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New Applications of “Abeer-AL-Tememe” Transformation in partial differential equations

Abeer F. Abaas [a]***, Ali H. Mohammed*** [b]

[a] *Mathematics Dep., Education College for women /Kufa University Iraq-Najaf, abeerf.alshabani@student.uokufa.edu.iq*

[b] *Mathematics Dep., Education College for women /Kufa University Iraq-Najaf, prof.ali57hassan@gmail.com*

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ABSTRACT

The Introduction of integral transforms as a partial differential equation solution technique results from the tremendous significance of differential equations and integral transforms in scientific domains. This study proposes a new integral transform Abeer Al- Tememe transform. The suggested transform applications has demonstrated its capacity to resolve partial differential equations.

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1. Introduction

An equation with the derivative of one or more functions is known as a differential equation in mathematics. An ordinary differential equation is a differential equation that has all of its derivatives with respect to one variable. A differential equation having derivatives with respect to several variables is called an n-degree partial differential equation, where n is the greatest derivative connected to the differential equation. The partial differential equation's solution is a function that results from utilizing the process of mathematics to solve it; when this function is introduced into the equation, it takes on a form of identity [5, 7] .

Because partial differential equations can generate mathematical formulas for real-world applications involving numerous variables, they are regarded as powerful tools. As a result, partial differential equation solving has attracted the attention of many mathematicians, and numerous methods have been put forth to find both approximate and exact solutions to these kinds of issues [14,15].

∗Corresponding author

Email addresses:

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Many scientific and technical problems can be solved with the help of integral transformations [6,8,10,11]. Over time, mathematicians have created a variety of integral transformations that are useful for solving partial differential equations [2,3,9,12,13].

In 2016 [4], the Al-Tememe transform defines certain fundamental concepts of differentiation and integration and is utilized in the solution of certain types of PDEs with variable coefficients .

In 2017[1] , the researchers use Al-Tememe transform to solve the linear systems of first and second order of partial differential equations with variable coefficients and solve linear systems of partial differential equations by matrices

2.Definition and Basic Concepts

2.1 Definition of Abeer-Al-Tememe transform

Abeer Al-Tememe transform is described as $AT(f(x))=\int_1^{\infty} x^{-\frac{s+1}{s}} f(x)dx$, $s > 0$ and we can write $AT\big(\mu(x,t)\big) = \int_1^{\infty} t^{-\frac{s+1}{s}} \mu(x,t)dt = w(x,s)$

Such that $\mu(x,t)$ is a function of x and t

2.2 Some properties of Abeer-Al-Tememe transform

2.2.1Linearity Property

The Linear property that is characteristic of this transformation is

$AT[a\mu_1(x,t) + b\mu_2(x,t)] = aAT[\mu_1(x,t)] + bAT[\mu_2(x,t)]$

where a, b are constants while the functions $\mu_1(x,t)$ and $\mu_2(x,t)$ are defined when ($t > 1$).

2.2.2 The partial derivatives of Abeer-Al- Tememe transform

If $AT(\mu(x,t)) = w(x,s)$, then

1. $AT\big[t\,\mu_t(x,t)\big] = -\mu(x,1) + \frac{1}{s}w(x,s)$

Proof:

$$AT\Big(t\,\mu_t(x,t)\Big) = \lim_{l\to\infty} \int_1^{l} t^{-\frac{s+1}{s}+1}\,\mu_t(x,t)dt$$

$$= \lim_{l\to\infty}\{t^{-\frac{s+1}{s}+1}\mu(x,t)\Big|_1^{l} - \int_1^{l}\Big(-\frac{s+1}{s}+1\Big)t^{-\frac{s+1}{s}}\mu(x,t)dt\}$$

$$= -\mu(x,1) + \frac{1}{s}w(x,s)$$

2. $AT\big[t^2\mu_{tt}(x,t)\big] = -\mu_t(x,1) - \Big(\frac{1-s}{s}\Big)\mu(x,1) + \frac{1-s}{s^2}w(x,s)$

