Mathematical Modeling of water surface at Unsteady Flow in Al-Msharah River

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ABSTRACT

This study is concerned with unsteady flow in open channels in boundary conditions that may exist in river or channel system. A software Hydrologic Engineering Center-River Analysis System (HEC-RAS) was used for a case study. The software contains a hydrodynamic model for simulating the unsteady flow in open channels based on a one-dimensional form of the Saint-Venant equations, by formulating a discrete form using the implicit finite difference scheme, then solving using the Newton-Raphson iteration procedure and the modified Gaussian elimination technique.

The case study adopted with the help of HEC-RAS application was applied to a reach of Al Msharah River (channel) between AL Msharah Barrage (upstream)and Al Malah Bridge (Downstream), for a length of (49 Km) divided to (52) cross sections.

Model runs and the comparison of results with actual field data indicate that the accuracy of the results obtained for(θ=1 and Δt=5 minute) is quite acceptable, where θ is a weighting parameter and Δt is the time interval.

The paper covers the case of normal depth of flow and observed water stage data within the period (from January 2006 to July 2006) .

The results demonstrate that the area at Al Am'arah city at distance 17.5 Km from upstream(cross section 30)could be subjected to flooding at High Flow, therefore, it is recommended to adjust cross sections to prevent the flooding in this area.

At last , Calibration of the hydrodynamic model is achieved in a study reach using the observed data (water stage) along AL Msharah River and show that a good agreement.

Keywords: Mathematical model, finite difference, unsteady flow, water surface.

الخلاصة

في هذه البحث تم دراسة الجريان غير الثابت في القنوات المفتوحة بأعتماد شــروط جريــان محــددة لمقطع النهر أو القناةبأستخدام برنامج (HEC-RAS)، هذا البرنامج ينفذ نموذج هيــدروديناميكي للجريــان غير الثابت في القنوات المفتوحة مستندا على صياغة أحادية البعد لمعادلات(Saint- Venant)باســتخدام مخطط الفروق المحدد الضمنية(finite difference) ومن ثم حل هذه المعادلات باستخدام طريقة-Netwon (Raphson)التكرارية وتقنية (Gauiss)المعدلة للاختزال.

في الدراسة الحالية تم تطبيق البرنامج على مجرى نهر المشرح للمنطقة المحصورة بين جسر المشرح عند مقدم النهر وبين جسر الملاح عند المؤخر ولمسافة (49 كم) .تم تقسيمها الى (52) مقطع. ولفتــرة بــين كانون الثاني 2006 ولغاية تموز 2006 . ونفذ نموذج هيدروديناميكي بالأعتمــاد علــى بيانــات مرصــودة وبمعامل موزون (Ө=1) وتغير بالزمن (Δt=5) دقيقة.

لقد تمت تناول حالة الجريان الطبيعي للمجرى المائي وتم حــساب وقيــاس العوامــل الهيدروليكيــة الاساسية في المجرى ورسم مقطع سطح الماء اضافة الى مقطع حجم الماء .

وقد بينت الدراسة على ان المساحة ضمن مدينة العمارة والواقع على مسافة 17,5 كم من مقدم النهــر عند (المقطع 30)تقع ضمن منسوبالفيضان عند حالة الجريان العادي ، مما يجب تحسين مقطع القناة وتغيير مناسيب الارض الطبيعية على طول المقطع لكي يمكن أستيعاب كميات المياه القادمــة اليــه ومنــع حــدوث الفيضانات .

واخيراً تم تقييم الانموذج المحسوب مع المقاس لمقاطع سطح الماء وكانت النتائج جيدة .

1- INTRODUCTION

The study of water surface of unsteady flow in natural channel is very important in water resources systems, design of hydraulic structures, analyses of river mechanics problems, the development of river control works and to the life and works of humans. Movement of a flood wave in a single channel or a network of channels is good example of unsteady non uniform flow. There are problems characterized by the time dependence of flow and cover a wide range of phenomena including surface runoff, tidal motions, reservoir regulation and flood movements.

Simulation of unsteady flows in open channels and shallow water bodies is an important, interesting, and difficult subject in hydraulic engineering, because many variables enter into the functional relationship and the differential equations cannot be integrated in closed forms except under very simplified conditions. The subject has gained an entirely new dimension since the advent of the high-speed digital computers which the numerical solution of unsteady flow equation attainable and practical especially in the one-dimensional flow field. As one of the several mathematical methods developed for unsteady flow computation, the method of characteristics has been studied and used among hydraulic engineers (Woolhiser & Morris, 1980).

In the present study the application of implicit finite difference scheme is simulated for unsteady flow in open channel(Al Mshara River) as case study were conducted. The Saint-Venant equations (a combination of both the continuity and momentum equations) are used to solve unsteady flow problems.

2. MATHEMATICAL MODEL

The law of conservation of mass (continuity equation for unsteady flow may) be established by considering the conservation of mass in an infinitesimal space between two channel sections in unsteady one dimensional flow the discharge changes with distance at a rate $\partial Q/\partial x$, and the depth changes with time at a rate of $\partial y/\partial t$. Referring to Figure (1).

The continuity equation of unsteady flow at the following form:- (Mohammed, A. Y., 1993)

$$\frac{\partial Q}{\partial x}+\frac{\partial A}{\partial t}=«q \qquad ...(1)$$

And the last form of the conservation of momentum equation should be:

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)-q\frac{Q}{A}=gA\left(S_o-S_f-\frac{\partial Y}{\partial x}\right) \quad ...(2)$$

The tow equations are named Saint Venant equations have been used for one dimensional unsteady flow to calculate water surface profile in single channel and looped channel (networks).

The four-point implicit scheme is sought over a discrete rectangular mesh of points on the (distance, time) plane. The mesh points are determined by the intersection of straight lines of location and time. The location lines are drawn with spacing Δx_i, which is parallel to t- axis. The t-axis may be used as the upstream channel boundary location, and the last line drawn parallel to the t-axis, to be designated the N^{th} line, can be used to represent the downstream channel boundary location. The time lines are drawn parallel to the x-axis with spacing Δt_j. Each point of the network is identified by two indices, a subscript (i) to designate the x-position of the point, and a superscript (j) to designate the t-value. Figure(1) shows the computational grid on the x-t plane to be used for the development of the numerical procedure.

