The Second Scientific Conference of Pure and Applied Sciences

Special Volume , Number 1, pp.23 - 44, , August 2010

SHEAR EFFECT ON THE ELASTIC CRITICAL BUCKLING LOAD OF NON-PRISMATIC COLUMNS

Dr. Hayder T. Nimnim
Assistant Professor

Manar Al Fadul
Assistant Lecturer

College of Engineering/ University of Kufa

ABSTRACT:

An analytical solution for determining the elastic critical buckling load for tapered members including shear effect are studied in this paper with different support conditions. The calculations are based on the exact stability function including shear effect for uniformly tapered beam-column having laced or battened build-up structural members, and approximate stability functions for a wide range of non-prismatic member including shear effect using the finite difference method. The modified stability functions for nonlinear tapered member including shear effect is also obtained in this paper by using the stability functions for a linearly tapered member in the form of modified shape factor ($\overline{m}$) and modified taper ratio ($\bar{u}$). Buckling of tapered columns is studied in many numerical examples. The results show that, inclusion the effect of shear deformation in the calculation of critical load can result in a significant reduction in the value of the critical load for both prismatic and non-prismatic laced columns. The effect of the shear is depending on the value of shear parameter μ_2 and tapering ratio u. The average ratio of reduction is (63.315%) and (64.7 to 65) % for prismatic and non-prismatic laced columns respectively. In the structures having prismatic and non-prismatic solid columns, the effect of shear deformation can have a significant effect with the range of (32.42 to 56.78) % on the value of critical load.

INTRODUCTION

Non-Prismatic members are used in many structural applications such as highway bridges, buildings, towers as well as in many mechanical components. However most of the past researches work have been neglected the effect of shear deformation, even though this effect is quite important and must be included in the analysis for the correct prediction of the response, especially in the lattice, trussed or open web structures that exhibit a relatively higher shear deformation than the solid and closed web structures. The effect of shear on tapered members is greater than that prismatic one [(1)].

The phenomenon of elastic stability, or buckling, has given rise to extensive theoretical and experimental investigations. As important stability problem that has not received a proportional amount of research attention is that of the non-prismatic beam-column with lattice or batten build up members and / or tapered sections, taking into consideration the effect of shear.

S. Z. Al-Sarraf [(2)] described a method which permits a rapid determination of elastic critical loads (ignoring the shear effect) of structures having uniformly linearly tapered members. Modified stability functions for uniformly tapered beam-column having wide flange, box section and other cross-sectional shapes are developed. Approximate formulae for the modified stability functions, which are simple functions for prismatic members, have been presented. These formulae are exact for solid circular or square cross-section members and with good accuracy for other cross-section members. AL-Damerchi, H.T.N. [(3)], made a theoretical basis for the large displacement elastic-plastic stability analysis of plane steel frames with linearly and nonlinearly tapered members. Exact and approximate modified stability functions were derived (ignoring the shear effect) for a wide range of tapered members. Later, S. Z. Al-Sarraf [(4)] studied the shear effect on the elastic stability of frames having prismatic members. Modified stability functions for prismatic beam-columns having any solid cross-sectional shapes, laced or battened built-up sections were developed in terms of shear flexibility and axial load parameter which make possible the rapid prediction of the elastic critical load of structures taking into consideration the effect of the shear force in the members by using a hand-computing method. These modified stability functions are simple functions of stability of prismatic members. The approximate formulas predict the accurate elastic critical load to within 1%.

Also, H. A. Al-Qurashi [(3)] developed these approximate formulae for the modified stability functions for non-prismatic members (which given by S. Z. Al-Sarraf [(2)]) including shear effect, which are simple functions for prismatic members including shear effect. Later, M.A. Al-Fadul [(5)] derived a new modified stability functions for non-prismatic members including shear effect. These modified stability functions are exact (for build up structures) and accurate approximate using the finite difference method (for a wide range of tapered members).

M. J. Iremonger [6] used the finite difference method to determine the buckling loads for tapered and stepped columns. The convergence towards the correct buckling load as the number of segment sub-division is increased may not be monotonic. It was concluded that, consistent and correct results can be obtained using only a small number of segment subdivisions coupled with extrapolation procedures.

D. Polyzois and I. G. Raftoyiannis [7] present study deals with the stability behavior of web-tapered beams subjected to bending loads. According to the current AISC specifications, a tapered member is treated as prismatic with modified length and cross section identical to that of the tapered member’s smaller end. Modification factors are used to determine the modified length and also to account for the moment gradient within the unbraced length and the restraining effect for adjacent spans. In this study, these factors are evaluated via a finite-element analysis and compared to the ones proposed by the current specifications. The current design specifications for steel tapered beams are re-examined, and modifications that cover a wider range of geometry and loading cases are proposed.

G.Ozay and A. Topcu [8], presented a more realistic and comprehensive static analysis technique for structures having non-prismatic members. A general stiffness matrix for non-prismatic members that is applicable to Timoshenko beam theory was derived in the proposed method. The stiffness coefficients was determined for constant, linear, and parabolic height variations of members, employing analytical and (or) numerical integration techniques. Uniform, triangular, and trapezoidal distributed loads over the entire member or along any part of it, concentrated loads, moments at points on the member, and any of these load combinations are taken into consideration to determine the fixed-end forces. The accuracy of the proposed analysis technique was verified by comparing the results of the numerical examples with those obtained from the general analysis program SAP90 using a large number of sub elements.

B.K. Lee et.al [9], dealt with the elastica of shear deformable tapered columns. Non-directional differential equations governing the elastica of buckled columns and their boundary conditions including the effect of shear deformation were derived according to the Bernoulli-Euler beam theory. These equations were solved numerically to obtain both the elastica and the buckling load of the columns. The Runge-Kutta method was used to integrate the differential equations, and the Regula–Fabi method was used to obtain the unknown initial value of the elastica and buckling load, respectively. Three types of cross sectional shapes with both hinged ends and both clamped ends were considered in the numerical examples.

L.Jin-Jun and L. Guo-Qiang [10], established a generalized finite element for buckling analysis of tapered columns with various cross sections by using Chebyshev polynomial approach to the governing differential equation. The proposed formulation involved the effects of shear deformation and is therefore applicable to lattice columns. Buckling of a tapered lattice column, with respective lace bars and batten plates, was studied in the numerical example. The effects of shear deformation on the buckling loads of tapered lattice columns were specified.

In the present study, the critical load for laced and solid non-prismatic members with linear and nonlinear taper taking into consideration the effect of shear are studied with different support conditions.

SHEAR FLEXIBILITY PARAMETER OF TAPERED MEMBERS:

The effect of shear on elastic stability analysis depends on the type of the structure whether it is open or closed web structure. The open web structure exhibits relatively higher shear deformation than the solid one. Therefore, a parameter which gives a sense about how much the effect of shear deformation will be occurred on this structure must be introduced. When this parameter has a large value, this means that the effect of shear defamation is high and vice versa.

The derivation of shear flexibility parameter for non-prismatic members is presented in this study as follows:

- Shear Flexibility Parameter of Solid Tapered Members:

Depending on the derivation procedure produced by Lin and Glouser [8] derived for a solid prismatic member, can be used it here, to calculate the shear flexibility parameter for non-prismatic one and as follows:

Consider an element of length L, of a structural member as shown in Fig. (1), acted upon by axial force Q, shearing force V, and bending moment M.

The deformation is separated into two parts; (1) bending, and (2) shear deformations. The change in the slope which produced by the bending in length (L) at end 2 is (see Fig. (2)):

$$\phi L = \frac{ML}{EI_2} \qquad (1)$$

And the change in the slope which produced by the shear at end 2 is:

$$\gamma = \frac{V}{GA_{V2}} \qquad (2)$$

Where:

EI_2, GA_{V2}: are the flexural and shear rigidities of a tapered member at end 2 as shown in Fig. (2).

A_{V2}: is the effective shear area at end 2, and equal to A_2 / B_2, which B_2 is the shear shape factor at end 2. (see Table (1)).

