IRAQI Academic Scientific Journals

University of Kirkuk Journal For Administrative and Economic Science (2024) 14 (4): 225-236

UKJAES

University of Kirkuk Journal For Administrative and Economic Science

ISSN 2222-2995
University of Kirkuk Journal For Administrative and Economic Science

Ahmed Hardi Gharib, Mohammed Aryan Omar & Taher Hindreen Abdullah. Forecasting Iraq's Monthly Oil Prices (2021-2023): A Comparative Study of GM (2,1) and GAO-GM (2,1) Models. *University of Kirkuk Journal For Administrative and Economic Science* (2024) 14 (4):225-236.

Forecasting Iraq's Monthly Oil Prices (2021-2023): A Comparative Study of GM (2,1) and GAO-GM (2,1) Models

Hardi Gharib Ahmed [1], Aryan Omar Mohammed [2], Hindreen Abdullah Taher[3]

[1] 1Department of Banking and Finance Science, University of Sulaimani - College of Commerce, Sulaimani, Iraq
[2] Department of International Commerce, University of Sulaimani - College of Commerce, Sulaimani, Iraq
[3] Department of Information Technology, University of Sulaimani - College of Commerce, Sulaimani, Iraq

hardi.ahmed@univsul.edu.iq[1]
aryan.omar@univsul.edu.iq[2]
hindreen.taher@univsul.edu.iq[3]

Abstract: This study evaluates the performance of two forecasting models, GM(2,1) and GAO-GM(2,1) for forecasting the oil price of Iraq, the monthly dataset in use is since (jan. 2021 to Dec. 2023), focusing on their predictive accuracy and efficiency. Accuracy Metrics such as Mean Squared Error (MSE), Root Mean Squared Error (RMSE), Akaike Information Criterion (AIC), and Bayesian Information Criterion (BIC) were used to compare the performance models. The results indicate that the GAO-GM (2,1) model significantly outperforms the GM(2,1) model, achieving a lower MSE of 62.384 compared to 86.092 and an RMSE of 7.898 versus 9.279, respectively. Additionally, GAO-GM (2,1) demonstrates a superior fit to the data, as reflected in its lower AIC and BIC values. The study also examines the efficiency of a Genetic Algorithm Optimizer (GAO) across multiple iterations, revealing a consistent convergence toward optimal solutions. The analysis includes a comparison of actual oil prices against predicted values, highlighting the model's accuracy and the importance of forecasting in decision-making processes. Statistical tests for normality confirm the validity of the model's residuals, further supporting the reliability of the predictions. Overall, the findings underscore the effectiveness of the GAO-GM (2,1) model in providing accurate and actionable forecasts, making it a valuable tool in contexts such as finance and economics.

Keywords: Grey system theory, Genetic algorithm optimizer, Oil price, GM (2,1) model.

التنبؤ بأسعار النفط الشهرية في العراق (٢٠٢١-٢٠٢٣): دراسة مقارنة لنموذجي GM(٢,١) وGAO-GM(٢,١)

م.م. هردي غريب أحمد[١]، م.م. أريان عمر محمد[٢]، م. هندرين عبد الله طاهر[٣]

[١] قسم العلوم المصرفية والمالية، جامعة السليمانية - كلية التجارة، السليمانية، العراق
[٢] قسم التجارة الدولية، جامعة السليمانية - كلية التجارة، السليمانية، العراق
[٣] قسم تكنولوجيا المعلومات، جامعة السليمانية - كلية التجارة، السليمانية، العراق

IRAQI Academic Scientific Journals

University of Kirkuk Journal For Administrative and Economic Science (2024) 14 (4): 225-236

المستخلص: تم تقيم أداء نموذجي GM (٢,١) وGAO-GM (٢,١) للتنبؤ بسعر النفط في العراق في هذه الدراسة، تم استخدام البيانات الشهرية للفترة (يناير ٢٠٢١ إلى ديسمبر ٢٠٢٣). ولقد إستخدامنا مقاييس الدقة مثل متوسط الخطأ التربيعي (MSE) وجذر متوسط الخطأ التربيعي (RMSE) ومعيار معلومات أكايكي (AIC) ومعيار المعلومات البايزي (BIC) لمقارنة الأداء نماذج. تشير النتائج إلى أن نموذج GAO-GM (٢,١) يتفوق بشكل كبير على نموذج GM (٢,١)، حيث حقق متوسط خطأ أقل قدره ٦٢,٣٨٤ مقارنة بـ ٨٦,٠٩٢ ومتوسط خطأ تربيعي متوسط قدره ٧,٨٩٨ مقابل ٩,٢٧٩، على التوالي. بالإضافة إلى ذلك، يُظهر نموذج GAO-GM (٢,١) ملاءمة فائقة للبيانات، كما ينعكس في قيم AIC وBIC المنخفضة. يتضمن التحليل مقارنة بين أسعار النفط الفعلية والقيم المتوقعة، مما يعكس الضوء على دقة النموذج وأهمية التنبؤ في عمليات صنع القرار. بشكل عام، تؤكد النتائج فعالية نموذج GAO-GM (٢,١) في توفير توقعات دقيقة وقابلة للتنفيذ، مما يجعله أداة قيمة في سياقات مثل التمويل والاقتصاد.

