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A Mathematical Model to Predict the Liquid Holdup in Vertical Slug Two-Phase Flow in Pipes

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Abstract:

In the present work, a model to predict the liquid holdup in slug, two-phase gas-liquid, upwardly flow has been developed. This model has compared with nine published methods by using experimental tests for two types of flow systems available in the literature. The comparison achieved using the relative performance factor (F_{PR}) which is representing an equivalent result of the most statistical tools as: error, average error and standard deviation. The results showed the best performance of the present model than the others for each system of flow separately.

Keywords: Two-phase, vertical pipe, liquid holdup, slug flow pattern, mathematical model

الخلاصة:

في هذه الدراسة تم استحداث موديل جديد لحساب نسبة احتجاز السائل في جريان ثنائي الطور من نظام غاز- سائل بالجريان العمودي.هذا الموديل تم مقارنته مع تسعة طرق منشورة سابقا" باستخدام بيانات عملية لنظامين من الجريان منشورة مسبقا". المقارنة تمت باستخدام معامل الأداء (F_{PR}) الذي يمثل نتيجة مكافئة لأغلب أدوات المقارنة الإحصائية مثل: نسبة الخطأ ، نسبة الخطأ النسبي ، الانحراف المعياري. النتائج بينت الأداء الأفضل للموديل الحالي بالمقارنة مع الطرق المنشورة الأخرى ولكل نظام جريان بصورة منفصلة.

Nomenclatures:

The Symbol	The specification	Units
g	Gravitational acceleration	m/s^2
D	Diameter of the pipe flow	m
H_L	Liquid holdup	-
L	Length	m
V	Velocity	m/s
P	Pressure	Pa
T	Temperature	^oC

The Dimensionless Groups:

The Symbol	The Specification
N_{vg}	Gas velocity number
N_{Lv}	Liquid velocity number
N_L	Liquid viscosity number
N_d	Diameter number
Re_m	Mixture Reynolds number
Fr	Froude Number

Greek Symbols:

The Symbol	The specification	The Units
δ	Liquid Film Thickness	m
μ	Viscosity	N. s / m
ρ	Density	N/m^3
σ	Surface Tension	N/m
λ	No-slip holdup	Less

Subscripts:

Symbol	**Specification**
sg	Superficial gas
sL	Superficial liquid
i	Inner
TB	Taylor Bubble
fs	Small film (here: part of paraboloid)
fb	Big film (here: cylindrical region)
av	Average
L	Liquid
meas.	Measured Value
cal.	Calculated Value

1. Introduction:

The liquid holdup represents an important matter in the two-phase, gas-liquid flow. It limits the shape of flow called "flow pattern" and consequently control the pressure gradient which is happened always accomplished with the flow in any stream. It is clarify that the slug flow pattern is widely spread in each investigations and situations in the two-phase flow, this type of flow represents the most dangerous pattern of flow comparing with others due to its configuration which is consisting of large bubble of gas phase called "Taylor Bubble" in additional to scattered bubbles of gas in the liquid phase as shown in **Fig.1** [Zhao (2005)].

Many investigators studied this factor empirically based on experimental data and these correlations are specialized really for their ranges of data among them Duns and Ros (1963), Hagedorn and Brown (1967), Orkiszweiksi (1967), Beggs and Brill (1973), Aziz et al. (1972) and Mukherjee and Brill (1985). It is found that the best methods among them is at same time disappears the worst in the prediction of its outrange of data. Other investigators developed models to predict this factor based on the analytical procedure which is called usually "mechanistic model",

