Weak Form of Alpha-Continuity and Precontinuity in Intuitionistic Fuzzy Special Topological Spaces

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Abstract:
The purpose of this paper is to study and introduce weak form of intuitionistic fuzzy special α -continuous and discuss relation it with precontinuous and semicontinuous . Some of their characteristic properties are establish, also weak form of α-open ,semiopen and preopen functions are studied.

Introduction:
After the introduction of fuzzy set by Zadeh there have been a number of generalizations of this fundamental concept.The notion of intuitionistic fuzzy set introduced by Atanassov is one a mong them .using the notion of intuitionistic fuzzy sets Coker [1]introduced the notion of intuitionistic fuzzy topological spacees. The concept is used to defined intuitionistic fuzzy special sets by Coker [5] .Insection 3 we define the notion of intuitionistic fuzzy special semiopen (resp. preopen ,α-open)functions and inveseigate relation among them .In section 4 we discuss characterizations of intuitionistic fuzzy special precontinuous(resp. semicontinuous, α-continuous)functions and relation among them.

Preliminaries:
We introduce some basic and results that are used in the sequel.

Definition2.1 [5]
Let X be a non-empty set. An intuitionistic fuzzy special set A is an object having the form $A=\langle x,A_1,A_2\rangle$ where $A_1$and A_2 are subsets of X satisfying $A_1\cap A_2=\phi$. The set A_1 is called the set of members of A, while A_2 is called the set of nonmembers of A.

Definition 2.2[5]
Let X be a non-empty set, and let $A=\langle x,A_1,A_2\rangle$ and $B=\langle x,B_1,B_2\rangle$ be intuitionistic fuzzy special sets respectively. Furthermore, let $\{A_i;i\in J\}$ be an arbitrary family of intuitionistic fuzzy special sets in X , where $A_i=\langle x,A_i^{(1)},A_i^{(2)}\rangle$, then :

1. $A\subseteq B\Leftrightarrow A_1\subseteq B_1\ \&\ B_2\subseteq A_2$,
2. $A=B\Leftrightarrow A\subseteq B\ \&\ B\subseteq A$,
3. The complement of A is denoted by $\overline{A}$ and defined by $\overline{A}=\langle x,A_2,A_1\rangle$,
4. $\bigcup A_i=\langle x,\cup A_i^{(1)},\cap A_i^{(2)}\rangle$, $\bigcap A_i=\langle x,\cap A_i^{(1)},\cup A_i^{(2)}\rangle$,
5. $\tilde{\Phi}=\langle\phi, X\rangle$, $\tilde{X}=\langle X,\phi\rangle$

The image and pre image of intuitionistic fuzzy special sets is an intuitionistic fuzzy special set defined as follows.

Defintion2.3 [5]
Let X and Y be two non-empty sets and f: X$\to$Y be a function.

a) If $B=\langle y,B_1,B_2\rangle$ is an intuitionistic fuzzy special set in Y, then the preimage(inverse image) of B under f is denoted by $f^{-1}(B)$ is an intuitionistic fuzzy special set in X and defined by $f^{-1}(B)=\langle x,f^{-1}(B_1),f^{-1}(B_2)\rangle$.

If $A=\langle x,A_1,A_2\rangle$ is intuitionistic fuzzy special set in X, then the image of A under f denoted by $f(A)$ is intuitionistic fuzzy special set in Y.

Proposition2.4[5]
Let A be an intuitionistic fuzzy special set in X And B an intuitionistic fuzzy special set in Y, and f:X $\to$Y,be afunction then,

1. $f^{-1}(\overline{B})=\overline{f^{-1}(B)}$
2. if f is surjective then $\overline{f(A)}\subseteq f(\overline{A})$and if furthermore f is injective , we have $\overline{(f(A))}=f(\overline{A}))$

Definition 2.5 [5]
An intuitionistic fuzzy special topology on a non-empty set X is a family T of an intuitionistic fuzzy special set in X satisfying the following conditions.

1- $\tilde{\Phi},\tilde{X}\in T$
2- T is closed under finite intersections.
3- T is closed under arbitrary unions.

The pair (X,T) is called an intuitionistic fuzzy special topological spaces. Any element in T is usually called intuitionistic fuzzy special open set.

NOTE; from now the word space means intuitionistic fuzzy special topological space and open set means intuitionistic fuzzy special open set for short in the whole work, also for other terminology.