الكلمات المفتاحية: نظرية النظام الرمادي، خوارزمية الجينية، سعر النفط، نموذج GM(٢,١).

Corresponding Author: E-mail: hardi.ahmed@univsul.edu.iq

Introduction

The monthly oil prices of Iraq from 2021 to 2023 reflect significant fluctuations influenced by a variety of factors, including global demand, geopolitical events, and changes in production levels. As one of the leading oil producers in the Organization of the Petroleum Exporting Countries (OPEC), Iraq's oil revenues are crucial for its economy, which heavily relies on hydrocarbon exports. In 2021, the oil market began recovering from the impacts of the COVID-19 pandemic, leading to an increase in prices, which averaged around $70 per barrel by mid-year (OPEC, 2022). This upward trend continued into 2022, driven by heightened demand and supply constraints exacerbated by geopolitical tensions, particularly the Russia-Ukraine conflict, which led to a surge in oil prices (International Energy Agency, 2023). However, by 2023, prices exhibited volatility due to fluctuating global economic conditions and strategic decisions made by OPEC+ regarding production cuts (World Bank, 2023). Understanding these trends is essential for stakeholders in the oil market, as they impact economic stability, investment decisions, and policy-making in Iraq. The Genetic Algorithm Optimized GM(2,1) model integrates the principles of Grey forecasting with genetic algorithms to enhance predictive capabilities in time series analysis. The GM(2,1) model, a prominent member of Grey model theory, excels in managing small sample sizes (short time series) and inherent uncertainties, making it suitable for diverse applications, including agriculture, economics, and environmental science (Wu et al., 2019). However, the performance of the GM(2,1) model can be further improved through optimization techniques, particularly genetic algorithms, which utilize natural selection principles to fine-tune model parameters (Chen et al., 2020). This hybrid approach not only improves forecasting accuracy but also enhances the model's adaptability to changing data patterns, addressing the challenges posed by volatile environments (Zhang et al., 2021). As research progresses, the GA-GM(2,1) model represents a significant advancement in predictive analytics, offering robust solutions for decision-makers requiring reliable forecasts in complex systems (Liu & Yang, 2022).

Literature Review

The GM(2,1) model, a type of Grey forecasting model, has gained attention in time series forecasting due to its ability to handle small sample sizes and uncertainty. Researchers have successfully applied the GM(2,1) model to various fields, including economics, environmental science, and agriculture. For example, Wu et al. (2019) demonstrated its efficacy in forecasting agricultural yields in China, highlighting its accuracy in predicting trends with limited data. However, the model's performance can be further enhanced through optimization techniques, such as genetic algorithms, which help in fine-tuning model parameters to achieve better forecasting accuracy.

The integration of genetic algorithms with the GM(2,1) model has shown promising results in improving prediction accuracy. In their study, Chen et al. (2020) introduced a Genetic Algorithm Optimized GM(2,1) model, which was tested against traditional forecasting methods. The study found that the GA-GM(2,1) model significantly reduced forecasting errors, particularly in volatile markets. The optimization process enabled the model to adapt more effectively to changes in data patterns, leading to improved robustness and reliability in predictions.

Numerous studies have compared the GM(2,1) model optimized by genetic algorithms to other forecasting models. For instance, Zhang et al. (2021) conducted a comparative analysis of the GA-GM(2,1) model against ARIMA and exponential smoothing methods in forecasting energy consumption. Their findings indicated that the GA-GM(2,1) model consistently outperformed the other methods in terms of Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE). This research underscores the model's adaptability and effectiveness, particularly in scenarios with fluctuating data trends.

The theoretical framework behind the GM(2,1) model and its optimization through genetic algorithms provides a robust foundation for various applications. Liu and Yang (2022) explored the mathematical principles governing the GM(2,1) model and its enhancements via genetic algorithms. Their research emphasizes the significance of parameter optimization in achieving accurate predictions, particularly in complex systems with inherent uncertainties. The study also discusses practical implications for industries such as finance and logistics, where precise forecasting is critical for strategic decision-making.

In conclusion, the GM(2,1) model, particularly when optimized with genetic algorithms, represents a promising approach to addressing the challenges of time series forecasting. Its proven adaptability and accuracy make it a valuable tool for researchers and practitioners alike, paving the way for more informed decision-making across various sectors. Continued research in this area holds the potential for further advancements, enhancing the reliability and applicability of forecasting models in an increasingly uncertain world.

1. Methodology

A. The GM (2,1) Models

The appropriate grey two-by-one model is for exponential pattern sequences; also, it is used to show changes in monotonic patterns. While for another non-monotonic wave, such as development sequences or satiate sequences that is sigmoid, we are able to use GM two by one [5,6].