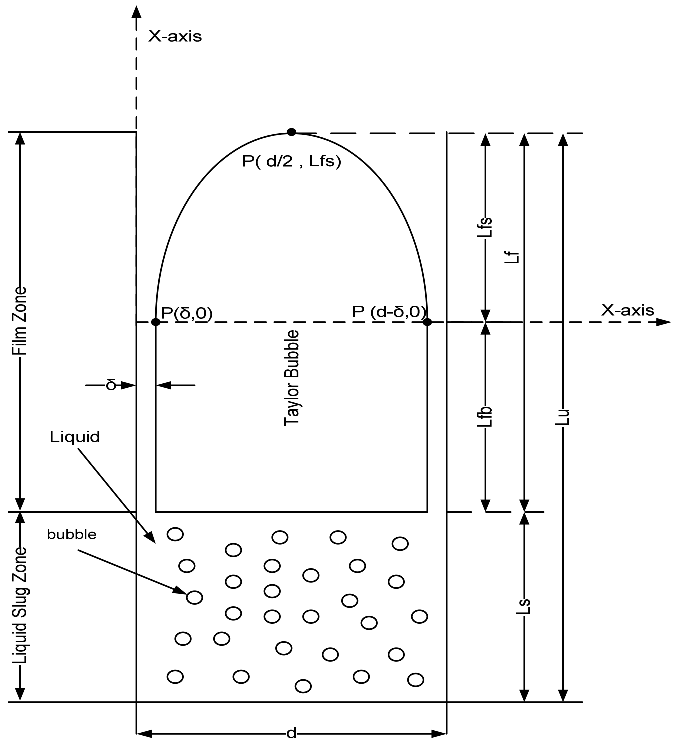

Figure 1. Configuration of Slug Unit

among them Barnea (1990), Hasan and Kabir (1988) and (1992), Ansari et al (1994), Petalas and Aziz (2000), Abdul-Majeed (1997), Oddie et al (2003), Xiaodong (2005), Clayton (2006), and lastly Kaji etal (2009). These works seem to be more reliable than the empirical correlations because they are depending on the physical concepts such as the continuity and momentum equations but these methods are still complicated and need to elongated procedure to obtain the required terms, this leads to more error. The present study adopts nine methods to estimate the liquid holdup in two-phase slug vertical upward flow. Some of them, really, had been developed in semi-empirical procedure and other is completely mechanistic model, these methods are displayed in **Table 1** and in details in **Appendix A**.

Table 1. The used methods

	Symbol	The method
1	A	Aziz et. al. (1972)
٢	BB	Beggs and Brill (1973)
٣	MB	Mukherjee and Brill (1985)
٤	H-K-1	Hasan and Kabir (1988)
٥	H-K-2	Hasan and Kabir (1992)
٦	An	Ansari (1994)
٧	B	Barnea (2000)
٨	P-A	Petalas and Aziz (2000)
٩	Cl	Clayton T. Crowe (2006)

2. The Developed Model:

As shown in **Fig.1** , the slug unit in upward vertical flow be symmetry and could be identified by two major zones; film and liquid slug zones. The first one configured by a big bubble of gas called Taylor's Bubble in shape similar to the upper part of the shot, while the second zone be similar to homogenous flow which is recognized by small bubbles of gas in continuous liquid.

In the present work, the film zone divided into two regions. The upper region treated as the small part cut from the vertical paraboloid and the lower region is treated as the cylindrical shape. The development procedure displayed in details in the next section.

Development Procedure:

Firstly: Three points assumed, using second degree polynomial to get the curve equation passed through these points, and lastly, using the single integral to revolve the curve about the axial line of the pipe to get the volume of the part of the paraboloid. This volume could be represented by:

$$\forall_1 = \frac{2\pi L_{fs}}{\left(\delta - \frac{d}{2}\right)} \left[\frac{d}{2}\left(\delta^2 - \delta d\right) X_1 + \frac{1}{2}\left(\delta^2 + \delta d - \frac{d^2}{2}\right) X_2 - \frac{d}{2} X_3 - \frac{1}{4} X_4 \right] \quad (1)$$

