ON $m-$Topological space

Rana B. Yasseen
Dep. of mathematics , College of education For women , University of Tikrit , Tikrit , Iraq
(Received / / 2007, Accepted / / 2008)

Abstract:
In this paper, we study the $m-$Compact on $m-$Topological spaces, and we introduce a same new $m-$separation axioms of $m-$Topological spaces $(m-T_0, m-T_1, m-T_2)$ and we proof all $m-$ separation axioms. are $m-$ hereditary and $m-$Topological property.

1- Introduction :

Let (X,τ_x) and (Y,τ_y) be topological spaces on which $m-$separation axioms $(T_\circ, T_1, T_2)$ are assumed unless explicity stated [3]. A sub class $\tau^* \subseteq p(x)$ is called supratopology on X if $X \in \tau^*$ and τ^* is closed under arbitrary union, let (X,τ^*) is called a supratopological space .

The members of τ^* are called supra open sets. We called τ^* asupratopology associated with τ_x if $\tau_x \subseteq \tau^*$ .let (X,τ_1^*) and (Y,τ_2^*) be asupratopological spaces. A function $f:(X,\tau_1^*)\to(Y,\tau_2^*)$ is an S^*-continuous function if the inverse image of each supra open set in Y is a supra open set in X [1]. let E be a subset of X , E is called an $m-$set with τ^* if $E\bigcap T\in\tau^*$ for all $T\in\tau^*$ Then the class τ_m of all $m-$sets with τ^* is contained τ^* called an $m-$ topology with τ^* and the members of τ_m are called $m-$ open sets. A subset B of X is called an $m-$closed set if the complement of B is an $m-$open sets. Thus the intersection of any family of $m-$closed set and the union of finitely many $m-$ closed sets is an $m-$closed set. in case τ_m is an $m-$ topology with τ^* on X the topological spaces (X,τ_m,τ^*) with τ^* be denoted by (X,τ_m) [5]. The $m-$closure (resp. $m-$interior) of a subset E of X will be denoted by $m-CL(E)$ (resp. $m-\text{int}(E)$) is the intersection of all $m-$closed subset of X containing E (resp. is the union of all $m-$open subsets of X whish is contained in E).We say that a function $f:(X,\tau_m)\to(Y,\mu_m)$ is called $m-$open function. If the image of any $m-$ open set in X is an $m-$open set in Y, we say that f is a S^* - homeomorphism if and only if f is bijective , f is supra open function and f is S^*-continous [5].let P be any property in X , if P is carried by S^*-home to another space Y we say P is a topological property. Let A be subset of X, A $m-$cover of A is a family of subsets of X whose union includes A. A $m-$ sub cover of A $m-$cover of A is a sub family of so A $m-$ cover of A.

Lemma 1.1.
Let $f:(X,\tau_m)\to(Y,\mu_m)$ be is S^*-continuous function, then function is ms - continuous.

2- $m-T_\circ$ - space induced by $m-$ Topology .

Definition 2.1.
Let (X,τ_m) be an $m-$ topological space, then (X,τ_m) is called $m-$ $T_\circ-$ space and denoted by $(m-T_\circ)$ if for any distinct pair of points x,y of X there exists one $m-$open set u_m in τ_m contains one of the points but not the other.

Example 2.2.
Let $X=\{a,b,c,e\}$ and
$\tau^*=\{X,\{a\},\{c\},\{a,c\},\{e,a\},\{b,c\},\{a,c,e\},\{a,b,c\}\}$ with ϕ then $\tau_m=\{X,\phi,\{a\},\{c\},\{a,c\},\{e,a\},\{b,c\},\{a,c,e\},\{a,b,c\}\}$ is $m-\tau_0$.
And we take τ^* is supratopology without empty set thus $\tau_m=\{X,\{a,b,c\}\}$ is not $m-\tau_0$.

Remark 2.3
Every $m-$open set on (X,τ_m) is asupraopenset on (X,τ^*) the converse is not true.

Example 2.4.
let $X=\{a,b,c,e\}$,
$\tau^*=\{X,\phi,\{a\},\{c\},\{e\},\{e,a\},\{a,c\},\{c,e\},\{a,b\},\{a,b,e\},$
$\{b,c\},\{e,b,c\},\{a,b,c\},\{a,c,e\}\}$ and
$\tau_m=\{X,\phi,\{a\},\{c\},\{e\},\{e,a\},\{a,c\},\{c,e\},\{a,b\}$
$,\{b,c\},\{a,b,c\},\{a,c,e\}\}$ notice that each of $\{a,b,c\},\{a,b,e\}$ is supropen set but not $m-$open set.

