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On the Total Domination and Vertex Covering of the Standard Chessboard

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ABSTRACT: The movement of pieces on chessboards has led to several problems. Many researchers have solved some of these problems, but many of them remain unsolved. Graph theory is a mathematical method used to describe and discover solutions to such problems. The problem of dominating the chessboard was found in topics pertaining to puzzles in mathematics and entertainment mathematics. In this paper, we survey some results concerning parameters domination, total domination, and covering vertices in chessboard. After converting the chessboard to graph.

Keywords: Graph theory, Chessboard, Domination, Total domination, Cover vertex.

1. INTRODUCTION

The multitude and varied applications of graph theory in various disciplines, including social science, biology, and engineering, have led to an increase in its popularity [1]. The mathematical representation of a network is another name for a graph. The term "network" refers to the same concept as a graph in most studies. Data on the relationships among several items can be described using a graph in which the items are represented by vertices, relations, and edges [2]. Many problems related to applications of graph theory have been mentioned in the literature frequently previously (see, e.g., [3,4]).

People have been captivated by the ancient board game of chess for centuries in numerous nations worldwide. One of the trickiest and most researched games on skill is chess. The fundamental elements of chess have given rise to numerous traditionally researched chessboard puzzles. Several chess-related problems have been studied using graph theoretical analysis [5]. One of the most popular games in the in the world is chess, which was first played in northwestern India in the sixth century. It was derived from an older Indian game called Chaturanga [6]. Chess is a strategy board game for two players, a checkered game square board organized in a grid 8x8. Despite its complexity, the goal of the game is to approach, threaten, and capture pieces until they reach their target [7].

Although chess has many problems, there are still many mysteries in the game that we do not yet know how to solve. Mathematically oriented challenges are a natural fit for chess because of their algorithmic structures. Chess puzzles have been solved by mathematicians and computer scientists using computer algorithms since the 20th century [5]. The study of dominating sets in graphs was greatly inspired by the old difficulties of covering chessboards with the fewest possible chess pieces [8].

Research on domination in graphs expanded in the late 1950s and the early 1960s. The topic dates back to De Jaenisch in 1862, who investigated the issues of determining how many queen pieces are required to control or cover a chessboard [9]. Claude Berge first presented the domination number of a graph or the concept of the (coefficient of external stability) in his graph theory book from 1958. The terms (domination number) and (dominating set) were first used by Oystein Ore in his graph theory book from 1962 [10]. The recognized notation $\gamma(G)$ was proposed by Cockayne and Hedetniemi in 1977 to represent the domination number [11]. The study of recreational mathematics led to the investigation of dominance in graphs. Mathematicians specifically looked at how to arrange pieces in the same kind way that they could attack or take control of each square on a checkerboard [12].

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Researchers examined the vertex cover parameter and its variations for many kinds of graphs. Vertex cover is also a crucial concept in chessboard graphs. Subset of vertices covering every edge in the graph. Otherwise, each vertex in the vertex cover is incident on at least one edge of the graph [13]. There are numerous real-world uses for the notion of a vertex covering set, including resource allocation, network design, and optimization. Moreover, determining a minimal vertex cover aids in reducing expenses or maximizing effectiveness [14].

In this paper, we applied some parameters such as domination, total domination, and covering vertices in a chessboard and calculated (size of the smallest dominating set γ(G), minimum cardinality of a total dominating set $\gamma_t(G)$, and minimum cardinality of a vertex cover $C(G)$ of a graph G concluded from the chessboard.

2. CHESSBOARD

The chessboard in this work is a square of size 8×8 with 64 square cells. There are six types of pieces. The chessboard Contains 32 pieces when starting a game, covering half of the standard 64 squares, 16 black pieces, and 16 white pieces. For every set of colors, there are eight pawns (P) , two rooks (R), two knights (N) , two bishops (B), one king (K), and one queen (Q) [5]. Five of the six classic chess pieces were used in this study: queen, king, rook, knight, and bishop. Although many fascinating mathematical problems are associated with the traditional game of chess, we are primarily concerned with the movement of pieces. Every piece can make a distinct set of movements [15].

A graph can be created from a chessboard using squares as vertices, where two vertices are adjacent if a piece of chess covers the other square [8]. The number of squares in the chessboard and the number of vertices and edges in the graph as shown in Figure (1), after converting the chessboard to a graph. When merging chess piece movements and graph theory. Graphs do a great job in depicting chessboard and piece movements. Every chess piece creates its own graph by utilizing edges to show the progress of the piece from one permitted square to another and vertices to represent a single square on the board [15].