Function K in the intervals i,i+1 and j, j+1 may be replaced by its weight average between these points. For the time interval $t_j \le t \le t_{j+1}$.

$$K(x_i,t)=\theta K_i^{j+1}+(1-\theta)K_i^j$$

$$K(x_{i+1},t)=\theta K_{i+1}^{j+1}+(1-\theta)K_{i+1}^j \quad ...(3)$$

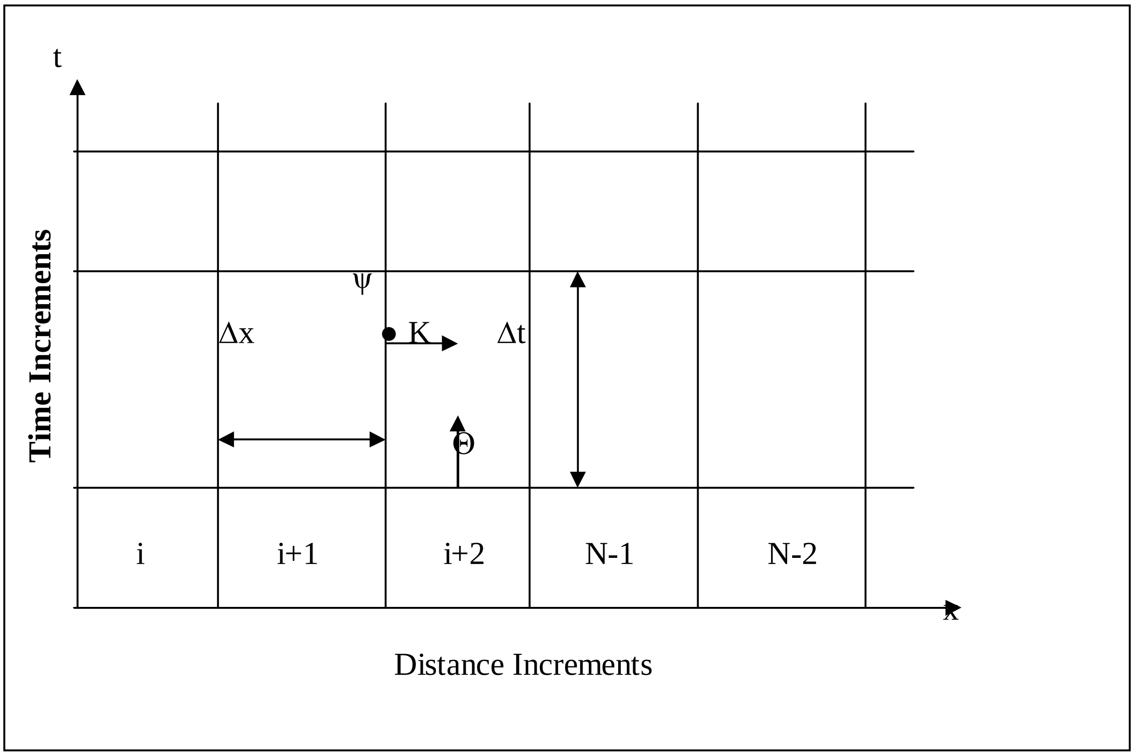


Figure (1)Network of Points on the (x-t)Plane

And for the space interval $x_i \leq x \leq x_{i+1}$

$$K(x,t_j) = \psi K_{i+1}^{j} + (1-\psi) K_i^{j}$$

$$K(x,t_{j+1}) = \psi K_{i+1}^{j+1} + (1-\psi) K_i^{j+1} \quad ...(4)$$

The time and space derivatives of K become,

$$\frac{\partial K}{\partial t} = \frac{K(x,t_{j+1}) - K(x,t_j)}{\Delta t}$$

$$= \frac{\psi\left(K_{i+1}^{j+1} - K_{i+1}^{j}\right) + (1-\psi)\left(K_i^{j+1} - K_i^{j}\right)}{\Delta t} \quad ...(5)$$

$$\frac{\partial K}{\partial x} = \frac{K(x_{i+1},t) - K(x_i,t)}{\Delta x}$$

$$= \frac{\theta\left(K_{i+1}^{j+1} - K_i^{j+1}\right) + (1-\theta)\left(K_{i+1}^{j} - K_i^{j}\right)}{\Delta x} \quad ....(6)$$

Clearly ,whole families of finite difference schemes may be obtained by varying the parameters ψ and θ, all such schemes, however, are four- point schemes.

When ψ =1/2 ,the system of equation (3) to (6) becomes the Preissmann four-point scheme whereby the time derivative i:

$$\frac{\partial K}{\partial t} = \frac{K_{i+1}^{j+1} + K_i^{j+1} - K_{i+1}^{j} - K_i^{j}}{2\Delta t} \quad ...(7)$$

and the function K is expressed as

$$K = \frac{\theta}{2}\left(K_{i+1}^{j+1} + K_i^{j+1}\right) + \frac{(1-\theta)}{2}\left(K_{i+1}^{j} + K_i^{j}\right) \quad ...(8)$$

Substitution of the finite difference approximations is defined by equations (6) to(8) for the derivative and non-derivative terms in the unsteady flow equations (1)and (2), the finite difference formulations are presented the continuity, F_i, and momentum, G_i, equations for the reach i. For the equations written in terms of the discharge Q and depthY, the formulations of these equations in Preissmann scheme (ψ = $^1/_2$) are as follows :

