Original Article

Mechanical Vibration Reduction of a Nonlinear Half-Car Model using Integral-Proportional Derivative (I-PD) Controller

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Abstract

Vehicles usually consist of several essential systems. The performance of the vehicle is evaluated through the efficiency of these systems to perform their duties. The suspension system is one of these systems dedicated to absorbing shocks arising from vehicles passing over road bumps, thus reducing vibrations and achieving passenger comfort while driving. This paper presents a study on enhancing ride comfort in a nonlinear half-car model using a modified Proportional-Integral-Derivative (PID) controller. In this study a half-car model is developed considering the nonlinearities in the suspension system components. A nonlinear half-car model was adopted to increase accuracy and make the overall system closer to reality. Instead of the feed-forward conventional PID controller gains, the proposed controller gains are formed by putting the proportional and derivative gains in the feedback path while keeping the integral gain in the feed-forward path to act as an I-PD controller. The proposed controller is integrated into the model to deal with these nonlinearities effectively and to achieve the optimal performance of the vehicle body. The overall system has been developed and simulated in the Matlab Simulink environment to show the dynamic response. Simulation results demonstrate the effectiveness of the I-PD controller in improving the ride comfort and handling stability of the nonlinear half-car model by reducing body acceleration and suspension deflection. A comparison with other study has been conducted to verify the effectiveness of the proposed controller.

Keywords: I-PD controller, Nonlinear half-car, Oscillation, Ride comfort, Suspension system.

1. Introduction

When vehicles cross over specific road bumps, they exhibit a common behavior due to the presence of suspension systems, causing the vehicle to oscillate up and down for a limited period of time. This undesirable oscillation is a relative movement among the components of the suspension system, the vehicle body, and the wheels. These oscillations are gradually absorbed or damped depending on the efficiency of the vehicle's suspension system.

In general, three suspension systems can be recognized passive, semi-active, and active [1]. The passive suspension system usually consists of a spring and damper connected between the vehicle body and the wheel. Although most manufacturers are trying to improve the characteristics and ability of suspension systems to deal with road disturbances in order to obtain greater stability for the vehicle's body while driving, there are some limits related to this issue. Therefore, another approach can be followed by making the suspension system active. The system is activated through the use of an active hydraulic actuator powered to generate the force necessary to control the oscillating movement. The actuator is mounted in parallel with the spring and damper to increase the absorption capacity of the suspension system and at the same time, it can be controlled using different control techniques. The control of these vehicles has become a famous task, requiring precise and reliable systems. Many researchers have tried to use and innovate many control methods to improve the

vehicle's performance with regard to suspension systems. Ekoru et al. [2] presented the design of a two-loop, force/suspension travel PID control system for a nonlinear half-car active vehicle suspension system. Kumar and Medhavi [3] analyzed and optimized an active full vehicle suspension model using advanced fuzzy logic controller to improve driver comfort, safety and road handling. Kumar et al. [4] proposed a hybrid intelligent control technique based on combination of neural network and fuzzy for hydraulic actuated active half-car suspension system. Mustafa et al. [5] presented an optimized sliding mode with particle swarm optimization algorithm for vibration control of active half-car suspension systems. Gandhi et al. [6] used a half car active suspension model with 4 degrees of freedom with different controllers such as proportional, integral and derivative, linear quadratic regulator, fuzzy and adaptive neuro fuzzy inference system. Khodadadi and Ghadiri [7] used PID, fuzzy logic and H_∞ controllers to control the car suspension system based on half-car. Also, a self-tuning PID controller based on fuzzy logic is developed to improve the performance of the system. Pedro et al. [8] presents a differential-evolution optimized, independent multi loop proportional-integral-derivative (PID) controller design for full-car nonlinear, electrohydraulic suspension systems. Liang et al. [9] designed an optimal vibration controller for vehicle active suspension systems. Yildiz [10] considered a nonlinear suspension design for half vehicle model by using particle swarm optimization technique for optimizing the vehicle vibrations. Khan et al. [11] used

feedback linearization and linear quadratic regulator controller with a half-car model. Dangor et al. [12] presented the design of proportional-integral-derivative (PID) controller for a nonlinear quarter-car active vehicle suspension system using evolutionary algorithms such as the particle swarm optimization, genetic algorithm and differential evolution. Mohamed et al. [13] designed and studied linear quadratic regulator optimal control and PID classic control to achieve half car performance such as ride comfort and road stability. Barethiye et al. [14] used linear and hybrid shock absorber models to analyze half-car performances. Tan et al. [15] designed a dual-loop proportion integration differentiation controller based on the particle swarm algorithm is designed to control full car suspension system. Yang et al. [16] studied the nonlinear dynamic characteristics of the autonomous vehicle passing through hybrid speed control humps to improve the safety and comfort of half-car model passing through the expressway. Pati et al. [17] designed a controller for a half-car suspension system based on sliding mode control using proportional-integral-derivative (PID) sliding surface. Nan et al. [18] proposed a fuzzy logic control strategy for active half-car suspension system which is utilized to generate counterforce to isolate vibration from the rough ground. Nagarkar et al. [19] implemented genetic algorithm (GA)-based optimization algorithm to tune PID parameters and FLC membership functions' range and scaling factors to control a nonlinear quarter-car suspension system. Dahunsi et al. [20] proposed multi-loop proportional + integral + derivative controllers' gains tuning with global and evolutionary optimization techniques for a nonlinear full-car electrohydraulic active vehicle suspension system. L. C. Félix - Herrán et al. [21] designed and applied a fuzzy-H∞ control, improved with weighting functions to a novel model of a one-half semi active suspension. Zhu et al. [22] developed a new hydraulically interconnected suspension with the using of fuzzy, PID and optimal linear quadratic regulator controllers to control vehicle body's roll motion. Pedro and Baloyi [23] designed a direct adaptive neural network controller to control a nonlinear half-car suspension system and improve ride comfort. Lan and Ni [24] applied fuzzy-PID controller to a half-car suspension system to enhance ride comfort by reducing the body acceleration and pitch angle. Guevara et al. [25] proposed a novel linear parameter varying (LPV) state-space (SS) model with a fictional input to represent nonlinear half-car active suspension system. Ekoru and Pedro [26] used an inner PID hydraulic actuator force control loop, in combination with an outer PID suspension deflection control loop, to control a nonlinear half-car. Hasbullah and Faris [27] proposed an active disturbance rejection control with input decoupling transformation for a half-car model. Kumar and Medhavi [28] designed an automobile suspension system to improve the performance of nonlinear half-car model using genetic algorithm. Wu and Liu [29] presented a novel controller design for half-cars suspension magneto-rheological by introducing a piecewise control approximation model.