Thus, within the length (L), depending on the ratio of change in the slope caused by the shear to that caused by the moment is defined as "shear flexibility parameter "and it is equal to[1,4]:

$$\mu_2 = \frac{P_{e2}}{GA_{V2}} \tag{3}$$

, is the Euler load of end 2. Where $p_{e2} = \pi^2 EI_2 / L^2$

• Shear Flexibility Parameter of Laced Tapered Members [1,5]:

As it was mentioned previously, the open web structures exhibits a relatively higher shear deformation than the solid one, however there are many types of laced structural members may be catalogued according to five different lacing configurations as shown in Fig. (3).

The parts of laced structural members consist of two main longitudinal elements, lacing (diagonal) elements, with or without strut (transverse) elements. The two longitudinal elements are connected in one, two or more planes by the lacing bars and strut elements which serve as web of the member. The shear flexibility parameter for laced non-prismatic members can be calculated as given in Table (2).

NONLINEAR TAPERED MEMBER:

The derivation of the modified stability functions for a linearly tapered members including shear effect is presented by authors [5], using exact method for a laced members and accurate approximate method (finite difference method) for all shape factors of tapered members. For a member having nonlinear taper either in one or two directions as shown in Fig.(4), the same procedure which is used by Al–Damerchi [3], can be adopted here to include the effect of shear in the calculations, and as follows:

The depth d_x may be expressed by

$$d_x = d_2 \left(\frac{x}{a} \right)^{\overline{\psi}} \tag{4}$$

where:

a: is the distance of end 2 from the origin "o", point of zero depth (see Fig.(2)).

d_2: is the depth at end 2.

From Eq. (4) the depth at end 1 can be obtained as:

$$d_1 = d_2 \left(\frac{b}{a} \right)^{\overline{\psi}} \tag{5}$$

Where b is the distance of end 1 from the origin "o", and:

$$u^{1/\overline{\psi}} = \frac{b}{a} \tag{6}$$

Where $u = d_1 / d_2$.

Equation (6) can be written as:

$$\overline{u} = \frac{b}{a} \tag{7}$$

Where u is the modified taper ratio and may be obtained as:

$$\overline{u} = u^{1/\overline{\psi}} \tag{8}$$

The moment of inertia of the cross-sectional area of a tapered member about the axis of bending may be expressed in the form:

$$I_x = I_2 \left(\frac{x}{a} \right)^{\overline{\Psi} m} \tag{9}$$

Where I_x is the moment of inertia at distance x from the origin "o". Eq. (9) can be written as:

$$I_x = I_2 \left(\frac{x}{a} \right)^{\overline{m}} \tag{10}$$

Where $\overline{m}$ is the modified shape factor and may obtained as :

$$\overline{m} = \overline{\Psi}\ m \tag{11}$$

Where m is the shape factor and may be obtained by using the following equation [2, 3];

$$m = \mathrm{Log}\,(I_1 / I_2) / \mathrm{Log}\,(u) \tag{12}$$

The effective cross-section area of a tapered member may be expressed in the form:

$$A_{vx} = A_{v2}\left(\frac{x}{a}\right)^{\overline{\Psi}\,\eta} \tag{13}$$

Where A_{vx} is the effective cross-sectional area at distance x from the origin "o". Equation (13) can be written as:

$$A_{vx} = A_{v2}\left(\frac{x}{a}\right)^{\overline{\eta}} \tag{14}$$

Where $\overline{\eta}$ is the modified shape factor and may be obtained as:

$$\overline{\eta} = \overline{\Psi}\,\eta \tag{15}$$

Where η is the shape factor and may be obtained by using the following equation [3, 5];

$$\eta = \log\left(A_{V1}/A_{V2}\right) / \operatorname{Log}(u)) \tag{16}$$

Substituting Eq.(13) into the following equation;

$$\frac{dy}{dx} = \frac{-dy_m}{dx} + \frac{V_x}{A_{Vx}G} \tag{17}$$

and equation (10) into the following equation;

$$\frac{d^2y}{dx^2} = \frac{-M(x)}{EI_x} + \frac{a^{\overline{\eta}}}{A_{V2}G}\left[\overline{\eta}\,x^{-\overline{\eta}-1}\left(\frac{M_1+M_2}{L}\right) - Q\,\overline{\eta}\,x^{-\overline{\eta}-1}\frac{dy}{dx} + Q\,x^{-\overline{\eta}}\frac{d^2y}{dx^2}\right] \tag{18}$$

yields the same solution of linearly tapered members [3, 5] but with the use of a modified shape factors and modified taper ratio. So, the modified stability functions for a nonlinear tapered member including shear effect can be obtained by using the stability functions for a linearly tapered member by using the modified shape factors $(\overline{m})$ and $(\overline{\eta})$ and modified taper ratio $(\overline{u})$.

THE EFFECT OF SHEARING FORCE AND TAPERING RATIO ON THE CRITICAL LOAD FOR NON-PRISMATIC MEMBERS:

When buckling occurs there will be shearing forces acting on the cross section of the member. The critical load for a laced column is always less than for a solid column having the same cross-sectional area and the same slenderness ratio l/r. This decrease in critical load is due to primarily to the fact that the effect of shear on deflections is much greater for a laced column than for a solid bar. The actual value of the critical load depends upon the detailed arrangement and dimensions of lacing bars. So, In the present study, the shear effect and tapering ratio on the critical load will be discussed for uniformly tapered laced columns with m=2 for case No.1 to case No.3, nonlinear tapered laced columns with m=4 (Case No.4) with different support conditions, uniformly tapered built-up columns with deferent tapering ratio, and a frame with non-prismatic columns with m=3 (Case No.6) as follows:

• Case 1- Cantilevered Tapered Beam-Column:

Consider the structure shown in Fig. (5). To find the elastic critical load of this structure of the sway mode, joint B is translated laterally by δB and rotated by θ_B. The end moments due to these deformations are:

$$\begin{bmatrix} M_B \\ V_B L \end{bmatrix} = \begin{bmatrix} K_2 \overline{S}_1 & -K_2\left(\overline{S}_1 + \overline{SC}\right) \\ -K_2\left(\overline{S}_1 + \overline{SC}\right) & K_2 Q \end{bmatrix} \begin{bmatrix} \theta_B \\ \delta / L \end{bmatrix} \tag{19}$$

Where:

$Q = \overline{S}_1 + \overline{S}_2 + 2\overline{SC} - \pi^2 \rho_2$,and $K_2 = EI_2 / L$

$\overline{S}_1, \overline{S}_2$,and $\overline{SC}$ are the exact modified stability functions including shear effect [5].

At the critical load $\sum M_B = \sum V_B = 0$. The load parameter ρ_2 which makes the determinant of the stiffness matrix equation (19) vanish is the critical load parameter;

$$K = \begin{vmatrix} K_2 \overline{S}_1 & -K_2\left(\overline{S}_1 + \overline{SC}\right) \\ -K_2\left(\overline{S}_1 + \overline{SC}\right) & K_2 Q \end{vmatrix} \tag{20}$$

Then; the ratio of elastic critical buckling load ($P_{cr}=\rho_2 * Pe_2$) to the Euler load of end 2 ($Pe_2=\pi^2 EI_2 / L^2$) is the load parameter ρ_2 can be drawing with shear flexibility parameter μ_2 as shown in Fig. (8).

• Case 2- Hinged Ends Tapered Beam-Column:

Consider the structure shown in Fig. (6). To find the elastic critical load, joint A and B are rotated by θ_A and θ_B respectively. The end moment due to these deformations are:

$$\begin{bmatrix} M_{BA} \\ M_{AB} \end{bmatrix} = \begin{bmatrix} K_2 \overline{S}_1 & K_2 \overline{SC} \\ K_2 \overline{SC} & K_2 \overline{S}_2 \end{bmatrix} \begin{bmatrix} \theta_B \\ \theta_A \end{bmatrix} \tag{21}$$

At the critical load the determinant of the stiffness matrix is reduced to:

$$\bar{S}_2\bar{S}_1 - S\bar{C}^2 = 0 \qquad (22)$$

Then; elastic critical load of this member can be calculated using Eq. (22) as in previous case.