Theorem 2.5.
An $m-$Topological space (X,τ_m) is $m-\tau_0$- space if and only if for each pair of distinct points x,y of X, $m-cl(\{x\})\neq m-cl(\{y\})$.

Proof :
Sufficiency. suppose that $x,y\in X$, $x\neq y$,Let $z\in X$ such that $z\in m-cl(\{x\})$ but $z\notin m-cl(\{y\})$.We claim that $x\notin m-cl(\{y\})$. for if $x\in m-cl(\{y\})$ then $m-cl(\{x\})\subset m-cl(\{y\})$. this contradiction the fact that $z\notin m-cl(\{y\})$. consequently $x\in(\ m-cl\{y\})^c$ to which y does not belong.

Necessity let (X,τ_m) be an $m-T_\circ$ -space and $x,y\in X, x\neq y, \exists$ $m-$open set $u_m \ni x\in u_m$ or $y\in u_m$ then u_m^c is an $m-$closed set which $x\in u_m$ and $y\in u_m^c$. Since $m-cl(\{y\})$ is the smallest $m-$closed set containing y [because $m-cl(E)=E\cup(m-\text{int}(E))$], if $m-cl(\{y\})\subset u_m^c$ and therefore $x\notin m-cl(\{y\})$. The $m-cl(\{x\})\neq m-cl(\{y\})$.

Definition 2.6:
Let (X,τ_m) be an $m-$topological space ,let E be a subset of X ,then the $\tau_{m_E}=\{(E\cap T_m)\in\tau_m\setminus T_m$ is $m-openset\}$, is called relative $m-$ topology space ($m-$subspace for short)

For Example.
Let $X=\{a,b,c,e\}$, τ^* is supratopology of with empty set and also $E=\{a,b,c\}$ then $\tau_{m_E}=\{E,\phi,\{a\},\{a,c\},\{b,c\},\{b\},\{a,b\}\}$ hence (E,τ_{m_E}) is called relative $m-$sub space.

Definition 2.7.
Let (X,τ_m) be any $m-$topological space if p is any property in X ,then we called p is $m-$hereditary if its appear in a relative $m-$topological space if no we say p is $non-m-$ hereditary.

Theorem 2.8.
Let (X,τ_m) be any $m-T_\circ$ —space, then the relative $m-$ topological space (E,τ_{m_E}) is $m-T_\circ$.

Proof:
Since (X,τ_m) be the $m-$topology space of $m-\tau_0$,let $e_1\neq e_2\in X$, $\exists$ an $m-$openset $u_m\subseteq X$ such that u_m containing one of e_1,e_2 but not both, since $E\subseteq X$ let $e_1,e_2\in E$, $e_1\neq e_2$ now we have $e_1\in E$, $e_1\in u_m$ then $e_1\in E\cap u_m=u_{m_E}$ or $e_2\in E$ and $e_2\in u_m$ then $e_2\in E\cap u_m=u_{m_E}$ hence is $m-T_\circ$ –space.

Definition 2.9
A function $f:(X,\tau_m)\to(Y,\mu_m)$ is $m-$homeomorphism if and only if f is abijective , $m-$open function and $ms-$continous.

Definition 2.10.
Let $f:(X,\tau_m)\to(Y,\mu_m)$ be an $m-$ homeomorphism, let p any property in X we say that p is ms^*-topological property if p is appear in Y. **Theorem 2.11.**
The property $m-T_\circ$ on $m-$topology space is topological property.

Proof:
Let $(X,\tau_m),(Y,\mu_m)$ be an $m-$topological spaces $f:(X,\tau_m)\to(Y,\mu_m)$ afunction be $m-$home ,let $y_1\neq y_2\in Y$ since f is abjective , $\exists x_1\neq x_2\in X$ such that $y_1=f(x_1), y_2=f(x_2)$ since (X,τ_m) is $m-T_\circ$-space ,then $\exists$ one $m-$openset u_m of X such that $x_1\in u_m$, $x_2\notin u_m$ or $x_1\notin u_m, x_2\in u_m$ and function $m-$open. then $f(x_1)\in f(u_m), \forall x_1\in f(u_m), x_1\in u_m$ hence (Y,μ_m) is $m-T_\circ$ –space.

3 - $m-T_1$ - space induced by $m-$ topology.