Where: $X_1 = \frac{d}{2} - \delta$, $X_2 = \frac{3d^2}{4} - 2\delta d + \delta^2$, $X_3 = \frac{7d^3}{8} - 3\delta d^2 + 3\delta^2 d - \delta^3$ and

$$X_4 = \frac{15 d^4}{16} - 4\delta d^3 + 6 d^2 \delta^2 - 4\delta^3 d + \delta^4$$

The cylindrical region, the volume of the gas core zone could be calculated from: $\forall_2 = \frac{\pi}{4} d_i^{\,2} L_{fb}$

and $d_i = d - 2\delta$ (2)

The total gas volume in the liquid film zone will be:

$$\forall_G = \forall_1 + \forall_2$$

Hence, the liquid holdup in the liquid film zone will be:

$$H_{Lf} = 1 - \frac{\forall_G}{\forall_p} \quad (3)$$

where: $\forall_p$ is the total volume of the pipe of liquid film portion. $\forall_p = \frac{\pi}{4} d^2 L_f$

To complete the task by predicting the total liquid holdup in slug unit, must using a method to predict the liquid slug holdup (H_{LLs}). To do this without giving up the simplicity of the present model, the following correlation will used:

$$H_{LLs} = 1 - \frac{V_{sg}}{C_1 + C_2 V_m} \quad (4)$$

The constants C_1 and C_2 are proposed continuously by many investigators as shown in table below:

The Author	C_1	C_2
Schmidt (1977)	0.033	1.25
Fernandes (1981)	0.425	2.65
Sylvester (1987)	0.425	2.65

This study used $C_1 = 0.425$ and $C_2 = 0.725$.

The estimation of liquid slug length (Ls) is displayed in the literature as shown in table below:

The Author	Ls
Fernandes et al. (1981)	20d
Dukler et. al. (1985)	16d to 45d
Ansari et al (1994)	30d

A value of (32d) is considered in the present study.

At last, the total holdup over the slug unit could be found by the following equation:

$$H_L = \frac{H_{Lf} L_f + H_{LLs} L_s}{L_u} \quad (5)$$

3. Fluid Properties:

By using the facilities correlations, properties of the fluids flow could be estimated. In the present work, the two-phase flow represented by two types of flow systems: Air-Water and Air-Kerosene. The properties for each fluid could be found as [Abdul-Majeed (1997)]:

1. Air:

$$\rho = Pav/[0.287(273 + Tav)]$$

$$\mu = 0.00001(1.7044 + 0.00613Tav - 0.0000314Tav^2)$$

2. kerosene:

$$\rho = 832.34 - 0.8333Tav$$

$$\mu = 0.001\exp(0.0664 - 0.0207Tav)$$

$$\sigma = 27.6 - 0.09Tav$$

3.Water:

$$\rho = 1000 \quad kg/m^3$$

$$\mu = 10^{-7}(1.77226 - 0.0557784Tav + 0.001026Tav^2 - 0.0000083Tav^3)$$

$$\sigma = 0.074 N/m$$

4. Experimental Tests:

No experimental apparatus has done in the present work, but all tests are conducted from published tests of Abdul-Majeed's (1997) work, it includes (45) tests in Air-Water and (35) tests in Air-Kerosene flow Systems, the flow ranges of these tests are shown in **Table 2** and **Table 3**.

Table 2. Flow Ranges of Air-Water system

	Variables	Minimum	Maximum
1	Liquid Velocity	0.003 m/s	3.00 m/s
2	Gas Velocity	0.07 m/s	6.00 m/s
3	Average Pressure	274 KPa	420 KPa
4	Average Tamp.	19 ^{o}C	30 ^{o}C
8	Liquid Holdup	0.25	0.78

Table 3. Flow Ranges of Air-Kerosene system

	Variables	Minimum	Maximum
1	Liquid Velocity	0.004 m/s	3.00 m/s
2	Gas Velocity	0.07 m/s	6.00 m/s
3	Average Pressure	240 KPa	410 KPa
4	Average Tamp.	18 ^{o}C	30 ^{o}C
8	Liquid Holdup	0.28	0.8