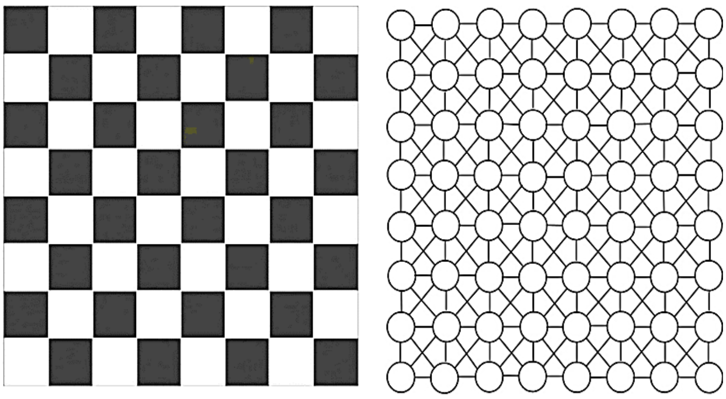

FIGURE 1. - Convert the chessboard to graph

3. PRELIMINARIE

Let $G = (V(G), E(G))$ be an undirected and simple graph, where $V(G)$ symbolizes the set of vertices, and $E(G)$ denotes the set of edges $E(G)$. If D subset of $V(G)$, D is called a dominating set of G if $\forall$ u $\in V(G)\setminus D$ $\exists$ v$\in$D, such that u and v are adjacent [12]. The size of the smallest dominating set denoted by γ(G), refers to the domination number of G. Thus, it has $1 \leq \gamma(G) \leq n$, where n is the number of vertices in graph G [16].

A subset D^t of $V(G)$, D^t is called a total dominating set of G if for every vertex v of G, there is a vertex $u \in D^t$ such that u dominates v. The total domination number of G is the minimal cardinality of a total dominating set of G and denoted by $\gamma_t(G)$ [9] [17]. The total dominating set D^t is not always the same as the dominating set D. Members of a dominating set D may be found inside D^t or next to its vertices. However, if there is a complete dominant set D^t, in each member of D^t must be next to a vertex in D^t [18]. A subset C of $V(G)$, C is called a vertex covering set of a graph G if each edge in G incident with a vertex in C. The vertex cover number of G is the minimum cardinality of a vertex cover of G, indicated by C(G) [13]. Next, we examined the role of chees pieces in the construction of the domination number and independence number for square chessboards, as well as the covering problem of chessboards after converting the chessboard to a graph.

4. MAIN RESULTS

The chessboard piece movement has sparked many puzzles, many of which remain unresolved. New problems arise to enact new ways of playing, even as old ones are overcome. Graph theory is a mathematical tool that we have at our disposal to describe and solve the current issues.

4.1 QUEENS IN CHESSBOARD

In the chess game, each player has only one queen. The strongest piece on the chessboard was the queen. The power of the queen is directly related to its movement. The rook's side-to-side, down-to-up moves and the bishop's diagonal moves are combined to form the queen's moves allowed to move as many squares of vacant squares. The only restriction of the queen is that it cannot jump over the pieces [10]. Any square to which a queen can move has an edge between the two squares or two vertices in the graph. See Figure (2a). Queen domination is a popular problem that has been studied and expanded into other chess pieces [5].

Determine the smallest number of queens that may be placed on a board such that any square can either have a queen occupying it initially or can be occupied by a queen in a single step. The chessboard is controlled by queens, which are referred to as dominating sets, regardless of whether the square is dominated by one or more pieces or not. In Figure (2b) the five queens' problem identifies that are displayed as controlling the entire chessboard. As a result, the domination number for queens on a chessboard is 5 i.e. $\gamma(\mathrm{GQ}) = 5$ [19].