Definition 3.1.
let (X,τ_m) be an $m-$topological space ,then (X,τ_m) is called $m-T_1$-space and denoted by ($m-T_1$) if for any distinct pair of points x,y of X there exists two $m-$ open sets u_m, v_m in τ_m such that, $x\in u_m$ $x\notin v_m$ and $y\in v_m$, $y\notin u_m$. for example
let $X=\{a,b,c,e\},\tau^*$ be asupratopology of X with empty set thus
$\tau_m=\{X,\phi,\{a\},\{b\},\{a,b\},\{e\},\{a,c\},\{a,b,c\},\{a,e\},\{b,c\},\{b,e\},$ $\{a,b,e\}\}$ it $m-T_1$ - space

Remark 3.2.
Every $m-T_1$ - spaces is $m-T_\circ$ –spaces but the converse is not true according. From example (2.2) (X,τ_m) is $m-T_\circ$ – spaces but not $m-T_1$.

Theorem 3.3.
An $m-$Topological space (X,τ_m) is $m-\tau_1$- space if and only if every singleton subset of X is $m-$closed.

Proof:
Suppose X is $m-\tau_1$- space and $x\in X$ we show that $\{x\}^c$ is $m-$open,let $y\in\{x\}^c$ .then $x\neq y$,so by $m-\tau_1$ there exist an $m-$openset G_x s . t $x\in G_x$ but $y\notin G_x$ hence $x\in G_x\subseteq\{x\}^c$ and $\{x\}^c=\bigcup\{G_x : x\in\{x\}^c\}$.
Conversely , suppose $\{x\}$ is $m-$closed for every $x\in X$ let $x\neq y\in X$ and $x\neq y$ implies $x\in\{y\}^c$ is an $m-$open set and $y\in\{x\}^c$ is an $m-$ open set . To show that (X,τ_m) is $m-\tau_1$- space, sines $\{x\}^c,\{y\}^c$ are $m-$open sets, $x\in\{y\}^c$ and $y\in\{x\}^c$ then $m-\tau_1$- space

Proposition 3.4.
Let (X,τ_m) beany $m-T_1$ — space, then any finite set is $m-$closed.
By Theorem3.3 easily we get the following Proposition

Theorem 3.5.
Let (X,τ_m) be any $m-T_1-$space, then the relative $m-$ topological space (E,τ_{m_E}) is $m-T_1$.

Proof :
since (X,τ_m) be an $m-$topology space of $m-T_1-$ space, let $e_1,e_2\in X$, $\exists$ two an $m-$ open sets u_m ,

$v_m \subseteq X$ such that $e_1 \in u_m, e_2 \notin u_m$ and $e_2 \in v_m$, $e_1 \notin v_m$ .since $E \subseteq X$, let $e_1 \neq e_2 \in E$ now we have $e_1 \in E, e_1 \in u_m$ then $e_1 \in E \cap u_m = u_{mE}$ and $e_2 \in E, e_2 \in v_m$ then $e_2 \in E \cap v_m = v_{m_E}$ thus (E, τ_{m_E}) is $m-T_1$

Theorem 3.6.

The property $m-T_1$-space is topological property.

Proof:

Let $(X,\tau_m),(Y,\mu_m)$ be $m-$topology spaces $f:(X,\tau_m)\to(Y,\mu_m)$. Be function is $m-$ home. let $y_1 \neq y_2 \in Y$,since f is abijective, $\exists x_1, x_2 \in X$ such that $y_1 = f(x_1), y_2 = f(x_2)$,since (X,τ_m) is $m-T_1$, $\exists$ two an $m-$open sets u_m, v_m of X such that $x_1 \in u_m, x_2 \notin u_m$ and $x_2 \in v_m, x_1 \notin v_m$. And $m-$open function then $f(x_1) = y_1 = f(u_m)$ is $m-$open and $f(x_2) = y_2 = f(v_m)$ is $m-$open hence (Y,μ_m) is $m-T_1$.

4- $m-T_2$ - space induced by $m-$ topology .

Definition 4.1.

let (X,τ_m) be an $m-$topological space, then (X,τ_m) is called $m-T_2$-space and denoted by ($m-T_2$) if for any distinct pair of points x, y of X there exists two disjoint $m-$ open sets u_m, v_m in τ_m contains then respectively. for Example

Let $X=\{a,b,c,e\}$, τ^* is supratopology of X with empty set

$\tau_m = \{X,\phi,\{a\},\{b\}\{c\},\{a,c\},\{b,a\},\{e,a\},\{b,a,e\},,\{b,c\},\{b,e\},\{b,c,a\}\}$ space .

Remark 4.2.

Every $m-T_2$ -space is $m-T_1$-space but the converse is not true.

Example4.3.