The primary objective of this study is to validate the effect of using the Integral-Proportional Derivative (I-PD) controller on a nonlinear half-car model and improve the performance of the vehicle's suspension system, reducing vibrations and providing a smoother ride.

2. Mathematical modeling of the system

The nonlinear half-car suspension system is modeled using a combination of mathematical equations and physical parameters. This type of modeling is crucial for understanding and analyzing the performance of vehicle suspension, which plays a vital role in ensuring passenger comfort and vehicle stability.

The main structure of a half-car model typically consists of one sprung mass (vehicle's body) and two unsprung masses (the wheels) with front and rear suspension systems. Each suspension system consists of damper and spring elements to isolate the road disturbances from the vehicle's body. The non-linearity in the half-car model represents the non-linearity in the characteristics of the suspension elements. These nonlinear characteristics are incorporated into the half-car model to accurately represent the real-world behavior of the system [14] and [23].

The half-car model used in this study has four degrees of freedom. The equation of motion of the physical model is derived from Newton's second law of motion, taking into consideration that the static equilibrium point is the origin for the displacement of the mass center and the angular displacement of the car body. The passive half-car model is shown in Fig. 1.
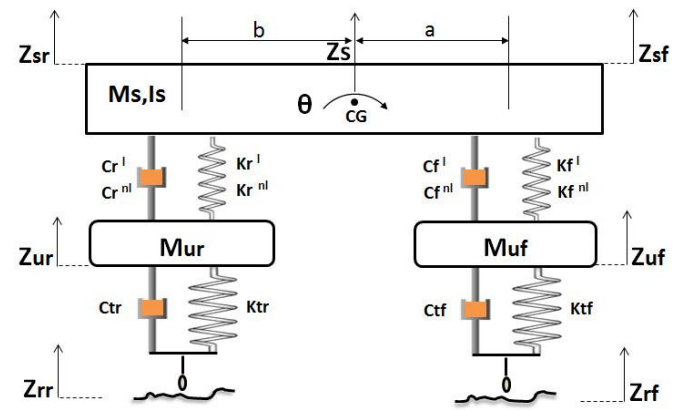

Fig. 1 Passive nonlinear half-car model.

To derive equations of motion for the passive nonlinear half-car model, the following assumptions are considered:

The pitch angle (θ) is small, springs and dampers have linear and nonlinear properties, tires have stiffness and damping properties, the effect of friction is neglected, and the tires are always in contact with the road surface. The forces considered in the model include the forces exerted by the springs and dampers and the forces resulting from the interaction between the sprung and unsprung masses.

As a result, the equation of motion for vehicle body (heave and pitch) and wheels (front and rear) can be derived and presented below in the form of multiple forces as [4].

$$M_s \ddot{Z}_s = -F_{kf} - F_{cf} - F_{kr} - F_{cr} \tag{1}$$

$$I_s \ddot{\theta} = -aF_{kf} - aF_{cf} + bF_{kr} + bF_{cr} \tag{2}$$

$$M_{uf} \ddot{Z}_{uf} = F_{kf} + F_{cf} - F_{ktf} - F_{ctf} \tag{3}$$

$$M_{ur} \ddot{Z}_{ur} = F_{kr} + F_{cr} - F_{ktr} - F_{ctr} \tag{4}$$

The linear and nonlinear force components resulted from the front and rear suspension systems can be expressed as [2] and [8].