The complement of an open set in a space (X,T) is called intuitionistic fuzzy special closed set (closed set, for short).

Definition 2.6 [5]

Let (X,T) be a space and let $A=\langle x,A_1,A_2\rangle$ be an intuitionistic fuzzy special subset(subset, for short) in a set X. The interior (intA) and closure (clA)of a subset A of X are defined:

$$int\,A=\bigcup\{G : G\subseteq A, G\in T\},$$

$$clA=\bigcap\{F : A\subseteq F, \overline{F}\in T\}$$

In other words: The intA is the largest intuitionistic fuzzy special open set contained in A, and clA is the smallest closed set contained A ie. intA⊆ A and A⊆ clA.

The following proposition gives a relation between the closure and interior and this relation is very useful in next chapters.

Proposition 2.7 [1]

Let (X,T) be a space and A be subset of X, then

1. $cl\left(\overline{A}\right)=\overline{int\,A}.$
2. $int\left(\overline{A}\right)=\overline{clA}.$

Proof: (1)

Suppose that A be any subset of X having the form $A=\langle x,A_1,A_2\rangle$. Since intA⊆ A , and intA is an open set ,so $\overline{int\,A}$ is closed set in X and $\overline{A}\subseteq\overline{int\,A}$,so $cl\overline{A}\subseteq cl\overline{int\,A}$…….(1).

From (Def.2.6)we have $\overline{cl\left(\overline{A}\right)}\subseteq\overline{\overline{A}}=A,$ so that $int\left(\overline{cl\left(\overline{A}\right)}\right)\subseteq int\,A$. Since $\overline{cl\left(\overline{A}\right)}$ is an open set, then $int\left(\overline{cl\left(\overline{A}\right)}\right)=\overline{cl\overline{A}}=int\,A$, so $\overline{int\,A}\subseteq cl\overline{A}$,…..(2) .

From (1) and (2) we get $cl\overline{A}=\overline{int\,A}.$

To prove (2) put $\overline{A}$ in stead of A in (1) we get the result.

Definition 2.8 [4]

Let (X,T) be a space and let A be subset of X. A is called:

1. An intuitionistic fuzzy special □-open set (□OS, for short) if $A\subseteq int(cl(int(A)))$.
2. An intuitionistic fuzzy special semi-open set (SOS, for short) if $A\subsetneq cl\,(int(A)).$
3. An intuitionistic fuzzy special pre-open set (POS, for short) if $A\subsetneq int\,(cl(A)).$

Remarks 2.9[3]

The complement of every semi-open, pre-open,□□open, is semi-closed, pre- closed,□□ closed denoted for short SCS,PCS,□□CS respectively.

Proposition 2.10

Let (X,T) be an intuitionistic fuzzy special topological spaces (space for short) and let A□X be an intuitionistic fuzzy special set (subset for short). Then,,

1- if A is open subset, then A is semi-open, pre-open,□-open

2- if A is □-open subset , then A is semi-open set.

3 - if A is □-open subset , then A is pre-open

The following examples shows that the converse of the above pro position is not true in general,

Example 2.11

Let $X=\{1,2,3\}, T=\{\phi,X,A\}$, where $A=\langle x,\{1\},\{2,3\}\rangle, Let\ B=\langle x,\{1\},\{2\}\rangle$..

B is not open set in X. But B is α-open, since $int\,cl\,int\,B=int\,clA=int\,X=X.$

Example 2.12

Let $X=\{a,b,c\}, T=\{\phi,X,A\}$, where $A=\langle x,\{a\},\{b\}\rangle.Let B=\langle x,\{a\},\{c\}\rangle$.

B is pre-open set in X since $int\,clB=int\,X=X.$ But B is not α-open, since $int\,cl\,int\,B=\phi$.