5. Statistical tool:

The comparison procedure carried out by using a parameter displayed by several investigators, this parameter is called the relative performance factor (F_{PR}). If a method has minimum value of the error tools which are displayed below, of course this method will be the best performance else it will be not. This factor is defined as:

$$F_{PR} = \frac{|E_1| - |E_{1min}|}{|E_{1max}| - |E_{1min}|} + \frac{E_2 - E_{2min}}{E_{2max} - E_{2min}} + \frac{E_3 - E_{3min}}{E_{3max} - E_{3min}} +$$

$$\frac{|E_4| - |E_{4min}|}{|E_{4max}| - |E_{4min}|} + \frac{E_5 - E_{5min}}{E_{5max} - E_{5min}} + \frac{E_6 - E_{6min}}{E_{6max} - E_{6min}} \quad (6)$$

It is observed that the relative performance factor depends on many parameters defined as in the follows:

1. Average Error:

$$E_1 = \frac{1}{n}\sum_{i=1}^{n} E_i$$

2. Absolute Average Error:

$$E_2 = \frac{1}{n}\sum_{i=1}^{n} |E_i|$$

3. Standard Deviation:

$$E_3 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (E_i - E_1)^2}$$

4. Average Percent of Error:

$$E_4 = \frac{1}{n}\sum_{i=1}^{n} PE_i$$

5. Absolute Average Percent of Error:

$$E_5 = \frac{1}{n}\sum_{i=1}^{n} |PE_i|$$

6. Percent Standard Deviation:

$$E_6 = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (PE_i - E_4)^2}$$

where

$$E_i = HL_{cal} - HL_{meas} \qquad \text{and} \qquad PE_i = \frac{HL_{cal} - HL_{meas}}{HL_{meas}} \times 100\,\%$$

According to the above equations, the range of this factor is limited between zero and 6. The zero value indicates the best performance, while the worst will have (F_{PR}) equals to 6 [Ansari et al. (1994), Abdul-Majeed (2000)].

6. Results and Discussion:

The published methods and the present model are tested by using the available data and for each group of these data which are given in **Table 2** and **Table 3** respectively, by using the (F_{PR}) who given in **eq.(15)**. **Table 4** shows that the best performance given by the present model where it have (F_{PR}=0), the performance of the methods: Aziz (1972), Hassan-Kabir (1988), Hassan-Kabir (1992), Barnea (2000) and Petalas-Aziz (2000) has closed results because of these methods are closed in assumptions. Beggs-Brill (1973) and Mukherjee-Brill (1985) gave bad result because they are fully empirical correlations. Ansari et al (1994) method has bad result due to it adopted some empirical correlations. Clayton (2006) have the worst result because of out of range of the flow conditions.

Table 4. Statistical Results for Air-Water system

	Methods	$E_1\times10^{-4}$	$E_2\times10^{-4}$	$E_3\times10^{-4}$	E_4	E_5	E_6	FPR
1	A	-4.01	4.01	26.6	-0.089	0.089	0.590	0.245
2	BB	-16.1	16.1	107	-0.358	0.358	2.374	2.63
3	MB	-28.3	28.3	187	-0.628	0.628	4.163	5.03
4	H-K-1	-3.96	3.96	26.3	-0.088	0.088	0.584	0.24
5	H-K-2	-4.04	4.04	26.8	-0.089	0.089	0.595	0.25
6	An	-17.2	17.2	114	-0.381	0.381	2.528	2.84
7	B	-4.26	4.26	28.3	-0.095	0.095	0.629	0.30
8	P-A	-6.59	6.59	43.7	-0.146	0.146	0.971	0.76
9	Cl	-33.2	33.2	220	-0.737	0.737	4.891	6
10	**Present**	**2.76**	**2.76**	**18.3**	**0.0613**	**0.0613**	**0.407**	**0**