Proof:

$$AT\big[t^2\mu_{tt}(x,t)\big] = \lim_{l\to\infty}\{\int_1^{l} t^{-\frac{s+1}{s}+2}\mu_{tt}(x,t)dt$$

$$= \lim_{l\to\infty}\{\, t^{-\frac{s+1}{s}+2}\mu_t(x,t)\Big|_1^{l} - \int_1^{l}\Big(-\frac{s+1}{s}+2\Big)t^{-\frac{s+1}{s}+2}\mu_t(x,t)dt\}$$

$$= -\mu_t(x,1) + \frac{1-s}{s}\Big[-\mu(x,1) + \frac{1}{s}w(x,s)\Big]$$

$$= -\mu_{t}(x,1) - \left(\frac{1-s}{s}\right)\mu(x,1) + \frac{1-s}{s^2}w(x,s)$$

3. $AT\left(t^3\mu_{ttt}(x,t)\right) = -\mu_{tt}(x,1) + \frac{1-2s}{s}\mu_{t}(x,1) - \frac{(1-2s)(1-s)}{s^2}\mu(x,1) + \frac{(1-2s)(1-s)}{s^3}w(x,s)$

Proof:

$$AT\left(t^3\mu_{ttt}(x,t)\right) = lim_{l\to\infty}\{\int_1^l t^{-\frac{s+1}{s}+3}\mu_{ttt}(x,t)dt\}$$

$$= lim_{l\to\infty}\{t^{-\frac{s+1}{s}+3}\mu_{tt}(x,t)\Big|_1^l - \int_1^l\left(-\frac{s+1}{s}+3\right)t^{-\frac{s+1}{s}+2}\mu_{tt}(x,t)dt\}$$

$$= -\mu_{tt}(x,1) + \frac{1-2s}{s}lim_{l\to\infty}[\int_1^l t^{-\frac{s+1}{s}+2}\mu_{tt}(x,t)dt$$

$$= -\mu_{tt}(x,1) - \frac{1-2s}{s}\mu_{t}(x,1) - \frac{(1-2s)(1-s)}{s^2}\mu(x,1) + \frac{(1-2s)(1-s)(1)}{s^3}w(x,s)$$

4. $AT\left[t^n\mu_{t}^{(n)}(x,t)\right] = -\mu_{t}^{(n-1)}(x,1) - \frac{(1-(n-1)s)}{s}\mu_{t}^{(n-2)}(x,1) - \frac{(1-(n-1)s)(1-(n-2)s)}{s^2}$

$\mu_{t}^{(n-3)}(x,t) - \cdots + \frac{(1-(n-1)s)(1-(n-2)s)\dots(1-s)}{s^n}w(x,s)$; $n = 1,2,\dots$

Proof:

$$A^T\left[t^n\mu_{t}^{(n)}(x,t)\right] = lim_{l\to\infty}\{\int_1^l t^{-\frac{s+1}{s}+n}\mu_{t}^{(n)}(x,t)dt\}$$

$$= lim_{l\to\infty}\{-t^{-\frac{s+1}{s}+n}\mu_{t}^{(n-1)}(x,t)\Big|_1^l - \int_1^l\left(-\frac{s+1}{s}+n\right)t^{-\frac{s+1}{s}+n-1}\mu_{t}^{(n-1)}dt\}$$

$$= -\mu_{t}^{(n-1)}(x,1) + \left(\frac{s+1}{s}-n\right)\left[-\mu_{t}^{(n-2)}(x,1) - \left(\frac{s+1}{s}-n-1\right)\mu_{t}^{(n-3)}(x,1) - \cdots + \left(\frac{s+1}{s}-n-1\right)\left(\frac{s+1}{s}-n-2\right)\dots\left(\frac{s+1}{s}-1\right)w(x,s)\right]$$

$$= -\mu_{t}^{(n-1)}(x,1) - \frac{(1-(n-1)s)}{s}\mu_{t}^{(n-2)}(x,1) - \frac{(1-(n-1)s)(1-(n-2)s)}{s^2}\mu_{t}^{(n-3)}(x,1) - \cdots + \frac{(1-(n-1)s)(1-(n-2)s)\dots(1-s)}{s^n}w(x,s).$$

Now that It is assumed that μ has an exponential order and is piecewise continuous.