Example 2.13

Let $X=\{a,b,c\}, T=\{\phi,X,A,B,C\}$, where $A=\langle x,\{a\},\{b,c\}\rangle, C=\langle x,\{b\},\{a\}\rangle\ and B=\langle x,\{a,b\},\phi\rangle$. Let $D=\langle x,\{a\},\{b\}\rangle$

D is semi-open set in X , since $cl\,int\,D=clA=\overline{C}$ but D is not α-open, since $int\,cl\,int\,D=A.$

Some kinds of weak open (Closed) functions between intuitionistic fuzzy special topological spaces

Definition3.1

Let f : X→Y be a function from a space (X,T) into a space (Y, ψ) ,then:

1. f is open(closed) function if f(A) is open set inY,for each open(closed) set A in X
2. f is semi-open (semi-closed) function if f(A) is semi-open (semi-closed)set of Y for each open(closed) set A in X
3. f is pre-open (pre-closed) function if f (A) is pre-open (pre-closed) set of Y for each open(closed) set in X
4. f is α -open(α-closed)function if f (A) is α-open (α-closed)set of Y for each open (closed)set in X.

Proposition 3.2

Let K={□, , pre, semi}, and Let f be a function from a space (X,T) in to a space (Y,ψ). If f isopen, then f is k-open.

The following examples show that the converse of (Prop.3.2) is not true in general.

Example 3.3

Let $X=\{a,b,c\}, T=\{\phi,X,A\}$, where $A=\langle x,\{a\},\{b,c\}\rangle$, and let $Y=\{1,2,3\}, \Psi=\{\phi,Y,B\}$, where

$B=\langle y,\{1\},\{2\}\rangle$. Defined a function f:X□Y as follows: f(a)=1,f(b)=2 and f(c)=3.Since $f(A)=\langle y,\{1\},\{2,3\}\rangle\notin\Psi$, so f is not open. But since intcl(f(A))=Y so, f is pre-open. And f is not □-open function since intclint(f(A))=ϕ.

We conclude from this example that :

1. f is not open function, but f is pre-open.
2. f is per-open function, but f is not □-open.

Example 3.4

Let $X=\{1,2,3,4\},T=\{\phi,X,A,B\}$, where $A=\langle x,\{1\},\{2,3\}\rangle,B=\langle x,\{2,3\},\{1\}\rangle$, and let $Y=\{a,b,c\},\Psi=\{\phi,Y,C,D,E\}$, where $C=\langle y,\{c\},\{a,b\}\rangle,D=\langle y,\{a\},\{b,c\}\rangle,E=\langle y,\{a,c\},\{b\}\rangle$. Defined a function f:X□Y as follows: f(1)=a,f(2)=b and f(3)=c.Since $f(B)=\langle y,\{b,c\},\{a\}\rangle\notin\Psi$, so f is not open and since clint(f(B))= $\overline{D}$ and so f is semi-open. But f is not □-open function since intclint(f(B))=C.

Weak Continuous functions between Intuitionistic fuzzy special topological spaces

Definition4.1 [2]

Let f : X→Y be a function from a space (X,T) into a space (Y, ψ) ,then:

1 .f is continuous function if $f^{-1}(B)$ is open set of X ,for each open set B in Y .

2 .f is semicontinuous if $f^{-1}(B)$ is semi-open set of X for each open set B in Y .

3. f is precontinuous if $f^{-1}(B)$ is pre-open set of X for each open set B in Y

4. f is α-continuous if $f^{-1}(B)$ isα- open set of X ,for each open set B in Y

Proposition 4.2

Let K={□, , pre, semi}, and Let f be a function from a space (X,T) in to a space (Y,ψ). If f is continuous, then f is k-continuous.

The following examples show that the converse of (Prop.4.2) is not true in general.

Example 4.3

Let $X=\{a,b,c\},T=\{\phi,X,A,B,C\}$, where $A=\langle x,\{a\},\{b,c\}\rangle,B=\langle x,\{b\},\{a\}\rangle,C=\langle x,\{a,b\},\phi\rangle$, and let $Y=\{1,2,3,4\},\Psi=\{\phi,Y,D,E,F\}$, where $D=\langle y,\{1,2\},\{3,4\}\rangle,E=\langle y,\{4\},\{1,2\}\rangle,F=\langle y,\{1,2,4\},\phi\rangle$. Defined a function f:X□Y as follows: f(a)=4,f(b)=1 and f(c)=2. f is semi- continuous, since $f^{-1}(D)$,is semi-open. But f is not □-continuous, since $int\,cl\,int\left(f^{-1}(D)\right)=B$.