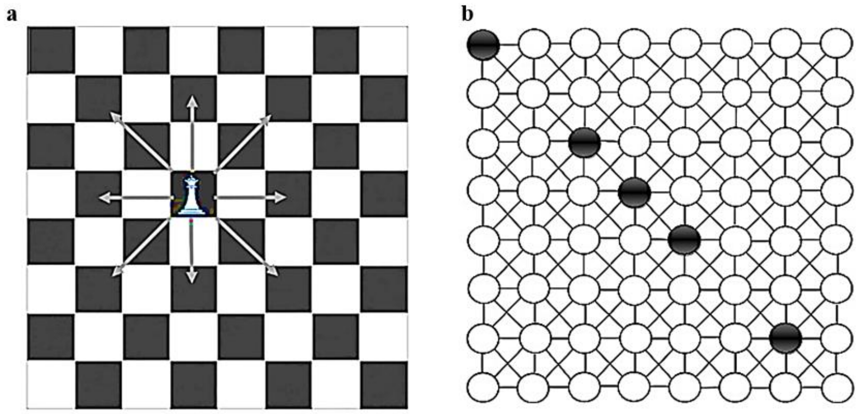

Figure 2.- (a) Queen move; (b) Queen domination

To find the total domination, add one queen to the dominating queens depicted in Figure (2b) thus, the dominating vertices are connected. Figure (3a) illustrates the notion of total domination on the chessboard. Hence, $\gamma_t(\mathrm{GQ}) = 7$. The concept of vertex cover is applied to the queen's problem on a chessboard by placing two pieces of the queen and distributing them according to Figure (3b). The rules of the game movements were followed for each queen. It should be noted that the entire chessboard was covered, and $C(GQ) = 2$.

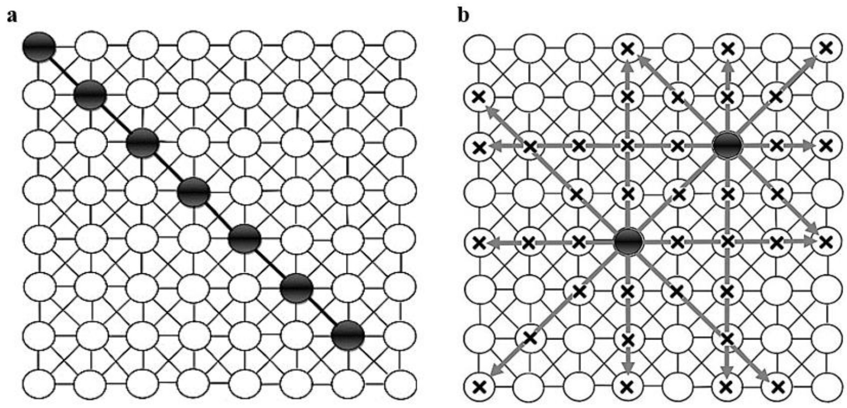

Figure 3.- (a) Total domination of queen; (b) Queens pieces covering the chessboard

4.2 KING IN CHESSBOARD

In chess, every player has only one piece of king. kings can advance by exactly one square in any direction. As long as they do not replace a friendly piece, Figure (4a) shows the king's movement [10]. Determine the smallest number of kings that may be placed on a board such that any square can either have a king occupying it or can be occupied by a king in a step. As a result, the chessboard is controlled by kings, which are referred to as dominating sets, as shown in Figure (4b). The domination number for kings on the chessboard is nine, that is, $\gamma(\mathrm{GK}) = 9$ [20].

a b

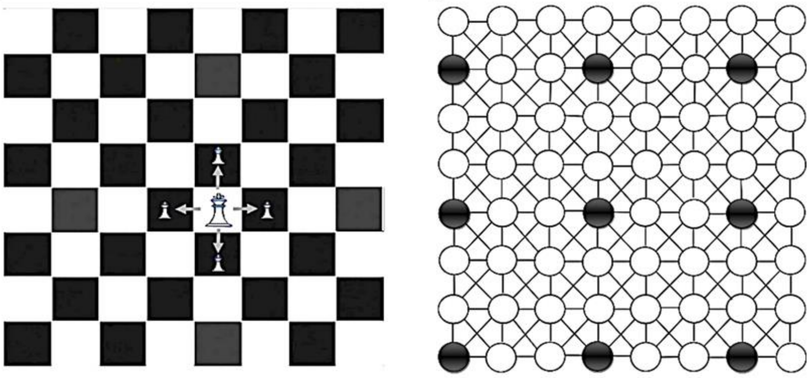

Figure 4.- (a) King move; (b) King domination

To find the total domination, vertices (pieces of king) are added to the set of dominated vertices in Figure (4a). To apply the concept of total domination by placing some of the kings' pieces on a chessboard, the dominant vertices are linked. Hence, $\gamma(GB) = 25$ as shown in Figure (5a). The concept of vertex cover is applied to the king's problem on a chessboard, placing four pieces of king and distributing them according to Figure (5b). The rules of the game's movement were followed for each piece of the king. Hince, we notice that the entire chessboard is covered, then $C(GR) = 4$.