Multiplied by $\Delta t_j / B_i^{j+1}$, the continuity equation (1) becomes

$$F_i = \frac{\Delta t_j}{\Delta x_i B_i^{j+1}}\left[\theta\left(Q_{i+1}^{j+1} - Q_i^{j+1}\right) + (1-\theta)\left(Q_{i+1}^{j} - Q_i^{j}\right)\right]$$

$$+ \frac{1}{2}\left[\frac{B_{i+1}^{j+1}}{B_i^{j+1}}\left(Y_{i+1}^{j+1} - Y_{i+1}^{j}\right) + \left(Y_i^{j+1} - Y_i^{j}\right)\right]$$

$$\text{«} \frac{\Delta t_j}{2B_i^{j+1}}\left[\theta\left(q_{i+1}^{j+1} + q_i^{j+1}\right) + (1-\theta)\left(q_{i+1}^{j} + q_i^{j}\right)\right] = 0 \quad ...(9)$$

and multiplied by (Δx_i/ g) the momentum equations becomes

$$G_i = \frac{\Delta x_i}{2g\Delta g_j}\left\{\left[\left(\frac{Q}{A}\right)_{i+1}^{j+1} + \left(\frac{Q}{A}\right)_{i}^{j+1}\right] - \left[\left(\frac{Q}{A}\right)_{i+1}^{j} + \left(\frac{Q}{A}\right)_{i}^{j}\right]\right\}$$

$$+\frac{1}{g}\left\{\theta\left[\left(\frac{Q^2}{A^2}\right)_{i+1}^{j+1} - \left(\frac{Q^2}{A^2}\right)_{i}^{j+1}\right] + (1-\theta)\left[\left(\frac{Q^2}{A^2}\right)_{i+1}^{j} - \left(\frac{Q^2}{A^2}\right)_{i}^{j}\right]\right\}$$

$$\pm\frac{\Delta t_i}{2g}\left\{\theta\left[\left(q\frac{Q}{A^2}\right)_{i+1}^{j+1} + \left(q\frac{Q}{A^2}\right)_{i}^{j+1}\right] + (1-\theta)\left[\left(q\frac{Q}{A^2}\right)_{i+1}^{j} + \left(q\frac{Q}{A^2}\right)_{i}^{j}\right]\right\}$$

$$+\left\{\theta\left[(Y+z)_{i+1}^{j+1} - (Y+z)_{i}^{j+1}\right] + (1-\theta)\left[(Y+z)_{i+1}^{j} - (Y+z)_{i}^{j}\right]\right\}$$

$$+\frac{\Delta x_i}{2}\left\{\theta\left[(S_f)_{i+1}^{j+1} + (S_f)_{i}^{j+1}\right] + (1-\theta)\left[(S_f)_{i+1}^{j} + (S_f)_{i}^{j}\right]\right\} = 0 \qquad ...(10)$$

where the friction slope S_f is computed from Manning 's equation for uniform flow:-

$$S_{fi} = \frac{n^2|Q_i|Q_i}{A_i^2 R_i^{4/3}}$$

Since R=A/P ,then:

$$S_{fi} = \frac{n^2|Q_i|Q_i}{A_i^{10/3}} P_i^{4/3} \qquad ...(11)$$

Equations (9) and (10)are nonlinear with respect to unknowns Y and Q at the points i and i+1 on the j+1 time line. All terms associated with the j^{th} time line are known either from the initial conditions or previous computations. This system of nonlinear algebraic equations cannot be solved directly since there are four unknowns , Y and Q at points i and i+1 on the j+1 time line , and only two equations. However, if these equations are applied to the (N-1)reaches between the upstream and downstream boundaries, a total of (2N-2) equations with 2N unknowns is obtained (N denotes the total number of nodes or cross sections). The two supplementary equations needed to close the system are provided from upstream and downstream boundary conditions. The resulting system of 2N equations with 2N unknowns may be symbolized as follows:

$$G_O(y_1, Q_1) = 0 \leftarrow \text{Upstream boundary condition}$$

$$\left.\begin{aligned}
&F_1(y_1, Q_1, y_2, Q_2) = 0 \\
&G_1(y_1, Q_1, y_2, Q_2) = 0 \\
&. \\
&. \\
&F_i(y_i, Q_i, y_{i+1}, Q_{i+1}) = 0 \\
&G_i(y_i, Q_i, y_{i+1}, Q_{i+1}) = 0 \\
&. \\
&. \\
&F_{N-1}(y_{N-1}, Q_{N-1}, y_N, Q_N) = 0 \\
&G_{N-1}(y_{N-1}, Q_{N-1}, y_N, Q_N) = 0
\end{aligned}\right\} \quad ...(12)$$

$$F_N(y_N, Q_N) = 0 \leftarrow \text{Downstream boundary condition}$$

The system (12) of nonlinear algebraic equations is solved by using the Newton-Raphson iterative method. The Newton-Raphson technique is derived from Taylor series expansion of the nonlinear function in which all terms of second and higher order are neglected. It should be noted that although the system of equations (12) involves 2N unknowns, each equation contains a maximum of four unknowns. This can be used to a great advantage in the computational schemes.

The computation needed data can be divided into two typies, Geometric Data and Unsteady Flow Data. The information needed to perform the mathematical model is shown in Figure (2).