B. The GM (2,1) Model

For raw data sequences $A^{(0)} = (a^{(0)}(1), a^{(0)}(2), \dots\dots, x^{(0)}(n))$, let it a generation accumulation and inverse accumulation generation be[5,7,8] $A^{(1)} = (a^{(1)}, a^{(1)}(1), a^{(1)}(2), \dots\dots, a^{(1)}(n))$ and $b^{(1)}A^{(0)} = (b^{(1)}a^{(0)}(2), \dots\dots, b^{(1)}a^{(0)}(n))$, where $b^{(1)}a^{(0)}(k) = a^{0}(k) - a^{(0)}(k-1), k = 2,3,\dots,n$ and the adjacent sequence of neighbor mean a generation of$A^{(1)}\ be\ Y^{(1)} = \left(y^{(1)}(2), y^{(1)}(3), \dots\dots, y^{(1)}(n)\right)$.

Then

$$\mathrm{b}^{(1)}\mathrm{a}^{(0)}(\mathrm{k}) + \mathrm{b}_1\mathrm{a}^{(0)}(\mathrm{k}) + \mathrm{b}_2\mathrm{y}^{(1)}(\mathrm{k}) = \mathrm{c} \qquad 2.\ 1$$

Is show that GM(2,1) model ;

$$\frac{d^2a^{(1)}}{dt^2} + \alpha_1\frac{da^{(1)}}{dt} + b_2a^{(1)} = c \qquad 2.2$$

Theorem: For the sequences$A^{(0)}A^{(1)}Y^{(1)}$ $and\ b^{(1)}a^{(0)}$, as defined above, let

$$B = \begin{bmatrix} -a^{(0)}(2) & -y^{(1)}(2) & 1 \\ -a^{(0)}(3) & -y^{(1)}(3) & 1 \\ \dots & \dots & \dots \\ -a^{(0)}(n) & -y^{(1)}(n) & 1 \end{bmatrix}$$

$$Y = \begin{bmatrix} b^{(1)}a^{(0)}(2) \\ b^{(1)}a^{(0)}(3) \\ \dots \\ b^{(1)}a^{(0)}(n) \end{bmatrix} = \begin{bmatrix} a^{(0)}(2) - a^{(0)}(1) \\ a^{(0)}(3) - a^{(0)}(2) \\ \dots \\ a^{(0)}(n) - a^{(0)}(n-1) \end{bmatrix}$$

Also, in the least squares parametric sequence$\hat{a}$ have estimated the $= [b_1, b_2, c]^T$ Of the G.M (2,1) is used as follows:

$\hat{b} = (C^T C)^{-1} C^T Y$ 2.3

Theorem: For the solution of the G.M two by one winterization equation depend the steps [9,10]:

1- If $A^{(1)^*}$ is a unique solution of $\frac{d^2a^{(1)}}{dt^2} + b_1\frac{da^{(1)}}{dt} + b_2a^{(1)} = b$ and $\bar{A}^{(1)}$the general solution of the corresponding homogeneous equation$\frac{d^2a^{(1)}}{dt^2} + b_1\frac{da^{(1)}}{dt} + b_2a^{(1)} = 0$, Then $A^{(1)} + \bar{A}^{(1)}$ represents a universal method for solving the whitening equation for GM(2,1).

2- The general solution to the preceding homogeneous differential equation requires satisfying the following three conditions: (i) if the defining equation for a $r^2 + c_1r + b_2 = 0$ has two distinct real roots r_1, r_2,

A(1)=$c1er1t$+$c2er2t$ 2.4

when the repeated root r is the characteristic equation,

A(1)=$ert(c1$+$c2t)$ 2. 5

when the two complex conjugate roots are the characteristic equation

$$r_1 = b + i\beta \ and\ r_2 = b - i\beta$$

$\bar{X}^{(1)} = e^{\alpha t}(c_1 cos\beta t + c_2 sin\beta t)$ 2.6

3- In particular, one of the following three scenarios may represent a solution to the winterization equation:

a. The characteristic equation root isn't zero,$A^{(1)^*} = C$.

b. In the characteristic equation, zero is one of the2 distinct roots of,$A^{(1)^*} = Ca$.

c. In the characteristic equation, zero is the only root of,$A^{(1)^*} = Ca^2$.

C. Genetic Algorithm Optimizer (GAO)

It is a heuristic optimization technique inspired by the principles of natural selection and genetics. It is particularly useful for solving complex optimization problems where traditional methods may be inefficient or infeasible. Here's a detailed overview of GAO, including its principles, components, processes, and applications. Genetic Algorithms (GAs) are optimization techniques inspired by natural selection, employing key concepts such as populations, fitness functions, and genetic operators. A population consists of a set of potential solutions, each represented as a chromosome. The fitness function evaluates these solutions based on how well they solve the given problem, guiding the selection process. Genetic operators play a crucial role in creating new solutions: selection involves choosing the fittest individuals for reproduction, while crossover combines two parent solutions to generate offspring. Mutation introduces random changes to maintain diversity within the population. The GAO process begins with the initialization of a random population, followed by evaluation, selection, crossover, mutation, and replacement. This iterative cycle continues until a stopping criterion, such as a maximum number of generations or a satisfactory fitness level, is achieved. The components work together to explore the solution space efficiently, allowing GAs to tackle complex optimization challenges.