Case 3- Fixed-Pinned Ends Tapered Beam-Column:

Consider the structure shown in Fig. (7). To find the elastic critical load for thus structure, joint B is rotated by θ_B .The end moments of the member due to θ_B are:

$$\begin{bmatrix} M_{BA} \\ M_{AB} \end{bmatrix} = \begin{bmatrix} K_2\bar{S}_1 \\ K_2 S\bar{C} \end{bmatrix} [\theta_B] \qquad (23)$$

At the critical load $M_{BA} = 0$ and it is equal to:

$$[M_{BA}] = [K_2\bar{S}_1][\theta_B] \qquad (24)$$

The load parameter ρ_2 which makes the determinant of the stiffness matrix equation (24) vanish is the critical load parameter.

From Fig.(8) to Fig.(10), it can be noted that the effect of shear will reduce the critical load for cantilever tapered members by 77% for u=4 and 56% for u=2 when shear flexibility parameter μ_2 are increased (from 0 to 2), and this reduction become to be more rabid for hinged and fixed-pinned members when the effect of shear force is included.

The lowering in critical force increases with the increasing of tapering ratio, so; this confirms that the effect of shearing force on non-prismatic member is greater than that of prismatic one.

• Case 4-Buckling of Nonlinear Tapered Columns for Different Support Conditions:

The elastic buckling load of non-linear tapered columns with three different ends conditions as given for cases No.1 to No.3 are studied. The value of the modified shape factor (m) is 4 (m=2 and ψ=2) with different values for tapering ratio. The calculations procedures are same as in the cases of No.1 to No.3. The results are shown in the figures from Fig. (11) to Fig. (13).

From the previous figures, it can be noted that the effect of shear will reduce the critical load for the same behavior of previous cases. So; it can be seen that the shearing force and tapering ratio has the same behavior on linear and nonlinear tapered members.

- Case 5-Buckling of Built-Up Columns:

A uniform cantilevered structure comprised of four circular chords tied together by horizontal and diagonal lacing as shown in Fig. (14) has been chosen as an example. The critical load was calculated for this structure, both in the absence of shear and with shear effect included.

In order to show the effect of tapering ratio on the value of the critical load, this uniform lacing column is tapered for a depth ratio (u = 1.5, 2, 2.5 and u=3) and for the same volume as shown in Table(3). Again, the critical loads are calculated for these cases, both with and without shear effect. The results are summarized in Table (4).

From the results in Table (4) and Fig.(15), it can be seen that the effect of shear deformation can cause a significant reduction in the value of critical load, its about (63.315)% for prismatic columns and (64.7-65)% for non-prismatic columns. From another side the increasing in tapering ratio from 1 to 2 and 3 lead to increase the value of the critical load by (20.89%) and (26.23%) in the absence of shear and (15.09%) and (21.45%) in the presence of shear respectively.

In general, if the cross-sectional areas A_b and A_d are small in comparison with the area of the chords, the critical load may be considerably lower than the Euler value. Thus a laced column may be considerably weaker than the solid column with the same EI, but since the amount of material used is less, the laced column may be more economical.

- Case 6-Buckling of Frame with Non-Prismatic Members:

This problem has been studied by Al-Sarraf[3] for computing the elastic critical load using the modified slope deflection method. In the present study the same method is used and the elastic critical load of the frame shown in Fig. (16) is to be found but with including the effect of shear by utilizing the stability functions of non-prismatic members including shear effect.

In this problem the beam is prismatic, whereas the stanchions are non-prismatic and the elastic critical load of the frame is calculated for m = 3, u = 3 and $K_{BB`} / (K_2)_{AB} = 8$. This example is solved for two values of shear flexibility parameter (μ_2 = 0.5, 0.2) for stanchions and (μ =0.265, 0.0454) for the beam.

To find the elastic critical load for (μ_2 =0.5) for stanchions and (0.265) for beam, of the sway mode, joint B is translated laterally by δ_B and joints B and B` rotate by θ_B. The end moments and forces of the members due to these deformations are:

$$\begin{bmatrix} M_{AB} \\ M_{BA} \\ M_{BB'} \\ V_{BA}\,L \end{bmatrix} = \begin{bmatrix} \left(K_2\, S\overline{C}\right)_{AB} & -\left(K_2\left(\overline{S}_1 + S\overline{C}\right)\right)_{AB} \\ \left(K_2\, \overline{S}_2\right)_{AB} & -\left(K_2\left(\overline{S}_2 + S\overline{C}\right)\right)_{AB} \\ \left(4.534K\right)_{BB'} & 0 \\ -\left(K_2\left(\overline{S}_2 + S\overline{C}\right)\right)_{AB} & \left(K_2 Q\right)_{AB} \end{bmatrix} \begin{bmatrix} \theta_B \\ \delta / L \end{bmatrix} \quad (25)$$

The equilibrium condition at joint B requires that:

$$\begin{bmatrix} \sum M_B \\ L\sum V_B \end{bmatrix} = \begin{bmatrix} \left(K_2\overline{S}_2\right)_{AB} + \left(4.534K\right)_{BB'} & -\left(K_2\left(\overline{S}_1 + S\overline{C}\right)\right)_{AB} \\ -\left(K_2\left(\overline{S}_2 + S\overline{C}\right)\right)_{AB} & \left(K_2 Q\right)_{AB} \end{bmatrix} \begin{bmatrix} \theta_B \\ \delta / L \end{bmatrix} \quad (26)$$

At the critical load $\sum M_B = \sum V_B = 0$. The load parameter ρ_2 which makes the determinant of the stiffness matrix equation (26) vanish is the critical load parameter.

By substitution the values of m, u, μ_2 and $K_{BB`} / (K_2)_{AB}$ in Eq.(26) gives the stability criteria:

$$K = \begin{vmatrix} \left(\overline{S}_2\right)_{AB} + 36.275 & -\left(\overline{S}_2 + S\overline{C}\right)_{AB} \\ -\left(\overline{S}_2 + S\overline{C}\right)_{AB} & \left(Q\right)_{AB} \end{vmatrix} \quad (27)$$

The load parameter ρ_2 making K vanish is found to be equal to (1.7431).

The elastic critical load of the frame is:

$2P = 2(1.7431)\ (Pe_2) = 3.486\ (Pe_2)_{AB}$

From the results in Table (5), it can be seen that the effect of shear deformation can have a significant effect on the value of critical load of the frames having solid prismatic and non-prismatic members. This effect increased from (32.42 to 56.78) % with the increasing of the value of shear parameter from 0.2 to 0.5.

CONCLUSIONS:

Based on the results obtained in the present study, several conclusions may be drawn. These may be summarized as follows:

1. The effect of shear deformation can play a major rule governing the behavior of the non-prismatic open web-structure under static loads. The effect of the shear is depending on the value of shear parameter μ_2 and tapering ratio u.

2. The exact stability functions that are derived by the authors are suitable for predicting the elastic critical buckling load including shear effect for structures having tapered lattice columns.

3. The modified stability functions for nonlinear tapered member including shear effect can be obtained by using the stability functions for a linearly tapered member but with using the modified shape factors ($\overline{m}$) and the modified taper ratio ($\overline{u}$).

4. Inclusion the effect of shear deformation in the calculation of critical load can result in a significant reduction in the value of the critical load for both prismatic and non-prismatic laced columns. The average ratio of reduction is (63.315%) and (64.7 to 65) % for prismatic and non-prismatic laced columns respectively.

5. In the structures having prismatic and non-prismatic solid columns with short lengths, the effect of shear deformation can have a significant effect on the value of critical load, its about (32.42 to 56.78) %.

REFERENCES:

1. Al-Qurashi, H.A., "Large Displacement Elastic Stability Analysis of Plane Frames Including Shear Effect", M.Sc. Thesis, University of Technology, Building and Construction Department, Baghdad, November 1999.

2. AL-Sarraf, S.Z., "Elastic Instability of Frames with Uniformly Tapered Member". The Structural Engineer, March, 1979, PP 18-24.

3. AL-Damerchi, H.T.N., "Large Displacement Elastic-Plastic Stability Analysis of Plane Steel Frames With Non-Prismatic Members". Ph.D. Thesis, University of Technology, Baghdad, 1999.