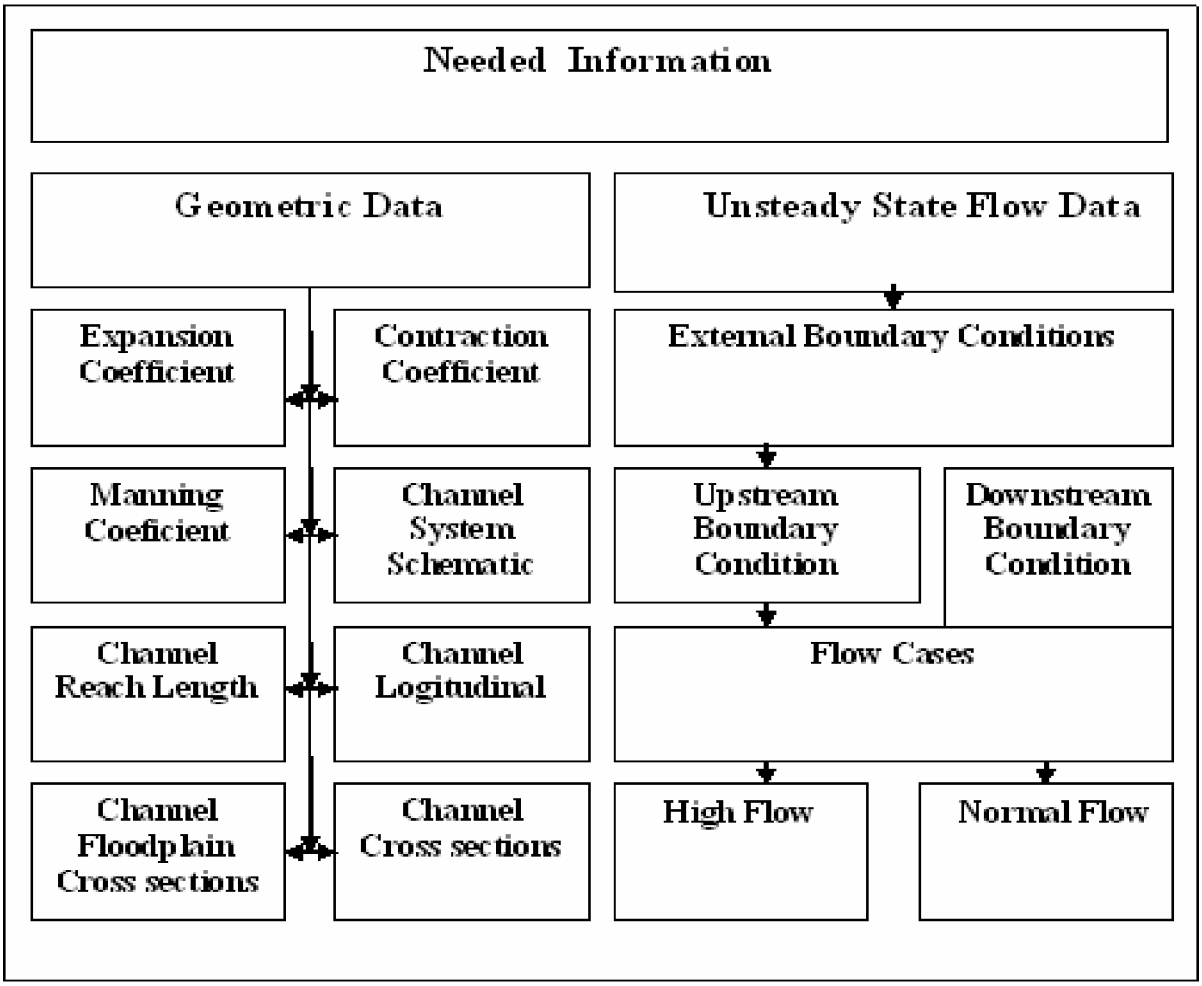


Figure (2)Needed Information to perform the Mathematical Model

3. HEC-Ras Package

The HEC-RAS has the interactive program features, such as multi windows facilities, which can treat any river system configurations with the ability of storing these configurations. It can provide the user with a report showing the river system data and presenting the computation results by charts and/or tables.

There are five main steps in creating a hydraulic model with HEC-RAS (U. S. Army Crops of engineers, 2007).

A. Starting a NEW Project:

The first step is to establish the directory of working and enter a title for the new project. To start anew project, go to the File Menu on the Main HEC-RAS window and select New Project.

B. Entering Geomagnetic Data:

The next step is to enter the necessary geometric data, which consists of connectivity information for the Stream System (River System Schematic),Cross-Section Data, and Hydraulic Structure Data. Geometric
Data are entered by selecting Geometric data from the Edit Menu on the main HEC-RAS window.

C. Entering Flow Data and Boundary Condition:

Once the Geometric Data are entered, the modeler can enter either Steady Flow or Unsteady Flow Data. The type of flow data entered depends upon the type of analysis to performed. The data entry form unsteady Flow Data is available under the Edit Menu bar option on the HEC-RAS main window(3.3).

4. Description of Al-Msharah River with application of the Program

Al-Msharah River length is 49 Km, discrete in to 52 cross sections. The measured distance between any two cross sections is referred to a reach length, Table (1) and Figure (3) demonstrate the reach length.

Al Msharah river extends from the center of Al M'arah city to the center of Al Msharah city of about 32.5 km, and from the center of Al Msharah city toward Al Huwayza marsh of about 16.5 km, this part is called Al Malah River. The last part of Al Msharah River which ends at Al-Sann'af Marsh close to Al-Sodda Bridge is called Al Abttar and has a length of 15km. Then discharge into Al Huwayza marsh Figure(3) is a general satellite image of Al Huwayza Marsh feeders.

Al-Sann'af Marsh and Al Msharah River outfall were treated as a one source point since Al Msharah River flows into Al Sann'af Marsh before Al Sann'af Marsh outfall at Al Huwayza Marsh, as shown in Figure(4).

Al Huwayza feeders can be classified into two types depending on the existence of water control structures. The first is Al Msharah, and Al Ka'hla River which are controlled by a head regulator located upstream of each one. Fifteen tributaries are branched from the right side bank of Al Msharah River and there are two hydraulic structures along its length, bridge and culvert. Lateral outflows were proposed as percentage of the total inflow, discharge is at the up stream end.

Average monthly inflow to Al Msharah River is shown in Figure (5) where these are three types of flow wet (Peak Flood),Normal and dry years, (Ministry of

Water Resources, Center for Restoration of Iraqi Marshlands, 2006). In the present study two hypothetical flow cases are considered, wet (Peak Flood) and Normal flow.

Boundary geometry for the analysis of flow in a natural stream is specified in terms of cross sections and the reach lengths. Cross sections are located at intervals along a stream to characterize the flow carrying capacity of the stream and its adjacent flood plain. Figures(6) to (9)demonstrate the importance of the cross sections geometry.

