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Effect of Curvature on Stability of Compressible Fluid Flow in circular Pipe Section

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Abstract :

In this paper a model of laminar flow for compressible fluid in curved tube and earth gravity effect was negligible. The basic differential equations which govern the flow were defined. The stability of this model was analyzed. It was found that the model is unstable under all conditions and in the compressible fluid the change in density make the fluid always unstable.

Model and Basic Differential Equations:

Introduction:

The subject of fluid mechanics is one of applied mechanics, where deals with fluid flow and static also study the equilibrium body and movements[1]

Mathematical Modeling is a study of ral phenomena using mathematical tools. A hypothesis of ideal and abstract study of model must not go far away from the problem which we study not to loss it's character. Mathematical modeling art devoted in a study of maximum factors which effect on the problem using simple tools.[2]

Al-Obaidi studied the effect of curvature on stability of fluid flow in tube. In this paper , we shall follow the same study with effect of earth gravity and the fluid compressible which are neglected by [3]

The model of study represents laminar fluid flow in tube with constant diameter. We choose the cylindrical coordinates system such that: the coordinate r represent a change of distance from the tube centre to the edge (the change of radius), the coordinate θ represent the change of flow angle and the coordinate z represent the change along the tube. The tube exposes to vertical curvature (figure 1).

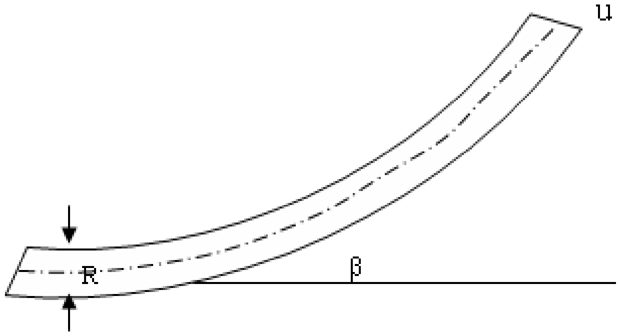

Figure (1): The model of study

R - tube radius

β - curvature angle

u - flow speed

Continuity Equation:

The continuity equation of flow given by [4] [5]

$$\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v_x)+\frac{\partial}{\partial y}(\rho v_y)+\frac{\partial}{\partial z}(\rho v_z)=0$$

and when it changes to the cylindrical coordinates becomes:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho ru)+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0 \quad \ldots(1.1)$$

using the hypothesis of solution for this model $v=w=0$ equation (1.1) becomes:

$$\frac{\partial\rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho ru)=0 \quad \ldots\ldots..(1.2)$$

by drive the equation (1.2) result :

$$\frac{\partial\rho}{\partial t}+\frac{\rho}{r}\frac{\partial(ur)}{\partial r}+u\frac{\partial\rho}{\partial r}=0$$

$$\frac{\partial\rho}{\partial t}+\frac{\rho u}{r}+\rho\frac{\partial u}{\partial r}+u\frac{\partial\rho}{\partial r}=0$$

and by multiplying the equation by u result:

$$u\frac{\partial\rho}{\partial t}+\rho u\frac{\partial u}{\partial r}+u^2\frac{\partial\rho}{\partial r}=-\rho\frac{u^2}{r} \quad \ldots\ldots(1.3)$$

Equation of motion:

The equation of motion in cylindrical coordinates given as: [4] [5]

$$\frac{\partial(\rho u)}{\partial t}+u\frac{\partial(\rho u)}{\partial r}=-\frac{\partial p}{\partial r}+F_r+\mu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right] \quad \ldots.(1.4)$$

$$\rho\frac{\partial u}{\partial t}+u\frac{\partial\rho}{\partial t}+\rho u\frac{\partial u}{\partial r}+u^2\frac{\partial\rho}{\partial r}=-\frac{\partial p}{\partial r}+F_r+\mu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right] \ldots(1.5)$$

by substations equation (1.3) into (1.5)

$$\rho\frac{\partial u}{\partial t}-\rho\frac{u^2}{r}=-\frac{\partial p}{\partial r}+F_r+\mu\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right] \quad \ldots.(1.6)$$

and using Euler equation of motion (figure 2) we have [6]:

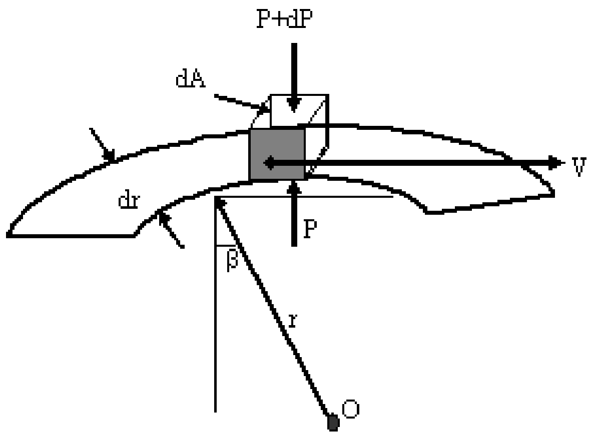

Figure (2)

$$\frac{dP}{dr}=\frac{\rho u^2}{s} \quad \ldots\ldots\ldots\ldots(1.7)$$

and representing the curvature radians s using the relation $s = \frac{L}{\beta}$ when L is the tube radiance, the equation (1.7) becomes:

$$\frac{dP}{dr} = \frac{\rho u^2 \beta}{L} \quad \text{………(1.8)}$$

substituting (1.8) in equation (1.6) and neglected the external forces $F_r = 0$ and $\mu = \rho \nu$ we obtain:

$$\frac{\partial u}{\partial t} - \frac{u^2}{r} = -\frac{u^2 \beta}{L} + \nu \left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} \right] \text{…(1.9)}$$