5. $AT\left[h(x)\mu_{x}(x,t)\right] = h(x)\frac{d}{dx}w(x,s)$

Proof:

$$A\left[h(x)\mu_{x}(x,t)\right] = \int_1^\infty h(x)t^{-\frac{s+1}{s}}\mu_{x}(x,t)dt$$

$$= \int_1^\infty h(x)\, t^{-\frac{s+1}{s}}\frac{\partial}{\partial x}\mu(x,t)dt$$

$$= h(x)\frac{d}{dx}\int_1^\infty t^{-\frac{s+1}{s}}\mu(x,t)dt$$

$$= h(x)\frac{d}{dx}w(x,s)$$

6. $AT\left[h(x)\mu_{xx}(x,t)\right] = h(x)\frac{d^2}{dx^2}w(x,s)$

Proof:

$$AT\left[h(x)\mu_{xx}(x,t)\right] = \int_1^\infty h(x)t^{-\frac{s+1}{s}}\mu_{xx}(x,t)dt$$

$$= \int_1^\infty h(x)t^{-\frac{s+1}{s}}\frac{\partial^2}{\partial x^2}\mu(x,t)dt$$

$$= h(x)\frac{d^2}{dx^2}\int_1^\infty t^{-\frac{s+1}{s}}\mu(x,t)dt$$

$$= h(x)\frac{d^2}{dx^2}w(x,t)$$

7. $AT\left[h(x)u_{xxx}(x,t)\right] = h(x)\frac{d^3}{dx^3}\, w(x,s)$

Proof:

$$A\left[h(x)\mu_{xxx}(x,t)\right] = \int_1^\infty h(x)t^{-\frac{s+1}{s}}\mu_{xxx}(x,t)dt$$

$$= \int_1^\infty h(x)\, t^{-\frac{s+1}{s}}\frac{d^3}{dx^3}\mu(x,t)dt$$

$$= h(x)\frac{d^3}{dx^3}\int_1^\infty t^{-\frac{s+1}{s}}\mu(x,t)$$

$$= h(x)\frac{d^3}{dx^3}w(x,s)$$

8. $A\left[h(x)\mu_x^{(n)}(x,t)\right] = h(x)\frac{d^n}{dx^n}\, w(x,s)\ ,\ n = 1,2,\ldots$

Proof:

$$A\left[h(x)\mu_x^{(n)}(x,t)\right] = \int_1^\infty h(x)t^{-\frac{s+1}{s}}\mu_x^{(n)}(x,t)dt$$

$$= \int_1^\infty h(x)\, t^{-\frac{s+1}{s}}\frac{\partial^n}{\partial x^n}\mu(x,t)dt$$

$$= h(x)\frac{d^n}{dx^n}\int_1^\infty t^{-\frac{s+1}{s}}\mu(x,t)dt$$

$$A\left[h(x)\mu_x^{(n)}(x,t)\right] = h(x)\frac{d^n}{dx^n}w(x,s)$$

9. $A\left[h(x)t\mu_{tx}(x,t)\right] = h(x)\left[-\mu_x(x,1) + \frac{d}{dx}\left(\frac{1}{s}\, w(x,s)\right)\right]$