Table(1) Reach Length Basic Information

Cross Section No.	Channel Distance (m)	Location
52	1000	Upstream River
51	700	Al Am'arah City
50	1000	
49	1000	
48	656	
47	843	
46	843	
45	843	
44	421.5	
43 A	421.5	Irrigation Channel
43	255	Al Am'arah City
42	889.5	
41	889.5	
40	889.5	
39	444.8	
38 A	444.8	Irrigation Channel
38	879	Al Amarah City
37	879	
36	879	
35	439.5	
34 A	439.5	Irrigation Channel
34	32.5	Al Am'arah City
33 A	32.5	Irrigation Channel
33	825.6	Al Am'arah City
32	825.6	
31	825.6	
30	1000	
29	1000	
28	1000	
27	1000	
26	1000	
25	494	
24 A	494	Irrigation Channel
24	2086	Al Am'arah City
23 A	2086	Irrigation Channel
23	40	Al Am'arah City
22 A	40	Al Msharah Bridge
22	397	Al Am'arah City
21	444.5	
20 A	444.5	Irrigation Channel

Continued Table(1)

Cross Section No.	Channel Distance (m)	Location
20	469.5	Al MSarah City
19 A	469.5	Irrigation Channel
19	65.5	Al Msarah City
18 A	65.5	Irrigation Channel
18	120.5	Al Msarah City
17 A	120.5	Irrigation Channel
17	2172	Al Msarah City
16	619.5	
15 A	619.5	Irrigation Channel
15	521.5	Al Msarah City
14 A	521.5	Irrigation Channel
14	127	Al Msarah City
13 A	127	Irrigation Channel
13	999	Al Msarah City
12	301.5	
11 A	301.5	Irrigation Channel
11	904	Al Msarah City
10	944	
9	1266	
8	867	
7	1175	
6	750	
5	825	
4	3	
3 A	3	Culvert
3	957	Al Msharah City
2	1518	
1	0	Al Malah Bridge

•Note: C.S. No. Cross Sections Number is presented .

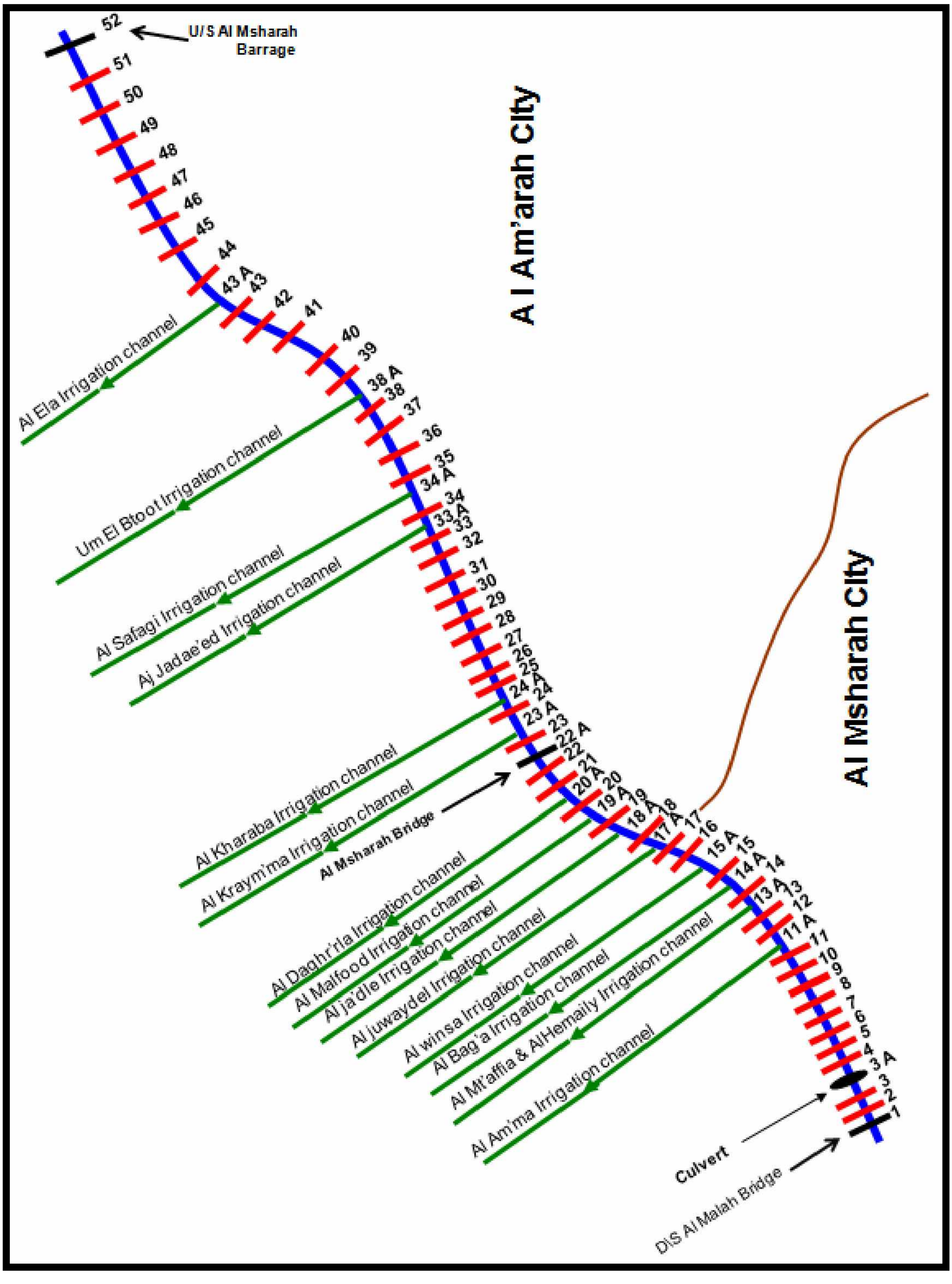


Figure (3) Location Map showing The Surreged Cross Sections For Al-Msharah River Between Al-Msharah Barrage And Al-Malah Bridge

5. Results And Discussions

Results of this study have been discussed for a normal depth, the normal depth has analyzed for most important flow parameters describing the flow characteristics of the study reach (between Al-Msharah bridge and Al-Malah bridge) .