The Genetic Algorithm Optimization (GAO) process involves several mathematical steps, each grounded in specific equations. Here's a breakdown of the main steps:

(1) Initialization: Generate an initial population of P individuals (chromosomes). Each individual is represented as a vector:

Population=$\{X_1, X_2, \ldots, X_P\}$

(2) Fitness Evaluation

- Evaluate the fitness of each individual using a fitness function f(X):
  $Fi = f(X_i)$ for i=1,2,…,P 2.7
- Normalize the fitness values if necessary to enhance selection pressure.

(3) Selection: Select individuals based on their fitness. Common methods include:

- **Roulette Wheel Selection**: Probability of selection for individual i:
  $P(X_i) = \frac{F_i}{\sum_{j=1}^{p} F_j}$ 2.8 (please investigate well about this formula and the above matrix)
- **Tournament Selection: Randomly select a subset and choose the best.**

(4) Crossover (Recombination): For selected pairs of parents X_α and X_b, generate offspring using crossover with a crossover rate r_c:

$$\text{Offspring} = \begin{cases} Crossover(X_\alpha, X_b) & if\ r < r_c \\ X_\alpha & if\ r \geq r_c \end{cases} \quad 2.9$$

where r_c is a random number (having uniform distribution with range) between 0 and 1.

(5) Mutation: Apply mutation to the offspring with a mutation rate r_m:

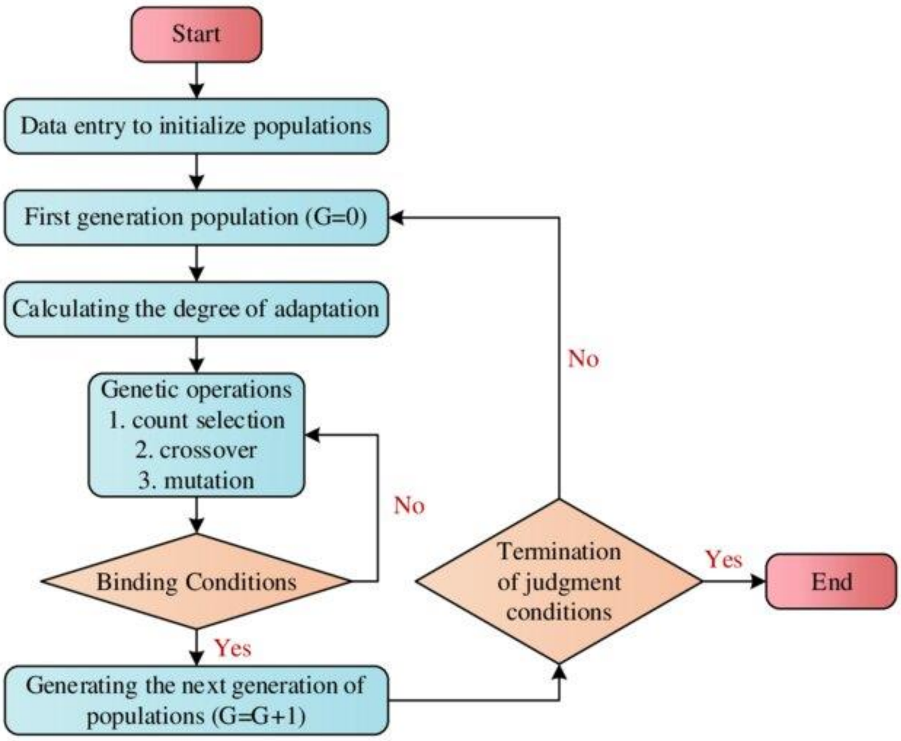
$$X_{new} = \begin{cases} Mutate & if\ r < r_c \\ X & if\ r \geq r_c \end{cases} \quad 2.10$$

The mutation process introduces small random changes to the individual.

(6) Replacement: Replace the old population with the new offspring to form the next generation:
New Population = $\{Offspring_1, Offspring_2, \ldots, Offspring_P\}$

(7) Termination Condition: Check for termination criteria, such as:

- A specified number of generations.
- A convergence threshold in fitness.
- Satisfactory solution found: If Termination Condition then stop.



Flowchart (1): represents the GAO steps

D. Evaluate Precision of Forecasting Models

To test the accuracy and the performance of the proposed model ,used some statistical tests and measurements, including, mean square error, root of mean square error, Akaike information criteria, and Bayesian information criteria.

(1) Mean Square Error (MSE)

Mean Squared Error (MSE) is a widely used metric for assessing the accuracy of a predictive model. It measures the average of the squares of the errors that is, the average squared difference between the actual values and the values predicted by the model.

$$\text{MSE} = \frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2 \qquad 2.11$$

where:

- n is the number of observations.
- y_t is the actual value.
- $\hat{y}_t$ is the predicted value.

A lower MSE indicates a better fit of the model to the data, as it suggests that the predictions are closer to the actual values. However, MSE can be sensitive to outliers because it squares the errors, which can disproportionately affect the overall score.