Let $X=\{a,b,c,e\}$, τ^* is supratopology of X with empty set and

$\tau_m = \{X,\phi,\{a\},\{e\},\{b,c\},\{b,a\},\{a,e\},\{a,e,c\},\{a,b,c\},\{b,e,c\},\{a,b,e\}\}$

(X,τ^*) is $m-\tau_1$ but not $m-\tau_2$

Theorem 4.4.

Let (X,τ_m) be any $m-T_2$ space, then the relative $m-$ topology space (E,τ_{m_E}) is $m-T_{2-}$ space

Proof:

Since (X,τ_m) be an $m-$topology space of $m-T_2$-space, let $e_1 \neq e_2 \in X$, $\exists$ two disjoint $m-$open sets u_m, v_m of X , such that $e_1 \in u_m, e_2 \notin u_m$ and $e_2 \in v_m, e_1 \notin v_m$ .let $E \subseteq X$, $e_1 \neq e_2 \in E$ now we have $e_1 \in E$, $e_1 \in u_m$ then $e_1 \in E \cap u_m = u_{mE}$ and $e_2 \in E, e_2 \in v_m$ then $e_2 \in E \cap v_m = v_{mE}$. To prove $u_{mE} \cap v_{mE} = \phi$ since $u_{mE} \cap V_{mE} = (E \cap u_m) \cap (E \cap v_m) =$ $E \cap (u_m \cap v_m) = E \cap \phi = \phi$ then (E,τ_{mE}) is $m-T_2$ -space.

Theorem 4.5.

The property $m-T_2$ -space is topological property.

Proof:

Let $(X,\tau_m),(Y,\mu_m)$ be an $m-$topology spaces, since $f:(X,\tau_m)\to(Y,\mu_m)$. a function is $m-$ home. let $y_1 \neq y_2 \in Y$,since f is abijective, $\exists x_1, x_2 \in X$, $x_1 \neq x_2$ such that $y_1 = f(x_1), y_2 = f(x_2)$, since (X,τ_m) is $m-T_2$, $\exists$ two disjoint $m-$open sets u_m, v_m of X ,contining the respectively. Sines $m-$open function then

$f(x_1) = y_1 \in f(u_m) = u^*{}_m$,

$f(x_2) = y_2 \in f(v_m) = v_m{}^*$ alos $f(u_m), f(v_m)$ are $m-$ open in Y ,sine's f^{-1} is $ms-$ continuous $u_m{}^* \cap v_m{}^* = f(u_m) \cap f(v_m)$ $= f(u_m \cap v_m) = f(\phi) = \phi$ .then (Y,μ_m) is $m-T_2$.

A $m-T_2$ is $m-$compact if each $m-$open covering has afinite $m-$sub covering.

Example 4.7.

Let $X=\{a,b,c,e\}$, τ^* is supratopology of X with empty set,

$\tau_m = \{X,\phi,\{a\},\{b\}\{c\},\{a,c\},\{b,a\},\{c,a\},\{e,c\},\{b,a,c\},,\{b,c,e\},\{e,c,a\}\}$ It is clearly every $m-T_2$ -space is $m-$ compact . hence $m-T_2$ is $m-$ compact if and only if is finite.

Theorem 4.8.

$m-$ compactness is a topological property.

Proof:

Let (X,τ_m) be an $m-$compact space. since $f:(X,\tau_m)\to(Y,\mu_m)$ be function is $m-$ home. To show that (Y,μ_m) is $m-$compact space. let $\{u_m\}$ be of an $m-$open cover Y . Then $Y \subseteq \bigcup u_m$,sine's f is $ms-$ continuous, $f^{-1}(u_m) = v_m$ hence v_m is $m-$ open sub set of X . Since X is $m-$ compact and $x \subseteq \bigcup_{i=1}^{n} v_{m_i}$, $y = f(x) \subseteq f\left(\bigcup_{i=1}^{n} v_{m_i}\right) = \bigcup_{i=1}^{n} u_{m_i}$

then (Y,μ_m) is $m-$compact space.

Theorem 4.9.

Let (X,τ_m) is $m-$compact space if and only if for each family $\{H_\alpha : \alpha \in i\}$ of $m-$closed sets in X

satisfying $\bigcap_{\alpha\in i} H_\alpha = \phi$,there is a finite sub family $H_{\alpha_1},...,H_{\alpha n}$ with $\bigcap_{i=1}^{n} H_{\alpha_i} = \phi$.