Computed water surface profile is shown in finger (10) . This may be attributed to the topography nature of the channel basin and longitudinal bank elevation was lower than water surface profile . Therefore cross sections should be adjusted to allow passing any flow , as well as changing the ground level with distance along the cross sections of the river .

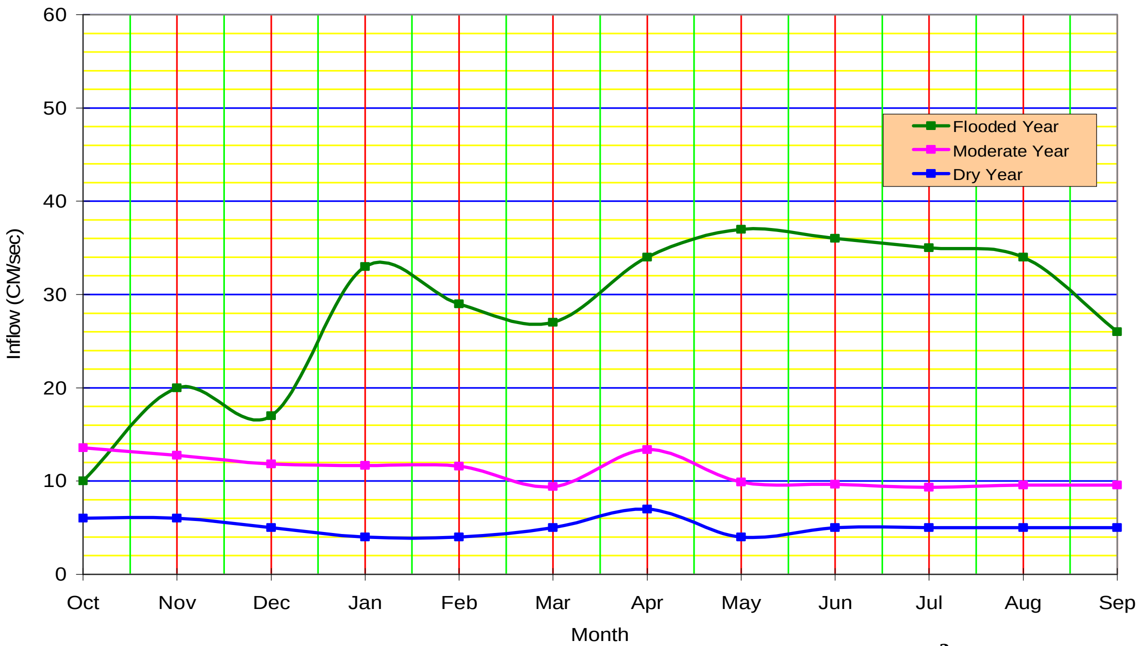
Fig(11) show the stage decreased form upstream toward downstream, and fig(12) show the water volume profile where it decrease from Al-Msharah Barrage station towards Al-Malah Bridge station because of water consumption and water losses along the study reach .

At last calibration of model computed with measured water surface profile is platted in fig (13) and show that a good agreement.

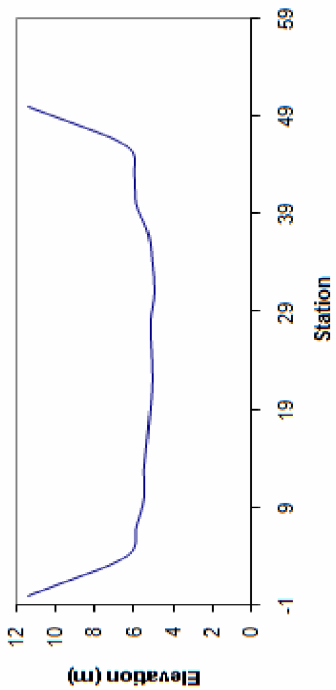
6. Conclusions

The most conclusion of this study show that the area at Am'arah city at distance 17.5 km from upstream (cross section 30) could be subjected to flooding at high flow, since its longitudinal bank elevation is lower than water surface, therefore, it is recommended to adjust cross sections to prevent the flooding in this area.

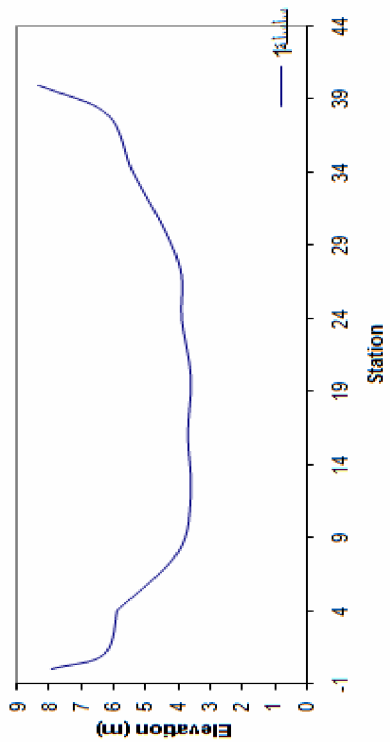
Figure (4) The location of AsSanna'f Marsh and Al Msharah River Outlet into the marsh