a b

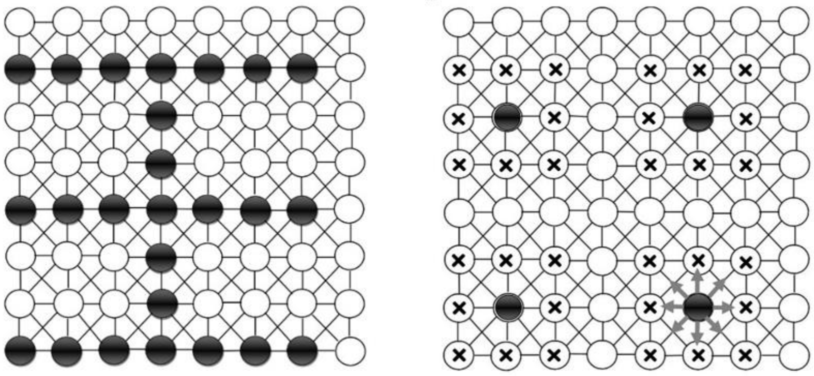

Figure 5.-(a) Total domination of king; (b) King pieces covering the chessboard

4.3 BISHOP IN CHESSBOARD

Game beings with two bishops pre each player. The bishop can advance any number of squares until it comes in contact with another piece or edge of the board. It can move along diagonals between squares but cannot leap over other pieces because it cannot move in a horizontal or vertical line. The piece consistently appears on the same-colored squares during the game, see Figure (6a) [10][20]. The bishops dominated the square chessboard when their pieces were arranged, as shown in Figure (6b). As a result, the chessboard is controlled by bishops, which are referred to as the dominating sets. Hence, the domination number for kings on the chessboard is eight, that is $\gamma(GK) = 8$ [20].

a b

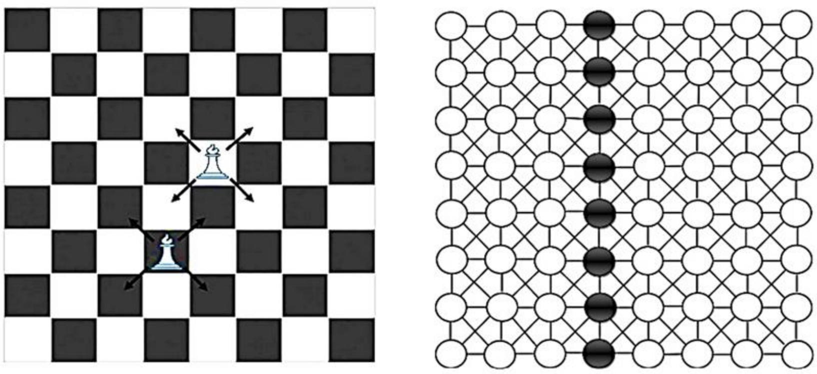

Figure 6.- (a) Bishop move; (b) Bishop domination

The concept of total domination is used to place bishop pieces on a chessboard in a certain manner, such as a single row or column, as shown in the graph output from the chessboard; thus, the dominating vertices are connected. Therefore, $\gamma_t(\mathrm{GB}) = 8$, as shown in Figure (7a). By applying the concept of vertex cover in bishop's problem on a chessboard, placing three pieces of bishop and distributing them according to Figure (7b). The rules of the game's movement were followed for each piece of the bishop. Hence, we notice that the entire chessboard is covered, then $C(GB) = 3$.

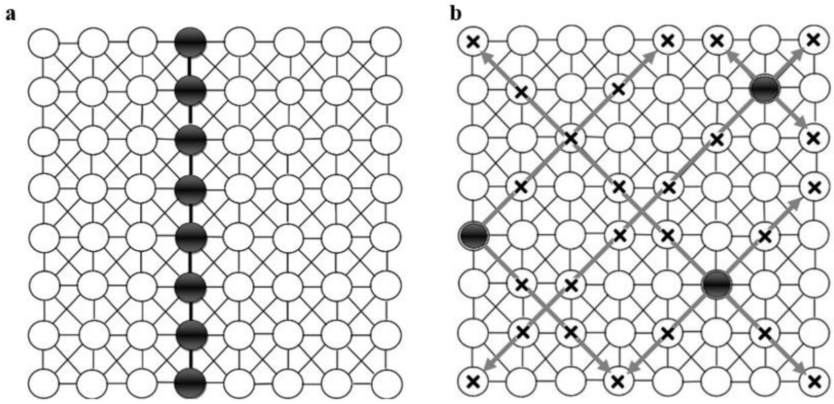

Figure 7.- (a) Total domination of bishop; (b) Bishop pieces covering the chessboard