(2) Square Root of Mean Square Error (RMSE)

The Root Mean Square Error (RMSE) is a commonly used metric to evaluate the accuracy of a predictive model, providing a measure of the model's prediction error. It represents the square root of the average squared differences between predicted and actual values, making it easier to interpret than the Mean Squared Error (MSE) because it is expressed in the same units as the original data.

$$\text{MSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(y_t - \hat{y}_t)^2} \qquad 2.12$$

(3) Akaike Information Criteria (AIC)

The Akaike Information Criterion (AIC) is a statistical measure used to compare the goodness of fit of different models while accounting for the complexity of each model. Developed by Hirotugu Akaike, AIC helps in model selection by balancing the trade-off between the accuracy of the model and the number of parameters used.

$$\text{AIC} = 2\,k - 2\ln(\mathscr{L}) \qquad 2.13$$

Where:

- n: is the number of observations.
- $\mathscr{L}$: is the log-likelyhood.
- k: is the number of explanatory variables in the model.

(4) Bayesian Information Criteria (BIC)

The Bayesian Information Criterion (BIC), also known as the Schwarz Information Criterion (SIC), is a statistical tool used for model selection among a finite set of models. It provides a criterion for choosing a model that balances goodness of fit with model complexity, incorporating a penalty for the number of parameters in the model.

$$\text{BIC} = k\,.\,\ln(n) - 2\ln(\mathscr{L}) \qquad 2.14$$

Where:

- n: is the number of observations.
- $\mathscr{L}$: is the log-likelyhood.
- k: is the number of explanatory variables in the model.

In conclusion the lower MSE, RMSE, AIC and BIC indicates a better fit of the model to the data, as it suggests that the predictions are closer to the actual values.

2. Applications

A. Data Description

This research utilizes Monthly data of Brent crude oil price, from January 1, 2021, to December 31, 2023. The data reflects the daily closing price in U.S. dollars per barrel.

B. Results and Discussions

In the discussion, we explore the implications of these findings, comparing both GM(2,1) and GAO-GM(2,1) models. Additionally, we address limitations of the study and propose directions for future research to further investigate this connection.

Table (1): shows the iteration of GAO for finding best estimator

	Performance	
Metrics	**GM(2,1)**	**GAO-GM(2,1)**
MSE	86.092	62.384
RMSE	9.279	7.898
AIC	162.395	150.799
BIC	163.979	152.382

The performance metrics for the two models, GM(2,1) and GAO-GM(2,1), demonstrate the comparative effectiveness of the latter. The Mean Squared Error (MSE) for GM(2,1) is 86.092, whereas the GAO-GM(2,1) model achieves a significantly lower MSE of 62.384, indicating that the GAO-GM(2,1) model provides more accurate predictions. This trend continues with the Root Mean Squared Error (RMSE), where the GAO-GM(2,1) shows an RMSE of 7.898 compared to 9.279 for GM(2,1), further confirming the improved accuracy of the GAO-GM(2,1) model. Additionally, the Akaike Information Criterion (AIC) is lower for GAO-GM(2,1) at 150.799 versus 162.395 for GM(2,1), suggesting that the GAO-GM(2,1) model not only fits the data better but also has fewer parameters relative to its goodness of fit. Similarly, the Bayesian Information Criterion (BIC) reflects this pattern, with GAO-GM(2,1) scoring 152.382 compared to 163.979 for GM(2,1). Then, these metrics collectively indicate that the GAO-GM(2,1) model outperforms the GM(2,1) model in terms of predictive accuracy and efficiency.

Table (2): shows the iteration of GAO for finding best estimator

Genetic Algorithm Optimizer								
Iterations	Mean	Best	Iterations	Mean	Best	Iterations	Mean	Best
iter1	-7136.368	-2197.336	iter35	-2441.737	-2146.959	iter69	-2349.269	-2121.255
iter2	-4448.186	-2197.336	iter36	-2262.315	-2146.959	iter70	-2575.207	-2121.255
iter3	-3396.129	-2197.336	iter37	-2275.705	-2133.482	iter71	-2362.925	-2121.255
iter4	-2966.859	-2197.336	iter38	-2800.549	-2133.482	iter72	-2243.762	-2121.255
iter5	-2807.231	-2197.336	iter39	-2276.383	-2133.482	iter73	-2198.827	-2121.255
iter6	-2753.624	-2197.336	iter40	-2804.511	-2130.627	iter74	-2447.067	-2121.255
iter7	-2995.416	-2184.685	iter41	-2366.912	-2130.627	iter75	-2536.83	-2121.255
iter8	-2858.746	-2184.685	iter42	-2285.532	-2130.627	iter76	-2303.32	-2121.255
iter9	-2662.629	-2184.685	iter43	-2215.939	-2130.627	iter77	-2273.96	-2121.255
iter10	-3098.343	-2184.685	iter44	-2573.769	-2130.627	iter78	-2292.228	-2121.255
iter11	-2582.61	-2184.685	iter45	-3203.014	-2130.627	iter79	-2357.741	-2121.255
iter12	-2694.805	-2184.685	iter46	-2714.684	-2122.735	iter80	-2885.109	-2121.255
iter13	-2504.255	-2184.685	iter47	-2277.161	-2122.735	iter81	-2558.79	-2121.255
iter14	-2550.511	-2168.893	iter48	-2210.82	-2122.735	iter82	-2494.067	-2121.255
iter15	-2644.826	-2168.893	iter49	-2496.084	-2122.735	iter83	-2217.348	-2121.255
iter16	-2444.107	-2168.893	iter50	-2245.428	-2122.735	iter84	-2570.699	-2121.255
iter17	-2367.686	-2168.893	iter51	-2307.078	-2122.735	iter85	-2295.714	-2121.255
iter18	-2313.261	-2168.893	iter52	-2264.004	-2122.735	iter86	-2257.034	-2121.255
iter19	-2299.709	-2168.893	iter53	-2296.04	-2122.735	iter87	-2165.143	-2121.255
iter20	-2602.73	-2168.893	iter54	-2462.483	-2122.735	iter88	-2351.396	-2121.255
iter21	-2553.113	-2168.893	iter55	-2428.742	-2122.735	iter89	-2269.774	-2121.255
iter22	-2618.452	-2168.893	iter56	-2453.779	-2122.735	iter90	-2417.117	-2121.255
iter23	-2579.644	-2168.893	iter57	-2265.136	-2122.735	iter91	-2292.125	-2121.255
iter24	-2394.059	-2146.959	iter58	-2265.607	-2122.735	iter92	-2260.097	-2121.255
iter25	-2356.186	-2146.959	iter59	-2197	-2122.735	iter93	-2811.703	-2121.255
iter26	-2465.637	-2146.959	iter60	-2333.75	-2122.735	iter94	-2534.14	-2121.255
iter27	-2407.601	-2146.959	iter61	-2251.66	-2122.735	iter95	-2379.367	-2121.255