Boundary Conditions:

The boundary conditions which governing our model:

1. $r = 0 \rightarrow u = u_\infty$
2. $r = R \rightarrow u = 0$
3. $0 \le \beta \le \pi/2$

Dimensionless Equations.

To generalize the model for measuring an effect we find dimensionless equations which governing the model without units for this goal we define some dimensionless variables as follows: [6]

$$\bar{u} = \frac{u}{u_\infty}, \quad \bar{r} = \frac{r}{d}, \quad \bar{P} = \frac{P - P_0}{P u_\infty^2}, \quad \bar{t} = \frac{t u_\infty}{d},$$

$$\bar{\beta} = \frac{\beta}{\pi/2}$$

and substituting these variables in equation of motion (1.9) we obtain:

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\bar{u}^2}{\bar{r}} = -\frac{\pi d^2 \bar{\beta}}{2L}\bar{u}^2 + \frac{\nu}{u_\infty}\left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{r^2}\frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}^2} \right] \text{…(1.10)}$$

assume that $a = \frac{\pi d \bar{\beta}}{2L}$, and $b = \frac{\nu}{u_\infty}$

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \frac{\bar{u}^2}{\bar{r}} = -a\bar{u}^2 + b\left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{r^2}\frac{\partial \bar{u}}{\partial \bar{r}} - \frac{\bar{u}}{\bar{r}^2} \right] \text{…(1.11)}$$

dimensionless boundary conditions becomes:

1. $\bar{r} = 0 \rightarrow \bar{u} = 1$
2. $\bar{r} = 1 \rightarrow \bar{u} = 0$

Stability Analysis:

Any system may be under external effects which can led to disturbance, this disturbance can cause existing of the system from its phase to anthor or it return to its phase.[8]

To analyze stability of this model we suppose that the system effected by disturbance and we suppose that $\bar{u}_1(\bar{r})$ represent the steady state flow and $\bar{u}_2(\bar{r},\bar{t})$ represent disturbance state which effect to the system.[8]

$$\bar{u}(\bar{r},\bar{t}) = \bar{u}_1(\bar{r}) + \bar{u}_2(\bar{r},\bar{t}) \text{……(2.1)}$$

substituting the equation (2.1) in the dimensionless equation (1.11) we obtain:

$$\frac{\partial(\bar{u}_1 + \bar{u}_2)}{\partial \bar{t}} - \frac{(\bar{u}_1^2 + \bar{u}_2^2)}{\bar{r}} = -a(\bar{u}_1^2 + \bar{u}_2^2) + b\left[\frac{\partial^2(\bar{u}_1 + \bar{u}_2)}{\partial \bar{r}^2} + \frac{1}{\bar{r}^2}\frac{\partial(\bar{u}_1 + \bar{u}_2)}{\partial \bar{r}} - \frac{(\bar{u}_1 + \bar{u}_2)}{\bar{r}^2} \right] \text{…….(2.2)}$$

$$\frac{\partial \bar{u}_1}{\partial \bar{t}} + \frac{\partial \bar{u}_2}{\partial \bar{t}} - \frac{\bar{u}_1^2}{\bar{r}} - \frac{\bar{u}_2^2}{\bar{r}} = -a\bar{u}_1^2 - a\bar{u}_2^2 + b\left[\frac{\partial^2 \bar{u}_1}{\partial \bar{r}^2} + \frac{\partial^2 \bar{u}_2}{\partial \bar{r}^2} + \frac{1}{\bar{r}^2}\frac{\partial \bar{u}_1}{\partial \bar{r}} + \frac{1}{\bar{r}^2}\frac{\partial \bar{u}_2}{\partial \bar{r}} - \frac{\bar{u}_1}{\bar{r}^2} - \frac{\bar{u}_2}{\bar{r}^2} \right] \text{……..(2.3)}$$

By separating the steady and disturbance states and neglecting the order (2) companies, we obtain: [9]

$$-\frac{\bar{u}_1^2}{\bar{r}} = -a\bar{u}_1^2 + b\left[\frac{1}{\bar{r}^2}\frac{\partial \bar{u}_1}{\partial \bar{r}} - \frac{\bar{u}_1}{\bar{r}^2} \right] \text{……(2.4)}$$

$$\frac{\partial \bar{u}_2}{\partial \bar{t}} = b\left[\frac{1}{\bar{r}^2}\frac{\partial \bar{u}_2}{\partial \bar{r}} - \frac{\bar{u}_2}{\bar{r}^2} \right] \text{…………(2.5)}$$