Example 4.4

Let $X=\{a,b,c\},T=\{\phi,X,A\}$, where $A=\langle x,\{a,b\},\{c\}\rangle$, and let $Y=\{1,2,3\},\Psi=\{\phi,Y,B\}$, where $B=\langle y,\{2\},\{1\}\rangle$. Defined a function f:X□Y as follows: f(a)=1,f(b)=2 and f(c)=3. f is not- continuous, since $f^{-1}(B)=\langle x,\{b\},\{a\}\rangle\notin T$, . But f is pre-continuous, since $int\,cl\left(f^{-1}(B)\right)=X$.

Example 4.5

Let $X=\{a,b,c\},T=\{\phi,X,A\}$, where $A=\langle x,\{a,b\},\{c\}\rangle$, and let $Y=\{1,2,3\},\Psi=\{\phi,Y,B\}$, where $B=\langle y,\{2\},\{1\}\rangle$. Defined a function f:X□Y as follows: f(a)=1,f(b)=2 and f(c)=3

.f is□,pre-continuous since $f^{-1}(B)=\langle x,\{b\},\{a\}\rangle$,is pre-open. ,but f is not □-continuous since $int\,cl\,int\left(f^{-1}(B)\right)=\phi$.

Theorem 4.6

Let f be a function from a space (X,T) in to a space (Y,ψ). Then the following assertions are equivalent.

i. f is semi-continuous.

ii. The inverse image of every closed set in Y is closed in X.

iii. $int\left(cl\left(f^{-1}(A)\right)\right)\subseteq f^{-1}\left(cl(A)\right)$ for every subset A in

Proof:

(i)□(ii):Let B be any closed set in Y, then $\overline{B}$ is open set in Y. Since f is semi-continuous, so $f^{-1}\left(\overline{B}\right)$ is semi-open set in X. But $f^{-1}\left(\overline{B}\right)=\overline{f^{-1}(B)}$,then $f^{-1}(B)$ is closed set in X.

(ii)□□(iii):Let A be any subset in Y, clA is closed set in Y. So $f^{-1}(clA)$ is semi-closed set in X by(ii). But $f^{-1}(A)\subseteq f^{-1}(clA)$ it follows that, $cl\left(f^{-1}(A)\right)\subseteq cl\left(f^{-1}(clA)\right)$, from this we have $int\left(cl\left(f^{-1}(A)\right)\right)\subseteq int\left(cl\left(f^{-1}(clA)\right)\right)\subseteq f^{-1}(clA)$.

(iii)□□i): Let A be any open set in Y. So $\overline{A}$ is closed set in Y, so $int\left(cl\left(f^{-1}\left(\overline{A}\right)\right)\right)\subseteq f^{-1}\left(cl\left(\overline{A}\right)\right)=f^{-1}(\overline{A})$ (cl $\overline{A}$ = $\overline{A}$).

This implies that

$$\overline{cl\left(int\left(f^{-1}(A)\right)\right)} = int\left(\overline{int\, f^{-1}(A)}\right) = int\left(cl\left(\overline{f^{-1}(A)}\right)\right)$$

$$= int\, cl\left(f^{-1}\left(\overline{A}\right)\right) \subseteq f^{-1}\left(\overline{A}\right) = \overline{f^{-1}(A)}$$

Thus $f^{-1}(A) \subseteq cl\left(int\left(f^{-1}(A)\right)\right)$. Hence $f^{-1}(A)$ is semi-open set in X. Therefore f is semi-continuous.

Theorem 4.7 [4]

Let f : X □□Y be a function from a space(X,T)into a space (Y,ψ),then the following assertions are equivalent.

i. f is pre-continuous.

ii. The inverse image of every closed set in Y is pre-closed in X.

iii. For each subset A in Y, $cl\left(int\left(f^{-1}(A)\right)\right) \subseteq f^{-1}\left(cl(A)\right)$.

Proof: (i)□□ii□

Let B be any closed subset in Y, then $\overline{B}$ is open set in Y . Since f is pre-continuous, then $f^{-1}\left(\overline{B}\right)$ is pre-open set in X .But $f^{-1}\left(\overline{B}\right) = \overline{f^{-1}(B)}$(Prop2.4. (1)). Then $f^{-1}(B)$ is pre-closed set in X.

(ii)□(iii): Let A be any open set in Y , then clA is closed set in Y. It follows from (ii) that $f^{-1}(clA)$ is pre-closed set in X. So that $f^{-1}(A) \subseteq f^{-1}(clA)$. It follows that $cl\left(int\left(f^{-1}(A)\right)\right) \subseteq cl\left(int\left(f^{-1}(clA)\right)\right) \subseteq f^{-1}(clA)$. Therefore, $cl\left(int\left(f^{-1}(A)\right)\right) \subseteq f^{-1}(clA)$.