Proof:

$$A\left[h(x)t\mu_{tx}(x,t)\right] = \int_1^\infty h(x)t^{-\frac{s+1}{s}+1}\mu_{tx}(x,t)dt$$

$$= h(x)\left[t^{-\frac{s+1}{s}+1}\mu_x(x,t)\Big|_1^\infty - \int_1^\infty\left(-\frac{s+1}{s}+1\right)t^{-\frac{s+1}{s}}\mu(x,t)\, dt\right]$$

$$= -h(x)\mu_x(x,1) + h(x)\frac{d}{dx}\left[\left(\frac{1}{s}w(x,s)\right)\right]$$

$$= h(x)\left[-\mu_x(x,1) + \frac{d}{dx}\left(\frac{1}{s}w(x,s)\right)\right]$$

10. $A\left[h(x)t^n\mu_{tx}^{(n)(m)}(x,t)\right] = h(x)\left[-\mu_{tx}^{(n-1)(m)}(x,1) + \frac{1+(n-1)s}{s}\left[-\mu_{tx}^{(n-2)(m)}\right.\right.$

$$(ẋ,1)-\left(\frac{1-ns}{s}\right)\mu_{ţẋ}^{(n-3)(m)}+\cdots+\frac{d^m}{dẋ^m}\left[\left(\frac{1-(n-1)s}{s}\right)\right.$$

$$\left.\left(\frac{1-ns}{s}\right)\left(\frac{1-(n+1)s}{s}\right)\ldots\left(\frac{1}{s}\right)w(ẋ,s)\right]\Bigg]$$

$n=1,2,\ldots$, and $m=1,2,\ldots$

Proof:

$$A\left[h(ẋ)ţ^n\mu_{ţẋ}^{(n)(m)}(ẋ,ţ)\right]=\int_1^\infty h(ẋ)ţ^{-\frac{s+1}{s}+n}\mu_{ţẋ}^{(n)(m)}(ẋ,ţ)dţ$$

$$=\int_1^\infty h(ẋ)ţ^{-\frac{s+1}{s}+n}\frac{\partial^m}{\partial ẋ^m}\mu_{ţ}^{(n)}(ẋ,ţ)\,dţ$$

$$=h(ẋ)\frac{d^m}{dẋ^m}\left[-\mu_ţ^{(n-1)}(ẋ,1)+\left(\frac{s+1}{s}-n\right)\left[-\mu_ţ^{(n-2)}(ẋ,1)-\left(\frac{s+1}{s}-n-1\right)\mu_ţ^{(n-3)}(ẋ,1)-\cdots+\right.\right.$$

$$\left.\left.\left(\frac{s+1}{s}-n-1\right)\left(\frac{s+1}{s}-n-2\right)\ldots\left(\frac{s+1}{s}-n\right)w(ẋ,s)\right]\right]$$

$$=h(ẋ)\left[-\mu_{ţẋ}^{(n-1)(m)}(ẋ,1)+\frac{1-(n+1)s}{s}\left[-\mu_{ţẋ}^{(n-2)(m)}(ẋ,1)-\left(\frac{1-ns}{s}\right)\mu_{ţẋ}^{(n-3)(m)}(ẋ,1)-\cdots+\right.\right.$$

$$\left.\left.\frac{d^m}{dẋ^m}\left[\left(\frac{1-(n+1)s}{s}\right)\left(\frac{1-ns}{s}\right)\left(\frac{1-(n-1)s}{s}\right)\ldots\left(\frac{1}{s}\right)w(ẋ,s)\right]\right]\right]$$

3. AT transform for basic functions:

No.	$f(ẋ)$	$A(f(ẋ))$
1	1	s
2	P	Ps
3	$ẋ^n$	$\frac{s}{1-ns}$
4	$\ln(ẋ)$	s^2
5	$(\ln(ẋ))^2$	$2s^3$
6	$(\ln(ẋ))^3$	$6s^4$
7	$(\ln(ẋ))^n$	$n!\,s^{n+1}$
8	$\cosh(a\ln(ẋ))$	$\frac{s}{1-a^2s^2}$
9	$\sinh(a\ln(ẋ))$	$\frac{as^2}{1-a^2s^2}$
10	$\cos(a\ln(ẋ))$	$\frac{s}{1+a^2s^2}$
11	$\sin(a\ln(ẋ))$	$\frac{as^2}{1+a^2s^2}$