$$F_{kj} = K_j^l (Z_{sj} - Z_{uj}) + K_j^{nl} (Z_{sj} - Z_{uj})^3 \qquad (5)$$

$$F_{cj} = C_j^l (\dot{Z}_{sj} - \dot{Z}_{uj}) + C_j^{nl} \sqrt{\left| \dot{Z}_{sj} - \dot{Z}_{uj} \right|}$$
$$* \operatorname{sgn}(\dot{Z}_{sj} - \dot{Z}_{uj}) - C_j^{sym} \left| \dot{Z}_{sj} - \dot{Z}_{uj} \right| \qquad (6)$$

where j denotes front and rear, respectively. Substituting $Z_{sf} = (Z_s + a\theta)$ for sprung mass displacement at front body and $Z_{sr} = (Z_s - b\theta)$ for sprung mass displacement at rear body [2] and [8], the equations of motion can be written as:

$$F_{kf} = K_f^l (Z_s - Z_{uf} + a\theta)$$
$$+ K_f^{nl} (Z_s - Z_{uf} + a\theta)^3 \qquad (7)$$

$$F_{kr} = K_r^l (Z_s - Z_{ur} - b\theta)$$
$$+ K_r^{nl} (Z_s - Z_{ur} - b\theta)^3 \qquad (8)$$

$$F_{cf} = C_f^l (\dot{Z}_s - \dot{Z}_{uf} + a\dot{\theta}) - C_f^{sym} \left| (\dot{Z}_s - \dot{Z}_{uf} + a\dot{\theta}) \right|$$
$$+ C_f^{nl} \left(\sqrt{\left| (\dot{Z}_s - \dot{Z}_{uf} + a\dot{\theta}) \right|} \right)$$
$$* \operatorname{sgn}(\dot{Z}_s - \dot{Z}_{uf} + a\dot{\theta}) \qquad (9)$$

$$F_{cr} = C_r^l (\dot{Z}_s - \dot{Z}_{ur} - b\dot{\theta}) - C_r^{sym} \left| (\dot{Z}_s - \dot{Z}_{ur} - b\dot{\theta}) \right|$$
$$+ C_r^{nl} \left(\sqrt{\left| (\dot{Z}_s - \dot{Z}_{ur} - b\dot{\theta}) \right|} \right)$$
$$* \operatorname{sgn}(\dot{Z}_s - \dot{Z}_{ur} - b\dot{\theta}) \qquad (10)$$

Due to the spring and damping properties of each tire, the generated forces acting on each wheel are written as [4].

$$F_{ktf} = K_{tf} (Z_{uf} - Z_{rf}) \qquad (11)$$

$$F_{ktr} = K_{tr} (Z_{ur} - Z_{rr}) \qquad (12)$$

$$F_{ctf} = C_{tf} (\dot{Z}_{uf} - \dot{Z}_{rf}) \qquad (13)$$

$$F_{ctr} = C_{tr} (\dot{Z}_{ur} - \dot{Z}_{rr}) \qquad (14)$$

To activate the system, the control unit and the hydraulic actuator are used. The forces F_{a1} and F_{a2} at the front and rear suspension systems are shown in Fig. 2. Thus, the equations of motion can be written as below [4]. The parameters of the nonlinear half-car model used for simulation are listed in Table 1.
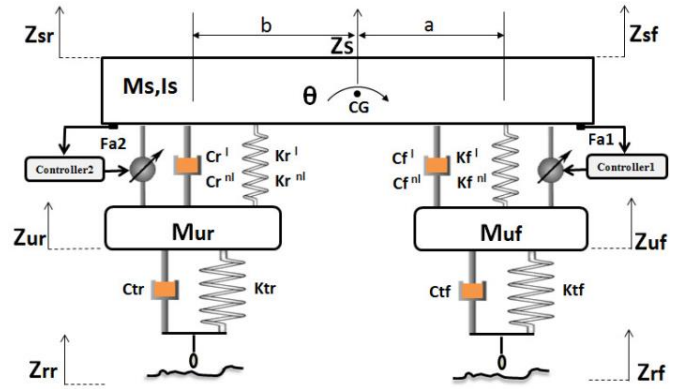

Fig. 2 Active nonlinear half-car model.

$$M_s \ddot{Z}_s = -F_{kf} - F_{cf} - F_{kr} - F_{cr} + F_{a1} + F_{a2} \qquad (15)$$

$$I_s \ddot{\theta} = -aF_{kf} - aF_{cf} + bF_{kr} + bF_{cr}$$
$$+ aF_{a1} - bF_{a2} \qquad (16)$$

$$M_{uf} \ddot{Z}_{uf} = F_{kf} + F_{cf} - F_{ktf} - F_{ctf} + aF_{a1} \qquad (17)$$

$$M_{ur} \ddot{Z}_{ur} = F_{kr} + F_{cr} - F_{ktr} - F_{ctr} + bF_{a2} \qquad (18)$$

Where, Z_s is the sprung mass displacement, Z_{uf} is the unsprung mass displacement at front body, Z_{ur} is the unsprung mass displacement at rear body, Z_{rf} is the input road profile to front wheel, Z_{rr} is the input road profile to rear wheel, θ is the vehicle rotational movement (rad), F_{a1} and F_{a2} are the front and rear actuator forces (Newton), F_{kf} and F_{kr} are the front and rear spring forces, F_{cf} and F_{cr} are the front and rear damping forces, F_{ktf} and F_{ktr} are the front and rear tire spring forces, F_{ctf} and F_{ctr} are the front and rear tire damping forces, respectively.

Table 1. System parameters for half-car model [2].