4. Al-Sarraf, S.Z., "Shear Effect on the Elastic Stability of Frames", the Structural Engineer, Vol. 64B, No. 2, June, 1986, pp. 43-46.

5. Al-Fadul, M.A.K., "Stability Functions for Non-Prismatic Members Including Shear Effect", M.Sc. Thesis, University of Kufa, Civil Engineering Department, Najaf, February, 2005.

6. Iremonger, M.J., "Finite Difference Buckling Analysis of Non-Uniform Columns". International Journal of Computer and Structures, Vol. 12, No. 5, 1980, PP. 741-748.

7. Polyzois D. and Raftoyiannis I.G., "Lateral-Torsional Stability of Steel Web-Tapered I-Beams". The Structural Engineering, Vol. 124, No. 10, October, 1998, PP 18-24.

8. Ozay G, and Topcu A., "Analysis of frames with non-prismatic members", Can. J. Civ. Eng. 27(1): 17–25 (2000) | doi: 10.1139/cjce-27-1-17 | © 2000 NRC Canada.

9. Lee B. K., Choi K. M., Lee T. E., and Kim Y., "Elastica of Shear-deformable Tapered Columns", KSCE Journal of Civil Engineering, Vol. 6, No. 2, June. 2002, pp 135-141.

10. Jin-Jun L., and Guo-Qiang L. , "Buckling Analysis of Tapered Lattice columns" using a generalized finite element, Journal of Communications in numerical methods in engineering, ISSN 1069-8299, 2004, Vol. 20, No.6, pp. 479-488 .

11. Timoshenko, S. P. and Gere, J. M., "Theory of Elastic Stability", McGraw-Hill Book Company, 2nd Edition, 1961.

Table (1) Typical Values of Shear Shape Factor for Solid Member [5].

Type of section	Shear shape factor (B)
Rectangular cross section	1.2
Circular cross section	1.11
I – Section bent about minor axis	$\approx 1.2A / A_f = (1.4 - 2.8)$
I – Section bent about major axis	$\approx A / A_w = (2 - 6)$

Where;

A_f = area of 2-flanges.

A_w = area of the web.

A = total cross-sectional area.

Table (2) The Governing Equations of Shear Flexibility Parameter for Laced Members [5].

Type	Shear Parameter
Pratt Bracing	$\frac{EI_2\pi^2}{L^2}\left[\frac{L_d\cos\alpha}{(\cos\phi+\sin\phi\tan\alpha)\cos(\phi-\alpha)A_d EL_C}+\frac{L_b}{A_b EL_C}\right]$
X-Bracing	$\frac{EI_2\pi^2}{L^2}\left[\frac{L_d\cos\alpha}{2E(\cos\phi+\sin\phi\tan\alpha)\cos(\phi-\alpha)A_d L_C}\right]$
Warren Bracing	$\frac{EI_2\pi^2}{L^2}\left[\frac{L_d\cos\alpha}{E(\cos\phi+\sin\phi\tan\alpha)\cos(\phi-\alpha)A_d L_C}\right]$
Howe Bracing	$\frac{EI_2\pi^2}{L^2}\left[\frac{L_d\cos\alpha}{E(\cos\phi+\sin\phi\tan\alpha)\cos(\phi-\alpha)A_d L_C}+\frac{L_b}{A_b EL_C}\right]$

Where;

α: is the angle of tapered.

Φ: is the angle between the horizontal and diagonal bars.

I2: is the moment of inertia with respect to end 2.

A_d, A_b: are the cross sectional areas of diagonal and batten bars respectively.

L_d, L_b, L_c: are the lengths of diagonal, batten and cord between hinges respectively.

L: is the total length of the tapered member.

Table (3) Properties of Cantilevered Laced Column of Case No. 5.

Tapering ratio (u)	d_1(in)	d_2 (in)	Φ	A_c (in^2)	A_b (in^2)	A_d(in^2)
1	58	58	54.0578	30.76	1.0	1.50
1.5	69.6	46.4	54.4162	30.76	1.048	1.506
2	77.333	38.667	55.0801	30.76	1.10	1.52
2.5	82.857	33.142	55.8033	30.76	1.155	1.532
3	88	28	56.6977	30.76	1.22	1.55

Table (4) Effect of Shear Deformation on the Critical Load for the Column of Case No. 5.

Tapering ratio (u)	Shear parameter (μ_2)	(P_2) Pcr with shear (Kips)	(P_1) Pcr without shear (Kips)	(P_1- P_2) / P_1 % Reduction due to shear
1	6.9038	12195.515	33244.580	63.315
1.5	4.1326	13353.151	37967.303	64.829
2	2.7443	14036.842	40192.663	65.076
2.5	1.9513	14476.725	41129.984	64.802
3	1.3529	14812.330	41965.483	64.703

Table (5) A Comparison Between the Results Obtained by the Present Study with Shear Effect and Ref. (2) without Shear Effect for Case No. 6.

(P_2) Pcr with shear Present Study		(P_1) Pcr without shear Ref. (3), μ = 0	(P_1-P_2/P_1) % Reduction due to Shear
μ_2 = 0.5	3.486 $(Pe_2)_{AB}$	8.067 $(Pe_2)_{AB}$	56.786
μ_2 = 0.2	5.451 $(Pe_2)_{AB}$		32.428

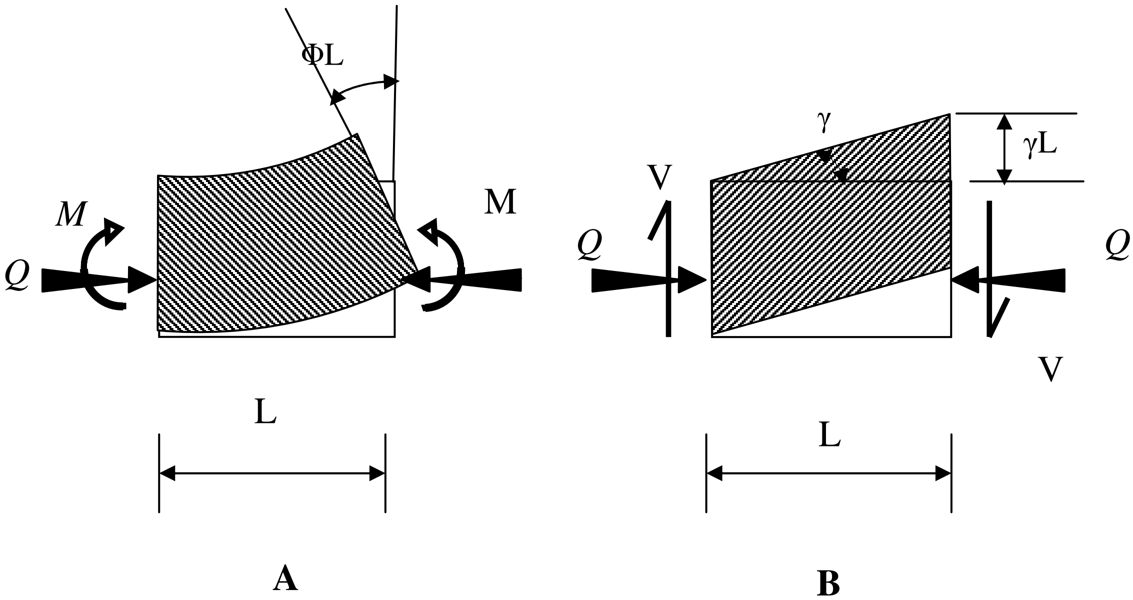

Fig. (1) Element of a Structural Member under; (A) Bending (B) Shear.

