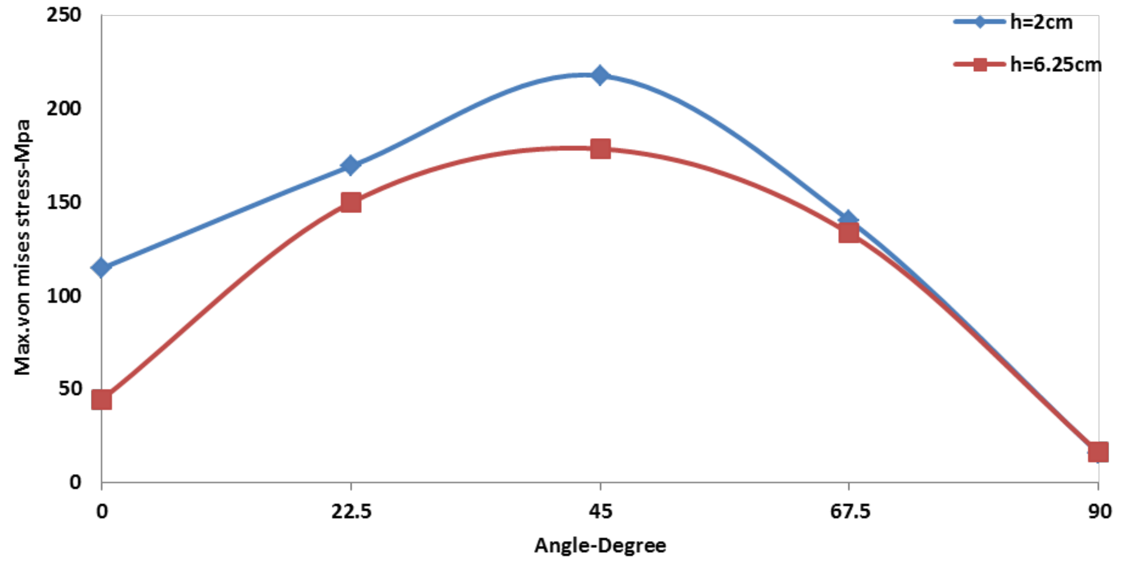
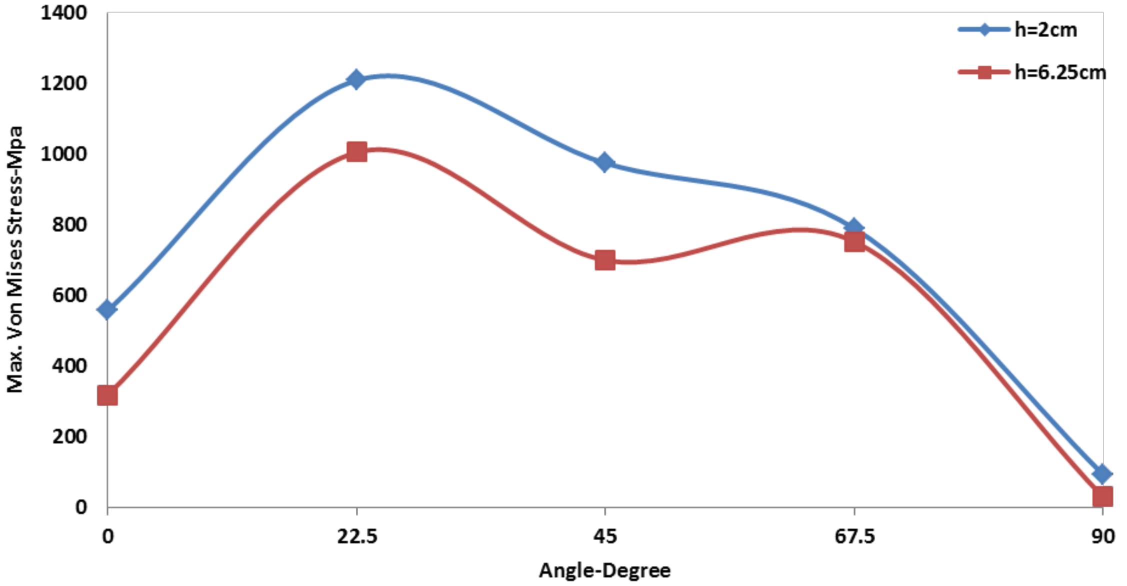
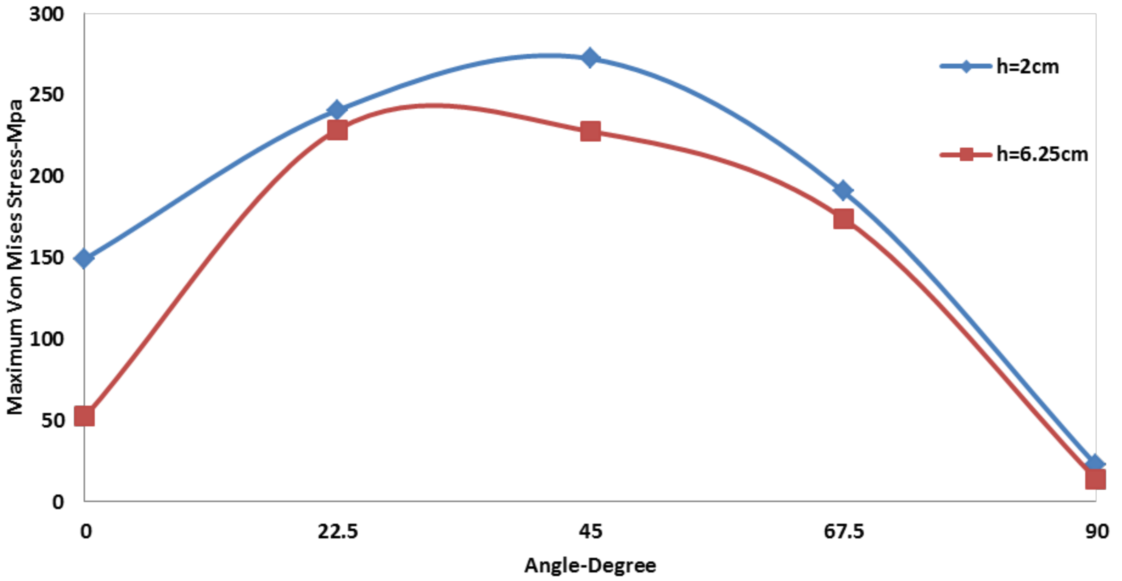
Static Load	
Harmonic Load	
Transient Load	

Fig. (10) Variation of Maximum Von Mises Stresses with Helix Angle for Different Types of Load.

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