Parameter	Description	Value	Unit
Ms	Sprung mass	580	kg
M_{uf}	Front unsprung mass	40	kg
M_{ur}	Rear unsprung mass	40	kg
I_s	Sprung mass pitch moment of inertia	1100	Kgm^2
K_f^l, K_r^l	Front and rear linear suspension stiffness	2.35×10^4	N/m
K_f^{nl}, K_r^{nl}	Front and rear nonlinear suspension stiffness	2.35×10^6	N/m
C_f^l, C_r^l	Front and rear linear suspension damping constants	700, 800	Ns/m
C_f^{nl}, C_r^{nl}	Front and rear nonlinear suspension damping constants	400	Ns/m
C_f^{sym}, C_r^{sym}	Front and rear symmetric suspension damping constants	400	Ns/m
K_{tf}, K_{tr}	Front and rear tire stiffness	1.9×10^5	Ns/m
C_{tf}, C_{tr}	Front and rear tire damping constants	70, 80	Ns/m
a	Distance from vehicle center of gravity (C.G.) to front axle	1	m
b	Distance from vehicle center of gravity (C.G.) to rear axle	1.5	m

2.1. Input road profile

In this study, the vehicle is excited by a sinusoidal upward bump on an otherwise smooth road. The front road profile disturbance Z_{rf} applies during the time period t_f when the front tire traverses the bump, while the rear road profile disturbance Z_{rr} applies during the time period t_r when the rear tire crosses the bump. After the front wheel starts to traverse the bump, a time delay t_d elapses before the rear wheel starts to cross the bump which can be computed in seconds. The vehicle forward velocity (V) was 20 m/s. The road profile at each wheel and time delay are given as below [2].

$$Z_{rf} = \begin{cases} \dfrac{a_1}{2}(1-\cos(\frac{2\pi Vtf}{\lambda})) & 1 \le t_f \le (1+\frac{\lambda}{V}) \\ \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

$$Z_{rr} = \begin{cases} \dfrac{a_2}{2}(1-\cos(\frac{2\pi Vtr}{\lambda})) & (1+t_d) \le t_r \le (1+t_d+\frac{\lambda}{V}) \\ \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

$$t_d = \frac{a+b}{V} \quad (21)$$

Where, Bump amplitudes (a_1 , a_2) = 0.11 m, Bump wavelength (λ) = 5 m

3. Control system

The control for the suspension system has been devised to enhance the damping capability of the hydraulic actuator in response to diverse road disturbances. In Fig. 2, the sensors on the car body transmit signals reflecting road disturbances. These signals are fed back to a reference set point for comparison with the actual signal to produce an error signal. As a result, the control unit adjusts the output variables to maintain them within acceptable ranges. To implement system control, the classical PID controller principle is adopted with a different arrangement for the purpose of improving suspension response. In the classical PID controller, the control loop has a single forward path as shown in Fig. 3. This controller is a well-established control technique that has been used for controlling various processes in various industries. However, it may not always meet the requirements [30] and [32]. Meanwhile, the structure of the I-PD controller is made up of dual loops. The inner loop consists of PD gains, while the outer loop represents the I gain, as shown in Fig. 4. The proportional and derivative actions exclusively act on the controlled variable, as opposed to the error, which is handled by the integral gain. The I-PD controller can be employed for unstable systems [31]. By employing the I-PD controller, enhanced capabilities for set point tracking and load disturbance rejection are achieved [33]. From [30] and [31], the time domain equations of the classical PID and I-PD controllers are given respectively by:

$$u(t) = K_p\, e(t) + K_i\, {}_0\!\int^t e(t)dt + K_d\, (de(t)/dt) \quad (22)$$

$$u(t) = -K_p\, y(t) + K_i\, {}_0\!\int^t e(t)dt - K_d\, (de(t)/dt) \quad (23)$$
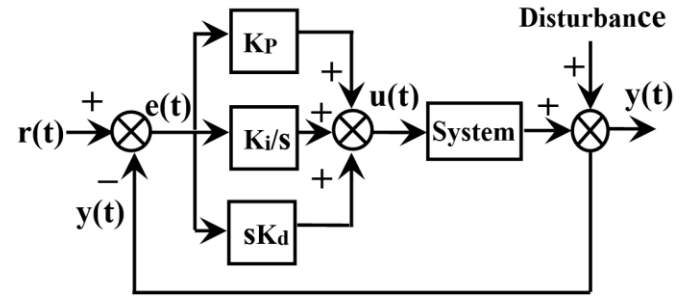

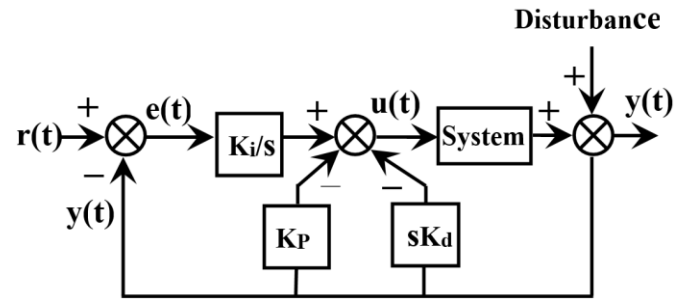
Fig. 3 Block diagram of classical PID controller.

Fig. 4 Block diagram of I-PD controller.

Where, $u(t)$: Control force, $e(t)$: Tracking error, $r(t)$: Desired output, $e(t) = r(t) - y(t)$, $y(t)$: Actual output, K_p: Proportional gain, K_i: Integral gain, and K_d: Derivative gain.