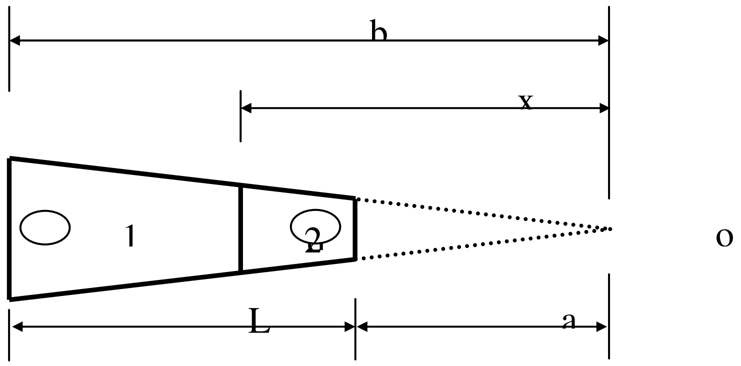

Fig. (2) Tapered Beam Column Element

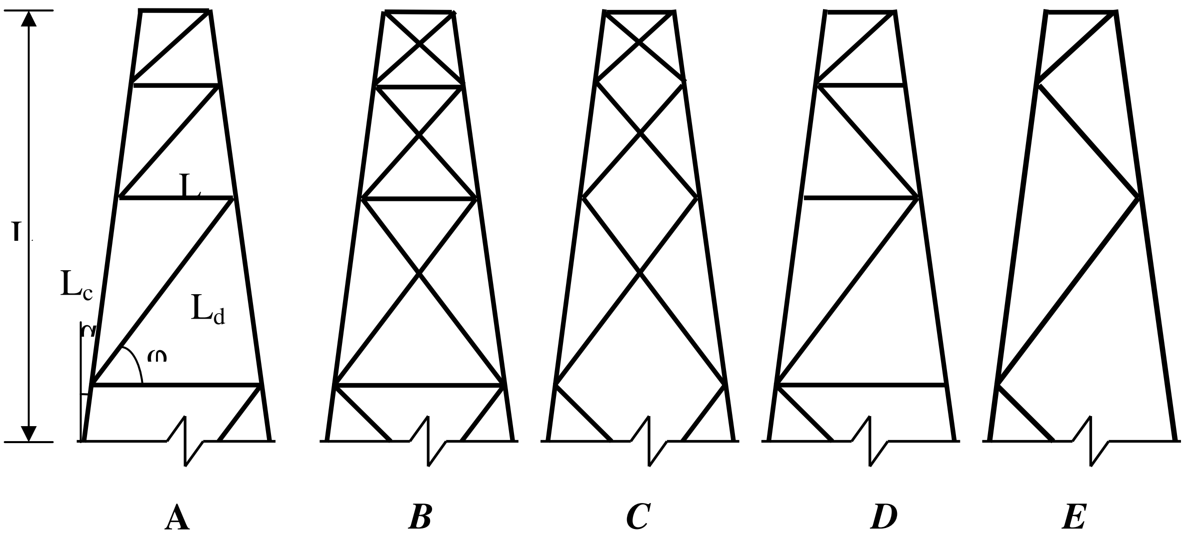

Fig. (3) Five Different Lacing Configuration for Laced Structural Members

(A) Pratt-Bracing ,(B) X-Bracing ,(C) Diamond Bracing ,(D) Howe Bracing,

and (E) Warren Bracing

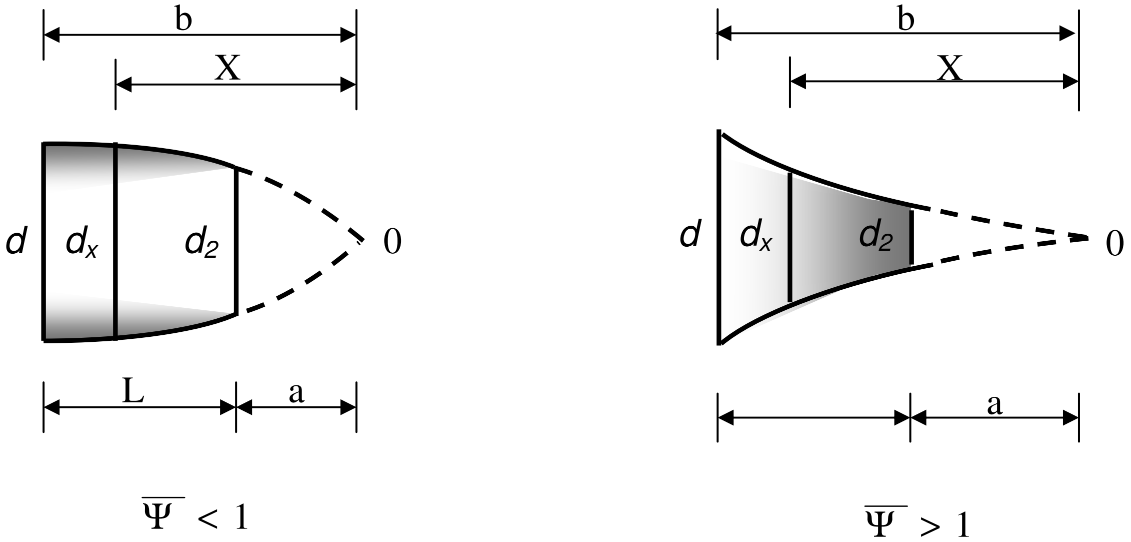

Fig. (4) Nonlinear Tapered Beam – Column Element.

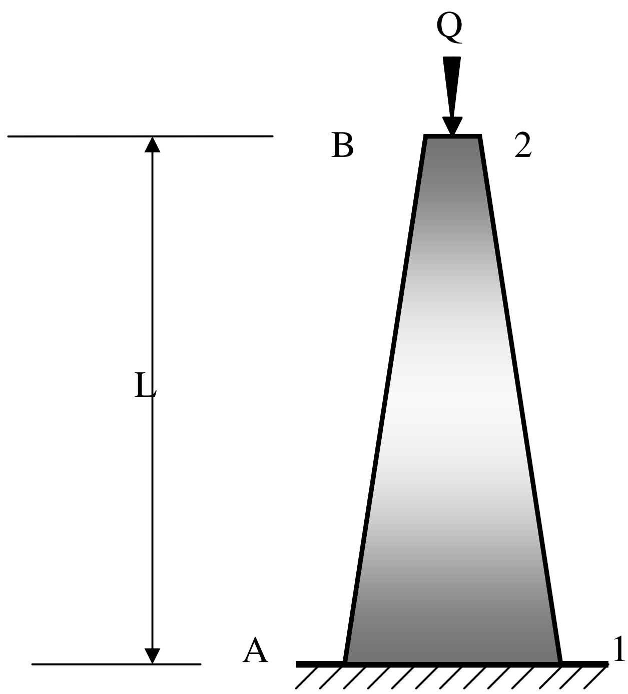

Fig. (5) Cantilevered Tapered Beam-Column.

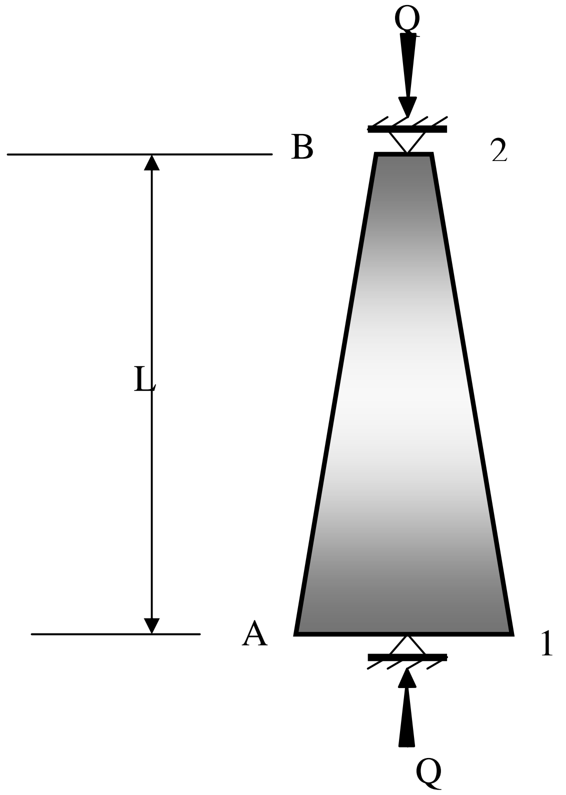

Fig. (6) Two-Hinged Tapered Beam-Column.