4.4 ROOK IN CHESSBOARD

Each player had two rooks at the beginning of the game. The rook is allowed to move as many squares as it wants in straight lines up and down or side to side, until it runs into another piece that blocks it or reaches the board's end. It cannot leap over other parts, as shown in Figure (8a) [10]. The chessboard is controlled by rooks, which are referred to as dominating sets. The rooks completely dominate a square chessboard when their pieces are arranged in a single row, single column, or diameter form, as shown in Figure (8b) [11]. Then, pieces of rook with domination equal to 8. Therefore $\gamma(GR) = 8$.

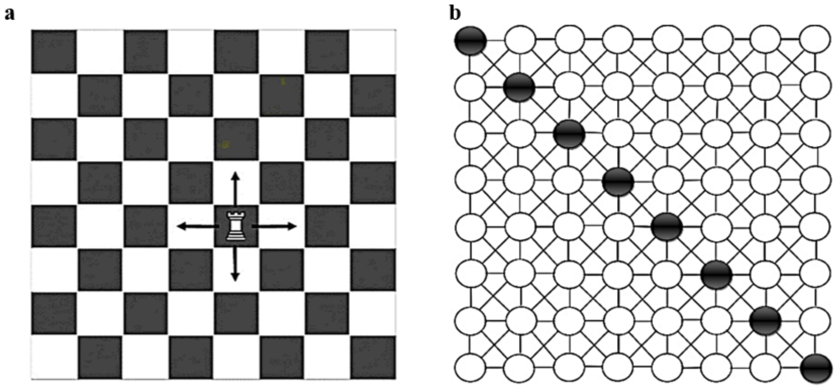

Figure 8.-(a) Rook move; (b) Rook domination

The concept of total domination is utilized in placing pieces of the rook on a chessboard in any column or row of the graph, such as a single row or column, as well as positioning arranged in a diagonal form in a graph output from the chessboard. Therefore $\gamma_t(\mathrm{GB}) = 8$, as shown in Figure (9a). The concept of vertex cover is applied in rook's problem on a chessboard, placing three rooks and distributing them according to Figure (9b). The rules of the game movements were followed for each piece of the rook. Hence, we observed that the entire chessboard was covered. Then $C(GR) = 3$.

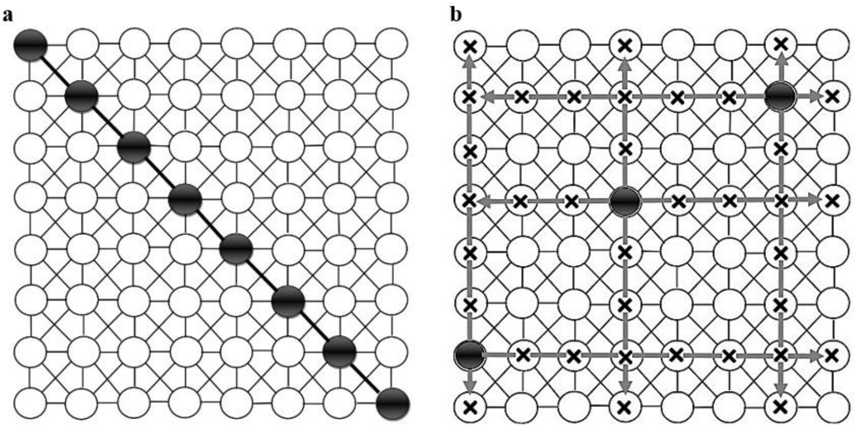

Figure 9.- (a)Total domination of rook; (b) Rook pieces covering the chessboard

4.5 KNIGHT IN CHESSBOARD

In chess, knights are permitted to travel one square in the opposite direction and two squares in the vertical or horizontal direction as long as they do not replace a friendly piece. The squares directly next to it are out of control, and the entire motion is similar to that of L. The game started with two knights for each player [20]. The exclusive ability of knights to leap over other pieces, whether friendly or enemy, sets them apart from other types of pieces. When the knight is in the middle of the board, he controls eight squares, as shown in Figure (10a) [10]. The knights completely dominate the square of the chessboard when their pieces are arranged as shown in Figure (10b). Then the pieces of knights with a domination number on the chessboard equal 12. Hence $\gamma(\mathrm{GN}) = 12$.