Steady State Equation:

The equation (2.4) represent the state of the model and solving this equation we obtain:

$$\bar{u}_1 = \frac{-1}{-\frac{1}{b}\bar{r} + \frac{1}{b} + \frac{a}{c}r^2 - \frac{2a}{c}r + \frac{2a}{c} - c_1 \exp(-r)} \text{…….(2.6)}$$

and using the dimensionless boundary conditions we obtain:

$$c_1 = \frac{2a}{c} + \frac{1}{c} + 1$$

$$c_1 = \frac{2a + 1}{c} + 1$$

$$c_1 = \frac{\pi d \bar{\beta} + L}{\nu L} u_\infty + 1$$

and substituting this value in equation (2.6) we obtain:

$$\bar{u}_1 = \frac{-1}{-\frac{1}{b}\bar{r} + \frac{1}{b} + \frac{a}{c}\bar{r}^2 - \frac{2a}{c}\bar{r} + \frac{2a}{c} - (\frac{2a+1}{c} + 1)\exp(-\bar{r})} \text{…….(2.7)}$$

and

$$\frac{\partial \bar{u}_1}{\partial \bar{r}} = \frac{\left[\frac{-1}{c}\bar{r} + \frac{2a}{c}\bar{r} - \frac{2a}{c} + (\frac{2a+1}{c} + 1)Exp(-\bar{r}) \right]}{[-\frac{1}{b}\bar{r} + \frac{1}{b} + \frac{a}{c}\bar{r}^2 - \frac{2a}{c}\bar{r} + \frac{2a}{c} - (\frac{2a+1}{c} + 1)\exp(-\bar{r})]^2} \text{…….(2.8)}$$

Disturbance State Equation:

The equation (2.5) represent a disturbance state equation of the model, and when solution this equation obtain:[10]

$$\bar{u}_2 = c_1 Exp\left[(\frac{1}{3}k^2 + \frac{1}{3})\frac{r^3}{c} \right] \times c_2 Exp\left[k^2 t \right]$$

assume $c_3 = c_1 \times c_2$

$$\bar{u}_2 = c_3 Exp\left[\frac{(k^4+k^2)}{3c} \times r^3 \times t\right] \text{......(2.9)}$$

where $k =$ constant

and when substituting in the equation (2.5) we obtain:

$$\frac{(k^4+k^2)}{3c} \times r^3 \times c_3 \times Exp\left[\frac{(k^4+k^2)}{3c} \times r^3 \times t\right] - \frac{c}{\bar{r}^2} \times \frac{(k^4+k^2)}{c} \times t \times c_3 \times Exp\left[\frac{(k^4+k^2)}{3c} \times r^3 \times t\right]$$

$$+ \frac{c}{\bar{r}^2} \times c_3 \times Exp\left[\frac{(k^4+k^2)}{3c} \times r^3 \times t\right] = 0 \text{.............(2.10)}$$

References:

١.الطحان، ياسين هاشم، و بكر، عبد الصبار ابراهيم، (١٩٩٠)، ميكانيك الموائع، دار الحكمة للطباعة والنشر، جامعة الموصل، العراق.

2. Ashmanov A.A (1981), “linear program”, Nayka, Moscow, USSR.
3. Al-Obaidi Mizal (2004), “Effect of Curvature on Stability of Fluid Flow in Tube”, unpublished thesis, Tikrit university, Iraq.
4. Bird R. (1960), “Transport Phenomena”, John Wiley & Sons.
5. Huphes, W.F. and Brighten, T.A., (1970), “Schaum’s out line of theory and problems of fluid dynamics” McGraw-Hill Book Company.
6. Fox, R.W. and McDonald A.T., (1981), “Introduction to fluid mechanics”, 5TH ed. John Wiley and Sons.
7. Al-obaidi Mahdi, (2001), “Conductions for stability of MHD Ecman Layer on porous plate”, J.Ed. vol.(48), pp. 164-168.
8. Helliwell, J.B. and Mosa, M.F., (1979), “Radiative Heat Transfer in Chanel Flow”, Int. Heat Transfer, 22, 657.
9. Al-Obaidi Mahdi, (200), “On the Stability of Heat Transfer through MHD Porous Medium”, Raf. Jour. Sci., vol.11 No.1, pp. 90-94.
10. Logan, J.D., “Applied Mathematics”, Joun Wiley and Sons.

تأثير الانحناء على استقرارية جريان مائع انضغاطي في أنبوب دائري المقطع

وفاء محي الدين طه

قسم الرياضيات، كلية التربية، جامعة تكريت، تكريت، جمهورية العراق

الملخص:

تم في هذا البحث دراسة نموذج جريان طباقي لمائع قابل للانضغاط في أنبوب دائري المقطع وقد وضعت المعادلات الأساسية التي تحكم هذا الجريان وتم تحليل الاستقرارية لهذا النموذج عندما تكون السعة ثابتة وقد تبين انه في هذه الحالة يكون النموذج غير مستقر دائما تحت كل الضروف وانه في المائع الانضغاطي تغير الكثافة يؤثر سلبا على استقرارية المائع.