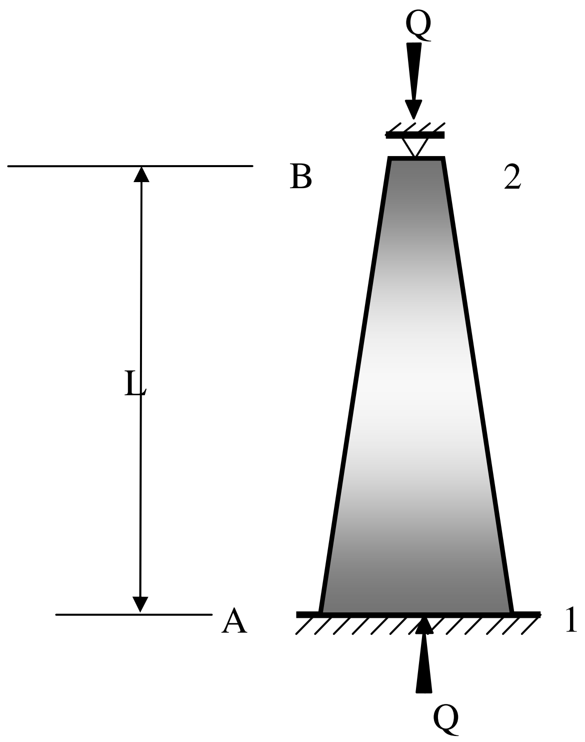

Fig. (7) Fixed-Pinned Tapered Beam-Column.

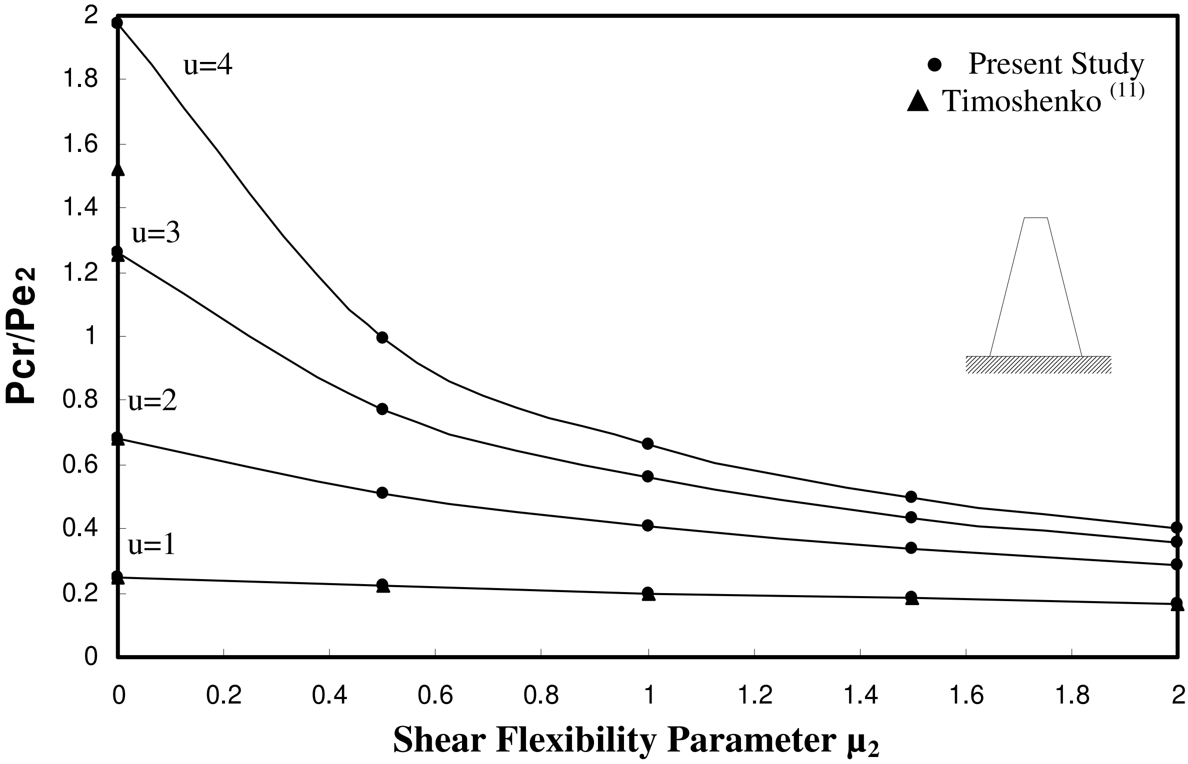

Fig.(8) Effect of Shear on the Critical Load for Linear Tapered Cantilevered Beam-Column.

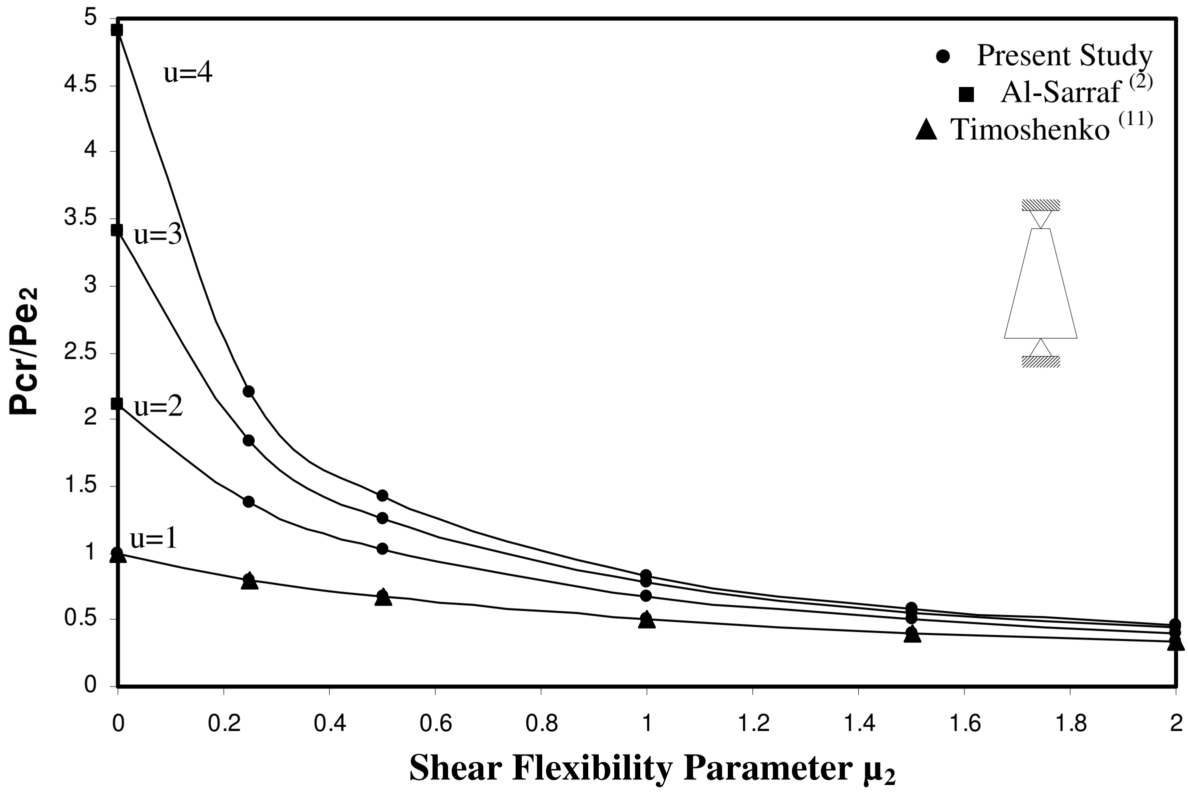

Fig.(9) Effect of Shear on the Critical Load for Linear Tapered Hinged Beam-Column.