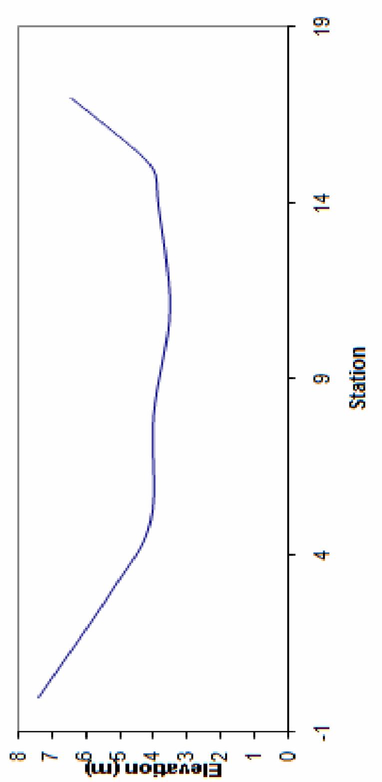
Figure(5)Al Msharah River average Monthly Flow, in m^3/sec Wet, Normal and Dry Years



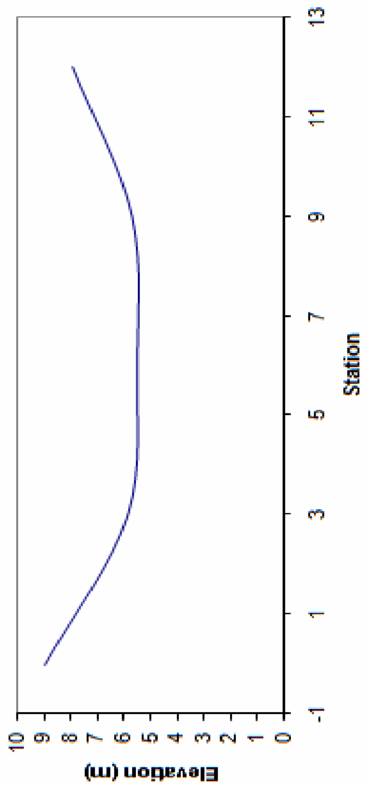
Figure(4.6)Cross Section Upstream Al Msharah River



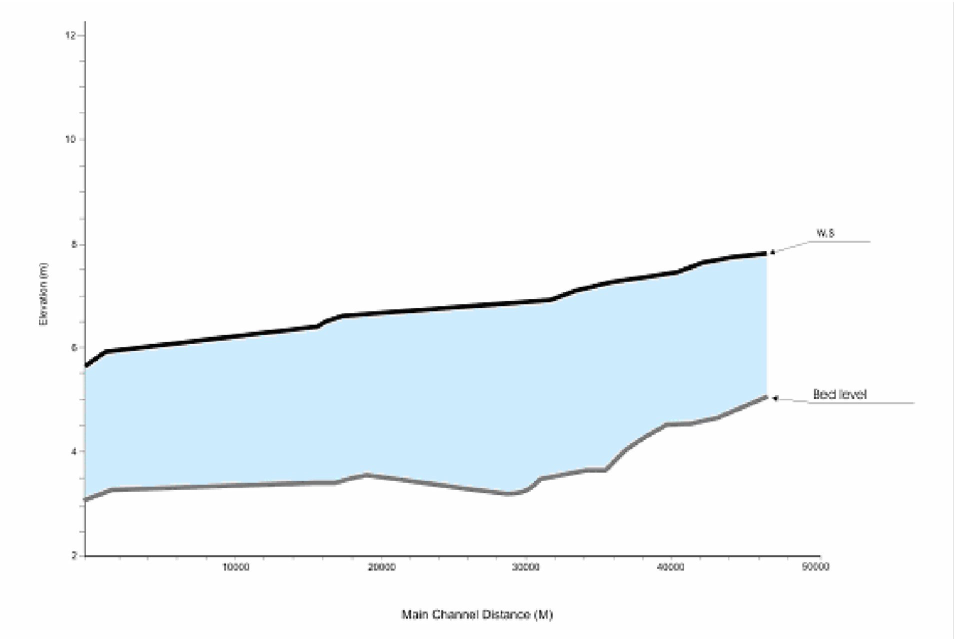
Figure(4.7)Cross section at 10 Km From Upstream(Al Am'arah City)



Figure(4.8)Cross Section at 37 Km From Upstream (AL Msharah City)