4. Computer simulation

The simulation procedure examines the nonlinear half-car suspension system incorporating passive and active I-PD controlled models. Utilizing mathematical model, the overall layout is developed through the Matlab Simulink environment as illustrated in Fig. 5. Initially, the simulation focuses on the passive half-car model to illustrate dynamic responses such as car body displacement, velocity, acceleration, and suspension deflection. Subsequently, a control methodology is implemented by using the proposed controller as shown in Fig. 6. To excite the half-car suspension system, the road bumps expressed in equations 19 and 20 are used and simulated as external disturbance sources. This can be seen in more details in the Fig. 7. The resulting simulated road profiles are shown in Fig. 8. The general subsystem block diagram for the active non-linear half-car model is created by simulink library as shown in Fig. 9. In this study, the vertical displacement of the front body (Z_{sf}) and rear body (Z_{sr}) are used as control variables, which are fed back as electronic signals to generate the desired hydraulic actuator forces.
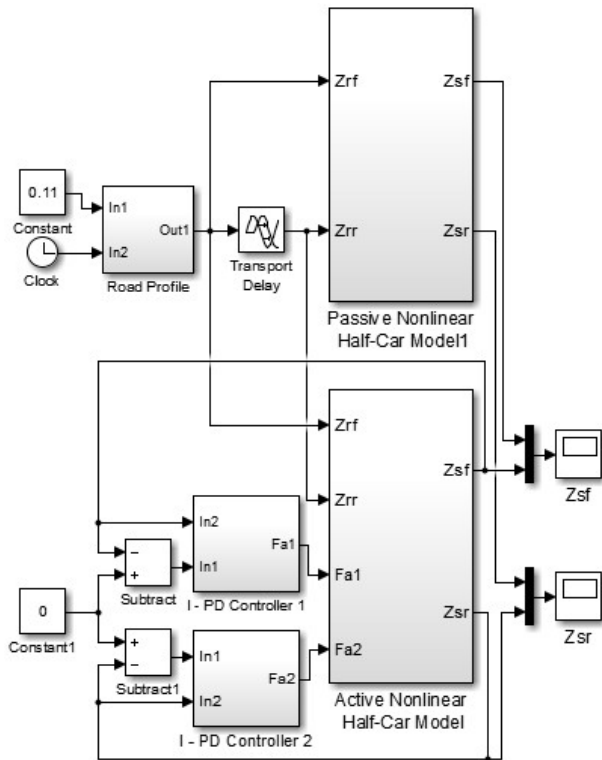
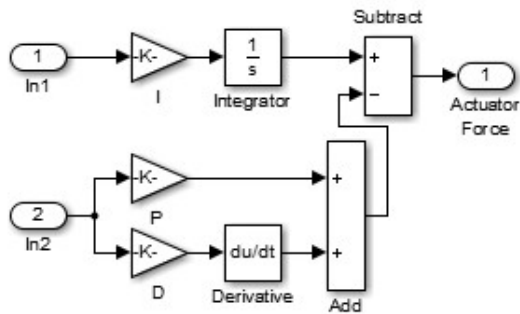**Fig. 5** Active nonlinear half-car model with I-PD controller.
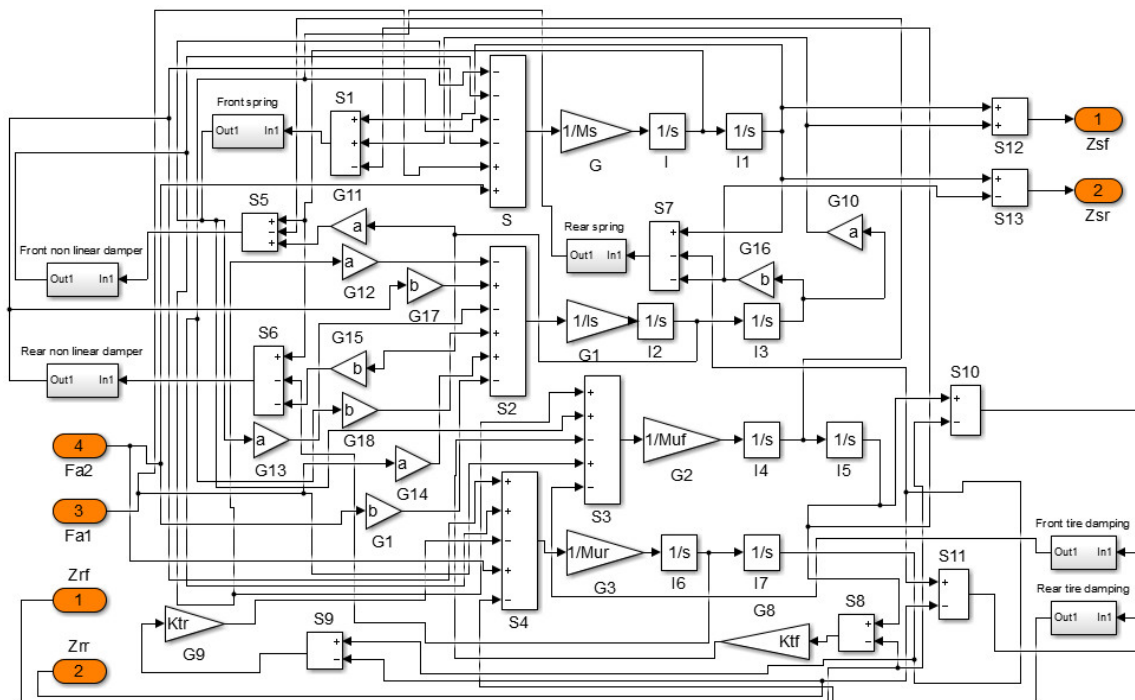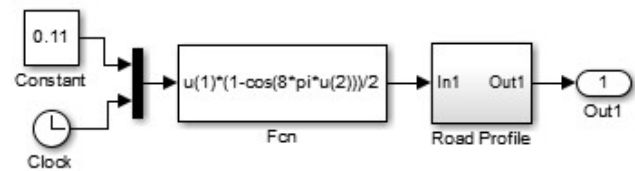

Fig. 6 Schematic of I-PD controller.