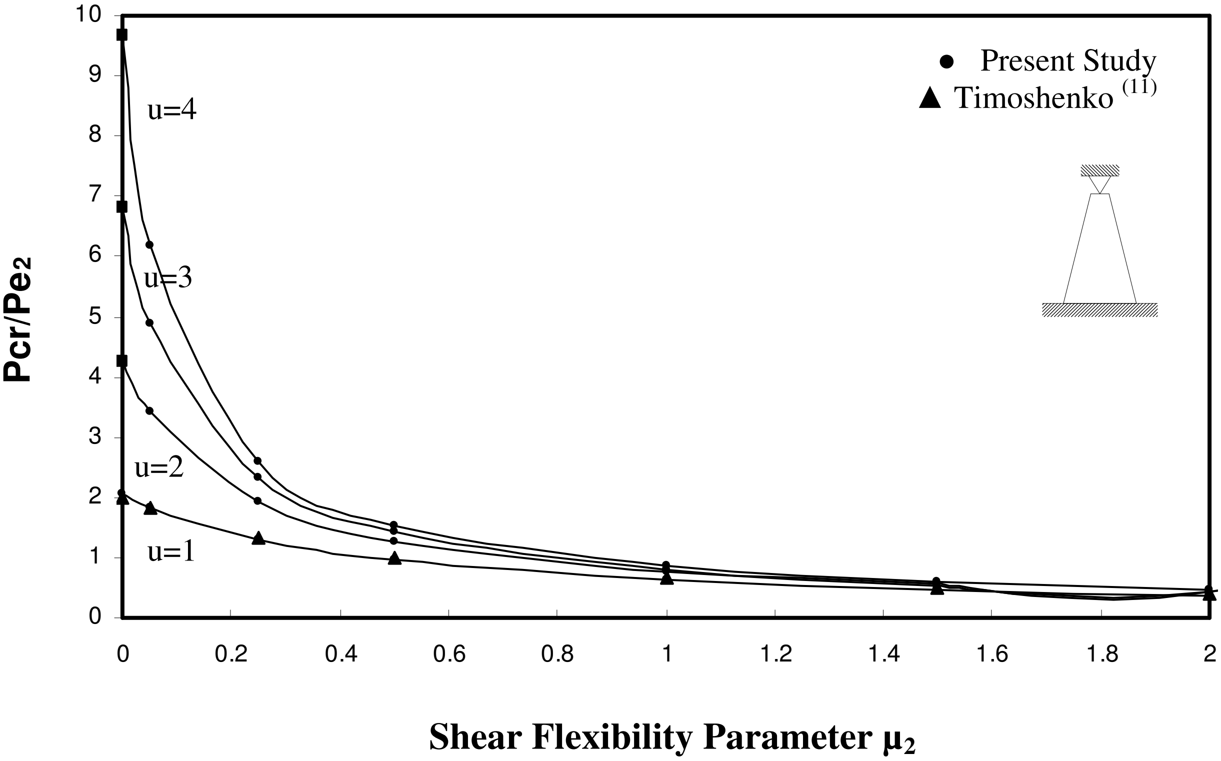

Fig.(10) Effect of Shear on the Critical Load for Linear Tapered Fixed-Pinned Beam-Column.

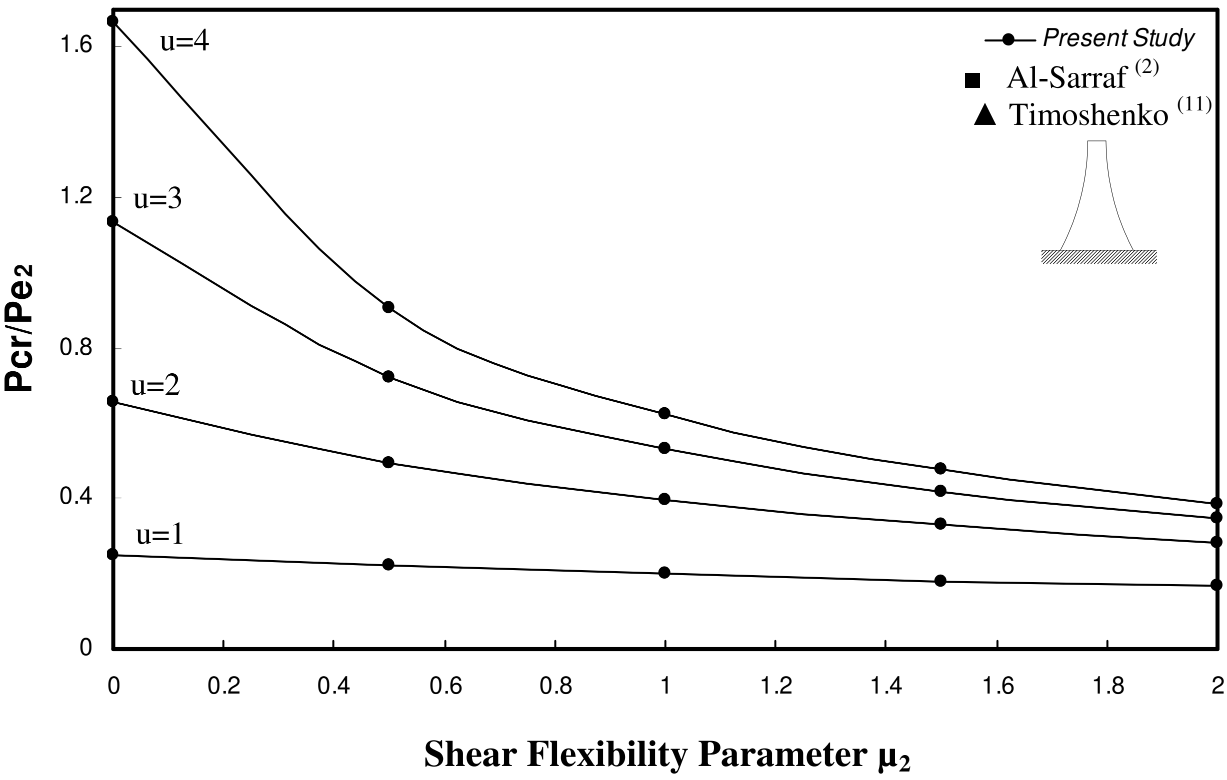

Fig.(11) Effect of Shear on the Critical Load for Nonlinear Tapered cantilevered Beam-Column, ψ=2.

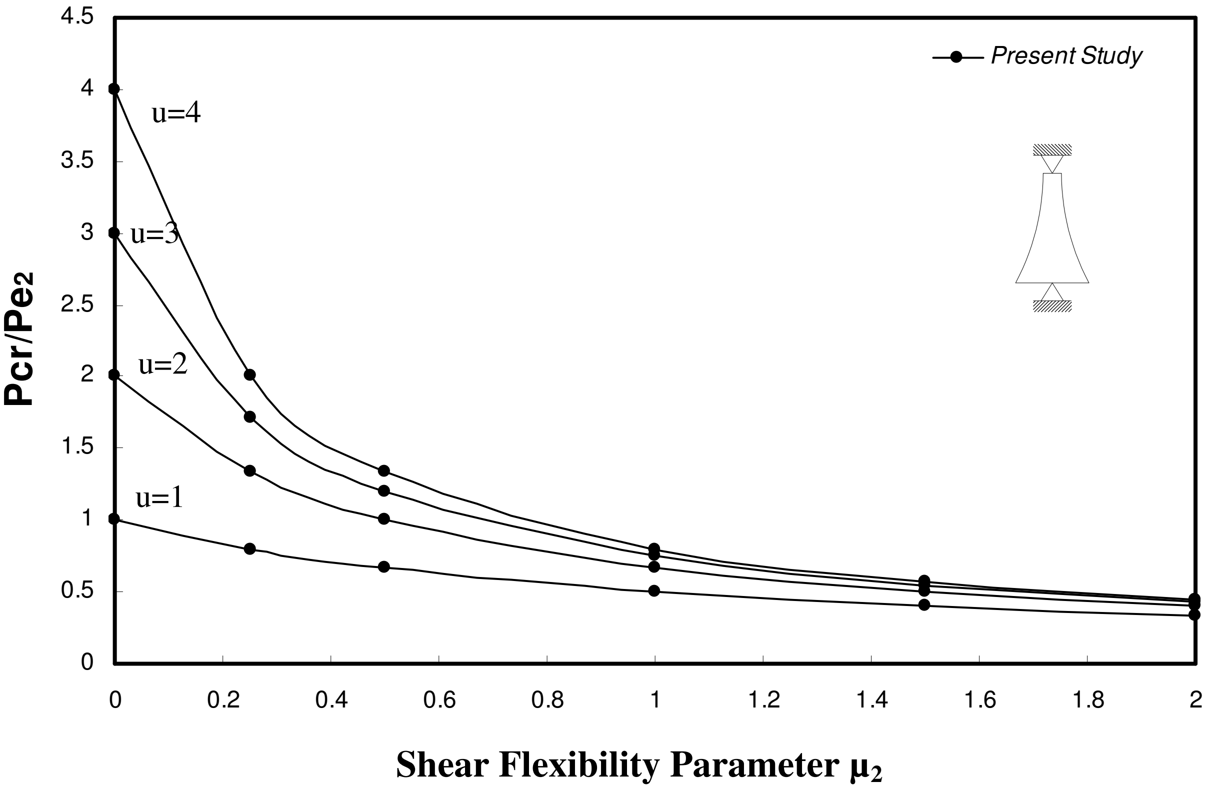

Fig.(12) Effect of Shear on the Critical Load for Nonlinear Tapered Hinged Beam-Column, ψ=2.