(iii)□(i): let A be any open set in Y ,then $\overline{A}$ is closed set in Y. So $cl\left(int\left(f^{-1}\left(\overline{A}\right)\right)\right) \subseteq f^{-1}\left(cl\left(\overline{A}\right)\right) = f^{-1}\left(\overline{A}\right) = \overline{f^{-1}(A)}$ .Thus by virtue of (Prop. 2.4(2)) we have $f^{-1}(A) \subseteq int\left(cl\left(f^{-1}(A)\right)\right)$. Therefore $f^{-1}(A)$ is pre-open set in X.

Theorem4.8

Let (X,T) , (Y,ψ) be two spaces. Let f:X□Y be a function . Then f satisfies $cl\left(int\left(cl\left(f^{-1}(B)\right)\right)\right) \subseteq f^{-1}\left(cl(B)\right)$ for every subset Bin Y, if and only if f is □-continuous.

Proof:

Let B be an open subset in Y then $\overline{B}$ is closed set in Y. So by hypotheses we get $cl\left(int\left(cl\left(f^{-1}\left(\overline{B}\right)\right)\right)\right) \subseteq f^{-1}\left(cl\left(\overline{B}\right)\right) = f^{-1}\left(\overline{B}\right)$ .It follows that

$$\overline{int\left(cl\left(int\left(f^{-1}\left(\overline{B}\right)\right)\right)\right)} = cl\left(\overline{cl\left(int\left(f^{-1}\left(\overline{B}\right)\right)\right)}\right) = cl\left(int\left(\overline{int\, f^{-1}(B)}\right)\right)$$

$$= cl\left(int\left(cl\,\overline{f^{-1}(B)}\right)\right) = cl\left(int\left(cl\left(f^{-1}\left(\overline{B}\right)\right)\right)\right) \subseteq f^{-1}\left(\overline{B}\right) = \overline{f^{-1}(B)}$$

So that $f^{-1}(B) \subseteq int\left(cl\left(int\left(f^{-1}(B)\right)\right)\right)$. This shows that $f^{-1}(B)$ is □-open set in X. Hence f is □-continuous.

Conversely: Suppose that f is □-continuous, then $f^{-1}(int\bar{B})$ is □-open set in X.

Thus $\overline{f^{-1}(cl(B)} = f^{-1}(int(\bar{B})) \subseteq int(cl(int(f^{-1}int(\bar{B}))))$ □

$$= int\left(cl\left(int\left(\overline{f^{-1}(cl(B))}\right)\right)\right)$$

$$= int\left(cl\left(\overline{cl(f^{-1}(cl(B))}\right)\right)$$

$$= \overline{int\left(int\left(cl\left(f^{-1}(cl(B))\right)\right)\right)}$$

$$= \overline{\left(cl\left(int\left(cl(f^{-1}(clB))\right)\right)\right)}$$

Then we have $cl\left(int\left(cl\left(f^{-1}(cl(B))\right)\right)\right) \subseteq f^{-1}(cl(B))$....(1)

Since $B \subseteq clB, then\ f^{-1}(B) \subseteq f^{-1}(cl(B))$.

Thus we can obtain $cl\left(int\left(cl(f^{-1}(B))\right)\right) \subseteq cl\left(int\left(cl\left(f^{-1}(cl(B))\right)\right)\right)$………..(2)

From1 & 2 we get $cl\left(int\left(cl(f^{-1}(B))\right)\right) \subseteq f^{-1}(cl(B))$

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شكل ضعيف للدوال الفا المستمرة والدوال المستمرة قبليا في الفضاءات التبولوجية المضببة الحدسية الخاصة

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الملخص:

ان الغرض من هذا البحث هو دراسة الشكل الضعيف للدوال الفا المستمرة وبحث علاقتها بالدوال المستمرة قبليا والدوال شبه المستمرة في الفضاءات التبولوجية المضببة الحدسية الخاصة . بعض الخواص لهذه الدوال درست ، كذلك تم دراسة الشكل الضعيف للدوال الفا المفتوحة والدوال المفتوحة قبليا والدوال شبه المفتوحة في الفضاءات التبولوجية المضببة الحدسية الخاصة .