Table 5. Statistical Results for Air-Kerosene System

	Methods	$E_1\times10^{-4}$	$E_2\times10^{-4}$	$E_3\times10^{-4}$	E_4	E_5	E_6	FPR
1	A	-2.25	2.25	20.0	-0.051	-0.051	0.445	0.38
2	BB	-9.06	-9.06	80.5	-0.201	0.201	1.79	2.7
3	MB	-17.7	-17.7	157	-0.393	0.393	3.50	5.6
4	H-K-1	-2.23	2.23	19.8	-0.050	-0.050	0.44	0.38
5	H-K-2	-2.31	2.31	20.5	-0.051	-0.051	0.457	0.4
6	An	-9.73	9.73	86.5	-0.216	0.216	1.92	2.9
7	B	-2.55	2.55	22.7	-0.057	-0.057	0.504	0.48
8	P-A	-4.32	4.32	38.4	-0.096	-0.096	0.854	1.08
9	Cl	-19.1	-19.1	169	-0.423	0.423	3.76	6
10	**Present**	**1.11**	**1.11**	**9.84**	**0.025**	**0.025**	**0.219**	**0**

By reviewing **Fig. 2** and **Fig. 3** , it is clear that the predicting of the present model is neither under-predicting nor over-predicting and the points are distributing near to the lines of error.