3.1 Definition of Inverse Abeer-Al-Tememe transform

Assuming a function μ(ӿ,ţ) where (ţ >1) and AT(μ(ӿ,ţ))= w(ӿ,s) then, μ(ӿ,ţ) is said to be the inverse for the Abeer Al-Tememe transformation and its written as: $(AT)^{-1}[w(ӿ,s)]=\frac{1}{2\pi i}\int_{\delta-i\tau}^{\delta+i\tau} ӿ^{\frac{s+1}{s}} w(ӿ,s)ds = \mu(ӿ,ţ)$

where $(AT)^{-1}$ returns the transform to the originally function.

4.Applications

Examples 4 .1

1.*To solve the PDE*

$ţ\,\mu_ţ + \mu_ӿ = ӿ \ln ţ \ , \ \mu(ӿ,1) = \mu(1,ţ) = 0$

By taking AT to both sides of the equation, So we get

$-\mu(ӿ,1) + \frac{1}{s} w + Dw = ӿ s^2 \ ; \ Dw = \frac{dw}{dӿ}$

$\left[\frac{1}{s} + D\right] w = ӿ s^2$

Now, to solve the ODE firstly we find the homogeneous solution.

$m + \frac{1}{s} = 0 \Longrightarrow m = \frac{-1}{s}$

$w_c = c_1(s) e^{-\frac{1}{s}ӿ}$

Since μ(1,t)=0 ⟹ AT [μ(1,t)] = w(1,s) = 0 ,then

$w_c = c_1(s) e^{\frac{-1}{s}ӿ} = 0 \Longrightarrow c_1(s) = 0$

Now, to find the particular solution

$\left(\frac{1}{s} + D\right) w_p = ӿ s^2$

$w_p = \frac{ӿ s^2}{\frac{1}{s}+D} = \frac{s^2}{\frac{1}{s}[1+sD]} \cdot ӿ$

$w_p = s^3[1 - sD] \cdot ӿ = s^3[ӿ - s]$

$w_p = s^3 ӿ - s^4$

By taking $(AT)^{-1}$ to both sides then we get

$\mu = \frac{ӿ(\ln ţ)^2}{2} - \frac{1}{6}(\ln t)^3$

Example 4.2 : To solve the PDE

$ӿ^2 \mu_{ӿӿ} + ӿ\mu_ӿ + ţ^2 \mu_{ţţ} = ӿţ$

$\mu(ӿ,1) = \mu_ţ(ӿ,1) = 0$ and $\mu(1,ţ) = \mu(2,ţ) = 0$

By taking AT to both sides of the equation, so we get

$ӿ^2 D^2 w + ӿDw - \mu_ţ(ӿ,1) - \left(\frac{1-s}{s}\right)\mu(ӿ,1) + \frac{1-s}{s^2} w(ӿ,s) = ӿ \cdot \frac{s}{1-s}$

$ẋ^2 D^2 w + ẋ D w + \left(\frac{1-s}{s^2}\right) w = \frac{ẋ s}{1-s}$

$\Longrightarrow \left(ẋ^2 D^2 + ẋ D + \frac{1-s}{s^2}\right) w = \frac{ẋ s}{1-s}$

Now, to solve the above ODE we assume

$Z = \ln ẋ$, thus $D_1 = ẋ D, D_1(D_1 - 1) = ẋ^2 D^2$ and $D_1 = \frac{d}{dz}$.