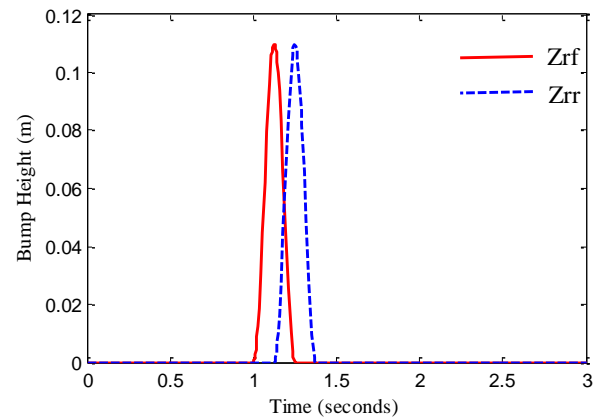**Fig. 7** Block diagram of input road profile.

Fig. 8 Input road profiles.

As a result, the system's response characterized by properties including rise time, overshoot, settling time, and steady-state error can be obtained. Using the manual tuning approach with the aid of Table 2, the values of the I-PD gains are determined as $K_p = 0.1$, $K_i = 0.01$, and $K_d = 2700$ for the front controller and $K_p = 0.05$, $K_i = 0.2$, and $K_d = 1800$ for the rear controller.

Table 2. Response of proportional, integral and derivative controller gains [34].

Closed loop response	Rise time	Overshoot	Settling time	Steady state error
K_p	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small change	Decrease	Decrease	Small change

Fig. 9 Subsystem block diagram for active nonlinear half-car model.

5. Results and discussion

In this section, the Matlab Simulink software is used to investigate the effect of the proposed I-PD control technique on the performance of the nonlinear half-car model. Improving and stabilizing the dynamic response are important features to reflect ride comfort and vehicle stability during driving. Different responses resulting from the simulation of the nonlinear half-car model such as vertical and angular displacement, vertical acceleration, front and rear suspension travels, as well as front and rear vehicle body displacement, are investigated. When running the simulation for five seconds, the system's dynamic behavior and response to the input reference are obtained in terms of overshoot and settling time. Some studies in the literature [2], [8], [12], [13], [23], and [26] focused on using the classical PID controllers to control nonlinear half-car models while other studies used other control techniques. The nonlinear half-car model is examined in this study for passive and active I-PD controllers. Reduction both vertical and angular body displacement reflects vehicle stability.

On the same time, the reduction of body acceleration reflects ride comfort. Settling time is another important parameter to show the overall period for system stability. The vehicle body displacement and acceleration for the passive and active I-PD controlled systems are shown in Fig. 10 and 11, respectively. It can be seen that a clear reduction in the amplitudes of the controlled system. Additionally, the settling time has been reduced. Figs. 12-18 highlighted noticeable enhancements in the response of the vehicle body when using I-PD controllers compared with the passive system. From all figures, the responses showed that the settling time for system oscillation have been reduced significantly which ensures high and fast passenger comfort. In addition, the peak overshoots are clearly damped, and good reference tracking is obtained, reflecting appropriate dynamic response. As a result, sufficient shock absorption and vehicle vibration reductions are achieved. To summarize the results, Table 3 presents the main peak amplitude values and settling time resulted from passive and active nonlinear half-car models.
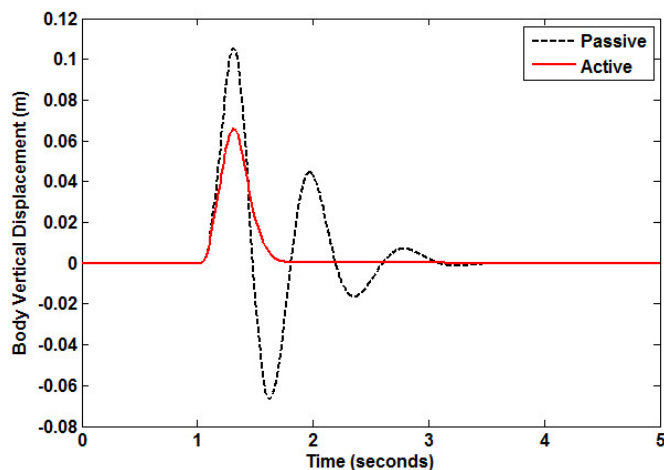