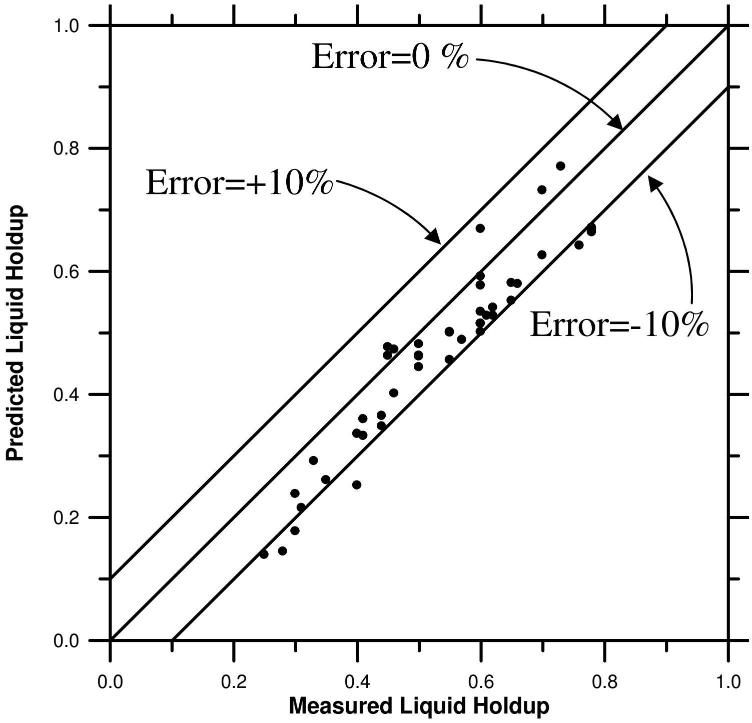

Figure 2. Predicted versus measured liquid holdup using Air-Water system

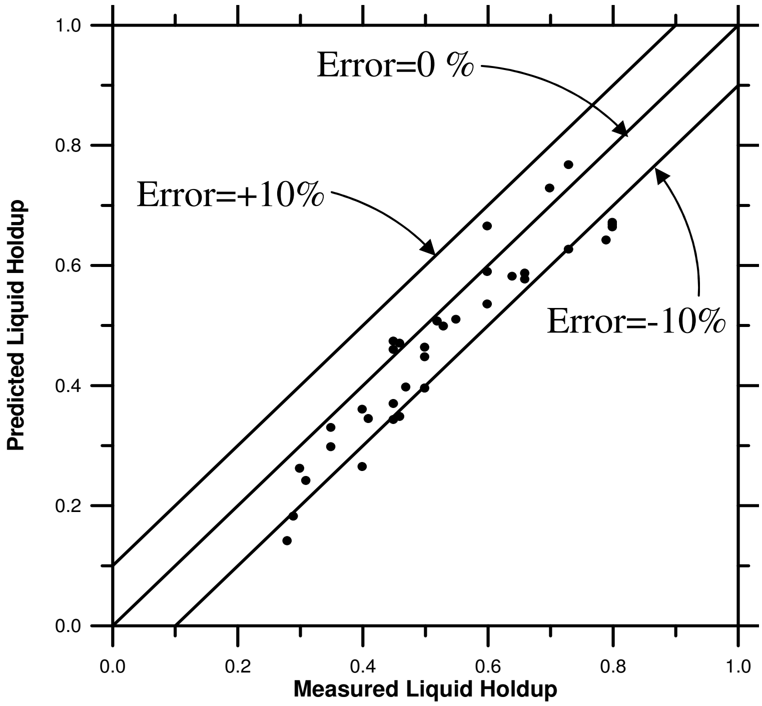

Figure 3. Predicted versus measured liquid holdup using Air-Kerosene system

7. Conclusions:

1. No one of the nine methods gives the real prediction for the whole tests in various fluids flowing.
2. The present model is easy to use than the other mechanistic models because of it has no iteration techniques.
3. The present model is in simplicity used to predict the liquid holdup in liquid film zone in slug flow when excluded the adopted assumption in obtaining the liquid slug holdup.
4. The present model is overestimation in the predicting the liquid holdup with the other methods are underestimation.
5. The present model based on mathematical analysis therefore it may be appropriated to operate for clear fluids (fluids of low density and viscosity).

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Appendix A

The used methods in the Comparison:

Aziz et al (1972) suggested the following:

$$H_L = 1 - \frac{V_{sg}}{1.2V_m + V_b} \quad \text{(A1)}$$

where: $V_b = C\sqrt{\frac{g\,d(\rho_L - \rho_g)}{\rho_L}}$

Also, $C = 0..345\left[1 - e^{-0.029Nv}\right]\left[1 - e^{(\frac{3.37 - NE}{m})}\right]$, $NE = \frac{g\,d^2(\rho_L - \rho_g)}{\sigma}$

M	Nv
10	≥250
$69Nv^{-0.35}$	<250 & >18
25	≤18

$$Nv = \frac{\sqrt{g\,d^3(\rho_L - \rho_g)\rho_L}}{\mu_L}$$

In (1973), Beggs and Brill developed a study to the liquid holdup in slug and plug flow pattern without considering the existence of liquid slug zone:

$$H_L = H_{L_0} \times \psi \quad \text{(A2)}$$

where: $H_{L0} = \frac{a\lambda_L{}^b}{Fr^c}$, $\psi = 1 + C\left[\sin(1.8\theta i - 0.333\sin^3(1.8\theta 1\right]$,

$C = (1-\lambda_L) \times \ln(2.96 \times \lambda L^{0.305} \times NLv^{-0.4473} \times Fr^{0.0978})$, $Fr = \frac{V_m{}^2}{g\,d}$ and

$$N_{Lv} = 1.938\,V_{sL}\sqrt[4]{\frac{\rho_L}{\sigma}}$$

Mukerjee and Brill (1987) suggested a procedure same to Beggs and Brill's correlation(1973), they deduced that:

$$H_L = \exp\left[(0.3902 + 2.3432\,N_L{}^2)\left(\frac{N_{gv}{}^{0.4757}}{N_{Lv}{}^{0.2887}}\right)\right] \quad \text{(A3)}$$

Hasan and Kabir (1988) proposed the following:

$$H_L = 1 - \frac{V_{sg}}{1.2\,V_m + V_s} \quad \text{(A4)}$$

Where: $V_s = 0.35\left[\frac{g\,d\,(\rho_L - \rho_g)}{\rho_L}\right]^{0.5}$

While, in (1992), they suggested the following equation:

$$H_L = C_{ob} - (1 - L_{sL})\frac{V_{sg}}{V_{TB}} - m\,(0.25\,V_{sg}) \quad \text{(A5)}$$