Proof :

Suppose (X,τ_m) is $m-$compact space, let $\forall$ $\{H_\alpha : \alpha \in i\}$ of $m-$closed sets in X , $\bigcap_{\alpha} H_\alpha = \phi$ .then by De Morgan's low, $X = \bigcup_{\alpha} H_\alpha^c$,so $\{H_\alpha^c\}$ is $m-$open cover of X ,sines each H_α is $m-$closed. But X is $m-$compact, hence $\exists\ H_{\alpha_1}^c, H_{\alpha_2}^c .....H_{\alpha_n}^c \in \{H_\alpha^c\}$ s.t $X = H_{\alpha_1}^c \cup H_{\alpha_2}^c \cup ... \cup H_{\alpha_n}^c$ thus by De Morgan's low, $\phi = \bigcap_{i=1}^{n} H_{\alpha_I}$.

Convesrty. Let $\{G_\alpha\}$ be an $m-$open cover of X , $X = \bigcup_{\alpha} G_\alpha$ by De Morgan's low $\phi = X^c = (\bigcup_{\alpha} G_\alpha)^c = \bigcap_{\alpha} G_\alpha^c$. Since each G_α is $m-$open, $\{H_\alpha^c\}$ is aclass of $m-$closed set. hence $\exists G_{\alpha_1}^c, \cap ... \cap G_{\alpha_n}^c = \phi$, thus by De Morgan's low, $X = \phi^c = \bigcup_{i=1}^{n} H_{\alpha_i}$.

Proposition 4.10.

Any $m-$closed subspace of $m-$compact space is $m-$compact .

Proposition 4.11.

Every $m-$compact subset of $m-T_2$ -space is $m-$closed.

Proof:

let K be an $m-$compact subset of $m-T_2$ -space of X ,let $x \in X \setminus K$ .For each $y \in K$, $\exists$ disjoint $m-$open u_y and v_x of Y and X respectively. then $\{u_y\}$ is an $m-$open cover K which to afinite sub covering $\{u_{y_i}\}_{i=1}^{n}$,say K is $m-$compact. let v_{x_i} be the $m-$open of X for $i = 1,2,..,n$ then $V = \bigcap_{i=1}^{n} v_{x_i}$ is $m-$open of X and $v \bigcap_{i=1}^{n} u_{y_i} = \phi, \forall i = 1,2,.....n$ implies that $x \in v \subseteq X \setminus K$ this $X \setminus K$ is $m-$open and K is $m-$closed.

Proposition 4.12.

The image of any K $m-$closed subset of $m-$ compact space is $m-$closed is $m-T_2$ -space under $ms-$continuous.

Proof

By **Proposition**(4.10) K is $m-$compact space if $f : (X,\tau_m) \to (X,\tau_m)$ is $ms-$continuous, then $f(K)$ is $m-$ compact by (4.11) hence $m-$closed, $m-T_2$ -space.

Theorem 4.13.[4]

Let (X,τ_m) be an $m-$compact, Y be $m-T_2$ -space and $f : (X,\tau_m) \to (Y,\mu_m)$ $ms-$continuous then f is $m-$closed map.

Proof:

Let $A \subseteq X$ be an $m-$closed it is $m-$compact and consequently so is $f(A)$ since Y is $m-T_2$ -space, then $f(A)$ is $m-$closed in Y .

References

1. A.S.Mashhor, A.A.Allam ,F.S.Mahmoud and F.H.Khedr ,on supratopological spaces , Indain J.pure Appl. Math 14(1983),502-510
2. J.Dugundji , topology , library of congress catalog card number :66-10940 , printed in the United States of America
3. M.Caldas, D.N.Geogiou and S.Jafari, characterizations of low separation axioms via α - open sets and α -closure operator , Bol. Soc. Paran ,Math(35)v211/2(2003),1-14 .
4. S.H. Han , w.K. min , H.S.Chang, on m-sets, Kangweon – Kyungki Math. Jour.7 (1999), no 2 , PP. 315-320.
5. W.K.Min, H.S.chang, on M-continuity, Kangweon – Kyungki Math. Jour.6 , no 2 (1998), 323 - 329.

حول الفضاءات التبولوجيه $-m$

رنا بهجت ياسين

قسم الرياضيات ، كلية التربية للبنات ،جامعة تكريت ،تكريت ،العراق

(تاريخ الاستلام: / /٢٠٠٧ ، تاريخ القبول: / / ٢٠٠٧)

الملخص

في هذا البحث درسنا تراص $-m$ على الفضاءات التبولوجية $-m$ وقدمنا تعريفا جديدا لبعض بديهيات الفصل $-m$ على الفضاءات التبولوجيه $-m$ $(m-T_0, m-T_1, m-T_2)$ وبرهنا بان كل بديهيات الفصل $-m$ تحقق الصفة الوراثية $-m$ والصفة التبولوجية $-m$