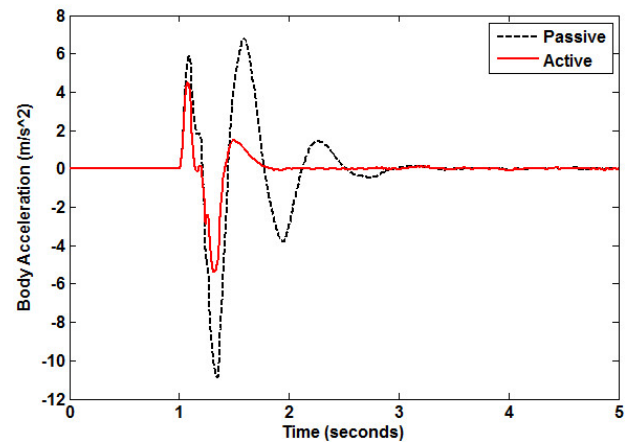**Fig. 10** Vertical body displacement.
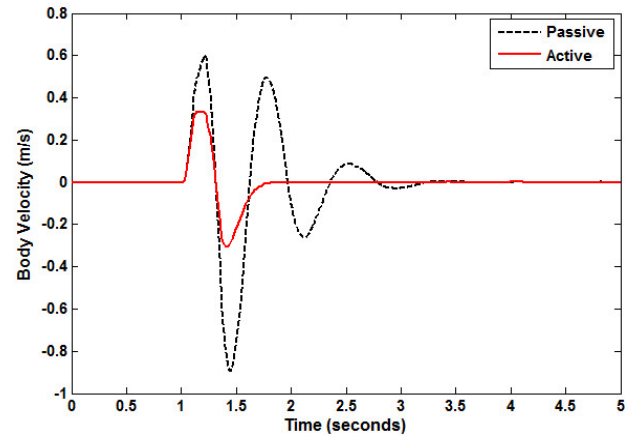

Fig. 11 Vertical body acceleration.

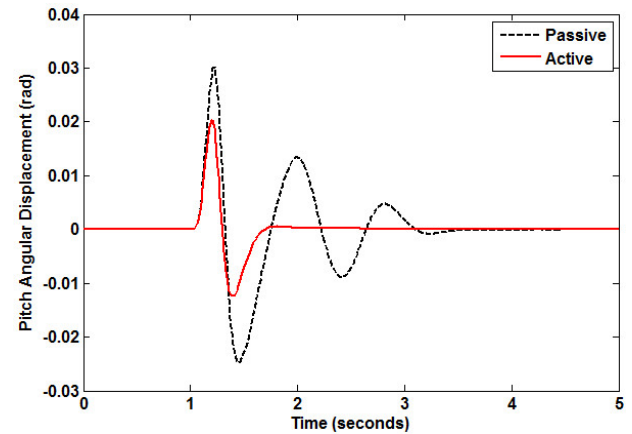**Fig. 12** Vertical body velocity.

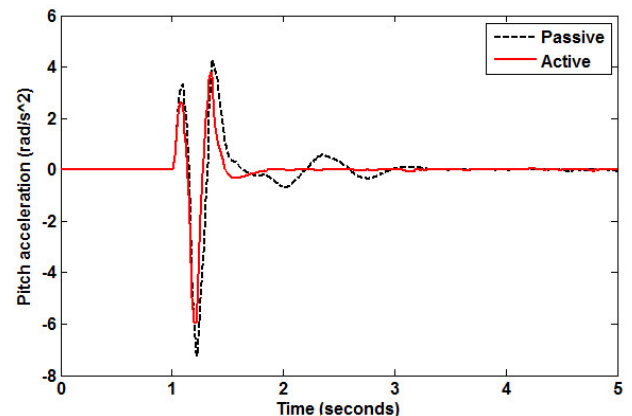**Fig. 13** Angular body displacement.

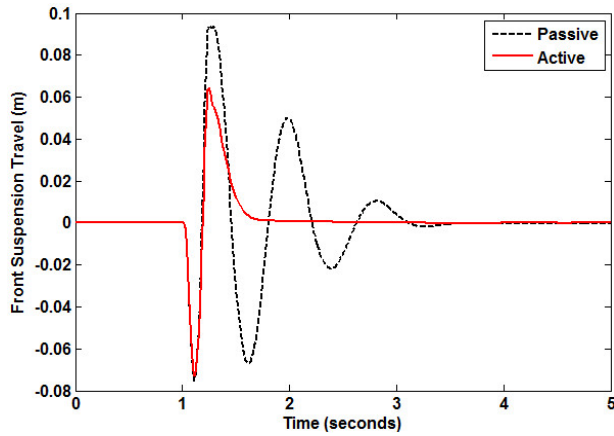**Fig. 14** Angular body acceleration.