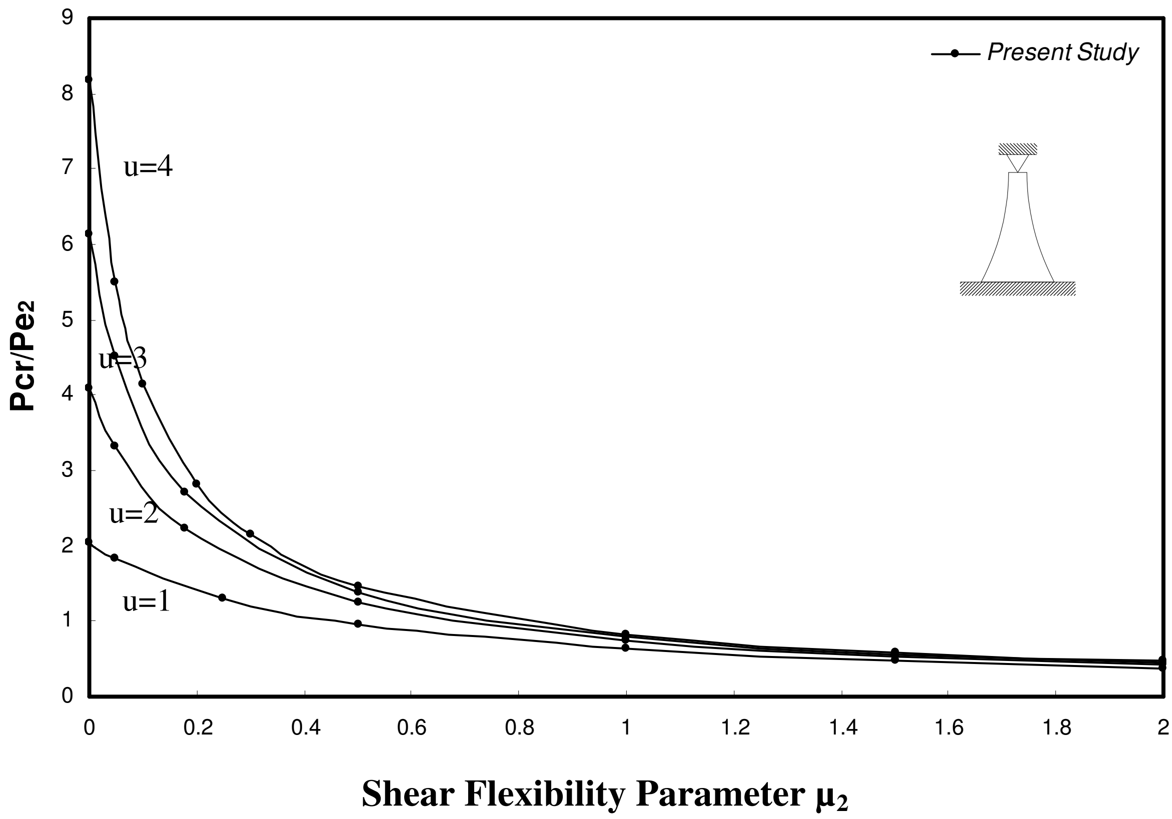

Fig.(13) Effect of Shear on the Critical Load for Nonlinear Tapered Fixed-Pinned Beam-Column, ψ=2.

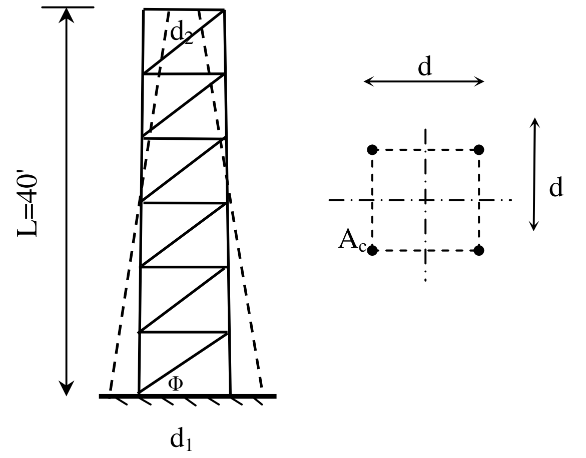

Fig. (14) Geometry and Loading Conditions of Case No. 5.

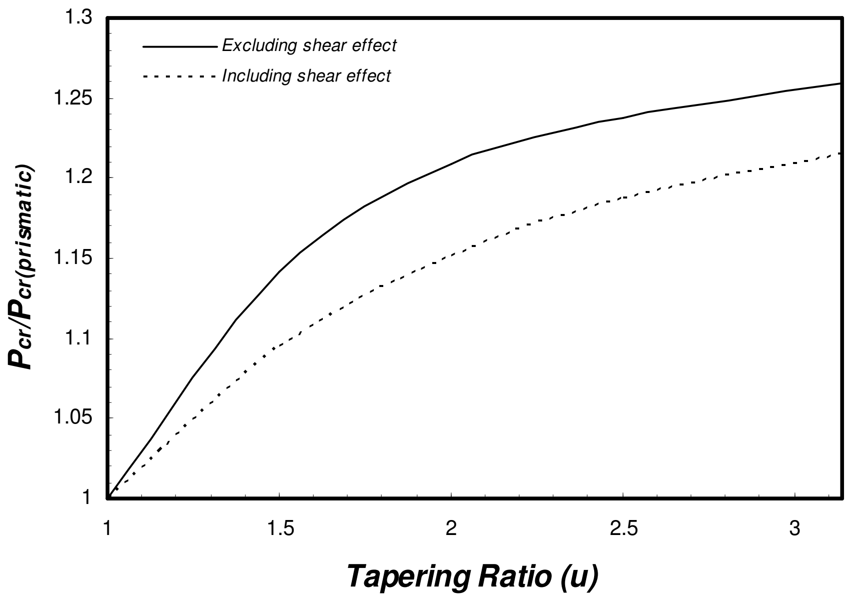

Fig. (15) The Effect of Shear and Tapering Ratio on the Critical Load for Case No. 5.

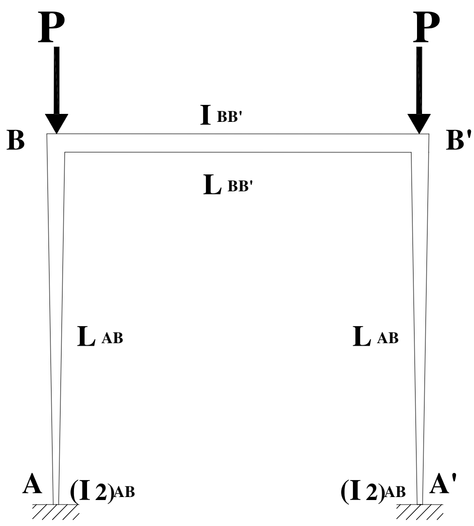

Fig. (16) Geometry and Loading Conditions for the Frame in Case No.