iter28	-2642.845	-2146.959	iter62	-2412.371	-2122.735	iter96	-2211.072	-2121.255
iter29	-2355.168	-2146.959	iter63	-2320.379	-2122.735	iter97	-2226.309	-2121.255
iter30	-2348.783	-2146.959	iter64	-2323.481	-2122.516	iter98	-2196.51	-2121.255
iter31	-2372.049	-2146.959	iter65	-2383.787	-2122.516	iter99	-2456.22	-2121.255
iter32	-2395.566	-2146.959	iter66	-2639.684	-2122.516	iter100	-2219.527	-2121.255
iter33	-2396.916	-2146.959	iter67	-2316.743	-2121.255			
iter34	-2494.643	-2146.959	iter68	-2245.332	-2121.255			

The data presented illustrates the performance of a Genetic Algorithm Optimizer (GAO) over multiple iterations, showcasing key metrics that evaluate its efficiency in finding optimal solutions. Each iteration is characterized by two primary metrics: the mean and the best values of the objective function. The mean indicates the average performance of all candidate solutions evaluated during that iteration, while the best value represents the highest-performing solution found at that point. As the iterations progress, we observe a general trend of decreasing mean and best values, suggesting that the GAO is effectively converging toward an optimal solution. Initially, there is significant fluctuation in the mean values, indicative of exploration within the solution space. However, as the algorithm matures through iterations, the mean stabilizes, reflecting improved convergence. By iteration 35 and beyond, the best values demonstrate a marked consistency, suggesting that the optimizer has successfully identified a satisfactory solution within a specific range.

Table (3): shows the optimized estimated parameters

GAO-GM(2,1)		
iteration 35	α	0.9956971
	b	55.07711

Table (4): represents the actual and predicted values of GAO-GM(2,1) model

Periods	Oil Price	Predicted	Residuals	Periods	Oil Price	Predicted	Residuals
1	54.77	55.077	-0.303	19	111.93	122.670	-10.744
2	62.28	54.771	7.505	20	100.45	111.976	-11.530
3	65.41	62.248	3.162	21	89.76	100.500	-10.735
4	64.81	65.396	-0.590	22	93.33	89.806	3.526
5	68.53	64.813	3.722	23	91.42	93.315	-1.899
6	73.16	68.514	4.650	24	80.92	91.428	-10.504
7	75.17	73.140	2.026	25	82.50	80.965	1.536
8	70.75	75.161	-4.414	26	82.59	82.493	0.092
9	74.49	70.769	3.720	27	78.43	82.590	-4.155
10	83.54	74.474	9.064	28	84.64	78.448	6.190
11	81.05	83.501	-2.451	29	75.47	84.613	-9.147
12	74.17	81.061	-6.890	30	74.84	75.509	-0.670
13	86.51	74.200	12.306	31	80.11	74.843	5.265
14	97.13	86.457	10.672	32	86.15	80.087	6.060
15	117.25	97.084	20.161	33	93.72	86.124	7.598
16	104.58	117.163	-12.588	34	90.60	93.687	-3.091
17	113.34	104.634	8.703	35	82.94	90.613	-7.673
18	122.71	113.303	9.408	36	77.63	82.973	-5.342

The table presents a detailed comparison of actual oil prices against predicted values, along with the residuals, which indicate the difference between these two. For each period, the actual oil price is shown alongside its predicted counterpart, derived from a forecasting model. The residuals reflect the accuracy of the predictions, also the table shows varied prediction performance across periods, with some residuals indicating good accuracy, while others reveal notable discrepancies.