Fig. 15 Front suspension travel.
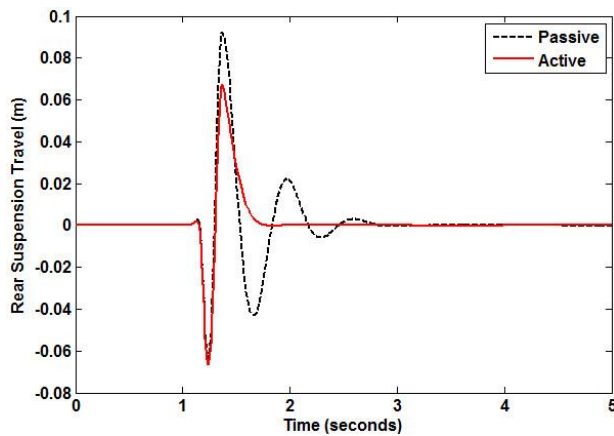

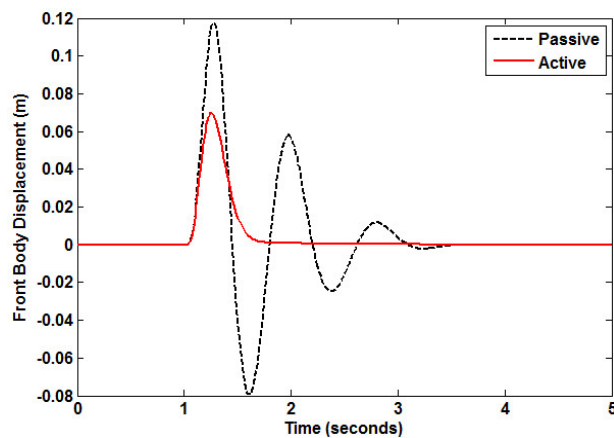**Fig. 16** Rear suspension travel.
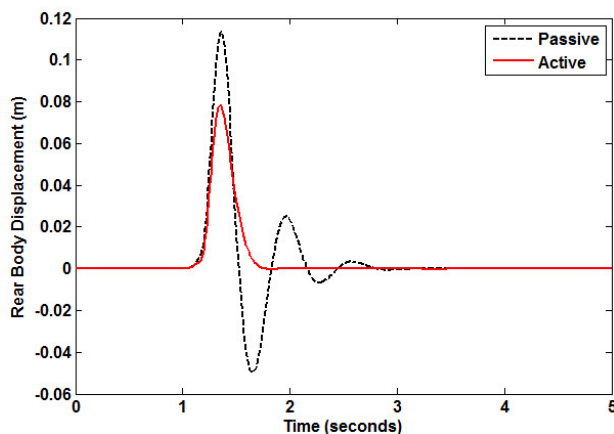

Fig. 17 Front body vertical displacement.

Fig. 18 Rear body vertical displacement.

Table 3. Peak value and settling time for nonlinear half-car model.

Parameter	Response	Passive	Active
Body acceleration	Peak value (m/s^2)	6	4.5
	Settling time (sec)	2.3	0.8
Body displacement	Peak value (m)	0.105	0.066
	Settling time (sec)	2.5	0.75
Body velocity	Peak value (m/s)	0.6	0.334
	Settling time (sec)	2.25	0.8
Pitch angle displacement	Peak value (rad)	0.03	0.02
	Settling time (sec)	2.75	0.75
Pitch angle acceleration	Peak value (rad/s)	3.33	2.6
	Settling time (sec)	2.5	0.8
Front suspension travel	Peak value (m)	0.094	0.0645
	Settling time (sec)	2.6	0.8
Rear suspension travel	Peak value (m)	0.0924	0.0673
	Settling time (sec)	1.85	0.75
Front body displacement	Peak value (m)	0.1178	0.0698
	Settling time (sec)	2.55	0.75
Rear body displacement	Peak value (m)	0.1137	0.0785
	Settling time (sec)	1.82	0.75

6. Comparison with other study

To verify the effectiveness of the proposed I-PD controller, the results obtained from this study compared with results from reference [2]. Due to the importance of vehicle's body acceleration which reflects ride comfort, it was selected as a main property for comparison purpose as shown in Fig. 19. Through the comparison, there is a clear reduction of both acceleration overshoot and settling time when using I-PD controller compared with the conventional PID controller used in the above reference. This demonstrates the effectiveness of the proposed controller to reduce vehicle's mechanical vibrations.
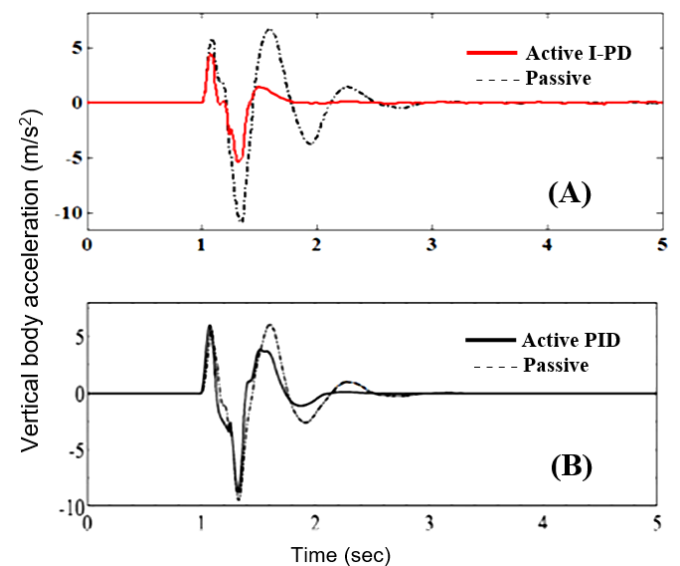

Fig. 19 Vertical body acceleration (A) current study, (B) reference [2].

7. Conclusion

This paper presented modeling, simulation, and control of a nonlinear half-car model using I-PD controller to show the effect of such controller on a passive model. The proposed controller addresses the limitations of the classical PID controller on some systems as mentioned in the literature and demonstrates performance improvement in terms of ride comfort and vehicle stability. According to the simulation analysis of the nonlinear half-car model, the controlled system showed smaller peak amplitudes. Regarding settling time, the I-PD controlled system showed faster settling which lead to faster suppression of vehicle body vibrations resulted from road disturbances.

The simple structure and the short time required to compute controller gains makes it easier to use compared with Fuzzy and Fuzzy-PID controllers or using more difficult control techniques. Based on the results, the obtained enhancement especially the reduction in vehicle body displacement, acceleration and suspension travel represents the suitable behavior and high efficiency of the proposed controller to damp the mechanical vibration of the model. From the above illustration, it can be concluded that the I-PD controller provides good passenger ride comfort and vehicle stability.

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