After we substitute the a above a assumption in the differential equation, we get

$\left(D_1^2 + \frac{1-s}{s^2}\right) w = \frac{ẋ s}{1-s}$

Firstly we find the homogenous solution

$m^2 + \frac{1-s}{s^2} = 0$

$m^2 = -\frac{1-s}{s^2} = \frac{s-1}{s^2} \Longrightarrow m = \mp \frac{\sqrt{s-1}}{s}$

Then the homogeneous solution

$w_c = c_1(s) e^{\frac{\sqrt{s-1}}{s} z} + c_2(s) e^{-\frac{\sqrt{s-1}}{s} z}$

$w_c = c_1(s) ẋ^{\frac{\sqrt{s-1}}{s}} + c_2(s) ẋ^{-\frac{\sqrt{s-1}}{s}}$

Since we have $\mu(2, ţ) = 0 \Longrightarrow AT(\mu(2, ţ)) = w(2, s) = 0$, then

$w_c = c_1(s) 2^{\frac{\sqrt{s-1}}{s}} + c_2(s) 2^{-\frac{\sqrt{s-1}}{s}}$

$c_1(s) 2^{\frac{\sqrt{s-1}}{s}} + c_2(s) 2^{-\frac{\sqrt{s-1}}{s}} = 0$ ... *(1)*

And since we have $\mu(1, ţ) = 0 \Longrightarrow AT(\mu(1, ţ)) = w(1, s) = 0$,then

$w_c = c_1(s) 1^{\frac{\sqrt{s-1}}{s}} + c_2(s) 1^{-\frac{\sqrt{s-1}}{s}} = 0$

$c_1 + c_2 = 0 \Longrightarrow c_1 = -c_2$

By substitute in (1) we get that $c_1 = c_2 = 0$

Now to find the particular solution of

$\left(D_1^2 + \frac{1-s}{s^2}\right) w(z, s) = \frac{e^z s}{1-s}$

We substitute (1) instead of (D_1), then to find the form of w_p

$w_p(z, s) = \frac{e^z s^3}{(1-s)(s^2-s+1)}$

$w_p(ẋ, s) = \frac{ẋ s^3}{(1-s)(s^2-s+1)}$

Taking $\frac{s^2}{(1-s)(s^2-s+1)} = \frac{A}{1-s} + \frac{Bs+C}{s^2-s+1}$

$= \frac{A}{1-s} + \frac{Bs+C}{\left(1-\frac{1}{2}s\right)^2 + \frac{3}{4}s^2}$

$\Longrightarrow A = 1 , B = 0, c = -1$

Then $\frac{\text{x̧} s^3}{(1-s)(s^2-s+1)} = \frac{\text{x̧} s}{(1-s)} - \frac{\text{x̧} s - \frac{1}{2}\text{x̧} s^2}{\left(1-\frac{1}{2}s\right)^2+\frac{3}{4}s^2} - \frac{\frac{1}{2}\text{x̧} s^2}{\left(1-\frac{1}{2}s\right)^2+\frac{3}{4}s^2}$

$$= \frac{\text{x̧} s}{(1-s)} - \frac{\text{x̧} s\left(1-\frac{1}{2}s\right)}{\left(1-\frac{1}{2}s\right)^2+\frac{3}{4}s^2} - \frac{\text{x̧}}{2}\frac{s^2}{\left(1-\frac{1}{2}s\right)^2-\frac{3}{4}s^2}$$

Now, we take $(\text{AT})^{-1}$ to both sides, so we get

$\mu = \text{x̧ţ} - \text{x̧}\, \text{ţ}^{\frac{1}{2}} \cos\frac{\sqrt{3}}{2}\ln \text{ţ} - \frac{\text{x̧}}{\sqrt{3}}\text{ţ}^{\frac{1}{2}} \sin\frac{\sqrt{3}}{2}\ln \text{ţ}$

Example 4.3: To solve the following PDF

$\text{ţ}^2\mu_{\text{ţţ}} + \text{ţ}\mu_{\text{ţ}} + \text{x̧ţ}\,\mu_{\text{ţx̧}} = \text{x̧} \cos \ln \text{ţ}$

Where $\mu(\text{x̧}, 1) = \mu_{\text{ţ}}(\text{x̧}, 1) = 0$ and $\mu(1, \text{ţ}) = 0$.