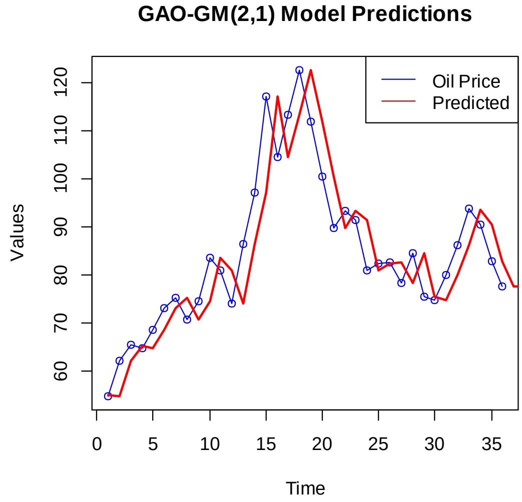


Figure (1): demonstrates the scatter plot of actual and predicted values.

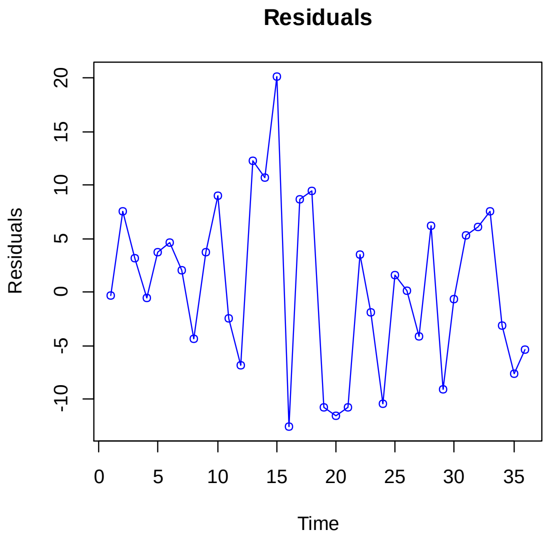


Figure (2): shows the scatter plot of residual values.

Table (5): demonstrates the normality test for the residuals of the GAO-GM(2,1) model

Tests of Normality						
Variable	**Kolmogorov-Smirnova**			**Shapiro-Wilk**		
Residuals	Statistic	df	Sig.	Statistic	df	Sig.
	0.072	36	0.200	0.975	36	0.579

The above table presents results from two statistical tests for normality applied to the residuals of a model, the Kolmogorov-Smirnov test and the Shapiro-Wilk test. Both tests assess whether the residuals are normally distributed, which is an important assumption in many statistical analyses, from their p-values which are both greater than 0.05 that means the residuals are distributed normally, so the assumption of normally distributed is achieved.

Table (6): represents the forecasted values for 12 periods

Periods	Forecasted	Confidence Interval 95%	
37	73.6577	68.0007	78.6065
38	71.0345	65.3775	75.9833
39	72.469	66.812	77.4178
40	79.952	74.295	84.9008
41	75.6291	69.9721	80.5779
42	80.443	74.786	85.3918
43	82.7952	77.1382	87.744
44	77.2811	71.6241	82.2299
45	73.5721	67.9151	78.5209
46	72.7692	67.1122	77.718
47	74.664	71.5897	77.7383
48	71.894	68.8197	74.9683

The table presents forecasted values for oil prices, each row corresponds to a period, indicating the forecasted value for that period and the range within which the true value is expected to fall with 95% confidence. For instance, in period 37, the forecasted value is approximately 73.66, with a confidence interval ranging from about 68.00 to 78.61. This means that based on the model used, we can be 95% confident that the actual value will lie within this range. Similarly, for period 38, the forecasted value is 71.03, with a confidence interval of 65.38 to 75.98. This format allows stakeholders to assess the reliability of the forecasts, as wider intervals indicate greater uncertainty. Monitoring these values over time can help in decision-making processes, especially in contexts like finance, economics, or supply chain management, where predictions are crucial for planning and strategy.

3. Conclusions

In conclusion, the comparative analysis of the GM(2,1) and GAO-GM(2,1) models highlights the superior predictive capabilities of the GAO-GM(2,1) model. The metrics demonstrate a clear advantage for the GAO-GM(2,1), with a significantly lower Mean Squared Error (MSE) of 62.384 compared to 86.092 for the GM(2,1), as well as a lower Root Mean Squared Error (RMSE) and improved AIC and BIC values. These findings indicate that the GAO-GM(2,1) model not only provides more accurate predictions but also achieves this with greater efficiency. The performance of the Genetic Algorithm Optimizer (GAO) over multiple iterations illustrates a systematic convergence toward optimal solutions, as evidenced by decreasing mean and best values. Additionally, the analysis of residuals indicates that the model meets the assumption of normality, further validating its robustness. Forecasting results, accompanied by 95% confidence intervals, provide valuable insights for stakeholders, emphasizing the reliability and practical utility of the model in decision-making processes. Overall, this comprehensive evaluation underscores the effectiveness of the GAO-GM(2,1) model in delivering precise and actionable forecasts in dynamic contexts such as oil price prediction.