Where: $L_{sL} = \frac{C_{oa}\,(1.2\,V_m + V_{BR})}{V_{sg}{}^n}$

And they assumed:

n=1 , C_{oa}=0.10, C_{ob}=0.9 and m=0 If $V_{sg} > 0.4$

n=0 , C_{oa}=0.25, C_{ob}=1.0 and m=1 If $V_{sg} \leq 0.4$

In (1994), Ansari et al. proposed:

$$\overline{H_L} = \frac{H_{LLs}\,L_s + H_{LTB}\,L_f}{L_u} \quad \text{(A6)}$$

where: $H_{LLs} = 1 - \frac{V_{sg}}{0.425 + 2.65 V_m}$, $H_{LTB} = 1 - H_{GTB}$,

H_{GTB} suggested to obtain by applying the method of Newton-Raphson to the following equation:

$$9.916\sqrt{g\,d}\,H_{LTB}\,(1-\sqrt{1-H_{LTB}})^{0.5} - V_{TB}(1-H_{LTB}) + H_{GLS}(V_{TB} - V_{GLs}) + V_m$$

Barnea (2000) developed the following equation:

$$H_L = \frac{H_{LLs} V_{TB} + V_{GLs}(1-H_{LLs}) - V_{sg}}{V_{TB}} \qquad \text{(A7)}$$

where: $V_{GLs} = 1.2\,V_m + V_{BR}$ and $V_{TB} = 1.2 V_m + 0.35\sqrt{g\,d}$

In (2000) also, Petalas and Aziz proposed the following set of equations:

$$H_L = \frac{H_{LLs} V_t + V_{Gdb}(1-H_{LLs}) - V_{sg}}{V_t} \qquad \text{(A8)}$$

Where: $V_{Gdb} = C_o V_m + V_b$, $V_t = C_o V_m + V_d$

$V_d = 0.316\,V_{d\infty}\sqrt{Re_\infty}$, $Re_\infty = \rho_L V_{d\infty}\, d/2\mu_L$

$V_{d\infty} = 0.345(1-e^{-\beta})\sqrt{g\,d\frac{(\rho_L - \rho_g)}{\rho_L}}$, $\beta = B_o\, e^{(3.278 - 1.424 \ln B_o)}$,

$B_o = (\rho_L - \rho_g)\sigma^{-1} g\, d^2$, $C_o = 1.76\,Re_{mL}^{\;-0.031}$ and $Re_{mL} = \frac{\rho_L V_m d}{\mu_L}$

Clayton (2006), put the following correlation:

$$H_L = \frac{C H_{LLs} + \lambda_L}{1 + C} \quad \text{(A9)}$$

Where: $C = (C_o - 1)\frac{V_b}{V_m}$, $\lambda_L = \frac{V_{sL}}{V_m}$,

C_o=2 for laminar flow while $C_o = \frac{\log Re_m + 0.089}{\log Re_m - 0.74}$ for turbulent flow

Also, $H_{LLs} = \frac{V_{mo} + V_{m1}}{V_{mo} + V_m}$ for $V_m \geq V_{m1}$ else $H_{LLs} = 1$

$$V_{mo} = \frac{480}{3(C_o - 1)} E_o^{-0.5} \left[\frac{g\sigma(\rho_L - \rho_g)}{\rho_L^2} \right]^{0.25} + \frac{V_b}{C_o - 1} \quad \text{and} \quad E_o = \frac{g d^2 (\rho_L - \rho_g)}{4\sigma}$$

There are other method used to predict the liquid holdup in vertical slug flow. Unfortunately, some of them are not available clearly in the literature, among them: Abdul-Majeed (2000), Oddie et al (2003) and Kaji et al (2009).