By using the AT transform to both sides of the equation, so we get

$-\mu\text{ţ}(\text{x̧}, 1) - \left(\frac{1-s}{s}\right)\mu(\text{x̧}, 1) + \frac{1-s}{s^2}w(\text{x̧}, s) - \mu(\text{x̧}, 1) + \frac{1}{s}w(\text{x̧}, s) + \text{x̧}D\left(\frac{1}{s}w\right) = \frac{\text{x̧}s}{1+s^2}\ ; \quad Dw = \frac{dw}{d\text{x̧}}$

$\frac{1-s}{s^2}w + \frac{1}{s}w + \frac{1}{s}\text{x̧}Dw = \frac{\text{x̧}s}{1+s^2}$

$\left(\frac{1}{s^2} + \frac{1}{s}\text{x̧}Dw\right) = \frac{\text{x̧}s}{1+s^2}$

Now, to solve the above ordinary differential equation we assume $\text{z} = \ln \text{x̧}$, thus

$D_1 = \text{x̧}D$ and $D_1 = \frac{d}{dz}$

After we substitute the above assumption in the differential equation, we get

$\left(\frac{1}{s^2} + \frac{1}{s}D_1\right) = \frac{\text{x̧}s}{1+s^2}$

Firstly we find the homogeneous solution

$\frac{1}{s^2} + \frac{1}{s}m = 0$

$\frac{1}{s}m = \frac{-1}{s^2} \Longrightarrow m = \frac{-1}{s}$

Then the homogeneous solution.

$w_c = c_1(s)\, e^{\frac{-1}{s}z}$

$w_c = c_1(s)\, \text{x̧}^{\frac{-1}{s}}$

Since we have $\mu(1, \text{ţ}) = 0 \Longrightarrow AT[\mu(1, \text{ţ})] = w(1, s) = 0$

Then

$w_c = c_1(s)1^{\frac{-1}{s}} = 0 \Longrightarrow c_1(s) = 0$

Now to find the particular solution, we substitute (1) instead of (D_1), then

$\left(\frac{1}{s^2}+\frac{1}{s}\right) w_p(z,s)=\frac{e^z s}{1+s^2}$

$(1+s)w_p(z,s)=\frac{e^z s^3}{1+s^2} \Longrightarrow w_p(z,s)=\frac{e^z s^3}{(1+s)(1+s^2)}$

$w_p(\text{x̧},s)=\frac{\text{x̧}s^3}{(1+s)(1+s^2)}$

Take $\frac{s^2}{(1+s)(1+s^2)}=\frac{A}{1+s}+\frac{Bs+C}{1+s^2}$

$=\frac{A+As^2+Bs+C+Bs^2+Cs}{(1+s)(1+s^2)}$

$\Longrightarrow A=\frac{1}{2},\ B=\frac{1}{2}, C=\frac{-1}{2}$

Then $\frac{\text{x̧}s^3}{(1+s)(1+s^2)}=\frac{\frac{1}{2}\text{x̧}s}{1+s}+\frac{1}{2}\frac{\text{x̧}s^2}{1+s^2}-\frac{\frac{1}{2}\text{x̧}s}{1+s^2}$

Now we take $(AT)^{-1}$ to both sides, we obtain

$\mu=\frac{\text{x̧}}{2}\text{ţ}^{-1}+\frac{\text{x̧}}{s}\sin\ln\text{ţ}-\frac{\text{x̧}}{2}\cos\ln\text{ţ}.$

5.conclusions

This essay presents and explains the Abeer-Al-Tememe transform of partial derivatives and demonstrates its application to a number of initial value problems. we were able to determine the right response quite rapidly.

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