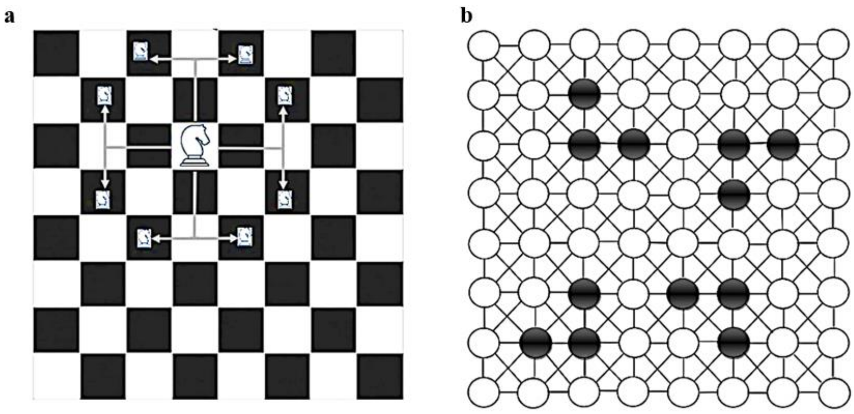

Figure 10.-(a) Knight move; (b) Knight domination

The concept of total domination is utilized in placing by add some of the knight's pieces on a chessboard as shown in Figure (11a), hence $\gamma t(\mathrm{Gk}) = 15$. The concept of vertex cover is It is applied to the knight problem on a chessboard, placing four knights and distributing them according to Figure (11b). The rules of the game's movements were followed for each piece of knight. Hence, we notice that the entire chessboard is covered, and then $C(GN) = 4$.

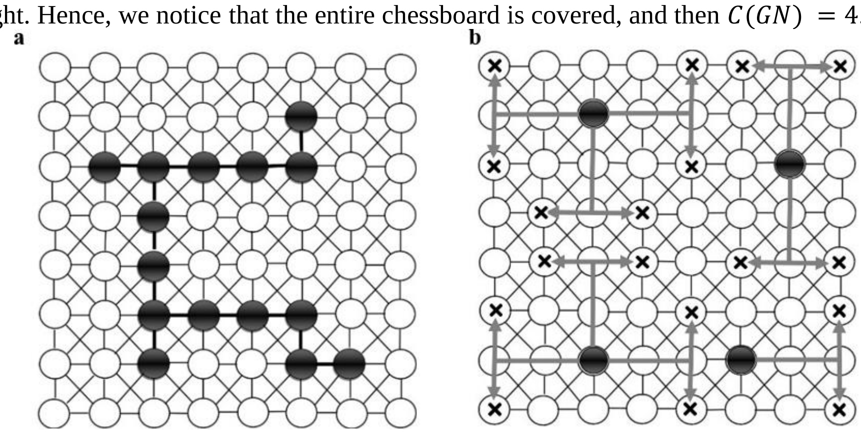

Figure 11.- (a) Total domination of knight; (b) knight pieces covering the chessboard

5. CONCLUSIONS

The concepts of domination, total domination, and covering vertices have real-world uses in different fields like resource allocation, network design, and optimization. In this paper, we discussed the application of these concepts, and we found the exact values and placement for the pieces of chess on square chessboard 8 according to the rules of the game related to the movement of each piece. The results are presented in the following Table (1):

Table 1. - Shows the results that obtained

No.	Pieces of cheeseboard	Domination number	Total domination number	Vertex covering number
1	Queen	$\gamma(\mathrm{GQ}) = 5$	$\gamma_t(\mathrm{GQ}) = 7$	$\mathrm{C(GQ)} = 2$
2	King	$\gamma(\mathrm{GK}) = 9$	$\gamma_t(\mathrm{GK}) = 25$	$\mathrm{C(GK)} = 4$
3	Bishop	$\gamma(\mathrm{GB}) = 8$	$\gamma_t(\mathrm{GB}) = 8$	$\mathrm{C(GB)} = 3$
4	Rook	$\gamma(\mathrm{GR}) = 8$	$\gamma_t(\mathrm{GR}) = 8$	$\mathrm{C(GR)} = 2$
5	Knight	$\gamma(\mathrm{GN}) = 12$	$\gamma_t(\mathrm{GN}) = 15$	$\mathrm{C(GN)} = 4$

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