Figure(4.9) Cross Section at Downstream AL Msharah River



Figure(10) Water Surface Profile at Al Msharah River at Normal

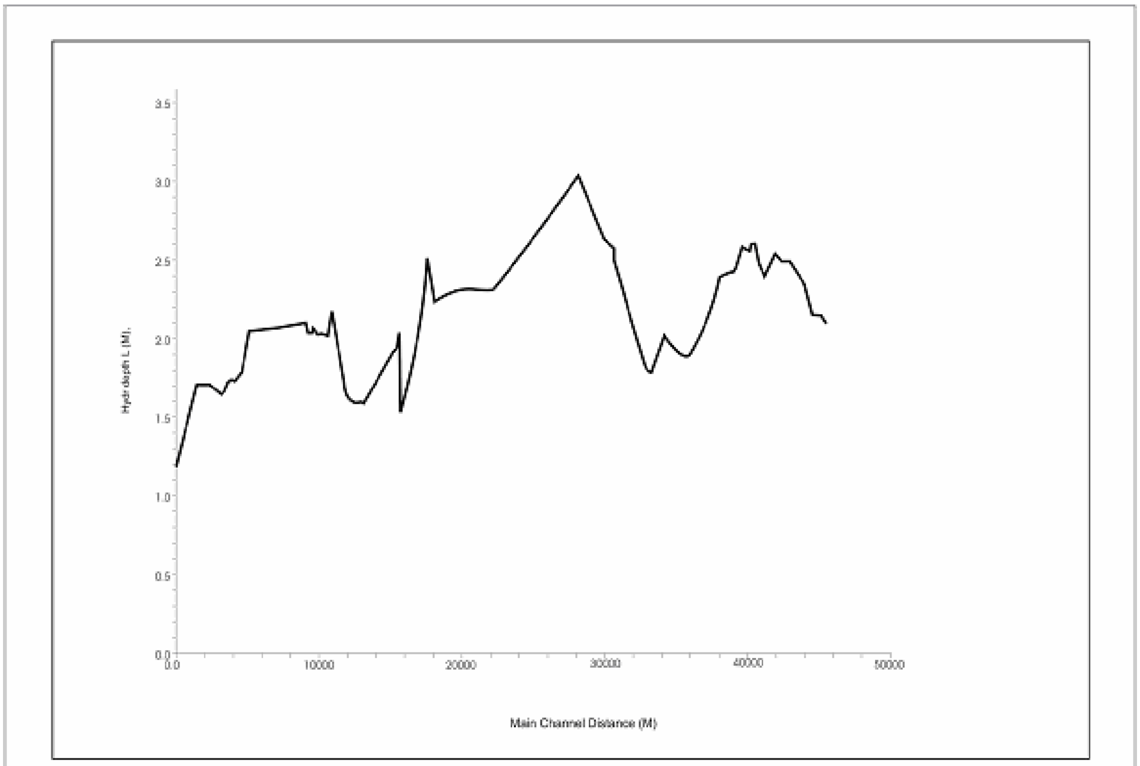


Fig (11) Stage Profile at Al Msharah River at Normal Flow

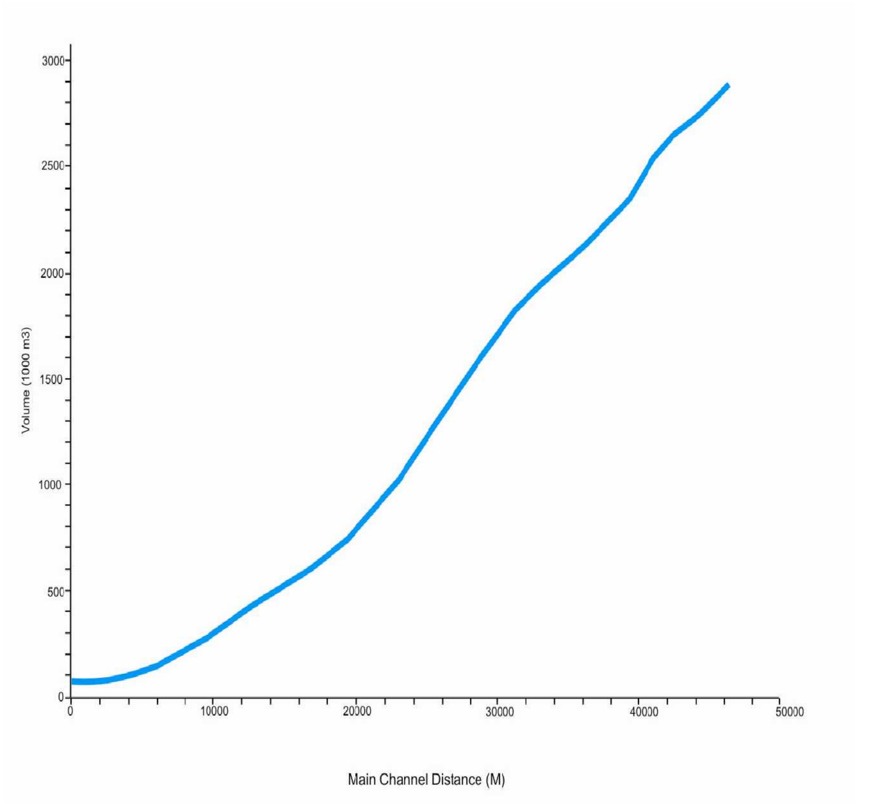


Figure (12) Water Volume Profile at Al Msharah River at Normal Flow

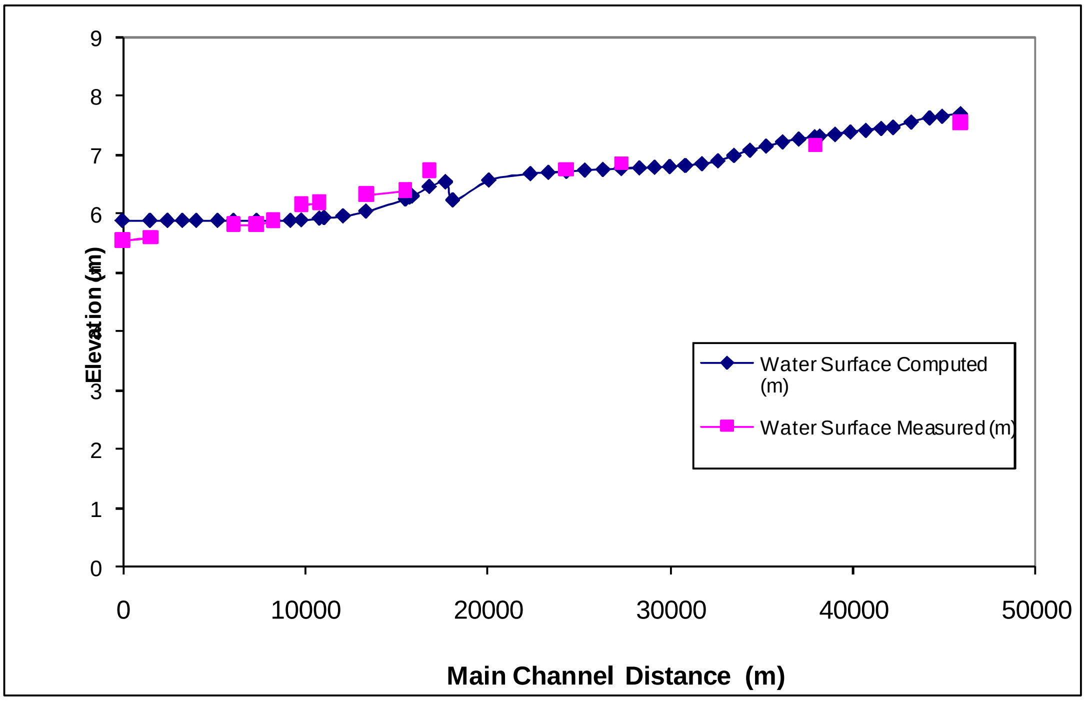


Fig.(13) calibration of model computed with Measured water surface profile

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