4. Limitations and Future works

A. Limitations

This study is constrained by its focus on a limited dataset covering only the period from January 2021 to December 2023, which may not fully capture longer-term trends or variations in oil prices. The GAO-GM(2,1) model’s effectiveness is assessed using specific performance metrics (MSE, RMSE, AIC, and BIC), which may not encompass all relevant factors affecting forecasting accuracy.

Additionally, external variables such as geopolitical events and economic shifts are not incorporated into the model, potentially limiting its ability to predict price changes under varying market conditions.

B. Future Study

Future research should aim to extend the dataset to include a broader historical range, enabling the analysis of longer-term trends and seasonal patterns in oil prices. Incorporating additional influential factors, such as global economic indicators and geopolitical risks, could enhance the model's predictive power. Furthermore, exploring the integration of hybrid models that combine GAO-GM(2,1) with other forecasting techniques may yield improved accuracy and robustness in predictions, while sensitivity analyses could help evaluate the impact of different modeling assumptions on outcomes.

References

1- Ahmed, B. K., Rahim, S. A., Maaroof, B. B., & Taher, H. A. (2020). Comparison Between ARIMA And Fourier ARIMA Model To Forecast The Demand Of Electricity In Sulaimani Governorate. *Qalaai Zanist Journal*, *5*(3), 908-940.
2- Ahmed, S. T., & Tahir, H. A. (2021). Comparison Between GM (1, 1) and FOGM (1, 1) Models for Forecasting The Rate of Precipitation in Sulaimani. *Journal of Kirkuk University for Administrative & Economic Sciences*, *11*(1), 301-314.
3- Asraa, A., Rodeen, W., & Tahir, H. (2018). Forecasting the Impact of Waste on Environmental Pollution. *International Journal of Sustainable Development and Science*, *1*(1), 1-12.
4- Aziz, A. A., Shafeeq, B. M., Ahmed, R. A., & Taher, H. A. (2023). Employing Recurrent Neural Networks to Forecast the Dollar Exchange Rate in the Parallel Market of Iraq. *Tikrit Journal of Administrative and Economic Sciences*, *19*(62), 2.
5- Chen, L., Zhou, T., & Liu, S. (2020). A Genetic Algorithm Optimized GM(2,1) Model for Time Series Forecasting. Journal of Forecasting, 39(5), 756-769.
6- International Energy Agency. (2023). *Oil Market Report*. Retrieved from IEA website.
7- Karim, A. J. M., & Ahmed, N. M. (2023). Vector Autoregressive Integrating Moving Average (Varima) Model of COVID-19 Pandemic and Oil Price. *International Journal of Professional Business Review: Int. J. Prof. Bus. Rev.*, *8*(1), 13.
8- Li, W., Han, Z. H., & Niu, D. X. (2008, April). Improved genetic algorithm-GM (1, 1) for power load forecasting problem. In *2008 Third International Conference on Electric Utility Deregulation and Restructuring and Power Technologies* (pp. 1147-1152). IEEE.
9- Liu, J., & Yang, Y. (2022). Theoretical exploration of GM(2,1) model and genetic algorithm optimization. Journal of Systems Science and Complexity, 35(3), 805-819.
10- Mahmood, H. O. F., Ali, M. A., & Ali, Z. J. R. (2023). Forecasting performance in iraqi stock exchange for the oil price throw the gm (1, 2) model and the impacts on economic growth. *3c Empresa: investigación y pensamiento crítico*, *12*(1), 311-322.
11- Niu, D. X., Li, W., Han, Z. H., & Yuan, X. E. (2008, October). Power Load Forecasting based on Improved Genetic Algorithm–GM (1, 1) Model. In *2008 Fourth International Conference on Natural Computation* (Vol. 1, pp. 630-634). IEEE.
12- OPEC. (2022). *Monthly Oil Market Report*. Retrieved from OPEC website.
13- Ou, S. L. (2012). Forecasting agricultural output with an improved grey forecasting model based on the genetic algorithm. *Computers and electronics in agriculture*, *85*, 33-39.
14- Rahim, S. A., Salih, S. O., Hamdin, A. O., & Taher, H. A. (2020). Predictions the GDP of Iraq by using Grey–Linear Regression.
15- Taher, H. A., Salih, K. K., & Alwan, A. S. (2024). Forecasting For Silver Closing Price and Modifying Predictions by Using GARCH (1, 1) Wavelet Transformation. *University of Kirkuk Journal For Administrative and Economic Science*, *14*(2).
16- World Bank. (2023). *Commodity Markets Outlook*. Retrieved from World Bank website.
17- Wu, J., Zhang, X., & Wang, Y. (2019). Application of GM(2,1) model in forecasting agricultural yield in China. Agricultural Systems, 175, 12-18.
18- Zhang, H., Li, Y., & Zhao, Q. (2021). Comparative study of forecasting models: GA-GM(2,1) versus ARIMA and exponential smoothing. Energy Reports, 7, 1234-1245.
19- Zhou, W., & He, J. M. (2013). Generalized GM (1, 1) model and its application in forecasting of fuel production. *Applied Mathematical Modelling*, *37*(9), 6234-6243.